String or $M$ Theory Axion as a Quintessence

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Abstract

A slow-rolling scalar field ($Q \equiv$ Quintessence) with potential energy $V_Q \sim (3 \times 10^{-3} \text{ eV})^4$ has been proposed as the origin of accelerating universe at present. We investigate the effective potential of $Q$ in the framework of supergravity model including the quantum corrections induced by generic (non-renormalizable) couplings of $Q$ to the gauge and charged matter multiplets. It is argued that the Kähler potential, superpotential and gauge kinetic functions of the underlying supergravity model are required to be invariant under the variation of $Q$ with an extremely fine accuracy. Applying these results for string or $M$-theory, we point out that the heterotic $M$-theory or Type I string axion can be a plausible candidate for quintessence if (i) it does not couple to the instanton number of gauge interactions not weaker than those of the standard model and (ii) the modulus partner $\text{Re}(Z) \equiv \text{Im}(Z) + 1$ has a large VEV: $\text{Re}(Z) \sim \frac{1}{2\pi} \ln(m_{3/2}^2 M_{\text{Planck}}^2 / V_Q)$. It is stressed that such a large $\text{Re}(Z)$ gives the gauge unification scale at around the phenomenologically favored value $3 \times 10^{16}$ GeV. To have a negative pressure, at present the quintessence axion is required to be at near the top of its effective potential. We discuss a scenario which would yield such present value of the quintessence axion in a natural manner.
I. INTRODUCTION AND SUMMARY

Recent measurements of the luminosity-red shift relation for Type Ia supernovae suggest that the Universe is accelerating and thus a large fraction of the energy density has negative pressure \[1\]. A nonvanishing cosmological constant (vacuum energy density) would be the simplest form of dark energy density providing the necessary negative pressure. However, there is a widespread prejudice that the correct, if exists any, solution to the cosmological constant problem \[2\] will lead to an exactly vanishing vacuum energy density. For instance, within the framework of the Euclidean wormhole mechanism \[3\] which has been proposed sometime ago as a solution to the cosmological constant problem, the vacuum energy density appears to exactly vanish if all fields (except for the spacetime metric) were settled down at their true vacuum expectation values (VEVs).

Recently quintessence in the form of slow-rolling scalar field has been proposed as an alternative form of dark energy density with negative pressure \[4\]. The true minimum of the quintessence potential is presumed to vanish, i.e. \((V_Q)_{\text{min}} = 0\), however the present value of \(Q\) is displaced from the true minimum, providing

\[
V_Q \sim (3 \times 10^{-3}\text{eV})^4, \tag{1}
\]

and a negative pressure with the equation of state:

\[
\omega = \frac{p_Q}{\rho_Q} = \frac{1}{2} \frac{\dot{Q}^2 - V_Q}{\dot{Q}^2 + V_Q} \lesssim -0.5, \tag{2}
\]

while obeying its equation of motion:

\[
\ddot{Q} + 3H\dot{Q} + \frac{\partial V_Q}{\partial Q} = 0. \tag{3}
\]

In order to have negative pressure, we then need the following slow-roll condition:

\[
\frac{\partial V_Q}{\partial Q} \lesssim \frac{V_Q}{M_P}, \tag{4}
\]

where \(M_P = M_{\text{Planck}}/\sqrt{8\pi} = 2.4 \times 10^{18}\ \text{GeV}\) denotes the reduced Planck scale. This slow-roll condition indicates that the typical range of \(Q\) is of order \(M_P\) or at least not far below \(M_P\).

Given the assumption of \((V_Q)_{\text{min}} = 0\), one can in principle compute the quintessence potential \(V_Q\) once the particle physics model for \(Q\) is given. However the required size of \(V_Q\) is extremely small compared to any of the mass scales of particle physics, and thus an utmost question is how such a small \(V_Q\) can arise from realistic particle physics models. In fact, the most natural candidate for a light scalar field whose typical range of variation is of order \(M_P\) is the string or \(M\)-theory moduli multiplets describing the (approximately) degenerate string or \(M\)-theory vacua. It is thus quite tempting to look at the possibility that \(Q\) corresponds to a certain combination of the string or \(M\)-theory moduli superfields. In this paper, we wish to explore this possibility and point out that the heterotic \(M\)-theory or Type I string axion can be a plausible candidate for quintessence if its modulus component has a large VEV.
To make the discussion more rigorous, we provide in section II a careful analysis of the low energy effective potential of generic quintessence\(^1\) in the framework of 4-dimensional effective supergravity model which is presumed to describe the dynamics of \(Q\) at high energy scales. We include both the perturbative and nonperturbative corections to \(V_Q\) induced by generic (nonrenormalizable) couplings of \(Q\) with the gauge and charged matter multiplets in the model. It is then argued that the Kähler potential \((K)\), superpotential \((W)\) and gauge kinetic functions \((f_a)\) of the underlying supergravity model are required to be invariant under the variation of \(Q\) with an extremely fine accuracy, which implies as one of its consequences that non-derivative couplings of \(Q\) are extremely suppressed and so there is no \(Q\)-mediated long range force. Also estimated are the size of \(Q\)-invariance breaking for various terms in \(K\), \(W\) and \(f_a\) which would produce the correct value of \(V_Q\).

Applying these results for string or \(M\)-theory, we show in section III that the heterotic \(M\)-theory or Type I string axion can be a plausible candidate for quintessence. The quintessence axion may correspond to the heterotic \(M\)-theory axion arising from the three form field of 11-dimensional supergravity on a manifold with boundary [6] or to the Type I string axion arising from the R-R two form field [7]. To avoid a too large potential energy, it is required that the quintessence axion does not to couple to the instanton number of gauge interactions not weaker than those of the standard model, particularly not to those of the QCD and even stronger hidden sector gauge interactions. Then the effective potential of heterotic \(M\)-theory axion is mainly induced by the membrane instantons wrapping the 2-cycle of the internal 6-manifold and stretched along the 11-th dimension, while that of Type I string axion is induced by \(D1\) or \(D5\) instantons wrapping the 2 or 6-cycle. The resulting axion potential is estimated as

\[
V_Q \sim e^{-2\pi \langle \text{Re}(Z) \rangle} m_{3/2}^2 M_P^2 \cos[2\pi \text{Im}(Z)],
\]

where \(\text{Re}(Z)\) is the modulus partner of the periodic quintessence axion \(\text{Im}(Z) \equiv \text{Im}(Z) + 1\) and \(m_{3/2}\) is the gravitino mass. Thus the quintessence potential energy (1) is obtained if (i) \(\text{Re}(Z)\) takes a VEV significantly larger than the self dual value of order unity:

\[
\langle \text{Re}(Z) \rangle \sim \frac{1}{2\pi} \ln(m_{3/2}^2 M_P^2 / V_Q) \sim 35,
\]

and (ii) (one-loop) threshold corrections to the gauge coupling constants not weaker than those of the standard model are \(Z\)-independent, which would assure that the quintessence axion does not to couple to the instanton numbers generating a too large \(V_Q\).\(^2\)

As will be discussed in section IV, when combined with \(\alpha_{GUT} = 1/25\) and \(M_P = 2.4 \times 10^{18}\) GeV, the quintessence axion potential (5) which is presumed to take the value of (1) fixes all the couplings and scales of the underlying model. Particularly it gives the gauge unification scale

\[
M_{GUT} \sim 1.3 \gamma \times 10^{16} \left( \frac{35}{\langle \text{Re}(Z) \rangle} \right)^{1/2} \text{GeV},
\]

\(^1\)Some aspects of the quintessence potential was discussed recently in [5].

\(^2\)Of course this is true only when the tree level gauge coupling constants are also \(Z\)-independent.
where $\gamma$ is a model-dependent constant of order unity which will be defined later. It is then interesting to note that $\langle \text{Re}(Z) \rangle \sim 35$ required for a quintessence axion determines $M_{\text{GUT}}$ at a value close to the phenomenologically favored $3 \times 10^{16}$ GeV.

