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Results of the recent comprehensive ALEPH measurement of τ decays involving kaons, yielding the total strange width $R_{\tau,S}$ and the strange spectral function, are used in a combination of strange and non-strange spectral moments to determine the strange quark mass and non-perturbative contributions to the strange hadronic width. Two methods with different theoretical uncertainties are employed, depending on whether or not the longitudinal spin zero hadronic component is included, yielding an average of $m_s(M_\tau^2) = (163_{-43}^{+34})$ MeV/ c^2 , where the errors account for experimental, theoretical and fitting uncertainties. The evolution down to 1 GeV gives $m_s(1 \text{ GeV}^2) = (217_{-57}^{+45})$ MeV/ c^2 . As a cross check, the running of the observables employed down to masses smaller than the τ mass is compared to the theoretical expectation.

1. INTRODUCTION

Semileptonic τ decays provide a clean environment for studying the physics of hadrons, which are produced via W exchange, *i.e.*, from the QCD vacuum. For this reason, the global properties of hadronic systems in τ decay are described by fundamental quantities, called spectral functions, which measure the transition probability to create hadrons out of the vacuum as a function of the hadronic mass. Due to their unambiguous theoretical and experimental definitions, τ spectral functions provide a clear-cut information on hadron dynamics in an interesting energy region dominated by resonances and leading into the simpler asymptotic QCD regime. Recently, ALEPH achieved a complete measurement of τ branching ratios involving kaons [1–4], giving insight into current properties and decay dynamics [5].

Being one of the free parameters of the Standard Model, the strange quark mass m_s appears in many phenomenological calculations. For example, it plays a crucial role in the predictions of the CP-violating kaon parameters, in particular ϵ'/ϵ [6]. It turns out that the strange τ decay rate is sensitive to m_s , which can be exper-

imentally determined [7]. Indeed, a breaking of chiral symmetry, induced by the relatively large strange quark mass, introduces a mass dependence into the perturbative QCD prediction of the total strange hadronic width of the τ . Using the total rates, as well as moments of the respective spectral functions, in a combination of strange and non-strange modes which cancels the dominant massless perturbative contribution, one can derive the strange quark mass by means of a combined fit with non-perturbative contributions. Particular attention is given to the theoretical prediction where an error in the calculation [8] of the second-order perturbative term has been recently pointed out [9]. After correction, a problematic convergence behaviour of the perturbative series is observed [9–11], leading to a larger theoretical uncertainty on m_s than had been previously assumed and a bias in the central value obtained [12]. Nevertheless, we believe that the determination of m_s from τ data is still very interesting because it is achieved through fits to pure inclusive data only.

The method adopted follows the line of the recent ALEPH $\alpha_s(M_\tau^2)$ determination from non-strange hadronic τ decays [13], and is based on a simultaneous fit of QCD parametrizations, including perturbative and nonperturbative components, to measured observables.

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2. THE STRANGE HADRONIC DECAY RATIO $R_{\tau,S}$

As previously demonstrated in Refs. [12], the inclusive τ decay ratio into strange hadronic final states,

$$R_{\tau,S} = \frac{\Gamma(\tau^- \rightarrow \text{hadrons}_{S=-1}^- \nu_\tau)}{\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)}, \quad (1)$$

can be used due to its theoretical prediction [7,8] in the framework of QCD and the Operator Product Expansion (OPE) [14] to determine $m_s(s)$ at the scale $s = M_\tau^2$. Since then it was shown in Ref. [9] that the perturbative expansion used for the massive term [8] was incorrect. After correction the series shows a problematic convergence behaviour [9–11].

Following Ref. [7] the theoretical prediction for the inclusive vector plus axial-vector hadronic decay rate is given by

$$R_{\tau,(S)}(M_\tau^2) = 12\pi S_{\text{EW}} \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \quad (2)$$

$$\times \left[\left(1 + 2\frac{s}{M_\tau^2}\right) \text{Im}\Pi^{(1+0)}(s) - 2\frac{s}{M_\tau^2} \text{Im}\Pi^{(0)}(s) \right]$$

with the two-point correlation functions $\Pi^{(J)} = |V_{ud}|^2 \Pi_{ud,V+A}^{(J)} + |V_{us}|^2 \Pi_{us,V+A}^{(J)}$, for the spin J of the hadronic final state. Equation (2) can be decomposed as

$$R_{\tau,(S)} = 3|V|^2 S_{\text{EW}} \left(1 + \delta^{(0)} + \delta'_{\text{EW}} + \delta^{(2-\text{mass})} + \sum_{D=4,6,\dots} \delta^{(D)} \right), \quad (3)$$

where $V = V_{ud}$ ($V = V_{us}$) for the non-strange (strange) case is the corresponding CKM matrix element. The residual non-logarithmic electroweak correction $\delta'_{\text{EW}} \simeq 0.0010$ [15] is omitted in the following. Throughout this analysis, all QCD observables are expressed in the $\overline{\text{MS}}$ renormalization scheme.

2.1. Perturbative contributions

Contributing terms in Eq. (3) are the perturbative part of mass dimension $D = 0$ given, *e.g.*,

in fixed-order perturbation theory (FOPT) using the $\overline{\text{MS}}$ renormalization scheme by the expansion [16]

$$\delta^{(0)} = a_s(M_\tau^2) + 5.202 a_s^2(M_\tau^2) + 26.37 a_s^3(M_\tau^2) + (K_4 + 78.00) a_s^4(M_\tau^2), \quad (4)$$

with $a_s(M_\tau^2) = \alpha_s(M_\tau^2)/\pi$ and the unknown N³LO coefficient K_4 .

