Quantum Corrections in the Hypermultiplet Moduli Space

Holger Günther¹, Carl Herrmann², Jan Louis¹

¹Martin–Luther–Universität Halle–Wittenberg, FB Physik, D-06099 Halle, Germany
²Centre de Physique Théorique, CNRS – Luminy Case 907, F-13288 Marseille Cedex 9, France

Abstract: Quantum corrections in the hypermultiplet moduli space of type IIA string theories compactified on Calabi-Yau threefolds are investigated.


String theories with $N = 2$ supersymmetry in four space-time dimensions ($d = 4$) can be constructed either by compactifying the type II string on a Calabi–Yau threefold $Y_3$ or the heterotic string on $K^3 \times T^2$. It is believed that the resulting string vacua all reside in the same moduli space and that any given string vacuum generically has 2 dual descriptions: either as a type II vacuum or as a heterotic vacuum [1, 2]. The spectrum and the low energy effective theory are strongly constrained by $N = 2$ supersymmetry but also depend on the specific ‘data’ of the Calabi–Yau compactification. The massless states come in $N = 2$ multiplets which are either vector multiplets $V$ containing a complex scalar, a vector and two Weyl fermions or hypermultiplets $H$ which contain four real scalars and two Weyl fermions. Apart from these two ‘standard’ $N = 2$ multiplets there also exist two further multiplets containing an antisymmetric tensor $B_{\mu\nu}$: the tensor multiplet [3] containing three real scalars, one $B_{\mu\nu}$ and two Weyl fermions as well as the vector tensor multiplet containing one real scalar, a vector, one $B_{\mu\nu}$ and two Weyl fermions [4, 5, 6]. In $d = 4$ an antisymmetric tensor is dual to a scalar and thus the latter multiplets can be dualized to a hyper- or vector multiplet, respectively.

In type IIA string vacua one has $h_{1,1}$ vector multiplets, $h_{1,2}$ hypermultiplets and one tensor multiplet [7]. The Hodge numbers $h_{1,1}$ and $h_{1,2}$ count the non-trivial $(1, 1)$ and $(1, 2)$ forms on $Y_3$ while the additional tensor multiplet is universal and contains the type IIA dilaton.¹ In heterotic string compactifications the situation is slightly more involved. The dilaton sits in a vector-tensor multiplet, the moduli of the $K3$ form hypermultiplets while

¹In type IIB vacua one finds $h_{1,2}$ vector multiplets, $h_{1,1}$ tensor multiplets and one universal multiplet which contains two antisymmetric tensors. So far there is no off-shell formulation of this multiplet known but it is likely to exist. In any case one can always dualize one of the antisymmetric tensors and obtain an additional, universal tensor multiplet containing the type IIB dilaton.
the moduli of $T^2$ come in vector multiplets. In addition there are generically moduli which arise from the gauge bundle and they can be part of either vector- or hypermultiplets.

$N = 2$ supergravity severely constrains the interactions among these multiplets. In particular, the complex scalars of the vector multiplets are coordinates on a special Kähler manifold $\mathcal{M}_V$ [8] while the real scalars of the hypermultiplets are coordinates on a quaternionic manifold $\mathcal{M}_H$ [9]. Locally the two spaces form a direct product [10], i.e.

$$\mathcal{M} = \mathcal{M}_V \otimes \mathcal{M}_H.$$  \hspace{1cm} (1)

In string compactifications both components obey a non-renormalization theorem [5, 11]. In heterotic vacua the dilaton is part of a vector-tensor multiplet or its dual vector multiplet. The fact that the dilaton organizes the string perturbation theory together with the product structure of the moduli space (1) implies that the moduli space of the hypermultiplets is determined at the string tree level and receives no further perturbative or non-perturbative corrections, i.e.

$$\mathcal{M}^\text{Het}_H = \mathcal{M}^{(0)\text{Het}}_H.$$ \hspace{1cm} (2)

In type IIA compactifications the dilaton resides in a tensor multiplet or its dual hypermultiplet and thus the moduli space of the vector multiplets is exact and not corrected either perturbatively or non-perturbatively

$$\mathcal{M}^\text{II}_V = \mathcal{M}^{(0)\text{II}}_V.$$ \hspace{1cm} (3)

The conjectured duality between type IIA and heterotic $N = 2$ string vacua implies that the low energy effective theories have to be identical when all quantum corrections are taken into account. In particular their moduli spaces have to coincide, i.e.

$$\mathcal{M}^\text{Het}_V = \mathcal{M}^{(0)\text{Het}}_V, \quad \mathcal{M}^\text{Het}_H = \mathcal{M}^{(0)\text{Het}}_H.$$ \hspace{1cm} (4)

Thus, the entire $\mathcal{M}^\text{Het}_V$ can be obtained by doing a tree level computation in type IIA while the entire $\mathcal{M}^\text{Het}_H$ can be obtained by doing a tree level computation in the heterotic string.