One potentially serious difficulty of the quintessence string or $M$-theory axion is that it requires a fine tuning of the present value of $\text{Im}(Z)$. As we will see, the canonical quintessence axion field is given by

$$Q \sim \frac{M_P \text{Im}(Z)}{\langle \text{Re}(Z) \rangle^2},$$

and so the slow-roll condition (4) applied for the axion potential (5) leads to

$$|2\pi \text{Im}(Z)|_{\text{present}} \lesssim \frac{1}{2\pi \langle \text{Re}(Z) \rangle} = \mathcal{O}(10^{-2}),$$

implying that at present the angular field $2\pi \text{Im}(Z)$ takes a value at near the top of its effective potential. At the end of section IV, we discuss a scenario yielding such present value of $\text{Im}(Z)$ in a rather natural manner, which is based on the modular and CP invariance of the moduli effective potential.

II. EFFECTIVE POTENTIAL OF GENERIC QUINTESSENCE

Our starting point is the 4-dimensional $N = 1$ effective supergravity model [8] which is presumed to describe the dynamics of $Q$ at high energy scales not far below $M_P$. The Kähler potential, superpotential and gauge kinetic functions of the model can be written as

$$K = K_0(Z_i, Z_i^*) + Z_{\alpha\beta}(Z_i, Z_i^*) C^\alpha C^\beta + \frac{1}{2} H_{\alpha\beta}(Z_i, Z_i^*) C^\alpha C^\beta$$

$$+ \frac{1}{2} X_{\alpha\beta\gamma}(Z_i, Z_i^*) C^\alpha C^\beta C^\gamma + \text{h.c.}] + \ldots,$$

$$W = W_0(Z_i) + \frac{1}{3!} Y_{\alpha\beta\gamma}(Z_i) C^\alpha C^\beta C^\gamma + \frac{1}{4!} \Gamma_{\alpha\beta\gamma\delta}(Z_i) C^\alpha C^\beta C^\gamma C^\delta + \ldots,$$

$$f_a = f_{a0}(Z_i) + \frac{1}{2} F_{aa\beta}(Z_i) C^\alpha C^\beta + \ldots,$$

where $C^\alpha$ denote the gauge-charged light matter superfields whose VEVs are far below $M_P$ and $Z_i$ stand for generic gauge-singlet light multiplets including the fields with large VEVs of $\mathcal{O}(M_P)$. $Z_i$ may correspond to the string or $M$-theory moduli superfields and/or some gauge-invariant composite superfields made of strongly interacting hidden sector fields and/or others whose typical range of variation is of $\mathcal{O}(M_P)$. The quintessence $Q$ is included in the model as a certain combination of $Z_i$ and $Z_i^*$:

$$Q \in \{Z_i, Z_i^*\}.$$

The ellipses stand for the terms of higher order in $C^\alpha$, and the leading non-renormalizable terms (in power countings of $C^\alpha$) are explicitly written to illustrate their effects on $V_Q$. Note that unless specified we are using the standard supergravity unit with $M_P = 1$ throughout this paper.
The superpotential \( W_0(Z_i) \) is assumed to be induced by some (nonperturbative) supersymmetry (SUSY) breaking dynamics, e.g. hidden sector gaugino condensation, which is already integrated out. The auxiliary \( F \)-components of certain \( Z_i \) develop non-zero VEVs:

\[
F_i = \langle e^{K_0/2} D_i W_0 \rangle = \mathcal{O}(m_{3/2} M_P),
\]

thereby breaking SUSY, where \( D_i W_0 = \partial_i W_0 + W_0 \partial_i K_0 \) is the Kähler covariant derivative with \( \partial_i = \partial/\partial Z_i \). Here we concentrate on the supergravity-mediated SUSY breaking scenario [8] yielding the soft SUSY breaking parameters (the soft scalar masses, gaugino masses, e.t.c.) of order the gravitino mass \( m_{3/2} = \langle e^{K_0/2} |W_0| \rangle \). For the case of gauge-mediation [9], the discussion will be modified but still leads to the same conclusion that \( K, W, \) and \( f_a \) should be invariant under the variation of quintessence, i.e. \( Q \)-independent, with an extremely fine accuracy. Note that unless constrained by some symmetries the coefficients functions in \( K, W, \) and \( f_a \) are generically \( Q \)-dependent, particularly when the terms of arbitrary order in \( Q \) are taken into account. Those coefficient functions determine the (nonrenormalizable) couplings of \( Q \) to the light gauge and charged matter multiplets. For instance, \( Q^n F_{\mu
u} F^{\alpha\beta}/M_P^n \) arises from a \( Q \)-dependent \( \text{Re}(f_a) \), \( Q^n F_{\mu
u} F^{\alpha\beta}/M_P^n \) from a \( Q \)-dependent \( \text{Im}(f_a) \), \( Q^n C^{\alpha\beta\gamma\delta}/M_P^n \) from a \( Q \)-dependent \( Y_{\alpha\beta\gamma} \), and so on. At any rate, once the \( Q \)-invariance condition is fulfilled, non-derivative couplings of \( Q \) are highly suppressed, and as a result \( Q \) does not mediate any macroscopic force.

Obviously the tree level potential of \( Q \) can be determined by \( K_0 \) and \( W_0 \) via the conventional supergravity potential:

\[
V_0 = e^{K_0} [K^{ij} D_i W_0 D_j W_0^* - 3 |W_0|^2] \sim m_{3/2}^2 M_P^2
\]

where \( K^{ij} \) is the inverse of the Kähler metric \( K_{ij} = \partial_i \partial_j K_0 \) and the typical range of \( V_0 \) under the variation of \( Z_i = \mathcal{O}(M_P) \) is estimated to be of order \( m_{3/2}^2 M_P^2 \). When quantum corrections are taken into account, generic (nonrenormalizable) couplings of \( Q \) to charged gauge and matter multiplets contribute to \( V_Q \). The leading quantum correction comes from the quadratically divergent one-loop potential [10] including the piece depending on \( Z_{\alpha\beta} \) and \( \text{Re}(f_a) \):

\[
\delta V_1 = \frac{\Lambda^2}{16 \pi^2} e^{K_0} D_i W_0 D_j W_0^* R^{ij} \sim \frac{1}{16 \pi^2} m_{3/2}^2 \Lambda^2
\]

where the cutoff scale \( \Lambda \) is of order the messenger scale of SUSY breaking, which is essentially of \( \mathcal{O}(M_P) \) for supergravity-mediated SUSY-breaking models,

\[
R^{ij} = K^{ij} K^{kl} \partial_i \partial_j [\ln(\det Z_{\alpha\beta}) - \ln(\det \text{Re}(f_{ab}))],
\]

where \( f_{ab} = f_a \delta_{ab} \), and the typical range of \( \delta V_1 \) is estimated to be of order \( \frac{1}{16 \pi^2} m_{3/2}^2 \Lambda^2 \) for the variation of \( Z_i = \mathcal{O}(M_P) \). This piece of one-loop potential arises from the well-known \( V_1 = \frac{1}{16 \pi^2} \Lambda^2 \text{Str}(M^2) \) where the mass matrix \( M^2 \) includes the soft SUSY breaking scalar and gaugino masses depending on \( Z_{\alpha\beta} \) and \( \text{Re}(f_a) \) [10].