The second term in Eq. (3) is the mass contribution of dimension $D = 2$, in practice only important for the strange quark mass. Using FOPT it is given by [8,9]

$$\delta_S^{(2-\text{mass})} = -8 \frac{m_s^2(M_\tau^2)}{M_\tau^2} \left[1 + \frac{16}{3} a_s(M_\tau^2) + 46.00 a_s^2(M_\tau^2) + \left(283.6 + \frac{3}{4} x_3^{(1+0)} \right) a_s^3(M_\tau^2) \right], \quad (5)$$

The third order coefficient $x_3^{(1+0)}$ occurs in the expansions of the massive $J = 1 + 0$ correlator [8, 17,18]

$$\Pi_{us,V/A}^{(1+0)(D=2)}(s) = -\frac{3}{4\pi^2} \frac{m_s^2(s)}{s} \left[1 + \frac{7}{3} a_s(s) + 19.58 a_s^2(s) + x_3^{(1+0)} a_s^3(s) \right], \quad (6)$$

while the $J = 0$ correlator is known up to third order

$$\Pi_{us,V/A}^{(0)(D=2)}(s) = \frac{3}{2\pi^2} \frac{m_s^2(s)}{s} \left[\frac{1}{a_s(s)} - \frac{5}{2} - 1.71 a_s(s) - 11.67 a_s^2(s) \right]. \quad (7)$$

Assuming naively the perturbative expansions to behave like geometric series, one can estimate the unknown coefficients to be $x_3^{(1+0)} \approx x_2^{(1+0)}(x_2^{(1+0)}/x_1^{(1+0)}) = 165 \pm 330$, using as error twice the estimated contribution. Setting $\alpha_s(M_\tau^2) = 0.334$ [13], Eq. (5) becomes

$$\delta_S^{(2-\text{mass})} = -8 \frac{m_s^2(M_\tau^2)}{M_\tau^2} [1 + 0.57 + 0.52 + (0.49 \pm 0.29)], \quad (8)$$

which converges badly, essentially due to the $J = 0$ contribution. An experimental separation of the

$J = 1$ and $J = 0$ parts does not improve the situation because the individual expansions converge even less than their sum. In addition, larger non-perturbative contributions are expected for less inclusive observables, making the m_s determination more delicate.

A so-called *contour-improved* FOPT [19,20] analysis (FOPT_{CI}) for the dimension $D = 2$ contribution has been presented in Refs. [10,11]. It consists of a direct numerical evaluation of the contour integral which corresponds to Eq. (2), using the solution of the renormalization group equation (RGE) to four-loops [21,22] as input for the running $\alpha_s(s)$ and $m_s(s)$. This provides a resummation of all known higher order logarithmic integrals and was observed to improve the convergence of the massless perturbative series [13,19]. The contour-improved prescription involves the logarithmic derivatives of the correlation functions (6) and (7), called Adler D -functions, which satisfy homogeneous renormalization group equations. With the notation of Ref. [10], the unknown coefficient $x_3^{(1+0)}$ of the correlator (6) translates into the unknown coefficient $d_3^{(1+0)}$ of the $D^{(1+0)}$ function as $d_3^{(1+0)} = x_3^{(1+0)} + 179.2$. With the above value for α_s , the contour-improved evaluation of the perturbative series reads

$$\delta_S^{(2-\text{mass})} = -8 \frac{m_s^2(M_\tau^2)}{M_\tau^2} [0.97 + 0.49 + 0.38 + (0.34 \pm 0.04)] , \quad (9)$$

with somewhat improved convergence compared to FOPT (8). Therefore, the results of the following analysis are based on the FOPT_{CI} approach.

It is interesting to note that the $J = 1 + 0$ part converges very well,

$$\delta_S^{(2-\text{mass})}(J = 1 + 0) = -8 \frac{m_s^2(M_\tau^2)}{M_\tau^2} [0.56 + 0.17 + 0.05 - (0.04 \pm 0.04)] , \quad (10)$$

while the expansion for the $J = 0$ component alone is ill-behaved,

$$\delta_S^{(2-\text{mass})}(J = 0) = -8 \frac{m_s^2(M_\tau^2)}{M_\tau^2} [0.41 + 0.32 + 0.32 + 0.38] . \quad (11)$$

A standard procedure to handle diverging asymptotic series is to truncate the expansion where the terms reach a minimum and to estimate the corresponding uncertainty as the full size of the last term retained. In the present case, it amounts to using the NNLO expansion for $J = 1 + 0$ with an uncertainty of 0.04, while it seems reasonable to truncate the $J = 0$ expansion after the leading order term, with 0.32 as the corresponding uncertainty. As discussed later in Section 8.7, a more detailed look at the $J = 0$ part in Eq. (2) supports this choice.

Following these observations, two methods are used in the following to determine $m_s(M_\tau^2)$:

- (i) in the *inclusive method*, the inclusive strange hadronic rate is considered and both $J = 1 + 0$ and $J = 0$ are included with their respective convergence behaviour taken into account in the theoretical uncertainties.
- (ii) the *'1+0' method* singles out the well-behaved $J = 1 + 0$ part by subtracting the experimentally determined $J = 0$ longitudinal component from data. The measurement is then less inclusive and the sensitivity to m_s is significantly reduced; however, the δ_2 perturbative expansion is under control and the corresponding theoretical uncertainty is reduced.