So far the geometry and duality properties of $\mathcal{M}_V$ have been extensively studied. However, $\mathcal{M}_H$ has been much less studied [12] - [18] and in particular the duality conjecture has only been verified in a very specific (and simple) example [2]. In this talk we study $\mathcal{M}_H$ in string perturbation theory of type IIA vacua.\footnote{Some specific non-perturbative corrections in $\mathcal{M}_H$ are discussed in refs. [13, 14, 18].} This is a necessary first step in order to further establish the type IIA – heterotic string duality. Apart from this aspect it is an interesting question in its own right and furthermore might also teach us more about the $N = 1$ string perturbation theory.

At the string tree level $\mathcal{M}^{(0)\text{II}}_V$ is not the most general quaternionic manifold but constrained by the c-map [19]. The $h_{1,2}$ complex scalars of the NS-NS sector by themselves span a special Kähler manifold which is characterized by a holomorphic prepotential [8]. In type IIA string vacua they pair up with $2 \times h_{1,2}$ real scalars from the R-R sector to form hypermultiplets; the combined geometry of these $4 \times h_{1,2}$ scalars is quaternionic.

In type IIA string perturbation theory we do not apriori know if the c-map is preserved. However, we do know the following generic facts [15]:

- $N = 2$ supersymmetry is unbroken in perturbation theory and thus the geometry of $\mathcal{M}_H$ must be quaternionic.
The dilaton $\phi$ resides in a tensor multiplet together with $B_{\mu\nu}$ and two real scalars $\xi_0, \tilde{\xi}_0$ from the R-R sector; $\phi$ organizes the string perturbation theory.

- There are $h_{1,2}$ hypermultiplets containing the scalars $(Z^a, \bar{Z}^\alpha, \xi_a, \bar{\xi}_a)$ where $a = 1, \ldots, h_{1,2}$. The complex $Z^a$ arise in the NS-NS sector and span a special Kähler manifold at tree level. The $\xi_0, \tilde{\xi}_0, \xi_a, \bar{\xi}_a$ arise in the R-R sector and thus enjoy a continuous Peccei-Quinn (PQ) symmetry in perturbation theory

$$\xi_i \rightarrow \xi_i + \gamma_i, \quad \tilde{\xi}_i \rightarrow \tilde{\xi}_i + \bar{\gamma}_i, \quad \gamma_i, \bar{\gamma}_i \in \mathbb{R}, \quad i = 0, \ldots, h_{1,2} . \quad (5)$$

- The scalars $\xi_i, \tilde{\xi}_i$ always appear in pairs in string amplitudes.

- In the large volume limit there is an additional continuous PQ symmetry which acts on $Z^a - \bar{Z}^\alpha$ according to

$$Z^a - \bar{Z}^\alpha \rightarrow Z^a - \bar{Z}^\alpha + \hat{\gamma}^a, \quad \hat{\gamma}^a \in \mathbb{R} , \quad (6)$$

and also transforms the $\xi_i, \tilde{\xi}_i$ in a way specified in ref. [20].

These features strongly constrain the perturbation theory and do lead in fact to a further (perturbative) non-renormalization theorem.

Let us first study the simple case of a Calabi–Yau compactification with $h_{(1,2)} = 0$ which implies that only the dilaton multiplet (and $h_{(1,1)}$ vector multiplets) are present. The tree level Lagrangian (in the string frame) for the dilaton multiplet reads [21, 22]

$$e^{-1}L^{(0)} = e^{-2\phi} \left( -\frac{1}{2} \mathcal{R} + 2(\partial_\mu \phi)^2 - \frac{1}{6} (H_{\mu
u\rho})^2 \right) - \partial^\mu C \partial_\mu \bar{C} - H^\mu (C \partial_\mu \bar{C} - \bar{C} \partial_\mu C) \quad , \quad (7)$$

where we defined $H_{\mu
u\rho} = \partial_\mu B_{\nu\rho}$, $H^\lambda = \frac{1}{6!} \epsilon^{\mu
u\rho\lambda} H_{\mu
u\rho}$, $C = \xi_0 + i \tilde{\xi}_0$ and we omitted the couplings of the vector multiplets. In type IIA vacua there is a one-loop correction to the Ricci scalar [23, 16] given by

$$e^{-1}(L^{(0)} + L^{(1)}) = -\frac{1}{2} (e^{-2\phi} + \hat{\chi}) \mathcal{R} + \ldots , \quad (8)$$

where $\hat{\chi}$ is up to normalization factors the Euler number $\chi$ of the Calabi–Yau $\hat{\chi} \sim \chi = 2(h_{1,1} - h_{1,2})$. $N = 2$ supersymmetry uniquely determines the one-loop correction of the dilaton multiplet to be

$$e^{-1}(L^{(0)} + L^{(1)}) = (e^{-2\phi} + \hat{\chi}) \left( -\frac{1}{2} \mathcal{R} - \frac{1}{6} (H_{\mu
u\rho})^2 \right) + 2(e^{-2\phi} + \hat{\chi})^{-1} e^{-4\phi} (\partial_\mu \phi)^2$$

$$- \partial^\mu C \partial_\mu \bar{C} - H^\mu (C \partial_\mu \bar{C} - \bar{C} \partial_\mu C) \quad . \quad (9)$$