\( H_{\alpha\beta} \) and \( X_{\alpha\beta\gamma} \) in \( K \) and the Yukawa couplings \( Y_{\alpha\beta\gamma} \) in \( W \) affect the potential energy at two-loop order. It has been noted that the two-loop supergraph of Fig. 1 leads to the quadratically divergent Yukawa-dependent potential energy [11]:
\[
\delta V_2 \sim \frac{\Lambda^2}{(16\pi^2)^2} e^{K_0} |DW_0|^2 |Y|^2 \sim \frac{|Y|^2}{(16\pi^2)^2} m_Z^2/2 \Lambda^2,
\]
(15)

where
\[
|DW_0|^2 = K^{i\bar{j}} D_i W_0 D_{\bar{j}} W_0,
\]
\[
|Y|^2 = Z^{a\bar{a}} Z^{\bar{\beta}\beta} Z^{\gamma\bar{\gamma}} Y_{\alpha\beta\gamma} Y_{\bar{a}\beta\bar{\gamma}}^*.
\]

for \(Z^{\alpha\bar{\beta}}\) being the inverse of \(Z_{\alpha\bar{\beta}}\). It has been noted also that the Kähler metric \((Z_{\alpha\bar{\beta}})\) and soft SUSY-breaking scalar mass \((m_Z^2)\) of \(C^\alpha\) receive quadratically divergent corrections depending upon \(H_{\alpha\beta}\) and \(X_{\alpha\beta\gamma}\) [12]:
\[
\delta Z_{\alpha\bar{\beta}} = -\frac{\Lambda^2}{16\pi^2} R_{\alpha\bar{\beta}},
\]
\[
\delta m_Z^2 = \frac{\Lambda^2}{16\pi^2} e^{K_0} |W_0|^2 R_{\alpha\bar{\beta}},
\]
(16)

where
\[
R_{\alpha\beta} = Z^{\beta\bar{\alpha}} Z^{\bar{\gamma}\gamma} X_{\alpha\beta\gamma} X_{\alpha\bar{\beta}\bar{\gamma}}^* - Z^{\gamma\bar{\beta}} K^{i\bar{j}} \partial_i H_{\alpha\gamma} \partial_{\bar{j}} H_{\bar{\beta}}^*.
\]

When combined together as the supergraphs of Fig. 2, these one-loop corrections lead to the following quartically-divergent two-loop correction to the potential energy:
\[
\delta V_2' \sim \left(\frac{\Lambda^2}{16\pi^2}\right)^2 e^{K_0} |W_0|^2 (|X|^2 + |\partial H|^2)
\]
\[
\sim \frac{|X|^2 + |\partial H|^2 m_Z^2/2 \Lambda^4}{(16\pi^2)^2 M_P^2},
\]
(17)

where
\[
|X|^2 = Z^{a\bar{a}} Z^{\bar{\beta}\beta} Z^{\gamma\bar{\gamma}} X_{\alpha\beta\gamma} X_{\bar{a}\beta\bar{\gamma}}^*,
\]
\[
|\partial H|^2 = Z^{a\bar{a}} Z^{\bar{\beta}\beta} K^{i\bar{j}} \partial_i H_{\alpha\beta} \partial_{\bar{j}} H_{\bar{\beta}}^*.
\]

Finally \(\Gamma_{\alpha\beta\gamma\delta}\) and \(F_{\alpha\beta}\) contributes to the potential energy through the 3-loop supergraph of Fig. 3:
\[
\delta V_3 \sim \frac{1}{(16\pi^2)^3} e^{K_0} (A^4 |DW_0|^2 |\Gamma|^2 + A^6 |\bar{D}W_0^* \partial F|^2)
\]
\[
\sim \frac{1}{(16\pi^2)^3} \left(\frac{m_Z^2/2 \Lambda^4}{M_P^2} |\Gamma|^2 + \frac{m_Z^2/2 \Lambda^6}{M_P^2} |\partial F|^2 \right),
\]
(18)

where \((\bar{D}W_0^* \partial F)_{\alpha\beta} = K^{i\bar{j}} \partial_i F_{\alpha\bar{a}} D_{\bar{j}} W_{0\bar{a}}^*\) and\n\[
|\Gamma|^2 = Z^{a\bar{a}} Z^{\bar{\beta}\beta} Z^{\gamma\bar{\gamma}} Z^{\delta\bar{\delta}} \Gamma_{\alpha\beta\gamma\delta} \Gamma_{\bar{a}\bar{\beta}\bar{\gamma}\bar{\delta}}^*,
\]
\[
|\partial F|^2 = K^{i\bar{j}} Z^{a\bar{a}} Z^{\bar{\beta}\beta} \partial_i F_{\alpha\beta} \partial_{\bar{j}} F_{\bar{a}\bar{\beta}}^*.
\]
(19)

So far, we have discussed the effective potential induced by the coefficient functions in \(K, W\) and \(f_a\) except for \(\text{Im}(f_{0a})\) whose VEV corresponds to the vacuum angle of the \(a\)-th gauge
group. In the effective supergravity model under consideration, possible nonperturbative hidden sector gauge interactions were already integrated out, and their effects are encoded in the effective superpotential $W_0$ which is a function of the holomorphic hidden sector gauge kinetic functions. This means that the effective potential of the both real and imaginary parts of the gauge kinetic functions of strongly interacting hidden sector are already included in (12).

Let us now consider the effective potential of the vacuum angles of the standard model gauge group. Due to the asymptotic freedom, the potential energy of the QCD vacuum angle $8\pi^2\text{Im}(f_{QCD})$ is induced mainly by the low energy QCD dynamics at energy scales around 1 GeV [13], and thus is unambiguously determined as

$$V_{QCD} \sim f^2_\pi m^2_\pi \cos[8\pi^2\text{Im}(f_{QCD})],$$

where $m_\pi$ and $f_\pi$ denote the pion mass and decay constant, respectively. However the potential energy of the electroweak vacuum angle $8\pi^2\text{Im}(f_{EW})$ is mainly due to the $SU(2)_L$ instantons at high energy scales around $M_{GUT}$. Since the multi-fermion vertex of the standard model fermions induced by the electroweak instanton violates both the baryon ($B$) and lepton ($L$) numbers, $\Delta B = \Delta L = 3$, the resulting potential energy is suppressed by the insertions of small $B$ and $L$-violating couplings in addition to the suppressions by the semiclassical factor $e^{-2\pi/\alpha_{GUT}}$ and the small SUSY breaking factor of order $m^2_{3/2}$. We thus have

$$V_{EW} \sim \epsilon e^{-2\pi/\alpha_{GUT}} m^2_{3/2} M^2_P \cos[8\pi^2 f_{EW}],$$

where the small factor $\epsilon$ includes $B$ and $L$ violating couplings obeying the selection rule $\Delta B = \Delta L = 3$ together with small Yukawa-type couplings and loop factors which are necessary to close all fermion zero modes [14]. The size of $\epsilon$ is highly model-dependent, particularly on the $B$, $L$, and fermion chirality-changing couplings available at $M_{GUT}$.

Summing all the contributions discussed so far, the typical size of the quintessence potential $V_Q$ is estimated as

$$V_Q \sim m^2_{3/2} M^2_P \left[ \frac{\delta \ln(K_0)}{\delta Q} + \frac{\delta \ln(W_0)}{\delta Q} + \frac{\Lambda^2}{16\pi^2 M^2_P} \left( \frac{\delta Z_{a\bar{\beta}}}{\delta Q} + \frac{\delta \text{Re}(f_{a})}{\delta Q} \right) \right]$$

$$+ \frac{\Lambda^2}{(16\pi^2)^2 M^2_P} \frac{\delta Y}{\delta Q} + \frac{\Lambda^4}{(16\pi^2)^2 M^2_P} \left( \frac{\delta |X|^2}{\delta Q} + \frac{\delta |\bar{H}|^2}{\delta Q} \right) + \frac{\Lambda^4}{(16\pi^2)^3 M^2_P} \frac{\delta |\Gamma|^2}{\delta Q}$$

$$+ \frac{\Lambda^6}{(16\pi^2)^3 M^2_P} \frac{\delta |\bar{F}|^2}{\delta Q} + \frac{f^2_\pi m^2_\pi}{m^2_{3/2} M^2_P} \frac{\delta \text{Im}(f_{QCD})}{\delta Q} + \epsilon e^{-2\pi/\alpha_{GUT}} \frac{\delta \text{Im}(f_{EW})}{\delta Q} \right] Q_{\text{typ}},$$

where $\delta G$ denotes the variation of the coefficient function $G$ for the quintessence variation $\delta Q$ and $Q_{\text{typ}}$ is the typical range of the quintessence variation. To produce the desired $V_Q \sim (3 \times 10^{-3} \text{ eV})^4$ without resorting to severe cancellation, we then need

$$\frac{\delta \ln(K_0)}{\delta Q} \lesssim 10^{-88} \frac{\kappa_{Q\kappa_{3/2}}}{\kappa_{Q\kappa_{3/2}}}, \quad \frac{\delta \ln(W_0)}{\delta Q} \lesssim 10^{-88} \frac{\kappa_{Q\kappa_{3/2}}}{\kappa_{Q\kappa_{3/2}}},$$

$$\frac{\delta Z_{a\bar{\beta}}}{\delta Q} \lesssim 10^{-81} \frac{\kappa_{Q\kappa_{3/2}}}{\kappa_{Q\kappa_{3/2}}}, \quad \frac{\delta |\bar{H}|^2}{\delta Q} \lesssim 10^{-74} \frac{\kappa_{Q\kappa_{3/2}}}{\kappa_{Q\kappa_{3/2}}},$$

$$\frac{\delta |\bar{F}|^2}{\delta Q} \lesssim 10^{-74} \frac{\kappa_{Q\kappa_{3/2}}}{\kappa_{Q\kappa_{3/2}}},$$