2.2. Nonperturbative contributions

The last term in Eq. (3) represents the nonperturbative contribution which, using the OPE, can be written as a sum of powers of M_τ^D

$$\delta^{(D)} = \sum_{\dim \mathcal{O}=D} C(s, \mu) \frac{\langle \mathcal{O}(\mu) \rangle}{(-M_\tau^2)^{D/2}} , \quad (12)$$

where the parameter μ separates the long-distance nonperturbative effects, absorbed into the vacuum expectation elements $\langle \mathcal{O}(\mu) \rangle$, from the short-distance effects which are included in the Wilson coefficients $C(s, \mu)$ [23].

Following Ref. [14], the first contribution to $R_\tau(s_0)$ beyond the $D = 0$ massless perturbative expansion and the $D = 2$ mass terms are the dimension $D = 4$ operators. They have dynamical contributions from the gluon condensate,

$\langle(\alpha_s/\pi)GG\rangle$, and the quark condensates, $m_u\langle\bar{u}u\rangle$, $m_d\langle\bar{d}d\rangle$ and $m_s\langle\bar{s}s\rangle$, of the light u, d, s quarks. Remaining $D = 4$ operators are the running quark masses to the fourth power.

The contributions from dimension $D = 6$ operators are rather complex. The occurring large number of independent operators of the four-quark type can be reduced by means of the vacuum saturation approximation [7,14] to leading order α_s . The operators are then expressed as products of scale-dependent two-quark condensates of the type $\alpha_s(\mu)\langle\bar{q}_i q_i(\mu)\rangle\langle\bar{q}_j q_j(\mu)\rangle$. Since the vacuum saturation approximation is a simplifying assumption, possible deviations are accounted for by introducing an effective scale independent operator of the form $\rho\alpha_s\langle\bar{q}q\rangle^2$ that is fit to the data.

For the theoretical prediction of $R_\tau(s_0)$ used here, the complete long and short distance parts of the dimension $D = 8$ contributions are absorbed into the scale invariant phenomenological $D = 8$ operator $\langle O_8 \rangle$.

Higher order contributions from $D \geq 10$ operators are neglected.

The formulae for the nonperturbative correlation functions used are taken entirely from Ref. [7].

3. EVIDENCE FOR THE EFFECT OF A MASSIVE STRANGE QUARK

From the ALEPH branching ratio analysis of τ decays into strange final states [5], one obtains

$$R_{\tau,S} = 0.1607 \pm 0.0066, \quad (13)$$

using the universality-improved value for the electronic branching ratio $B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) = (17.794 \pm 0.045)\%$ [13]. With Eqs. (3) and (4) one finds the QCD prediction for a massless strange quark neglecting the non-perturbative contributions to be

$$R_{\tau,S}^{(0)} = 0.1809 \pm 0.0036, \quad (14)$$

where the quoted error mainly reflects the uncertainties on $\alpha_s(M_\tau^2)$ and V_{us} . Comparing Eq. (13) with (14) reveals a discrepancy of about 2.7σ between both values, in the direction predicted for a non-zero m_s value.

This is evidence for the effect of a massive strange quark in a single observable which is essentially predicted by perturbative QCD, providing the basis for the determination of the running mass m_s at the scale of the τ mass.

4. SPECTRAL MOMENTS

While the gluon and the quark condensates are approximately known, the former from QCD fits and the latter from theoretical arguments, the dimension $D = 6, 8$ contributions are unknown. In analogy to the $\alpha_s(M_\tau^2)$ analysis [13] one uses the spectral functions to construct the moments,

$$R_{\tau,(S)}^{kl} \equiv \int_0^{M_\tau^2} ds \left(1 - \frac{s}{M_\tau^2}\right)^k \left(\frac{s}{M_\tau^2}\right)^l \frac{dR_{\tau,(S)}}{ds} \quad (15)$$

with $R_{\tau,(S)}^{00} = R_{\tau,(S)}$ for the non-strange and the strange cases, respectively. The factor $(1 - s/M_\tau^2)^k$ suppresses the integrand at the crossing of the positive real axis where the validity of the OPE is uncertain and the experimental accuracy is statistically limited. Its counterpart $(s/M_\tau^2)^l$ projects out higher energies. The theoretical prediction of the moments (15) follows the line described above and the modified Eq. (3) reads

$$R_{\tau,(S)}^{kl} = 3|V|^2 S_{\text{EW}} \left(A^{kl} + \delta^{kl(0)} + \delta_S^{kl(2-\text{mass})} + \sum_{D=4,6,\dots} \delta_{(S)}^{kl(D)} \right). \quad (16)$$

The perturbative and nonperturbative terms are taken from Ref. [24]. The perturbative dimension $D = 2$ mass-term for the moments using FOPT becomes

$$\delta_S^{kl(2-\text{mass})} = \frac{m_s^2(M_\tau^2)}{M_\tau^2} [c_0^{kl} + c_1^{kl} a_s(M_\tau^2) + c_2^{kl} a_s^2(M_\tau^2) + c_3^{kl} a_s^3(M_\tau^2)], \quad (17)$$

with the coefficients, c_i^{kl} , given in Table 1 for the pairs of (k, l) moments used in this analysis (see also the results given in Ref. [11]). For the $l \geq 1$ moments, the $x_3^{(1+0)}$ dependence starts only at order α_s^4 . Analogously to Eq.(9), the spectral moments (16) are numerically calculated (FOPT_{CI}).

