The scale of homogeneity in the Las Campanas Redshift Survey
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ABSTRACT

We analyse the Las Campanas Redshift Survey using the integrated conditional density (or density of neighbors) in volume-limited subsamples up to unprecedented scales (200 Mpc/h) in order to determine without ambiguity the behavior of the density field. We find that the survey is well described by a fractal up to 20-30 Mpc/h, but flattens toward homogeneity at larger scales. Although the data are still insufficient to establish with high significance the expected homogeneous behavior, and therefore to rule out a fractal trend to larger scales, a fit with a CDM-like spectrum with high normalization well represents the data.

Subject headings: galaxies: clusters: general - large-scale structure of universe

Following seminal work of Pietronero and coworkers, (see e.g. Pietronero et al. 1996), the possibility of a large-scale fractal distribution of the galaxies has been investigated by various authors. In the current literature, however, there are several conflicting estimates of the largest scale at which the galaxy distribution can be approximated by a fractal, ranging from a few Megaparsecs (e.g. Peebles 1993), to 20 Mpc/h (Davis 1996), to 40 Mpc/h (Cappi et al. 1998), up to more than 100 Mpc/h (e.g. Pietronero et al. 1996). A fractal distribution with dimension $D$ is characterized by the property that the correlation function

$$ g(r) = 1 + \xi(r) $$

(1)

decreases as a power law, $\sim r^{3-D}$. Consequently, the average density $\rho_c$ of galaxies at distance $r$ from another galaxy, or conditional density, also decreases as $\sim r^{3-D}$ since, by the definition of correlation function, $\rho_c = \rho_0 [1 + \xi(r)]$, where $\rho_0$ is the cosmic average density.

Naturally, one can infer the scale at which fractality gives way to homogeneity from several other observations, like the cosmic backgrounds, although the conclusions are bound to be model dependent. The availability in recent years of deep redshift surveys allows finally to study the matter distribution directly from its primary tracers, the galaxies. The deepest galaxy redshift survey so far published is the Las Campanas Redshift Survey (LCRS), Schectman et al. (1996). LCRS contains 23,697 galaxies with an average redshift $z = 0.1$, distributed over six $1.5^0 \times 80^0$ slices. In this paper we determine the behavior of $\hat{g}(r)$, the volume integral of $g(r)$, and of the
fractal dimension $D(r)$, in volume-limited subsamples of LCRS up to $r = 200$ Mpc/$h$, the largest scale so far investigated with such a statistics, by making use of purposely designed cells. This scale is more than four times the scale previously reached by Cappi et al. (1998) using SSRS2.

Let us begin by discussing why the statistics $g(r)$ is particularly convenient for our purposes. By far the most popular two-point estimators that have been used to investigate the clustering of the galaxies are the correlation function $\xi(r)$ and the power spectrum $P(k)$, a Fourier conjugate pair. However, the simplest statistics to use to determine the fractal properties of a distribution is $g(r)$, or $\hat{g}(r) = V^{-1} \int_0^r g(r') dV$, in terms of which a fractal is defined. There is also another reason to use $\hat{g}(r)$ to study the galaxy clustering. The spectrum of a finite survey is actually the convolution of the true power spectrum with the survey geometry. As a consequence, both the shape and the amplitude of the estimated spectrum are different from the true spectrum. In particular, the estimation of the average density from the sample itself forces the spectrum to vanish for $k \to 0$, so that the detection of a turnaround in the spectrum is often suspect (see for instance Sylos Labini & Amendola 1996). A similar problem occurs with the correlation function: the integral constraint (Peebles 1980) forces the correlation function to become negative at some scale, distorting its shape. In fact, using a subscript $s$ to denote quantities estimated in a finite sample of size $R_s$, and employing the fact that the density $\rho_s$ in a sample around an observer is a conditional density, the following relation holds

$$\hat{\xi}_s(r) = \frac{\xi(r) - \hat{\xi}(R_s)}{1 + \hat{\xi}(R_s)},$$

which shows explicitly the integral constraint $\int \xi_s dV_s = 0$. This problem is clearly absent from the statistics $g(r)$ because $g_s$ and $g$ are simply proportional, $g_s(r) = g(r)/\hat{g}(R_s)$. It follows that, contrary to what happens for $\xi$ and $P(k)$, the slope of $g_s$ is an unbiased estimator of the slope of the clustering trend, i.e. of its fractal dimension. In fact, we can evaluate the fractal dimension as

$$D = 3 + \frac{d \log \hat{g}}{d \log r},$$

and verify that $D_s = D$. Therefore, to test the claim of fractality it is necessary to check whether $g(r)$ can be approximated by a power-law, in which range of scales, and with which slope. The approach to homogeneity is then characterized by $D \to 3$, that is, to a flattening of $g(r)$.