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\[
\frac{\delta |X|^2}{\delta Q} \lesssim \frac{10^{-74}}{\kappa_Q \kappa^2 \kappa_{3/2}}, \quad \frac{\delta |Y|^2}{\delta Q} \lesssim \frac{10^{-79}}{\kappa_Q \kappa^2 \kappa_{3/2}}, \\
\frac{\delta |\Gamma|^2}{\delta Q} \lesssim \frac{10^{-72}}{\kappa_Q \kappa^2 \kappa_{3/2}}, \quad \frac{\delta |\partial F|^2}{\delta Q} \lesssim \frac{10^{-67}}{\kappa_Q \kappa^2 \kappa_{3/2}}, \\
\frac{\delta \text{Re}(f_a)}{\delta Q} \lesssim \frac{10^{-81}}{\kappa_Q \kappa^2 \kappa_{3/2}}, \quad \frac{\delta \text{Im}(f_{QCD})}{\delta Q} \lesssim \frac{10^{-42}}{\kappa_Q \kappa^2 \kappa_{3/2}},
\]

(23)

where \( \kappa_Q = \frac{Q_{\text{top}}}{M_P}, \) \( \kappa_A = \frac{(\Lambda / 10^{16} \text{GeV})^2}{\kappa_{3/2}} \) \( = \frac{(m_{3/2}^2 / \text{TeV})^2}{\kappa_{3/2}} \), and all other quantities are defined in the supergravity unit with \( M_P = 1 \), e.g. \( Q, K_0, W_0, \) and \( X_{\alpha \beta \gamma} \) correspond to \( Q/M_P, K_0/M_P^2, W_0/M_P^2, \) and \( M_P X_{\alpha \beta \gamma} \), respectively. Note that at least one of the variations is required to saturate its upper limit to produce the desired quintessence potential energy.

When expanded in powers of \( C^\alpha \) and also written in the unit with \( M_P = 1 \), all gauge-invariant coefficient functions in \( K, W, \) and \( f_a \) are expected to be of order unity except for \( W_0 \) which is of order \( m_{3/2}^2 \). Furthermore none of \( \kappa_Q, \kappa_A, \) and \( \kappa_{3/2} \) can be small enough to significantly compensate the extremely small numerators in (23). Then the limits of (23) on the variations of coefficient functions imply that \( K, W \) and \( f_a \) must be invariant under the variation of \( Q \), i.e. \( Q \)-independent, with an extremely fine accuracy. Note that terms of higher order in \( C^\alpha \) which are not explicitly discussed here are similarly constrained to be \( Q \)-independent. At any rate, if this \( Q \)-invariance is satisfied, \( Q \) behaves like a (pseudo) Goldstone boson having only derivative couplings and thus does not mediate any macroscopic force.

A possible breaking of the \( Q \)-invariance whose size was not estimated in (23) is the variation of the electroweak vacuum angle \( 8\pi^2 \text{Im}(f_{EW}) \) which is constrained as

\[
\frac{\delta \text{Im}(f_{EW})}{\delta Q} \lesssim \frac{10^{-20}}{\epsilon \kappa_Q \kappa_{3/2}}.
\]

(24)

Although quite sensitive to the physics at \( M_{\text{GUT}} \), generically \( \epsilon \) is suppressed by many powers of small Yukawa couplings, loop factors, and also small \( B \) and/or \( L \)-violating couplings, and so it can be easily smaller than \( 10^{-20} \) [14]. This would suggest that \( \text{Im}(f_{EW}) \) can have a sizable \( Q \)-dependence, and so a sizable \( Q \)-coupling to the electromagnetic \( F_{\mu\nu}F^{\mu\nu} \) which may leads to an interesting observable consequence [15]. However in view of the gauge coupling unification, i.e. \( \langle \text{Re}(f_{QCD}) \rangle = \langle \text{Re}(f_{EW}) \rangle \), it is highly unlikely that \( \text{Im}(f_{EW}) \) has a sizable \( Q \)-dependence, while \( \text{Im}(f_{QCD}) \) is \( Q \)-independent as in (23).

Let us close this section with a brief discussion of how plausible are some of the frequently used forms of \( V_Q \) [4] in view of our discussions above.

(a) Exponential potential: \( V_Q \sim M^4 e^{-Q/f_0} \). This form of \( V_Q \) may arise from some nonperturbative dynamics when \( Q \) corresponds to a dilaton [18]. At certain level, one

\[ \text{A possible loophole [16] for this argument is the possibility of massless up quark which has been argued to be phenomenologically viable [17]. In this case, the effective potential of the QCD vacuum angle \( \text{Im}(f_{QCD}) \) is suppressed by \( z = m_u/m_d \), i.e. \( V_{QCD} \sim z f_0^2 m_z^2 \), and then the upper bound on \( \delta \text{Im}(f_{QCD})/\delta Q \) in Eq. (23) should be read as } 10^{-42}/z \kappa_Q \kappa_A. \text{ Thus for } |z| \lesssim 10^{-40}, \text{ Im}(f_{QCD}) \text{ can have a sizable } Q \text{-dependence and so } Q \text{ can couple to both } (\bar{F}F)_{QCD} \text{ and } (\bar{FF})_{EW}. \]
may adjust the dynamics of the model to make this exponential potential to be of order $(3 \times 10^{-3} \text{eV})^4$. However usually the gauge coupling constants in the model are determined by the dilaton VEV, which means $\delta \text{Re}(f_a)/\delta Q$ is of order unity. Then the supergravity loop effects depending on $\delta \text{Re}(f_a)/\delta Q$ (see Eq.(22)) yield a dilaton potential energy of order

$$\frac{1}{16\pi^2} m_{3/2}^2 \Lambda^2$$

which is too large to be a quintessence potential energy.

(b) Inverse power law potential: $V_Q \sim M^{4+k}/Q^k$. This form of $V_Q$ may arise from a nonperturbative dynamics when $Q$ corresponds to a composite degree of freedom [18]. An attractive feature of this potential is that $M$ does not have to be too small compared to the particle physics mass scales, e.g. $M \gtrsim 10$ GeV for $k \geq 3$ and $Q \sim M_P$. However the inverse power law behavior is so easily upset by the Planck scale physics which would generate a non-renormalizable contribution $\delta V_Q \sim m_3^2 M_P^n Q^n / M_P^n$ which is too large to be the quintessence potential for $Q \sim M_P$.

(3) Pseudo-Goldston boson potential: $V_Q \sim M^4 \cos(Q/f_Q)$. This form of $V_Q$ arises when $Q$ corresponds to a (pseudo) Goldstone boson, which is perhaps the most plausible possibility in view of the $Q$-invariance conditions of (23). Obviously then the $Q$-invariance corresponds to the nonlinearly realized global symmetry associated with the Goldstone boson $Q$. Still the remained question is how the explicit breaking of the non-linear global symmetry can be so tiny as in (23), which is very nontrivial to achieve in view of that global symmetries are generically broken by Planck scale physics [19]. In the next section, we argue that heterotic $M$-theory or Type I string axion can be a plausible candidate for the quintessence Goldstone boson with the tiny $Q$-invariance breaking induced by the membrane or $D$-brane instantons.