Table 1

Coefficients of the FOPT expansion (17) for the $D = 2$ mass contribution to the moments (k, l) .

(k, l)	c_0^{kl}	c_1^{kl}	c_2^{kl}	c_3^{kl}
(0,0)	-8	-42.67	-368.0	$-2269 - 6x_3^{(1+0)}$
(1,0)	-7.5	-44.75	-424.4	$-3178 - 6x_3^{(1+0)}$
(2,0)	-7.2	-46.48	-469.8	$-3945 - 6x_3^{(1+0)}$
(1,1)	-0.3	1.73	45.42	766.8
(1,2)	-0.1	0.243	6.96	94.83

Fitting the τ decay rate and the spectral moments allows one to obtain simultaneously $m_s(M_\tau^2)$ and the nonperturbative contributions of dimensions $D = 6$ and $D = 8$.

5. DIFFERENCE OF MOMENTS

A convenient way to reduce the theoretical uncertainties is to consider the difference between non-strange and strange spectral moments, properly normalized with the respective CKM matrix elements,

$$\Delta_\tau^{kl} \equiv \frac{1}{|V_{ud}|^2} R_{\tau, S=0}^{kl} - \frac{1}{|V_{us}|^2} R_{\tau, S=-1}^{kl}, \quad (18)$$

for which the massless perturbative contribution vanishes so that the theoretical prediction now reads (setting $m_u = m_d = 0$)

$$\Delta_\tau^{kl} = 3S_{\text{EW}} \left(-\delta_S^{kl(2-\text{mass})} + \sum_{D=4,6,\dots} \tilde{\delta}^{kl(D)} \right). \quad (19)$$

The flavour independent gluon condensate disappears in the difference. Since effective s -independent operators $\tilde{\delta}^{kl(D)} = h^{kl} \langle \tilde{O}_D \rangle / M_\tau^D$ (see Ref. [13] for the values of the coefficients h^{kl}) are used for the dimension $D = 6$ and $D = 8$ terms, and only one operator for each dimension is fitted, the $\tilde{\delta}^{kl(D)}$ contain the full nonperturbative contributions to the difference of non-strange and strange moments. On the other hand, in Ref. [13], the non-strange nonperturbative contributions to the inclusive $V + A$ hadronic decay ratio have been found to be very small, giving only a marginal contribution to $\tilde{\delta}^{kl(D)}$ for $D = 6, 8$, so that one has approximately $\tilde{\delta}^{kl(D)} \approx -\delta_S^{kl(D)}$.

Table 2

Measured spectral moments Δ_τ^{kl} (left hand side) and their experimental correlations (right hand side).

(k, l)	Δ_τ^{kl}	(1,0)	(2,0)	(1,1)	(1,2)
(0,0)	0.413 ± 0.126	0.99	0.95	0.97	0.88
(1,0)	0.392 ± 0.086	1	0.99	0.91	0.78
(2,0)	0.378 ± 0.064	-	1	0.83	0.68
(1,1)	0.014 ± 0.026	-	-	1	0.96
(1,2)	0.008 ± 0.010	-	-	-	1

6. EXPERIMENTAL DETERMINATION OF THE MOMENTS

The experimental results on the spectral moments (18) and their correlations are given in Table 2. For the CKM matrix elements the values $|V_{ud}| = 0.9751 \pm 0.0004$ and $|V_{us}| = 0.2218 \pm 0.0016$ [25] are used, while the errors are part of the theoretical uncertainties discussed in the next section. All (k, l) moments are obtained from the same spectral function (see Ref. [5]) and therefore suffer from large correlations. This reduces the independent information used in the fit and thus introduces strong correlations between the adjusted parameters.

Figure 1 shows ALEPH data of the weighted integrand of the lowest moment Δ_τ^{00} (see Eq. (15)), which is the difference of the Cabibbo corrected invariant mass spectra, as a function of the invariant mass-squared. Also shown is the expectation from perturbative QCD for which the contribution vanishes.

7. THEORETICAL PARAMETERS AND UNCERTAINTIES

For the strong coupling constant, the recent ALEPH value, $\alpha_s(M_\tau^2) = 0.334 \pm 0.022$ [13] is used.

The dependence on the renormalization scheme applied ($\overline{\text{MS}}$) is already included in the theoretical error on $\alpha_s(M_\tau^2)$ and thus not further added in order to avoid double counting.

The renormalization scale (μ) dependence of the $D = 2$ prediction has been studied in Ref. [10]. When varying $\mu \neq M_\tau$, additional logarithms en-

Figure 1. Integrand of Eq. (15) for $(k = 0, l = 0)$, *i.e.*, difference of the Cabibbo corrected strange and non-strange invariant mass spectra (without single pion and kaon poles). Massless perturbative QCD vanishes in the difference.

ter the series and due to its truncation at finite order, a residual dependence on μ is left. To estimate the associated uncertainty, μ is varied from 1.3 to 3.6 GeV/ c^2 .