This action can be put into a more familiar form by dualizing $B_{\mu\nu}$ to a scalar field $a$ and perform a Weyl rescaling to the Einstein frame. One obtains

$$e^{-1}L = -\frac{1}{2} \mathcal{R} - \frac{|\partial_\mu S - 2\bar{C} \partial_\mu C|^2}{(S + S + 2\hat{\chi} - 2\bar{C}C)^2} - \frac{2|\partial_\mu C|^2}{(S + S + 2\hat{\chi} - 2\bar{C}C)} \quad , \quad (10)$$

where $S = e^{-2\phi} + i a + C \bar{C}$. The metric of the scalar fields is a Kähler metric with a Kähler potential

$$K = -\ln(S + S + 2\hat{\chi} - 2\bar{C}C) . \quad (11)$$
For $\hat{\chi} = 0$ one recovers the well known tree level manifold $SU(2,1)/U(2)$ [19, 21]. For $\hat{\chi} \neq 0$ we obtain precisely the metric conjectured by Strominger [15]. Expanding the metric of eq. (10) around large $S$ (weak coupling) it appears to have contributions at all orders in perturbation theory. However, as we just showed this is an artefact of the dualization and the definition of the $S$ field. In the field basis where the antisymmetric tensor is used (which is the appropriate basis of string perturbation theory) this correction is manifestly one-loop.$^3$

Furthermore, one can show that there is a perturbative non-renormalization theorem in that the action of eqs. (9), (10) is exact in perturbation theory and does not receive any perturbative corrections beyond one-loop [26].$^4$ This can be seen from the last term in (9) which cannot be multiplied by any power of $e^{-2\phi}$ (as would be necessary for higher loop corrections) without violating the Peccei-Quinn symmetry of eq. (5).$^5$ Alternatively one can show that the action (10) is the unique action compatible with the quaternionic geometry and all other perturbative properties enumerated above [26].

Let us now come to the general case where in addition to the dilaton multiplet also $h_{[1,2]}$ hypermultiplets are present. This situation is presently under investigation and we only indicate our preliminary results here [26]. The known loop corrections are [23, 16]

$$e^{-1}\mathcal{L} = -\frac{1}{2} (e^{-2\phi} + \hat{\chi}) \mathcal{R} - (e^{-2\phi} - \hat{\chi}) G_{ab}\partial_\mu Z^a \partial_\mu \bar{Z}^b + \frac{1}{4} H^\mu \left( \xi^i \partial_\mu \xi_i - \xi^i \partial_\mu \bar{\xi}_i - 2(\hat{\chi} V_\mu) \right) + \ldots \quad (12)$$

where $G_{ab}$ is the tree level Kähler metric of the $Z^a$ and $V_\mu = K_a \partial_\mu Z^a - K_{\bar{a}} \partial_\mu \bar{Z}^{\bar{a}}$. Thus, the loop correction is universal in that there is a universal correction to $G_{ab}$ and the Calabi-Yau geometry is not modified. Furthermore, the presence of the $H^\mu$ coupling together with the perturbative PQ-symmetries seem to imply a non-renormalization theorem: The geometry of $\mathcal{M}_H$ has a universal correction at one-loop but no higher perturbative corrections. More details will be presented in [26].

Acknowledgements

This work is supported in part by: GIF – the German–Israeli Foundation for Scientific Research (J.L.), the DAAD – the German Academic Exchange Service (C.H.) and the Landesgraduiertenförderung Sachsen-Anhalt (H.G.).

We thank I. Antoniadis and B. de Wit for usefull conversations. C.H. thanks Jan Louis and his group for their warm hospitality during a stay in Halle where this collaboration was initiated. J.L. thanks the organizers of the conference for providing such a pleasant and stimulating atmosphere.

References


$^3$This solves the apparent puzzle in [15] where compactification of $R^4$ terms in M-theory [24, 25] were used to identify the one-loop correction in the $S$-basis. The resulting metric was not quaternionic which lead Strominger to conjecture his “all-loop” formula. However, since $N = 2$ supersymmetry is unbroken one does expect a quaternionic metric at each loop order. What we just showed is that Stromingers all-loop formula is in fact only one-loop in the appropriate string basis.

$^4$Of course, non-perturbative corrections do appear [13, 14, 18].

$^5$In some sense this is a four-dimensional version of the non-renormalization theorem discussed in [25].
   K. Becker and M. Becker, hep-th/9901126.