### III. HETEROTIC $M$ OR TYPE I STRING AXION AS QUINTESSENCE

The most natural candidate for a light scalar field whose typical range of variation is of order $M_P$ is the string or $M$-theory moduli multiplets describing the (approximately) degenerate string or $M$-theory vacua. It is thus quite tempting to look at the possibility that $Q$ corresponds to a certain combination of the string or $M$-theory moduli superfields. In this section, we examine this possibility and point out that the heterotic $M$-theory or Type I string axion can be a plausible candidate for quintessence. The moduli superfields of our interests are $Z_I = (S, T_i)$ including the axion components $\text{Im}(Z_I)$ together with the modulus components $\text{Re}(Z_I)$ which correspond to the string dilaton or the length of the 11-th segment and the Kähler moduli describing the size and shape of the internal 6-manifold. Explicit computations show that generically

$$\frac{\delta K_0}{\delta \text{Re}(Z_I)} = O(1),$$

and so the effective potential of the modulus components are of order $m_{3/2}^2 M_P^2$ and the modulus masses are of order $m_{3/2}$ in view of the analysis in the previous section. It thus appears that the modulus components $\text{Re}(Z_I)$ can not provide a quintessence. However as we will see still quintessence can arise as a linear combination of the axion components $\text{Im}(Z_I)$ if the corresponding modulus component has a large VEV $\sim 35$.

Let us first consider the heterotic $M$-theory on a 11-dimensional manifold with boundary which is invariant under the $Z_2$-parity [6]:
\[ C \rightarrow -C, \quad x^{11} \rightarrow -x^{11}, \]  
(26)

where \( C = C_{ABC}dx^A dx^B dx^C \) is the 3-form field in the 11-dimensional supergravity. When compactified to 4-dimensions, axions arise as the massless modes of \( C_{\mu \nu 11} \) and \( C_{mn11} \):

\[
\epsilon^{\mu \nu \rho \sigma} \partial_\nu C_{\rho \sigma 11} = \partial_\mu \eta S, \quad C_{mn11} = \sum_i \eta_i (x^\mu) \omega_i^{mn},
\]  
(27)

where \( \omega^i \) (\( i = 1 \) to \( h_{1,1} \)) form the basis of the integer (1, 1) cohomology of the internal 6-manifold and \( \mu, \nu \) are tangent to the noncompact 4-dimensional spacetime. In 4-dimensional effective supergravity, these axions appear as the pseudo-scalar components of chiral multiplet:

\[
S = (4\pi)^{-2/3} \kappa^{-4/3} V + i \eta S, \\
T_i = (4\pi)^{-1/3} \kappa^{-2/3} \int_C \omega \wedge dx^{11} + i \eta_i,
\]  
(28)

where \( \kappa^2 \) denotes the 11-dimensional gravitational coupling, \( V \) is the volume of the internal 6-manifold with the Kähler two form \( \omega \), and the integral is over the 11-th segment and also over the 2-cycle \( C_i \) dual to \( \omega^i \). Here the axion components are normalized by the discrete Peccei-Quinn (PQ) symmetries:

\[
\text{Im}(S) \rightarrow \text{Im}(S) + 1, \quad \text{Im}(T_i) \rightarrow \text{Im}(T_i) + 1,
\]  
(29)

which are the parts of discrete modular symmetries involving also the dualities between large and small Re(\( S \)) or Re(\( T_i \)).

Holomorphy and the discrete PQ symmetries imply that in the large Re(\( S \)) and Re(\( T_i \)) limits the gauge kinetic functions can be written as:

\[
4\pi f_a = k_a S + \sum_i l_i T_i + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T_i}),
\]  
(30)

where \( k_a \) and \( l_i \) are model-dependent quantized real constants and the exponentially suppressed terms are possibly due to membrane or 5-brane instantons. For a wide class of compactified heterotic \( M \)-theory, we have [21–24]

\[
4\pi f_{E_8} = S + \sum_i l_i T_i + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T_i}), \\
4\pi f_{E_8}' = S - \sum_i l'_i T_i + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T_i}),
\]  
(31)

where \( l_i T_i \) corresponds to the one-loop threshold correction in perturbative heterotic string terminology with the quantized coefficients \( l_i \) determined by the instantons numbers on the hidden walls and also the orbifold twists.\(^5\)

\(^4\)This feature of the gauge kinetic function was first noted in [20].

\(^5\)For compactifications on smooth Calabi-Yau space, \( l_i \) can be determined either in the perturbative heterotic string limit [25,21] or in the 11-dimensional supergravity limit [26] and they take half-integer values [23], while they can be generic rational numbers for orbifolds [24].
Let \( Q \) denote a linear combination of \( \text{Im}(T_i) \) for which the combination \( \sum_i l_i T_i \) is \textit{invariant} under

\[
U(1)_Q : \quad Q \rightarrow Q + \text{constant.}
\]

(32)

Note that such \( Q \) exists always as long as \( h_{1,1} > 1 \). In this regard, models of particular interest are the recently discovered threshold-free models with \( l_i = 0 \) [24] for which any of \( \text{Im}(T_i) \) can be identified as \( Q \). At any rate, once such \( Q \) exists, we have

\[
\frac{\delta f_a}{\delta Q} = O(e^{-2\pi T}),
\]

(33)

where the internal 6-manifold is assumed to be isotropic and thus \( \text{Re}(T_i) \approx \text{Re}(T) \) for all \( T_i \). Holomorphy and discrete PQ symmetries imply also

\[
\frac{\delta Y_{\alpha\beta\gamma}}{\delta Q} = O(e^{-2\pi T})
\]

(34)

for the Yukawa couplings (and also higher order holomorphic couplings) in the superpotential of (10). If \( W_0 \) is induced by hidden sector dynamics described by \( Q \)-\textit{independent} gauge, Yukawa, and higher order holomorphic couplings, we have

\[
\frac{\delta W_0}{\delta Q} = \frac{\delta W_0}{\delta f_a} \frac{\delta f_a}{\delta Q} + \frac{\delta W_0}{\delta Y_{\alpha\beta\gamma}} \frac{\delta Y_{\alpha\beta\gamma}}{\delta Q} + ... = O(e^{-2\pi T}W_0).
\]

(35)

About the \( Q \)-invariance of the Kähler potential, it is convenient [23] to go to the limit in which the 11-th length \( \pi \rho \gg V^{1/6} \gg \kappa^{2/9} \) so that the physics below the Kaluza-Klein scale of the internal 6-manifold, i.e. below \( V^{-1/6} \), is described by a 5-dimensional supergravity with the gravitational coupling \( \kappa_5^2 = \kappa^2 / V \). In this limit, we have \( \text{Re}(S) \gg 1 \) and \( \text{Re}(T_i) \gg 1 \), and we may keep \( \text{Re}(T_i) \approx \text{Re}(S) \) for \( l_i \neq 0 \), while we may allow \( \text{Re}(T_i) \gg \text{Re}(T) \) for the threshold-free case with \( l_i = 0 \). (See Eq.(28) and Eq.(31).) In the resulting 5-dimensional effective supergravity, \( T_i \) correspond to the coordinates of a special Kähler manifold whose Kähler geometry is determined by the holomorphic prepotential [27]. Then the discrete PQ symmetries applied for the holomorphic prepotential imply that the Kähler potential is given by [23]

\[
K = \tilde{K}(T_i + T_i^*, S, S^*, C^\alpha, C^{\alpha*}) + O(e^{-2\pi T_i}),
\]

(36)

and so

\[
\frac{\delta K}{\delta Q} = O(e^{-2\pi T}).
\]

(37)

When applied for the effective potential analysis of the previous section, Eqs. (33), (34), (35), and (37) assure that in the large \( \text{Re}(T) \) limit the quintessence axion potential is exponentially suppressed as

\[
V_Q \sim e^{-2\pi(\text{Re}(T))} m_{3/2}^2 M_P^8 \cos[2\pi \text{Im}(T)]
\]