The uncertainty stemming from the truncation of the perturbative series for the dimension $D = 2$ term is handled as follows: the estimated error is the full size of the last term retained in the expansion, namely of order α_s^3 and α_s for the $J = 1 + 0$ and $J = 0$ series, respectively.

It is a conservative measure to add all three listed perturbative errors which are of common origin as independent uncertainties, implying some double counting.

The intrinsic ambiguity between FOPT and FOPT_{CI} is unresolvable at present. An estimate of this ambiguity is obtained from the difference between the results obtained using both approaches.

For the dimension $D = 4$ nonperturbative contribution, the products of quark condensates

Table 3

Theoretical uncertainties on $m_s(M_\tau^2)$. The uncertainty from the truncation of the perturbative series for the $D = 2$ mass term are estimated from adding or subtracting the value of the last retained term in the QCD expansion. The unequal positive and negative errors are averaged.

Error source	Range	$\Delta m_s(M_\tau^2)$ incl. '1+0'	
$\alpha_s(M_\tau^2)$	0.334 ± 0.022	3	3
truncation		15	10
R-scale μ	$(1.3 - 3.6) \text{ GeV}/c^2$	9	5
$m_s\langle\bar{s}s\rangle$	$-(0.0016 \pm 0.0003) \text{ GeV}^4$	4	5
S_{EW}	1.0194 ± 0.0040	1	1
$ V_{us} $	0.2218 ± 0.0016	13	20
Total errors		21	24

Table 4

Correlations from theoretical uncertainties between the moments Δ_τ^{kl} (FOPT_{CI}) for the inclusive method.

ALEPH	(0,0)	(1,0)	(2,0)	(1,1)	(1,2)
(0,0)	1	0.99	0.96	-0.70	0.19
(1,0)	-	1	0.99	-0.80	0.29
(2,0)	-	-	1	-0.88	0.39
(1,1)	-	-	-	1	-0.67
(1,2)	-	-	-	-	1

and quark masses are taken from the PCAC relations $(m_u + m_d)\langle\bar{u}u + \bar{d}d\rangle \simeq -2f_\pi^2 m_\pi^2$ and $m_s\langle\bar{s}s\rangle \simeq -C_{\bar{s}s/\bar{u}u}f_\pi^2(m_K^2 - m_\pi^2)$, with the pion decay constant $f_\pi = (92.4 \pm 0.3) \text{ MeV}$ [25] and $C_{\bar{s}s/\bar{u}u} = (0.85 \pm 0.15)$ [27,26].

No errors are introduced for the higher dimensional operators since they are fixed experimentally. Finally, a small uncertainty from the error on S_{EW} is included.

Table 3 gives the above theoretical uncertainties on $m_s(M_\tau^2)$ using FOPT_{CI} for the inclusive and '1+0' methods. The main error contributions stem from the truncation of the perturbative series. The corresponding theoretical correlation matrix is given in Table 4.

8. SEPARATION OF $J = 0, 1$ COMPONENTS

In principle, the identification of the $J = 0$ part in the strange spectral function would allow one to avoid the difficulties related to Eq. (11) by using only the $J = 1 + 0$ term in Eq. (2). Although the $J = 0$ component has not been thoroughly investigated in the data, it is clear that it is dominated by the single K pole. Other contributions can be identified: (i), off-shell vector K^* resonances can generate a 0^+ component [28], which is however too small to have a significant effect in this analysis, and, (ii), production of the scalar $K_0^*(1430)$ resonance is expected as discussed below.

The $K_0^*(1430)$ state decays almost exclusively into $K\pi$ with a branching ratio of $(93 \pm 10)\%$ [25] and an ALEPH fit to the $K^-\pi^0$ mass distribution as given by S. Chen [5] provides the branching ratio $B(\tau \rightarrow \nu_\tau K_0^*(1430)) = (0.0 \pm 2.5) \times 10^{-4}$. It is therefore clear that 0^+ contributions are strongly suppressed ($< 1.6 \times 10^{-2}$) compared to the dominant 1^- production. This is expected from chiral symmetry breaking, with contributions reduced by $\sim (m_K/M_\tau)^4$ as discussed in Ref. [28], not including the τ kinematic factor which suppresses higher mass states.

No direct estimate of extra 0^- components beyond the single K can be made from the present data in the $\bar{K}\pi\pi$ modes. This is due to the lack of statistics and to the background subtraction, especially in the high mass region where some evidence exists for a pseudoscalar state with a large width [25] at $1460 \text{ MeV}/c^2$. A contribution equal to the one of 0^+ , which is actually an upper limit, is assumed in this case.

The evaluation of the $J = 0$ contributions to $R_{\tau,S}$ obtained from integration of Eq. (2), using the ALEPH value $f_K = (111.5 \pm 0.9 \pm 2.3) \text{ MeV}$ [5], reads

$$\begin{aligned} R_{\tau,S}^{(0)}(K) &= -0.00615 \pm 0.00026, \\ R_{\tau,S}^{(0)}(\text{other } 0^-, 0^+) &= -0.0015 \pm 0.0015, \end{aligned}$$

leading to

$$\Delta_\tau^{00(0)} = 0.155 \pm 0.031, \quad (20)$$

where the $J = 0$ contributions in the nonstrange

Figure 2. Measurement of the longitudinal $\Delta_\tau^{00(0)}$ and expectation from QCD when the expansion is truncated after the term of order n . The value of $m_s(M_\tau^2)$ is chosen to be $150 \text{ MeV}/c^2$. The errors on the theoretical prediction follow the rule described in Section 2.1, *i.e.*, the uncertainty on the QCD sum is taken to be equal to the value of the last retained term (of order n) in the expansion.

part have been computed to be -0.00076 for the π pole (using the corresponding ALEPH branching ratio [29]) and safely assumed to be negligible in the other modes.

