Higgsinos in SUSY Models with Gaugino Mass Unification

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Abstract

In the MSSM, the assumptions of a common sfermion mass parameter $m_0$ and a common gaugino mass parameter $m_{1/2}$, along with the requirements from radiative electroweak symmetry breaking, lead to relatively large values of the Higgs mixing parameter $\mu$, and consequently to a gaugino-like lightest neutralino $\tilde{\chi}^0_1$. Lifting the requirement that the Higgs mass parameters $m_{H_D}$ and $m_{H_U}$ unify with the sfermion masses is known to allow for smaller $\mu$. We show that a $\mu$ parameter sufficiently small to yield a Higgsino-like neutralino $\tilde{\chi}^0_1$ requires a precise adjustment of the Higgs mass parameter $m_{H_U}$. Consequently a gaugino-type neutralino is still preferred in SUSY models with gaugino mass unification.
The composition of the lightest neutralino affects both its phenomenology and cosmology. The couplings of the light neutralino and chargino states, as well as the relationship between the two lightest neutralino and lightest chargino masses, depend on their Higgsino content. Thus the production rates and branching ratios for charginos and neutralinos, and the expected SUSY signatures associated with ino production, vary with the ino compositions, and studies of SUSY searches at future experiments need to include separate analyses for Higgsino and gaugino-like \( \tilde{\chi}^\pm \) and \( \tilde{\chi}^0 \) \[1\]. The cosmology of neutralinos is even more sensitive to their composition, as a Higgsino-like neutralino tends to have a significantly lower relic abundance than its gaugino-like counterpart, and searches for charginos, neutralinos and Higgs bosons at LEP have all but excluded Higgsinos as giving a dominant contribution to the energy density of the universe \[2\]. Further, the interaction rates of relic neutralinos with nuclei depend sensitively on the neutralino composition. It is therefore interesting to know what restrictions one may place on Higgsino content of the neutralino. In particular, we consider the standard lore, that relaxing the scalar mass unification condition to allow for non-universal Higgs mass parameters \( m_{H_D} \) and \( m_{H_U} \) permits the neutralino \( \tilde{\chi}^0 \) to be either Higgsino-like or gaugino-like. We will show that while it is true that the Higgs mixing mass \( \mu \) may be chosen small enough to provide a Higgsino-like LSP, it is at the price of choosing a very particular narrow range in \( m_{H_U} \).

It is well known that in models with both gaugino and scalar mass unification, the lightest neutralino tends to be a gaugino, and in particular a bino. \( |\mu| \) is fixed at the electroweak scale by the Higgs potential minimization condition, which at tree level reads

\[
|\mu|^2 = \frac{m_{H_D}^2 - m_{H_U}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{m_Z^2}{2}. \tag{1}
\]

Since \( m_{H_D}(M_X) \) and \( m_{H_U}(M_X) \) are fixed by the scalar mass unification condition, their low-energy values are determined by their RGE evolution to the electroweak scale. The dominant parts of the one-loop RGEs for \( m_{H_D} \) and \( m_{H_U} \) are given by

\[
\frac{d m_{H_D}^2}{d t} = \frac{1}{8 \pi^2} \left( -3 g_2^2 M_2^2 - g_1^2 M_1^2 + 3 h_2^2 (m_{Q_L}^2 + m_{R}^2 + m_{H_D}^2 + A_b^2) - \frac{1}{2} g_1^2 S \right)
\]

\[
\frac{d m_{H_U}^2}{d t} = \frac{1}{8 \pi^2} \left( -3 g_2^2 M_2^2 - g_1^2 M_1^2 + 3 h_1^2 (m_{Q_L}^2 + m_{R}^2 + m_{H_U}^2 + A_t^2) + \frac{1}{2} g_1^2 S \right), \tag{2}
\]

where

\[
S = m_{H_U}^2 - m_{H_D}^2 + \sum_{i=1}^{3} m_{Q_i}^2 - m_{L}^2 - 2 m_u^2 + m_d^2 + m_e^2. \tag{3}
\]

The terms proportional to \( S \) are the D-term contributions \[3, 4\] to the running of the scalar mass\(^2 \) parameters. In mSUGRA, \( S \) vanishes at the unification scale, and as \( S = 0 \) is a fixed
point of the RG equations[3], it remains 0 at all scales. Symmetries force $S$ to vanish in many models of interest, and consequently the D-term contribution to the scalar mass $\beta$ functions do not appear in many compilations of the SUSY 1-loop RGEs. Since we will be breaking scalar mass unification, we need to keep the $S$ terms on the right-hand side of (2).