The statistics $g(r)$ is a differential quantity. When applied to surveys with a relatively small density, as are the volume-limited slices of Las Campanas, it tends therefore to be very noisy. In this case, one can smear out the noise integrating over cells, obtaining the integrated correlation $\hat{\xi}(r)$. Here an important problem arises. Consider a distribution of particles described by a statistically isotropic correlation function $\xi(r)$. When we evaluate the expected total number of neighbors within a distance $r$, we are performing the integral

$$\hat{\xi}(r) = \frac{3}{r^3} \int_0^r \xi(r') r'^2 dr'.$$
If the cell is not spherical, the integral becomes 

\[ \hat{\xi} = \int \xi(r')W(r', \theta, \phi)dV, \]

where the window function \( W(r, \theta, \phi) \) is defined to be constant inside the cell and zero outside and is normalized to unity. Of course, only the part of the cell completely contained within the survey has to be considered. However, if the cells are not spherically symmetric, the value of \( \hat{\xi}(r) \) depends on the exact form of \( W(r, \theta, \phi) \), and in general is different from the definition in (4). Moreover, \( W(r, \theta, \phi) \) may vary from cell to cell. The obvious solution to this problem is to restrict the analysis to spherical cells. However, this limits the scale to the largest sphere contained within the survey boundaries which, for most surveys, can be very small: e.g., less than 10 Mpc/ \( h \) by radius for LCRS. As a matter of fact, so far all the works who used the \( \hat{g} \) statistics adopted spherical cells, thereby limiting the scales to less than 50 Mpc/ \( h \). Since at this scale the fractal behavior is more or less within the standard description (e.g., CDM), it is crucial to extend the analysis to larger scales. The simplest way to do so is to consider radial cells, that is, cells whose window function can be factorized in a radial and an angular function, both normalized to unity

\[ W(r, \theta, \phi) = W_r(r)W_{\Omega}(\theta, \phi). \] (5)

In this case in fact the function \( \hat{\xi}(r) \) is the same as in spherical cells, since the angular factor can be integrated out. The advantage is that one can design a radial cell such as to maximize the scale \( r \), still fitting it within the survey. As elementary as it is, this method has never been used in the literature on the galaxy clustering.

Before going to the data analysis, let us estimate the variance of \( \hat{g} \). Let us first assume that the three-point correlation function can be written as (Peebles 1980) \( \varsigma_{ijk} = Q[\xi_{ij}\xi_{jk} + \xi_{ij}\xi_{ik} + \xi_{ik}\xi_{jk}] \), where \( Q \) is independent of the spatial coordinates. Then, the variance of \( \hat{g} \) evaluated in \( N_c \) independent cells containing on average \( N_0 \) galaxies is (Peebles 1980; Amendola 1998)

\[ \sigma_g^2 \equiv N_c^{-1}\left[ N_0^{-1}(1 + \hat{\xi}) + \sigma^2 - \hat{\xi}^2 + Q(\hat{\xi}^2 + 2K_2) \right], \] (6)

where \( \sigma^2 = \int W_1dV_1W_2dV_2\xi_{12} \). The first term in Eq. (6) is the Poisson noise. Inserting the power spectrum we have

\[ \sigma^2 = (2\pi)^{-1}\int P(k)W_c(k)k^2dk, \] (7)

where \( W_c(k) \) equals \( W(k)^2 \) for spherical cells, but has to be evaluated numerically in the more general case that we study here (see Amendola 1998). The last term in the expression (6) can be written as (Peebles 1980) \( K_2 = \int W_1dV_1W_2dV_2\xi(r_1)\xi(r_{12}) \). In the important case in which \( \xi \) is a power law, it can be shown that a very good approximation is \( K_2 \sim \sigma^2\hat{\xi} \). For instance, if \( \xi \sim r^{-1} \), it turns out that \( K_2 = 1.04\sigma^2\hat{\xi} \). In the following, we will always approximate \( K_2 \) in this way.

Another problem arises in practice, namely that the cells we use are not independent, both because they are partially overlapping and because the clustering scale may be larger than the distance from cell to cell. The effect of the correlation is to reduce the number \( N_c \) at the denominator in Eq. (6). For instance, if the cells oversample the volume by a factor of two, it means that a cell out of two is redundant, and the effective number of cells can be taken as \( N_c/2 \).
In general, the number of effective independent cells may be approximated as $N_e = \min(N_c, V/V_c)$, where $V_c$ is the cell volume, although of course even this is an overestimation of the independent cells. Naturally, we could use $N$–body simulations to estimate the errors including the cell-to-cell correlation, but then we should generate a different simulation for any model we want to compare with; moreover, any finite $N$–body will inevitably cut large scale power, which in the case of testing fractals is a particularly severe limitation. Some comparisons with $N$–body shows that Eq. (6) underestimates the errors only by 30% at most.