(38)
even when all possible quantum corrections are taken into account. Thus what is necessary and sufficient for this axion potential to be of order \((3 \times 10^{-3} \text{eV})^4\) is

\[
\langle \text{Re}(T) \rangle \sim \frac{1}{2\pi} \ln(m_{3/2}^2 M_P^2 / V_Q) \sim 35, \tag{39}
\]

where \(m_{3/2} \sim 1 \text{ TeV}\) are used for numerical estimate.\(^6\) As we will see in the next section, this value of \(\text{Re}(T)\) can be compatible with \(\alpha_{\text{GUT}} \sim 1/25\) only in the heterotic \(M\)-theory limit, not in the perturbative heterotic string limit. Although the above quintessence axion potential has been obtained by the macroscopic analysis based on supersymmetry and discrete PQ symmetries, one can easily identify its microscopic origin by noting that

\[2\pi \text{Re}(T_i) = \left(4\pi\right)^{-1/3} \kappa^{-2/3} \int_{C_i} \omega \wedge dx^{11} \]

corresponds to the Euclidean action of the membrane instanton wrapping \(C_i\) and stretched along the 11-th segment \([21]\). When extrapolated to the perturbative heterotic string limit, such membrane instanton corresponds to the heterotic string worldsheet instanton wrapping the same 2-cycle \([29]\). Explicit computations then show that the Kähler potential and/or the gauge kinetic functions are indeed corrected by worldsheet instantons, yielding \(\delta K_0 = \mathcal{O}(e^{-2\pi T})\) and \(\delta f_0 = \mathcal{O}(e^{-2\pi T})\) \([30]\). These corrections can be smoothly extrapolated back to the heterotic \(M\)-theory limit and identified as the corrections induced by stretched membrane instantons. When integrated out, the correction to the hidden sector gauge kinetic function leaves its trace in the effective superpotential as \(\delta W_0 = \mathcal{O}(e^{-2\pi T} W_0)\). The axion potential \((38)\) is then obtained from the supergravity potential \((12)\) with \(\delta K_0 = \mathcal{O}(e^{-2\pi T})\) and/or \(\delta W_0 = \mathcal{O}(e^{-2\pi T} W_0)\).

Let us now turn to Type I string axions. The Type I axions (again normalized by the discrete PQ symmetries of \((29)\)) correspond to the massless modes of the R-R two form fields \(B_{\mu\nu}\) and \(B_{mn}\), and form 4-dimensional chiral multiplets together with the string dilaton \(e^D\) and the internal space volume \(V\):

\[
S = (2\pi)^{-6} \alpha'^{-3} e^{-D} V + i\eta_S,
\]

\[
T_i = (2\pi)^{-2} \alpha'^{-1} e^{-D} \int_{C_i} \omega + i\eta_i. \tag{40}
\]

Here we include \(D_9\) and \(D_5\) branes in the vacuum configuration, and consider the 4-dimensional gauge couplings \(\alpha_9\) and \(\alpha_{5i}\) defined on \(D_9\) branes wrapping the internal 6-manifold and \(D_5\) branes wrapping the 2-cycles \(C_i\), respectively \([31]\). Again holomorphy and discrete PQ symmetries imply that the corresponding gauge kinetic functions can be written as \((30)\). A simple leading order calculation gives \(\alpha_9 = 1/\text{Re}(S)\) and \(\alpha_{5i} = 1/\text{Re}(T_i)\), and so \([31]\)

\[
4\pi f_9 = S, \quad 4\pi f_{5i} = T_i. \tag{41}
\]

However this leading order result can receive perturbative and/or non-perturbative corrections. Generic perturbative corrections can be expanded in power of the string coupling

\(^6\)About stabilizing \(\text{Re}(T)\) at this large value, it has been shown in \([28]\) that both \(\text{Re}(S)\) and \(\text{Re}(T)\) can be stabilized at large VEVs of order \(1/\alpha_{\text{GUT}}\) by the combined effects of the multi-gaugino condensations and the membrane instantons wrapping the 3-cycle of the internal 6-manifold.
\(e^D\), the string inverse tension \(\alpha'\), and also the inverse tension \(e^D \alpha'^k\) of \(D_{2k-1}\) branes [7]. Generically they scale as

\[
e^n D \alpha'^m \propto [\text{Re}(T)]^{n+m}[\text{Re}(S)]^{-\frac{3n+m}{2}}. \tag{42}
\]

Combined with the leading order result (41) and also the general form of gauge kinetic function (30) dictated by holomorphy and discrete PQ symmetries, this scaling behavior implies that \(f_9\) can receive a \(T_i\)-dependent correction at order \(\alpha'^2\), while there is no perturbative correction to \(f_{5i}\), and so

\[
4\pi f_9 = S + l_i T_i + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T_i}),
\]

\[
4\pi f_{5i} = T_i + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T_i}). \tag{43}
\]

Similarly to the case of heterotic \(M\)-theory axion, the quintessence \(Q\) can arise as a linear combination of \(\text{Im}(S)\) and \(\text{Im}(T_i)\), however its explicit form depends on how the gauge couplings not weaker than those of the standard model are embedded in the model. Note that for such not-so-weak gauge couplings both \(\text{Re}(f_a)\) and \(\text{Im}(f_a)\) are required to be \(Q\)-independent as in (23), while for the weaker gauge interactions with \(\alpha_a(M_{\text{GUT}}) \lesssim 1/35\), \(\text{Im}(f_a)\) are allowed to have a sizable \(Q\)-dependence. Here are some possibilities. If all of the not-so-weak gauge couplings are embedded in \(\alpha_{5i}\), \(\text{Im}(S)\) can be a quintessence when \(\langle \text{Re}(S)\rangle \sim 35\) which is necessary for \(V_Q \sim (3 \times 10^{-3} \text{eV})^4\), and also the associated gauge coupling \(\alpha_9(M_{\text{GUT}}) \lesssim 1/35\) which is necessary to avoid a too large \(V_Q\) induced by the gauge instantons of \(\alpha_9\). If not-so-weak couplings are embedded in \(\alpha_9\), any of \(\text{Im}(T_i)\) can be a quintessence when \(\langle \text{Re}(T_i)\rangle \sim 35\). If the vacuum does not include any \(D_5\) brane wrapping the particular 2-cycle, e.g. the \(i\)-th cycle, the corresponding axion \(\text{Im}(T_i)\) can be a quintessence independently of the embedding when \(\langle \text{Re}(T_i)\rangle \sim 35\).

Once the quintessence component satisfying the constraints on gauge kinetic functions is identified, the \(Q\)-dependence of the Kähler potential and superpotential is suppressed by \(e^{-2\pi Z}\) (\(Z = S\) or \(T\)) as in the case of heterotic \(M\)-theory, and so the quintessence axion potential is again given by (38) where now \(T\) can be either \(S\) or \(T_i\). Microscopic origin of this Type I axion potential can be easily identified also by noting that \(2\pi \text{Re}(S)\) corresponds to the Euclidean action of \(D5\) brane instanton wrapping the internal 6-manifold and \(2\pi \text{Re}(T_i)\) is of \(D1\) string instanton wrapping the 2-cycle \(C_i\).

**IV. COUPLINGS AND SCALES WITH QUINTESSENCE AXION**

In the previous section, it was noted that the heterotic \(M\) or Type I string axion can provide a quintessence dark energy when its modulus partner has a VEV \(\sim 35\). In this section, we discuss the couplings and scales associated with this modulus VEV.

**Couplings and Scales in Heterotic \(M\)-theory**: For simplicity, let us consider a model without \(T\)-dependent threshold corrections, i.e. the quanitized coefficients \(l_i = 0\) in Eq. (31). We then have [26,21]

\[
\frac{1}{\alpha_{\text{GUT}}} = (4\pi)^{-2/3}\kappa^{-4/3}V = \text{Re}(S),
\]
\[ \omega = \frac{(4\pi)^{1/3} \kappa^{2/3}}{\pi \rho} \sum_i \text{Re}(T_i) \omega^i, \]

\[ V = \frac{1}{3!} \int \omega \wedge \omega \wedge \omega \approx \frac{4\pi \kappa^2}{(\pi \rho)^{3}} \sum_{ijk} C_{ijk} \text{Re}(T)^3 \]

(44)

where \( \pi \rho \) is the length of the 11-th segment, \( \omega \) is the Kähler form of the internal 6-manifold, and \( C_{ijk} = \int \omega^i \wedge \omega^j \wedge \omega^k \) are the intersection numbers of the integer (1, 1) cohomology basis \( \omega^i \). Here we have used the isotropy condition \( \text{Re}(T_i) \approx \text{Re}(T) \). Combining these with

\[ M_P^2 = \pi \rho V / \kappa^2, \]

one easily finds

\[ M_{KK} \equiv \frac{\gamma}{V^{1/6}} = \frac{\gamma M_P}{2 \sqrt{\pi} (\sum C_{ijk}/6)^{1/6} [\text{Re}(S)]^{1/2} [\text{Re}(T)]^{1/2}} \]

\[ \kappa^{-2/3} (\pi \rho)^{3} = (4\pi)^{1/3} \left( \frac{\sum C_{ijk}}{6} \right) \frac{[\text{Re}(T)]^3}{\text{Re}(S)} \]

(45)

where \( M_{KK} \) denotes the Kaluza-Klein threshold mass scale for the internal 6-manifold with volume \( V \). Here \( \gamma \) is a constant of order unity whose precise value depends on the details of compactification and also on the regularization of the threshold effects due to massive Kaluza-Klein modes. (For instance, \( \gamma \) is expected to be around \( 2\pi \) for toroidal compactifications, however it can be smaller for more complicate compactifications, e.g. about \( \sqrt{2\pi} \) for the compactification on \( S^6 \).)