It is the positive terms proportional to the top Yukawa coupling which drive $m^2_{H_u}$, negative and allow (1) to be satisfied in the standard scenario. Since the squark mass $\beta$ parameters receive a large contribution in their evolution proportional to the square of the gluino mass, and since $A_t$ approaches its quasi-fixed-point value proportional to $m_{1/2}$, the right-hand side of (2), and hence the changes in $m^2_{H_D}$ and $m^2_{H_U}$ as they are evolved to the electroweak scale, scale with the gaugino masses $\beta$. Neglecting the small $m^2_Z$ term, the final value of $\mu$ given by (1) is then simply proportional to $m_{1/2}/2$. The inclusion of radiative corrections to (1) do not significantly alter this result, although they can move the regions where scaling is violated. An example of the scaling between $\mu$ and the gaugino mass is given in Fig. 1, where the minimal supergravity (mSUGRA) solution for $\mu$ is shown in the $\{M^2, \mu\}$ plane at fixed $m_0 = 100$ GeV, with $A_0 = 0$. At intermediate and large $\tan \beta$, $A_t$ can of course stray far from the quasi-fixed point when $|A_0| \gg m_{1/2}$. Large $|A_t|$ increases the effect of the top Yukawa coupling in (2) and can subsequently increase $|\mu|$. This effect is limited by the fact that for $A_0$ too large, the right stop mass $\beta$ parameter is driven negative; however, a significant fractional increase in $|\mu|$ can still be produced, particularly for large $m_0$. By contrast, it is difficult to reduce $|\mu|$ by any significant amount, as the magnitude of $A_t$ is $\sim \mathcal{O}(m_{1/2})$ over most of its evolution, even if it doesn’t closely approach the quasi-fixed point. In Fig. 1, $|\mu|$ takes its minimum value for $A_0 \approx -2m_{1/2}$, where it is reduced from its $A_0 = 0$ value by $\sim 10\%$. The light shaded band shows the possible range of $\mu$, varying over allowed $A_0$.

In general, the neutralinos are linear combinations of the neutral gauginos and Higgsinos,  

$$
\chi_i = \beta_i \tilde{B} + \alpha_i \tilde{W} + \gamma_i \tilde{H}_1 + \delta_i \tilde{H}_2, \quad i = 1, \ldots, 4 \tag{4}
$$

In this notation, the gaugino purity of a neutralino $\chi_i$ is defined to be $p_i = \sqrt{\alpha_i^2 + \beta_i^2}$, and its Higgsino purity $\sqrt{1 - p_i^2} = \sqrt{\gamma_i^2 + \delta_i^2}$. In the $(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)$ basis, the neutralino mass matrix takes the form

$$
\begin{pmatrix}
M_1 & 0 & -M_Z s_0 c_\beta & M_Z s_0 s_\beta \\
0 & M_2 & M_Z c_0 c_\beta & -M_Z c_0 s_\beta \\
-M_Z s_0 c_\beta & M_Z c_0 c_\beta & 0 & -\mu \\
M_Z s_0 s_\beta & -M_Z c_0 s_\beta & -\mu & 0
\end{pmatrix}, \tag{5}
$$

$^1$In our sign conventions, the mixing term in the stop mass matrix is $-m_t(A_t + \mu \cot \beta)$. 