In the case of an exact fractal, the expected value of $\hat{g}$ inside a spherical region of radius $r$ embedded in a larger box of size $R_0$ is (Coleman & Pietronero 1992)

$$\hat{g}(r) = (r/R_0)^{D-3}. $$

(8)

It is not difficult to show that, neglecting $\sigma^2$ and $\hat{\xi}$ with respect to $\hat{\xi}^2$, i.e. in the limit of $R_0 \gg r$, and neglecting the Poisson noise, the variance in a fractal is

$$\sigma^2_{\hat{g}} \simeq N_c^{-1}\left[Q\left(1 + \frac{2DJ_2(\gamma)}{3}\right) - 1\right]\hat{g}^2, $$

(9)

where $\gamma = 3 - D$, and $J_2(\gamma) = 72/[(3 - \gamma)(4 - \gamma)(6 - \gamma)2^\gamma]$. Notice that the relative error on $\hat{g}$ is independent on the scale, as indeed is found numerically (Amici & Montuori 1998). For instance, for $Q = 1$ and $D = 2$, as some observations suggest, $\sigma^2_{\hat{g}} = N_c^{-1}(8/5)\hat{g}^2$.

An important consequence of Eq. (9) is that the relative error of the conditional density measured in a single cell can be very large for a fractal, more than 100%. Then, the average density in a sample of galaxies around us, a conditional density, has such a large variance, in a fractal, that it gives in practice no information. A further consequence is that the variance of the amplitude of the correlation function, and of related quantities as $r_0$ and $\sigma_8$, makes the use of the correlation amplitude, as opposed to its slope, useless in the case of fractals. This problem applies also to the case in which the density of a sample $n(r)$ is measured as a function of the distance from the observer, without averaging over several cells.

If we define the scale of homogeneity as the scale at which $\hat{g}$ flattens so that $D \geq 2.9$, then we can quantify it in any given CDM model. Clearly, this scale will be larger the higher is the normalization $\sigma_8$. It turns out that if $\sigma_8 \approx 1.5$, as observed for bright galaxies in SSRS2 (Benoist et al 1996) the CDM homogeneity scale can be as large as 50 Mpc/$h$, and reach $\approx 70$ Mpc/$h$ for clusters ($\sigma_8 \approx 2$): therefore, only at scales larger than 50 Mpc/$h$ the gap between the pure fractal model and the standard scenarios begins to be significant.

The $\hat{g}$ statistics in spherical cells has been applied to several galaxy surveys (Pietronero, Montuori & Sylos Labini 1997, Sylos Labini and Montuori 1997, Sylos Labini et al. 1998, Cappi et al. 1998). Here we summarize only the results from the deepest of such surveys, SSRS2 (Da Costa et al. 1994) . SSRS2 includes 3600 galaxies in 1.13 sr of the southern sky, down to an apparent magnitude of 15.5. The results of the analysis in Cappi et al. (1998) indicate that the conditional density decreases as $r^{-1}$ from 1 to 40 Mpc/$h$ in all the volume limited samples considered,
implying fractality with $D = 2$ on these scales, with no indication of flattening. Similar results are presented in Sylos Labini and Montuori (1997) for the APM-Stromlo survey and in Sylos Labini et al. (1998) for CfA2 and SSRS2 surveys. These are the largest scales that can be probed via this method. In Cappi et al. (1998) the conditional density from the observer alone, i.e. from the vertex of the sample, has also been studied, in order to extend the range. The results are that the sample is fractal up to 40-50 Mpc/h and tends to flatten above this scale. However, the errors, as expected, are quite large; within 2$\sigma$ the deepest samples include all values between $D = 2.7$ and $D = 4$. The errors quoted in Cappi et al. (1998), moreover, do not include the ensemble variance that, for fractals, is very large, as already mentioned.