The experimental result (20) can be compared to the QCD expectation for the longitudinal part. It was shown in Section 2.1 that the perturbative expansion in the δ_2 mass term diverges and a procedure to deal with this problem was proposed. The QCD prediction for $\Delta_\tau^{00(0)}$ is shown order by order in Fig. (2), using the δ_2 and δ_4 terms and ignoring higher order nonperturbative contributions. Theory and data are in agreement only when a limited number of terms are kept in the δ_2 perturbative expansion. In fact, truncating the expansion at the minimum ($n = 1$) as proposed in Section 2.1 is well supported by this

Table 5

Results of the fits for the inclusive and ‘1+0’ methods.

	inclusive	‘1+0’
$m_s(M_\tau^2)$	$153^{+22_{\text{exp}}+17_{\text{th}}}_{-26_{\text{exp}}-24_{\text{th}}} \pm 6_{\text{fit}}$	$184^{+33_{\text{exp}}+19_{\text{th}}}_{-41_{\text{exp}}-27_{\text{th}}} \pm 7.5_{\text{fit}} \pm 11_{J=0}$
$\tilde{\delta}^{(6)}$	$0.032 \pm 0.009_{\text{exp}} \pm 0.011_{\text{th}} \pm 0.002_{\text{fit}}$	$0.042 \pm 0.016_{\text{exp}} \pm 0.014_{\text{th}} \pm 0.004_{\text{fit}}$
$\tilde{\delta}^{(8)}$	$-0.019 \pm 0.008_{\text{exp}} \pm 0.006_{\text{th}} \pm 0.002_{\text{fit}}$	$-0.024 \pm 0.012_{\text{exp}} \pm 0.008_{\text{th}} \pm 0.003_{\text{fit}}$
$\chi^2/2ndf$	0.3	0.2

Table 6

Correlation matrix according to the fit results in Table 5 (FOPT_{CI}) for both methods applied.

Inclusive	$m_s(M_\tau^2)$	$\tilde{\delta}^{(6)}$	$\tilde{\delta}^{(8)}$	‘1+0’	$m_s(M_\tau^2)$	$\tilde{\delta}^{(6)}$	$\tilde{\delta}^{(8)}$
$m_s(M_\tau^2)$	1	0.65	-0.87	$m_s(M_\tau^2)$	1	0.78	-0.94
$\tilde{\delta}^{(6)}$	-	1	-0.78	$\tilde{\delta}^{(6)}$	-	1	-0.89
$\tilde{\delta}^{(8)}$	-	-	1	$\tilde{\delta}^{(8)}$	-	-	1

test. Including calculated higher order diverging terms would be inconsistent with the measured $J = 0$ part $\Delta_\tau^{00(0)}$ for $m_s(M_\tau^2)$ values in the 100–200 MeV/ c^2 range.

9. RESULTS

The fit minimizes the χ^2 of the differences between measured and fitted quantities contracted with the inverse of the sum of the experimental and theoretical covariance matrices taken from Tables 2 and 4. Due to the large correlations and the statistical limitation of the spectral functions at the end of the τ phase space, higher moments $l > 2$ do not add significant information to the fit. The results for $m_s(M_\tau^2)$ and the nonperturbative contributions to Δ_τ^{00} for the two methods (inclusive and ‘1 + 0’) are given in Table 5. The errors are separated according to their experimental, theoretical and “fit” origins. The latter stems from an intrinsic bias in the χ^2 minimization, due to the large correlations of the input observables, and includes the total difference between the fit results with and without correlations, while the results without correlations are given as central values. The theoretical errors contain the difference between the results found for FOPT and FOPT_{CI} which is for the strange quark mass $m_s^{\text{FOPT}_{\text{CI}}}(M_\tau^2) - m_s^{\text{FOPT}}(M_\tau^2) = 4 \text{ MeV}/c^2$. The

correlation matrix corresponding to the fit results in Table 5 is given in Table 6.

The results of the two methods are strongly correlated (79%). Uncorrelated errors include the truncation uncertainty of the $J = 0$ expansion and the uncertainty in the determination of the $J = 0$ part of $R_{\tau,S}$. The renormalization scale errors turn out to be anticorrelated due to the opposite behaviour of the inclusive and $J = 1 + 0$ rates as a function of μ [10]. Taking these effects into account, one finds that the two results on $m_s(M_\tau^2)$ are consistent with a $\chi^2/\text{d.o.f} = 1.4/1$ and they are combined to give

$$m_s(M_\tau^2) = (163^{+34}_{-43}) \text{ MeV}/c^2. \quad (21)$$

The errors quoted in Eq. (21) take into account the uncertainties given in Table 3 and the correlation between the two determinations. They are rescaled by a factor of 1.2 which is the square root of the χ^2 value quoted above to account for the inconsistency.