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where \( s_\theta (c_\theta) = \sin \theta_W (\cos \theta_W) \), \( s_\beta (c_\beta) = \sin \beta (\cos \beta) \), and where gaugino mass unification implies \( M_1 = 5/3 \tan^2 \theta_W M_2 \approx 0.4 \, m_{1/2} \). In the limit \( |\mu| \gg M_1 \), the lightest neutralino is gaugino-like, specifically a \( \tilde{B} \), with mass \( m_{\tilde{\chi}} \approx M_1 \). Only if \( |\mu| \lesssim M_1 \) will the lightest neutralino be Higgsino-like, with mass \( m_{\tilde{\chi}} \approx |\mu| \). Superimposed in Fig. 1 are contours of constant 99% and 97% bino purity \( |\beta_1| \), showing that the lightest neutralino in mSUGRA tends to be quite pure gaugino. The dark thick contour, which separates the gaugino-like from Higgsino-like neutralinos and corresponds to \( p_1 = \sqrt{1 - p_1^2} = 1/\sqrt{2} \), lies well to the left of the mSUGRA contour. Thus the prejudice towards a bino as the lightest neutralino.

The current LEP lower limit of 95 GeV on the mass of the lightest chargino is shown as the thin solid line in Fig. 1. The displayed chargino mass includes the full 1-loop corrections [5], which give it only a very mild dependence on \( m_0 \) (here taken to be 100 GeV). The chargino mass constraint provides an absolute lower bound on \( \mu \). Further, the intersection of the chargino bound with the Higgsino/gaugino threshold lies at \( M_2 \sim 180 \, \text{GeV} \), implying that a Higgsino-like neutralino is excluded by the chargino limits alone for \( m_{1/2} < 220 \, \text{GeV} \), for this value of \( \tan \beta \) and sign of \( \mu \). Similar corresponding bounds apply for other \( \tan \beta \) and for \( \mu < 0 \).

For sufficiently large \( m_0 \gg m_{1/2} \), the scalar mass contributions to the squark and Higgs mass parameters in (1) and (2) remain important, and scaling can break down. This is shown
in Fig. 2, where contours of constant $\mu$ are plotted in the $\{m_{1/2}, m_0\}$ plane for $\tan \beta = 10$; note the expanded scale of the $m_0$ axis compared to the $m_{1/2}$ axis. In all the figures we display, we use two loop RGEs\cite{4} to evolve the dimensionless couplings and the gaugino masses, and one loop RGEs\cite{3, 8} for the other soft masses, and we include one-loop SUSY corrections to $\mu$\cite{6} and to the top and bottom masses\cite{7}. For large $m_{1/2}/m_0$, $(\mu/m_{1/2}) \sim 1.25 - 1.35$. Note that the contours in Fig. 2 are almost vertical for $m_0 \leq m_{1/2}$, but deviate from vertical for $m_0 \gg m_{1/2}$. For sufficiently large $m_0$, $|\mu|^2$ determined by (1) becomes negative, and the vacuum does not exhibit correct electroweak symmetry breaking. Since large cancellations are required to allow $\mu = 0$, the location of the line at which this happens is sensitive to the radiative corrections to (1), as well as to the loop order at which the masses and couplings are run. However, it always is positioned at large $m_0 \gg m_{1/2}$. This region is marked by dark shading.

In Fig. 2, there are small regions at large $m_0$, near to where $|\mu|^2$ vanishes in (1), where $\mu$ is much smaller than that given by the naïve scaling relation. However, the size of the region with $|\mu| < 0.4m_{1/2}$ is tiny, and it is further reduced by the LEP chargino mass constraint. If we want a Higgsino LSP, hence small $|\mu|$, we must either live in the tiny fringe areas at large $m_0$, and content ourselves with a very heavy scalar spectrum, or we must break either the scalar or gaugino mass unification condition. Clearly, if $m_{H_D}$ and $m_{H_U}$ are free parameters in (1), $\mu$ can take any value, and it is part of the standard lore that breaking scalar mass

Figure 2: Contours of constant $\mu$, for $\tan \beta = 10, A_0 = 0$. The upper shaded region does not admit correct electroweak symmetry breaking.
unification for the Higgs mass parameters allows for a completely general set of Higgs sector masses and mixings. It is this standard claim we wish to examine in this paper.

