On Non-Supersymmetric CFT in Four Dimensions

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We show that the $\mathcal{N} = 0$ theories on the self-dual $D3$-branes of Type 0 string theory are in the class of the previously considered \emph{tadpole-free orbifolds} of $\mathcal{N} = 4$ theory (although they have $SO(6)$ global symmetry) and hence have vanishing beta function in the planar limit to all orders in 't Hooft coupling. Also, all planar amplitudes in this theory are equal to those of $\mathcal{N} = 4$ theory, up to a rescaling of the coupling.

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Vanishing of the planar beta function. Recently the non-supersymmetric gauge theories were studied in the framework of string theory/gauge theory correspondence [1][2][3]. The case [4] of $\mathcal{N} = 4$ super-Yang-Mills theory can be modified in several ways leading to the theories with lower supersymmetry. One possibility is to restrict the set of fields of $\mathcal{N} = 4$ theory according to the following principle: take a discrete subgroup $\Gamma$ of the $R$-symmetry group $SO(6)$ and let it act on the color indices in some representation $\mathcal{R}$:

$$g \mapsto \gamma_g \in U(\dim \mathcal{R}).$$

Let $\Phi^\alpha$ denote a field of the $\mathcal{N} = 4$ theory. The index $\alpha$ belongs to the representation of the $R$-symmetry group, and both color and Lorentz indices are omitted. Impose the invariance condition:

$$\Phi^\alpha = g_\beta^\alpha [\gamma_g^{-1} \Phi^\beta \gamma_g]$$

for any $g \in \Gamma$. It is known [5][6] that if the representation $\mathcal{R}$ is such that for any group element $g \neq 1$

$$\text{Tr}_\mathcal{R} \gamma_g = 0$$

then the planar graphs in the resulting theory are equal to those of the parent $\mathcal{N} = 4$ theory up to a rescaling of coupling. All these theories occur in the studies of the Type IIB theory compactified on $\mathbb{R}^{1,3} \times \mathbb{R}^6 / \Gamma$ [7] and they were suggested as examples of large $N$ conformal theories dual to certain string theory backgrounds [8]. They are known to be one-loop finite for all $\Gamma$ [9] while for higher loops they are known to have vanishing gauge beta functions in the planar limit (see [9] for two loops in the $\mathcal{N} = 1$ case, [5][6] for general case).

I. Klebanov and A. Tseytlin [2][3] suggested to study the $D$-branes in the Type 0B theory [10]. This theory has in the low-energy spectrum no fermions, the same NS bosons as Type IIB theory but the doubled set of RR bosons. The doubled set of RR fields leads to the doubled set of D-branes. In particular, there are two types of D3-branes: electric and magnetic ones. It turns out that the low-energy theory on the stack of $N$ D3-branes is a truncated version of the $\mathcal{N} = 4$ theory. In the case of pure electric or pure magnetic D3-branes one has a theory of $U(N)$ gluons coupled to six adjoint scalars while in the case of $(1, 1)$ branes one has $U(N) \times U(N)$ gauge group, six adjoint scalars for each gauge factor and two sets of bi-fundamental fermions $(N, \bar{N})$ and $(\bar{N}, N)$. Those fermions correspond to the open strings connecting electric and magnetic branes [11][2]. In the paper [3] the two-loop finiteness of this self-dual theory was proven in the leading-$N$ approximation.
The purpose of our paper is to point out that this theory has in fact vanishing planar beta function to all orders in 't Hooft coupling, just like any other orbifold theory studied in [8][9][5][6].

The point is that both the electric and self-dual theories are also orbifolds of $\mathcal{N} = 4$ theory. The slightly subtle point is that the R-symmetry group of $\mathcal{N} = 4$ theory is the spin cover of $SO(6)$, i.e. $SU(4)$. This group has a center $Z \approx \mathbb{Z}_4$. The group $\Gamma = \mathbb{Z}_2 \subset Z$ can also be used for orbifolding just like any other subgroup of $SU(4)$.

By inserting projectors on the $\Gamma$-invariant fields (1) into the 't Hooft diagrams one immediately sees that if the representation of $\Gamma$ in the gauge group obeys (2) then the planar gauge coupling beta function vanishes.

For the self-dual theory one starts with $U(2N)\mathcal{N} = 4$ theory and represents $\Gamma = \{1, \omega\}$ in the $U(2N)$ as follows:

$$\gamma_\omega = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

where the blocks are $N \times N$. Clearly this representation obeys (2).

For the purely electric theory one starts with $U(N)\mathcal{N} = 4$ gauge theory and takes the trivial representation of $\Gamma$ which does not obey (2). Hence the theory has non-trivial beta function even in the large $N$ limit.

**A puzzle.** Among the theories $\mathcal{N} = 0$ which were not explicitly considered in [9] there are also theories with $\Gamma = \mathbb{Z}_4$. We would start with $U(4N)\mathcal{N} = 4$ gauge theory and represent $\Gamma = \{1, \omega, \omega^2, \omega^3\}$ as $\gamma_g = \text{diag} (1, g, g^2, g^3) \otimes 1_{N \times N}$.

The field content of the resulting theory is: gauge group $U(N)^4$, six adjoint scalars per each group factor, four sets of bi-fundamental fermions: $(N_i, \bar{N}_{i+1}), i = 0, 1, 2, 3, 4 \equiv 0$.

It is interesting to find whether this theory (which clearly has vanishing planar beta function and hence would have dilaton-free dual $AdS_5 \times S^5$ background) has a string theory realization. We believe that recently developed $K$-theoretic tools will be helpful in the corresponding analysis [12][13][14].

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**Note added.** As this note was ready for publication a paper [15] appeared which studies further aspects of self-dual threebranes in type 0 string theory. Our result shows that in the large $N$ limit they should coincide with those of threebranes in type IIB theory.
References