The result (21) can be evolved to the scale of 1 GeV using the four-loop RGE γ -function [22]:

$$m_s(1 \text{ GeV}^2) = (217^{+45}_{-57}) \text{ MeV}/c^2. \quad (22)$$

The fitted m_s value using the inclusive method corresponds to a contribution $\delta_S^{(2)} = -(0.088 \pm 0.036)$ to $R_{\tau,S}$. No operator of dimension $D = 4$

has been fitted, but the small quartic strange mass corrections, for a total contribution $\delta_S^{(4)} = -(0.030 \pm 0.005)$, dominated by the strange quark condensate and where the error given is almost of pure theoretical origin. The $D = 6$ and the $D = 8$ strange contributions are $\delta_S^{(6)} = -0.031 \pm 0.015$ and $\delta_S^{(8)} = 0.018 \pm 0.010$, which add to $\delta_S^{(6,8)} = -(0.013 \pm 0.010)$. They are both fairly large compared to the non-strange case $\delta^{(6)} = 0.001 \pm 0.004$ and $\delta^{(8)} = -0.001 \pm 0.001$ [13].

10. TEST OF THE RUNNING OF $m_s(s)$

As shown for the non-strange case in Ref. [13], one can simulate the physics of a hypothetical τ lepton with mass $\sqrt{s_0}$ smaller than M_τ by replacing M_τ^2 everywhere in Eq. (2) by s_0 . Repeating the $m_s(s_0)$ analysis for $s_0 \leq M_\tau^2$ provides a direct test of the OPE approach adopted here, since the energy dependence of the theoretical prediction of $R_{\tau,S}(s_0)$ or $\Delta_\tau^{00}(s_0)$ is determined once the parameters are fixed. On the other hand, under the assumption of quark-hadron duality, the evaluation of the observables as a function of s_0 constitutes a test of the running of $m_s(s_0)$, governed by the renormalization group equation. Fig. 3 shows the running observables $R_{\tau,S}(s_0)$ and $\Delta_\tau^{00}(s_0)$ compared to the corresponding theoretical predictions using FOPT_{CI} and the fitted parameters in Table 5. Also shown is $m_s(s_0)$ obtained from $\Delta_\tau^{00}(s_0)$ when fixing the nonperturbative dimension $D = 6$ and $D = 8$ operators to the values in Table 5. The experimental errors decrease when s_0 becomes smaller due to the better statistical precision of the strange spectral function at lower energies (see Figure 16 in Ref. [5]). On the contrary, the theoretical errors blow up at small s_0 , as a consequence of the relatively large and imprecise nonperturbative terms and the uncertainty from the unknown higher order perturbative coefficients. Despite the expected breakdown of the perturbative approach at lower scales, it is remarkable that the prediction agrees with data within the theoretical uncertainties used in the analysis.

Figure 3. The observables $R_{\tau,S}(s_0)$ and $\Delta_\tau^{00}(s_0)$ (upper and middle plots) as a function of the “ τ -mass”-squared s_0 . The curves are plotted as one-standard deviation error bands to emphasize their strong point-to-point correlations. Also shown are the theoretical predictions using FOPT_{CI} for the fit parameters in Table 5. The lower plot shows $m_s(s_0)$, obtained from the comparison of $\Delta_\tau^{00}(s_0)$ and its theoretical prediction, within the experimental and theoretical error bands.

11. COMPARISON WITH OTHER DETERMINATIONS OF m_s

The present determination of m_s relies on a complete set of data for providing the sum of the vector and axial vector strange spectral functions. The detailed theoretical description of the τ strange hadronic width in terms of perturbative and nonperturbative QCD contributions allows a direct comparison with the measured spectral function, hence yielding $m_s(M_\tau^2)$.

Other determinations of m_s have been obtained by analyses of the divergences of the vector and axial-vector current two-point function correlators [26,27,30–37]. The phenomenological infor-

mation on the associated scalar and pseudoscalar spectral functions is reconstructed from phase-shift resonance analyses which are yet incomplete over the considered mass range and need to be supplemented by other assumed ingredients, in particular the description of the continuum. As a result, the values obtained differ significantly between the various analyses. Moreover, the quoted systematic uncertainties span a large range, which does not cover the observed differences, indicating that systematic effects have only been partially taken into account.

Another approach [39] considers the difference between isovector and hypercharge vector current correlators as related to the $I = 1$ and $I = 0$ spectral functions accessible in e^+e^- annihilation into hadrons at low energy. A recent reanalysis [38] points out the possibility of large corrections from isospin breaking, leading to significant deviations for the extracted m_s value.

Finally, lattice QCD calculations of m_s are available [40–43]. Their values also show quite large a spread.

The present determination of m_s is directly compared with the other derivations in Fig. 4. All values are given at a mass scale of 1 GeV. While a reasonable agreement is found, it is observed that the present, more direct, determination provides an interesting information for the phenomenology of strange particles. Its precision is unfortunately still limited. As an example, the Standard Model prediction for the CP-violation parameter ϵ'/ϵ is strongly dependent on the value of m_s and the present determination translates into a range of ϵ'/ϵ values spanning a factor of four [6].

12. CONCLUSIONS

Using total decay rates, as well as spectral moments obtained from a combination of strange and non-strange spectral functions which cancel the dominant massless perturbative QCD contribution, a global QCD fit to data measured by the ALEPH Collaboration is performed. This provides a simultaneous determination of the strange quark mass and non-perturbative power terms using the framework of the Operator Product Expansion. The value obtained is $m_s(M_\tau^2) =$

Figure 4. The ALEPH determination of $m_s(1\text{GeV}^2)$ compared to the results of other approaches. Detailed are given in the text.