Accordingly, we now relax the mSUGRA unification constraints assumed in Fig. 2 and allow $m_{H_D}$ and $m_{H_U}$ to vary freely. Fig. 3 displays contours of constant $\mu$ in the $\{m_{H_U}, m_{H_D}\}$ plane, for two values of fixed $m_{1/2}$ and sfermion mass $m_0$, and for three values of tan $\beta$. Plots for $\mu < 0$ are very similar. The shaded regions have either a tachyonic stop or stau (typically for large $m_{H_D}$) or do not permit electroweak symmetry breaking (for large $m_{H_U}$). The mSUGRA point is marked by a triangle. Gaugino purities of 0.95 and 0.71 (the purity value at which the lightest neutralino is half gaugino and half Higgsino) are denoted by dashed contours. The Higgsino region is a thin little strip, tucked up against the excluded shaded region. The hatched area is the part of the Higgsino region which satisfies the chargino mass constraints\(^2\). This graphically demonstrates the careful adjustments required to achieve a Higgsino-like lightest neutralino.

The position of the Higgsino region varies with $m_0$ and $m_{1/2}$, but to a good approximation, for $m_{H_D} = m_{H_U}$, the top end of the allowed Higgsino region lies at

$$m_{H_U}^2 \approx 2.6 m_{1/2}^2 + 1.0 m_0^2,$$

for $4 \lesssim \tan \beta \lesssim 30$. We’ve taken $A_0 = 0$ in Fig. 3; while the position of the region does vary some with $A_0$, the size of the hatched region remains small. Since $\mu$ is more sensitive to $m_{H_U}$ than $m_{H_D}$ in the allowed Higgsino region, we now for simplicity fix $m_{H_D} = m_{H_U}$, and we plot in the $\{m_{1/2}, m_0\}$ plane the percentage of the experimentally allowed $m_{H_U}$ parameter space which contains a Higgsino-like $\tilde{\chi}_1^0$; i.e. we display in Fig. 4 contours of constant $(m_{H_U}^{\text{max}} - m_{H_U}^{\text{min}})/m_{H_U}^{\text{max}}$. In the shaded areas, the Higgsino regions are either entirely excluded by the LEP chargino constraints or yield a tachyonic stau. Typically less than 5\% of the experimentally available range of $m_{H_U}$ yields a Higgsino-like lightest neutralino. The dashed contours are for $A_0 = 0$; taking $A_0 = -2 m_{1/2}$ yields the dotted contours of the tan $\beta = 10$ panel of Fig. 4, while taking $A_0 > 0$ makes the allowed Higgsino regions smaller than for $A_0 = 0$. The areas are insensitive to the sign of $\mu$. We note also that the entire allowed areas also satisfy the current LEP2 Higgs mass constraints.

We parenthetically note that the particular choice $m_{H_D} = m_{H_U}$ is convenient, because it allows the D-term contributions to the running of the scalar mass\(^2\) parameters to vanish at one loop. For large scalar mass non-universalities, the effect of the $S$ term in the RGEs can

\(^2\)The chargino bounds are weakened when the chargino is sufficiently degenerate with the neutralino, as in the pure Higgsino limit, for large enough $|\mu|$. We verify that $m_{\tilde{\chi}^\pm} - m_{\tilde{\chi}} > 10 \text{ GeV}$, so that the chargino limit is not reduced.
Figure 3: Contours of constant $\mu$ (solid lines) in the $\{m_{H_u}, m_{H_d}\}$ plane. Dashed lines are contours of gaugino purity $p_1 = 0.95$ and $p_1 = 0.71$. The hashed area is the part of the Higgsino region allowed by the LEP chargino mass constraint. The shaded areas are excluded either because they do not allow correct electroweak symmetry breaking (dark) or they yield a tachyonic sfermion (light). The mSUGRA point is marked by a triangle.
Figure 4: The fraction of the allowed $m_{H_u}$ parameter space which yields a Higgsino-type lightest neutralino, for $\tan \beta = \{4, 10, 30\}$ and $\mu > 0$. The fractional areas for $\mu < 0$ are similar. In the shaded areas, the entire Higgsino region is excluded either by the LEP chargino bound or by the presence of tachyonic sfermions. We’ve fixed $m_{H_D} = m_{H_U}$ and $A_0 = 0$. The light dotted contours for $\tan \beta = 10$ show the effect of taking $A_0 = -2m_{1/2}$.
be substantial, and the direction of the effect depends on the pattern of mass differences. This can lead to tachyonic sfermions in some cases where, in the universal case, the masses are well behaved. When $m_{H_U} = m_{H_D}$, and the other scalar masses are universal, $S$ vanishes, and this complication is avoided. In practice, taking $m_{H_D} = m_0$ instead of $m_{H_D} = m_{H_U}$ produces only a small downward shift in the contours in Fig. 4, but some of the allowed Higgsino regions in Fig. 4 at small $m_0$ now contain a tachyonic stau.