In heterotic \( M \)-theory, \( M_{KK} \) can be identified as the unification scale of gauge couplings on the boundary [26]:

\[ M_{GUT} = M_{KK}. \]

Also the VEVs of \( \text{Re}(S) \) and \( \text{Re}(T) \) can be determined by the two phenomenological inputs: \( \alpha_{GUT} = 1/25 \) and \( V_Q = (3 \times 10^{-3} \text{eV})^4 \), yielding

\[ \text{Re}(S) = \frac{1}{\alpha_{GUT}} \sim 25, \]

\[ \text{Re}(T) = \frac{1}{2\pi} \ln(m_{3/2}^2 M_P^2 / V_Q) \sim 35. \]

Applying these moduli VEVs to (45), we find

\[ M_{GUT} = \frac{1.3\gamma}{(\sum C_{ijk}/6)^{1/6}} \times 10^{16} \text{ GeV}, \]

(46)

which is very close to the phenomenologically favored value \( 3 \times 10^{16} \text{ GeV} \). We stress that the large \( \langle \text{Re}(T) \rangle \sim 35 \) required for the quintessence axion is essential for \( M_{GUT} \) to have a value close to the phenomenologically favored value. Large \( \text{Re}(T) \) leads also to the length of the 11-th segment significantly bigger than the 11-dimensional Planck length:

\[ \pi \rho \sim 10 \kappa^{2/9}, \]

(47)
implying that we are indeed in the 11-dimensional heterotic $M$-theory limit corresponding to the strong coupling limit of the heterotic $E_8 \times E_8$ string theory.

**Couplings and Scales in Type I string with D9 and D5-branes:** Again for simplicity consider a model without $T$-dependent threshold correction to $f_9$ in (43). We then have [7]

$$M_P^2 = 2(2\pi)^{-7}\alpha'^{-4}e^{-2D}V,$$

$$\frac{1}{\alpha_9} = (2\pi)^{-6}\alpha'^{-3}e^{-D}V = \text{Re}(S),$$

$$\frac{1}{\alpha_{5i}} = (2\pi)^{-2}\alpha'^{-1}e^{-D}\int_{C_i} \omega = \text{Re}(T_i),$$

and so

$$e^{2D} = \frac{\text{Re}(S)}{(\sum_{ijk} C_{ijk}/6)[\text{Re}(T)]^3};$$

$$M_{KK} \equiv \frac{\gamma}{V^{1/6}} = \frac{\gamma M_P}{2\sqrt{\pi}(\sum C_{ijk}/6)^{1/6}[\text{Re}(S)]^{1/2}[\text{Re}(T)]^{1/2}};$$

$$M_{\text{string}} \equiv \frac{1}{\sqrt[4]{\alpha'}} = \left[\frac{\sum C_{ijk}/6)^{1/4}[\text{Re}(S)]^{1/4}[\text{Re}(T)]^{3/4}}{\sqrt{\pi} M_P}\right].$$

In Type I case with a quintessence axion $\text{Im}(T)$ or $\text{Im}(S)$, we also have

$$M_{\text{GUT}} = M_{KK} \text{ or } M_{\text{string}},$$

$$\frac{1}{\alpha_{\text{GUT}}} = \text{Re}(S) \text{ or } \text{Re}(T) \sim 25,$$

$$\frac{1}{2\pi} \ln(m_{3/2}^2 M_P^2/V_Q) = \text{Re}(T) \text{ or } \text{Re}(S) \sim 35,$$

depending upon how the standard model gauge couplings are embedded [31]. For these moduli VEVs,

$$e^{2D} \ll 1, \quad \alpha' M_{KK}^2 = O(1),$$

indicating that we are in a domain of weakly coupled string but of strongly coupled worldsheet sigma model. It is also straightforward to see that they also give $M_{\text{GUT}}$ which is very close to the phenomenologically favored value, similarly to the case of heterotic $M$-theory.

**Quintessence Axion Scale and the Slow-Roll Condition:** The Kähler potential of the quintessence axion multiplet $Z = T$ or $S$ is given by

$$K_0 \approx -c \ln(Z + Z^*),$$

where $c = 1$ for $Z = S$, $c = 3$ for $Z = T$, and it is a constant of order unity in generic case. When all massive moduli (including $\text{Re}(Z)$) are integrated out, the effective lagrangian of the superlight quintessence axion is given by

$$\mathcal{L}_{\text{eff}} = \frac{c M_P^2}{4(\text{Re}(Z))^2} [\partial_\mu \text{Im}(Z)]^2 + m_{3/2}^2 M_P^2 e^{-2\pi \langle \text{Re}(Z) \rangle} \cos[2\pi \text{Im}(Z)]$$

$$= \frac{1}{2} (\partial_\mu Q)^2 + m_{3/2}^2 M_P^2 e^{-2\pi \langle \text{Re}(T) \rangle} \cos(Q/f_Q)$$

(52)
where the canonical quintessence axion $Q$ and its decay constant $f_Q$ are given by

$$Q = \sqrt{\frac{c}{2}} \frac{M_P}{\langle \text{Re}(Z) \rangle} \text{Im}(Z), \quad f_Q = \sqrt{\frac{c}{8\pi^2}} \frac{M_P}{\langle \text{Re}(Z) \rangle}.$$  

(53)

In order to have negative pressure, $Q$ must roll slowly and so satisfy

$$\frac{\partial V_Q}{\partial Q} \lesssim \frac{V_Q}{M_P}.$$  

(54)

When applied for (52), this slow-roll condition leads to

$$|2\pi \text{Im}(Z)|_{\text{present}} \lesssim \frac{1}{2\pi \langle \text{Re}(Z) \rangle} = \mathcal{O}(10^{-2}),$$  

(55)

implying that at present the angular field $2\pi \text{Im}(Z)$ takes a value at near the top of its effective potential.

The above slow-roll condition requires a fine-tuning of the present value of $\text{Im}(Z)$ and thus appears to be a serious difficulty of the quintessence axion. Here we present a scenario which would resolve this difficulty in a rather natural manner. The gauge symmetries of string or $M$-theory include discrete modular group [7] under which $Z$ and other generic moduli $\Phi$ transform as

$$\text{Re}(Z) \rightarrow \frac{1}{\text{Re}(Z)}, \quad \text{Im}(Z) \rightarrow \text{Im}(Z) + 1, \quad \Phi \rightarrow \Phi', \quad (56)$$

and also CP [32] under which

$$Z \rightarrow Z^*, \quad \Phi \rightarrow \Phi^*. \quad (57)$$

Here we take the simplest form of the $Z$-duality ($Z = S$ or $T$), i.e. $\text{Re}(Z) \rightarrow 1/\text{Re}(Z)$ with the self-dual value $\text{Re}(Z) = 1$, however our discussion is valid for other forms of the $Z$-duality transformation as long as the self-dual value is of order unity.