We applied our method of the radial cells to the Las Campanas redshift survey, LCRS, the deepest redshift survey so far studied. LCRS contains fields which includes galaxies with magnitude between 16.0 and 17.3, and fields with limits 15.0 and 17.7. Every field $i$ has associated a sampling factor $0 < f_i < 1$ which is the fraction of the galaxies randomly chosen out of the total number in the field within the magnitude limits; the weights $1/f_i$ must be taken into account in the statistics. We considered the only slice with all fields of large magnitude range (slice at -12$^h$). We evaluated the conditional density integrated in radial cells with a shape and orientation such as not to intersect the survey boundaries. We cut the sample into four VL subsamples, denoted as VL147-297, VL 190-330, VL280-410, and VL224-437 (see Fig. 1 and Table I) where the numbers give the lower and upper cutoff distance (the two cuts are necessary because LCRS has two limiting magnitudes). The use of volume-limited samples avoids the uncertainties connected to the radial selection functions. The correlation function in the full magnitude-limited sample is evaluated in Tucker et al. (1997). The results for $\hat{g}(r)$ are shown in Fig. 2. The errors are calculated from Eq. (6) using two models, the standard CDM with galaxy normalization $\sigma_8 = 1.5$ and $\Gamma = 0.3$, and the pure fractal with $D = 2$ and $Q = 1$. For comparison, we evaluated the errors from a simulation of a standard CDM model, and found that our theoretical estimate of the errors is approximate to better than 30%. It can be seen that there is an approximate $D \approx 2$ fractality on small scales, up to 20 or 30 Mpc/h, just as in the SSRS2 case, followed by a flattening of the slope. The results from SSRS2 of Cappi et al. (1998; sample with magnitude cut at -20) obtained with full spheres are reported in Fig. 2 along with VL147-297, the LCRS sample with average magnitude closer to SSRS2. As we can see, the scales we reach here are the largest scales ever reached for the $\hat{g}(r)$ statistics. In the case of VL280-410, the trend is decreasing, albeit with a change in slope, down to more than 100 Mpc/h, while for VL224-437 (the most sparse sample) a very noisy flattening is reached already at 40 Mpc/h. In Fig. 3 we plot the fractal dimension $D$ as a function of $r$, evaluated as least square slope of sets of five contiguous points of $\hat{g}(r)$. The tendency to $D = 3$ is very clear. It is also seen that the dimension is not really constant in any range, although the scatter is too large to infer a clear trend.

From these results we can conclude that there is a tendency to homogenization around 50-100 Mpc/h, as expected from a CDM model. However, we remark that we did not detect a clear homogeneous behavior, that is $\hat{g}(r) = 1$, not even at more than 100 Mpc/h, except for the most
sparse sample. This leaves again space for a fractal behavior to larger scales, especially in view of the large errors associated to a fractal. The dimension would be however closer to 3 at large scales.

We compared our results to the CDM-like power spectrum

\[ P(k) = A k T^2(\Gamma, k) G(\Omega_m, \Omega_\Lambda, \sigma_{8g}, \sigma_{8m}, \sigma_v), \]

where \( T^2(\Gamma, k) \) is the transfer function of Bond & Efstathiou (1984), \( G \) includes the redshift correction of Peacock & Dodds (1994) and the non-linear correction of Peacock & Dodds (1996), \( \sigma_v \) is the line-of-sight velocity dispersion, and the subscripts \( \Lambda, g \) and \( m \) refer to the cosmological constant, the galaxies and the total matter, respectively. In Fig. 2 we compare LCRS with a tentative model with \( \Gamma = 0.24, \sigma_{8g} = 1.4 \), assuming, for the other parameters, the values \( \sigma_{8m} = 1, \Omega_{tot} = 1, \Omega_m = 0.4, \Omega_\Lambda = 0.6, h = 0.6, \sigma_v = 300 \text{ km/sec.} \) Finally, we find the best fit to \( \hat{g} \) by varying \( \Gamma \) and \( \sigma_{8g} \) for scales larger than 30 Mpc/\( h \), in order to avoid the non-linear corrections at smaller scales. The results are listed in Table I. The average for all four samples is \( \sigma_{8g} = 1.5, \Gamma = 0.3 \).

To summarize, in this Letter we have applied the technique of radial cells to LCRS, pushing the analysis of fractality to 200 Mpc/\( h \). We have shown that a proper treatment of the errors is important in order to compare alternative models, and is particularly crucial for a fractal, in which the variance tends to be very large. We have also shown that the use of radial cells allows very large scales to be probed. The conclusion is that LCRS is well described by a CDM model with a high normalization, up to \( \sigma_8 \approx 1.6 \) for the deepest, and thus brightest, sample. The trend can be also approximated as a \( D \approx 2 \) fractal up to 20-30 Mpc/\( h \), but shows a clear flattening afterward.

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Fig. 1.— Two of the volume limited subsamples of LCRS that we analyse in this work.

Fig. 2.— The function $\hat{g}(r)$ in various VL samples of LCRS with the errors expected in a CDM model and in a fractal $D = 2$ model (dotted lines). The thin line is the tentative CDM fit (see text). In the upper right panel we also report for comparison $\hat{g}(r)$ from a sample of SSRS2, from Cappi et al. (1998)

Fig. 3.— The function $D(r)$ for the four samples, obtained as the least square slope of sets of five contiguous values of $\hat{g}(r)$

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Table I:

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