$(163_{-43}^{+34}) \text{ MeV}/c^2$, which evolved to 1 GeV becomes $m_s(1\text{GeV}^2) = (217_{-57}^{+45}) \text{ MeV}/c^2$, in agreement with other determinations. A test of the evolution of $m_s(s)$ to $s \leq M_\tau^2$ using “running” experimental observables compared to the theoretical prediction showed consistent results.

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REFERENCES

1. ALEPH Collaboration, *The Europ. Phys. J.* **C1** (1998) 65
2. ALEPH Collaboration, *The Europ. Phys. J.* **C4** (1998) 29
3. ALEPH Collaboration, *to be published soon*
4. ALEPH Collaboration, *Z. Phys.* **C74** (1997) 263
5. S. Chen, “*Tau Decays to Kaons from ALEPH and physics implications*”, Talk given at the TAU’98 Conference, Santander, Spain, 1998.
6. A. Buras et al., *Phys. Lett.* **B389** (1996) 749

7. E. Braaten, S. Narison and A. Pich, *Nucl. Phys.* **B373** (1992) 581
8. K.G. Chetyrkin and A. Kwiatkowski, *Z. Phys.* **C59** (1993) 525
9. K. Maltman, “*Problems With Extracting m_s from Flavour Breaking in Hadronic τ Decays*”, Report YU-PP-K/M-98-1 (1998), hep-ph/9804298
10. A. Pich, J. Prades, “*Perturbative Quark Mass Corrections to the Tau Hadronic Width*”, Report FTUV-97-47A (1998), hep-ph/9804462
11. K.G. Chetyrkin, J.H. Kühn, A.A. Pivovarov, “*Determining the Strange Quark Mass in Cabibbo Suppressed Tau Lepton Decays*”, Report TTP-98-18 (1998), hep-ph/9805335
12. M. Davier, *Nucl. Phys. C* (Proc. Suppl.) **55** (1997) 395;
S. Chen, *Nucl. Phys. B* (Proc. Suppl.) **64** (1998) 265
13. ALEPH Collaboration, *Eur.Phys.J.* **C4:409**, 1998
14. M.A. Shifman, A.L. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B147** (1979) 385, 448, 519.
15. E. Braaten and C.S. Li, *Phys. Rev.* **D42** (1990) 3888
16. L.R. Surguladze and M.A. Samuel, *Phys. Rev. Lett.* **66** (1991) 560;
S.G. Gorishny, A.L. Kataev and S.A. Larin, *Phys. Lett.* **B259** (1991) 144
17. K.G. Chetyrkin, *Phys. Lett.* **B390** (1997) 309
18. K.G. Chetyrkin and J. Kühn, *Phys. Lett.* **B406** (1997) 102
19. F. Le Diberder and A. Pich, *Phys. Lett.* **B286** (1992) 147
20. A.A. Pivovarov, *Sov. J. Nucl. Phys.* **54** (1991) 676, *Z. Phys.* **C53** (1992) 461
21. S.A. Larin, T. van Ritbergen and J.A.M. Vermaseren, *Phys. Lett.* **B400** (1997) 379
22. S.A. Larin, T. van Ritbergen and J.A.M. Vermaseren, *Phys. Lett.* **B405** (1997) 327
23. K.G. Wilson, *Phys. Rev.* **179** (1969) 1499.
24. F. Le Diberder and A. Pich, *Phys. Lett.* **B289** (1992) 165
25. R.M. Barnett et al., Particle Data Group, *The Europ. Phys. J.* **C3** 1998 1
26. K.G. Chetyrkin et al., *Phys. Lett.* **B404** (1997) 337
27. M. Jamin and M. Münz, *Z. Phys.* **C66** (1995) 633
28. M. Finkemeier and E. Mirkes, *Z. Phys.* **C72** (1996) 619
29. ALEPH Collaboration, *Z. Phys.* **C70** (1996) 579
30. C. Becchi et al., *Z. Phys.* **C8** (1981) 335
31. C.A. Dominguez and E. de Rafael, *Ann. Phys.* **NY 174** (1987) 372
32. C.A. Dominguez et al., *Phys. Lett.* **B253** (1991) 241
33. M. Jamin, *Nucl. Phys. B* (Proc. Suppl.) **64** (1998) 250
34. A.L. Kataev et al., *Nuovo. Cim.* **76A** (1983) 723
35. P. Colangelo et al., *Phys. Lett.* **B408** (1997) 340
36. S. Narison, *Phys. Lett.* **B216** (1989) 191
37. K.G. Chetyrkin et al., *Phys. Rev.* **D51** (1995) 5090
38. K. Maltman, *Phys. Lett.* **B428** (1998) 179
39. S. Narison, *Phys. Lett.* **B358** (1995) 113
40. R. Gupta and T. Bhattacharya, *Phys. Rev.* **D55** (1997) 7203
41. B.J. Gough et al., *Phys. Rev. Lett.* **79** (1994) 1662
42. C.R. Allton et al., *Nucl. Phys.* **B431** (1994) 667
43. N. Eicker et al., SESAM-Collaboration, hep-lat/9704019