Obviously the above discrete gauge symmetries are spontaneously broken at generic points on the moduli space except for the the self-dual point with $\text{Re}(Z) = 1$ and also the CP-invariant points with $\Phi = \Phi^* \text{ or } \Phi'^*$ and $\text{Im}(Z) = 0 \text{ or } \frac{1}{2}$. (Note that $\text{Im}(Z) \equiv \text{Im}(Z) + 1$, and so $\text{Im}(Z) = \frac{1}{2}$ is CP-invariant also.) Their spontaneous breaking at the string or $M$-theory scale is described by the moduli VEVs minimizing the modular and CP-invariant effective potential:

$$V_{\text{eff}}(\Phi, \Phi^*, \text{Re}(Z), e^{2\pi i \text{Im}(Z)})$$

$$= V_{\text{eff}}(\Phi', \Phi'^*, \frac{1}{\text{Re}(Z)}, e^{2\pi i \text{Im}(Z)})$$

$$= V_{\text{eff}}(\Phi^*, \Phi, \text{Re}(Z), e^{-2\pi i \text{Im}(Z)}). \quad (58)$$

One of the key assumptions for the existence of the quintessence axion is that the above moduli potential has the global (or at least a local) minimum at $\text{Re}(Z) \sim 35$ for which the $Z$-duality is spontaneously broken. However independently of the validity of this assumption, if the moduli potential is differentiable at the invariant point, the self-dual point with $\text{Re}(Z) = 1$
1 corresponds to a stationary point of the gauge-invariant moduli potential.\(^7\) It is then quite possible that this stationary point at \(\text{Re}(Z) = 1\) is in fact a local *minimum* with large positive potential energy.

About the CP violation, if the moduli VEVs at the minimum of the moduli potential are *not* CP-invariant, e.g. \(\langle \Phi^* \rangle \neq \langle \Phi \rangle\) and also \(\langle \Phi^* \rangle \neq \langle \Phi' \rangle\), CP would be spontaneously broken at the string or \(M\)-theory scale by the complex \(\langle \Phi \rangle\). Since Yukawa couplings are generically \(\Phi\)-dependent, this would result in complex Yukawa couplings which may explain the observed CP violation in the neutral Kaon system. However in this case soft SUSY breaking parameters are complex in general and then we may have a phenomenological difficulty to have a too large electric dipole moment of the neutron, i.e. the SUSY CP problem. Note that soft SUSY breaking parameters are determined also by the moduli VEVs [34]. Thus in view of the SUSY CP problem, a more interesting possibility is that the gauge-invariant moduli potential \((58)\) is minimized at the CP-invariant point. In this case, CP is *not* spontaneously broken by the moduli VEVs, *but* by the VEVs of some matter superfields at energy scales far below \(M_P\). This type of CP violation can give the complex Yukawa couplings of the ordinary quarks and leptons through the mixing with the massive fermions which is induced by the CP-violating VEVs of matter superfields [35]. However it does not significantly affect the CP-conserving soft parameters which are induced at high energy scales around \(M_P\), except for the small renormalization group effects [36].

Repeating the above discussions, generically the modular and CP-invariant points,

\[
\text{Re}(Z) = 1, \quad \text{Im}(Z) = 0 \text{ or } \frac{1}{2}, \quad \Phi = \Phi^* \text{ or } \Phi'^*,
\]

(59)
correspond to the stationary points of the full moduli potential if the potential is differentiable at the invariant point. It is then quite possible that \(Z = 1\) is in fact a local minimum with a large positive potential energy \(V_{\text{inf}}\), which will be called the inflationary minimum. At this inflationary minimum, all the moduli masses have the same order of magnitude:

\[
m_{\text{Re}(Z)} \sim m_{\text{Im}(Z)} \sim m_\Phi \sim \sqrt{V_{\text{inf}}/M_P^2}.
\]

(60)

Note that the mass of \(\text{Im}(Z)\) has the same order of magnitude as the other moduli masses for \(\langle \text{Re}(Z) \rangle = \mathcal{O}(1)\).

In order for \(\text{Im}(Z)\) to be a quintessence, it is needed that \(\text{Re}(Z) \sim 35\) corresponds to the global (or at least another local) minimum of the moduli potential which we will call the present minimum. For this large value of \(\langle \text{Re}(Z) \rangle\), the masses of \(\text{Re}(Z)\) and \(\text{Im}(Z)\) are given by

\[
m_{\text{Re}(Z)} \sim m_{3/2}, \quad m_{\text{Im}(Z)} \sim e^{-\pi\langle \text{Re}(Z) \rangle} m_{3/2}.
\]

(61)

It is assumed that the potential energy is exactly vanishing at the true minimum, however the quintessence axion \(\text{Im}(Z)\) is displaced from the true minimum, providing the dark potential energy density \(\sim (3 \times 10^{-3} \text{ eV})^4\).

\(^7\)This feature of the invariant points, i.e. the points with enhanced symmetry, and also some of its phenomenological virtues were discussed in [33].
About the location of the minimum in the axion direction, we have just two possibilities if CP is not spontaneously broken by the moduli VEVs as suggested by the (approximately) CP-invariant soft parameters. Either $\text{Im}(Z) = 0$ or $\text{Im}(Z) = 1/2$ is the minimum. Of course, if $\text{Im}(Z) = 0$ is the minimum, $\text{Im}(Z) = 1/2$ corresponds to the maximum, and vice versa. Now an interesting possibility is that $\text{Im}(Z) = 0$ was the minimum for the inflationary modulus value $\text{Re}(Z) = 1$, however it becomes the maximum for the present modulus value $\text{Re}(Z) \sim 35$. Note that the coefficient of the cosine potential of $\text{Im}(Z)$ is a function of $\text{Re}(Z)$, and so its sign can be changed when $\text{Re}(Z)$ varies from the inflationary value to the present value.

Given the properties of the moduli potential discussed above, the cosmological scenario yielding $\text{Im}(Z) \sim 0$ at present goes as follows. There was an inflationary period in the early universe during which $\text{Re}(Z)$ and other moduli were settled down near at the modular and CP-invariant local minimum of the potential, i.e. at $\langle \text{Re}(Z) \rangle_{\text{inf}} = 1$, $\langle \text{Im}(Z) \rangle_{\text{inf}} = 0$, and $\langle \Phi \rangle_{\text{inf}} = \langle \Phi^* \rangle_{\text{inf}}$. At this local minimum, all moduli masses including that of $\text{Im}(Z)$ are of order $\sqrt{V_{\text{inf}}/M_P^2}$ and thus are of order the Hubble expansion rate $H$. As a result, all the moduli are settled down at the values sufficiently near at the minimum if the number of inflation efoldings are large enough, e.g. bigger than 10 [37]. This inflation will be over during the period when $\text{Re}(Z)$ moves toward the global minimum at $\text{Re}(Z) \sim 35$. When $\text{Re}(Z)$ becomes somewhat bigger than the unity, the curvature of the effective potential in the axion direction becomes much smaller than the expansion rate $H$,

$$
\frac{1}{M_P} \sqrt{\frac{\partial^2 V_{\text{eff}}}{\partial \text{Im}(Z)^2}} \sim e^{-\pi \text{Re}(Z)} H \ll H,
$$

and so $\text{Im}(Z)$ does not move [37] from its location at the inflationary phase, i.e. $\text{Im}(Z) = 0$. However the coefficient of the axion potential which is a function of $\text{Re}(Z)$ changes its sign during this process, and thus $\text{Im}(Z) = 0$ which was the minimum at the inflationary phase becomes the maximum of the present axion potential, providing the desired quintessence potential energy $V_Q \sim (3 \times 10^{-3} \text{eV})^4$ under the assumption that there is a hidden mechanism leading to the precisely vanishing vacuum energy density when all fields including $\text{Im}(Z)$ (but still except for the spacetime metric) are settled down at their true VEVs. We stress that this scenario for $\text{Im}(Z) \sim 0$ at present does not require the fine-tuning of the parameters in the moduli potential. It just corresponds to one possibility among several different possibilities, which can be realized for certain range of the parameters in the full moduli potential which are calculable in the framework of the underlying string or $\mathcal{M}$-theory.

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[31] For a recent discussion of Type I string phenomenology with D-brane configurations, see L. E. Ibanez, C. Munoz and S. Rigolin, hep-ph/9812397.
FIG. 1. Two loop supergraph inducing the quadratically-divergent potential energy of $Y_{\alpha\beta\gamma}$.

$|\tilde{\partial}H|^2$ or $|X|^2$

FIG. 2. Quartically-divergent two loop supergraph for the potential energy of $H_{\alpha\beta}$ and $X_{\alpha\beta\gamma}$.

FIG. 3. Three loop supergraph for the potential energy of $\Gamma_{\alpha\beta\gamma\delta}$ and $F_{\alpha\beta\gamma}$. For $F_{\alpha\beta\gamma}$, two lines denote the gauge multiplets while the others are the charged matter multiplets.