The Higgsino regions typically lie at $m_{H_U} > m_0$, and for $m_{1/2} \gg m_0$, (6) gives $m_{H_U} \gg m_0$. This is in contrast to expectations for the pattern of scalar mass non-universality coming, for example, from evolution of the soft masses from $M_P$ to $M_{GUT}$ in models where the soft masses unify at $M_P$ rather than $M_{GUT}$ [9]. These effects in fact tend to reduce $m_{H_U}$ from its unified value, rather than enlarge it, and the concomitant generic increase in $|\mu|$ and consequent decrease in the neutralino Higgsino content are documented in [9]. However, once universality is violated at the GUT scale, D-terms associated with the reduction in rank of the gauge group to the SM can provide additional non-universal contributions to the soft masses, and these can in principle yield a sufficiently large $m_{H_U}$ [9, 10]. Rather than study the effect of non-universality patterns from specific models, it is common (see e.g. [13, 16, 19]) to parameterize the GUT scale non-universalities by introducing factors $\delta_i$, so that, e.g. $m_{H_D}^2 = (1 + \delta_{H_d})m_0^2$ and $m_{H_U}^2 = (1 + \delta_{H_u})m_0^2$, and to allow the range $-1 \leq \delta_{H_d}, \delta_{H_u} \leq 1$, in accord with expectations from [9]. This spread of $\delta_i$ is chosen to comfortably encompass the expected deviations from universality from GUT scale effects. Comparing to (6), we see that this permits a Higgsino-type neutralino for $m_0 \gtrsim m_{1/2}$.

It is useful to compare our results with previous discussions of fine-tuning in mSUGRA and the MSSM [11–18]. It has been observed [11] that mSUGRA itself exhibits fine-tuning, in part due to the condition (1), which requires a large cancellation between the soft mass terms when $m_{1/2}$ is much larger than $m_Z$. It is common to introduce the fine-tuning parameters $\Delta_a$[11], which describe the sensitivity of the electroweak scale to variations of each parameter $a$ in the model, and where

$$\Delta_a = \left| \frac{a}{m_Z^2} \frac{\partial m_Z^2}{\partial a} \right|. \tag{7}$$

The largest $\Delta_a$ gives the degree of fine-tuning in the model, for a fixed parameter set. Minimal SUGRA is typically characterized by large $\Delta_\mu$, since small changes in $\mu$ upset the delicate cancellations in (1) and produce large (fractional) changes in $m_Z$. Relaxing the scalar mass unification condition[13–18] for $m_{H_D}$ and $m_{H_U}$ can somewhat ameliorate the

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3An alternate definition, where $m_Z^2$ is replaced by $m_Z$, is a factor of 2 smaller. See [12] for an alternate tuning measure which compares the parameter sensitivity to an “average” sensitivity. See also [16] and [17] for other tuning measures.
fine-tuning problem, precisely because their low-energy values can be adjusted to yield a smaller value for $\mu$ in (1). These studies are germane to our problem, as small $\mu$ is required for a Higgsino-like LSP. However, no one to date has specifically addressed the issue of the conditions required to obtain a Higgsino-like LSP.

In fact, $\Delta_\mu$ is much smaller in the Higgsino regions in Figs. 2 and 3 than in the generic regions of Fig. 2. However, the largest of the sensitivity parameters always remains large. In Fig. 2, $\Delta_{m_0} > 100$ in the region of interest, and in Fig. 3, $\Delta_{m_{H_U}}$ is typically $> 80$. Furthermore, the size of the allowed Higgsino regions are reduced, as discussed above, by the lower bound on the mass of the lightest chargino, which forbids the smallest values of $|\mu|$. This fact is not reflected in the sensitivity parameter $\Delta$, which is essentially a local function of the MSSM parameters. At very large $\tan \beta \sim 50$, one can find Higgsino-like neutralinos with lower values of $\Delta$ [13]. However, one still finds similar percentage allowed areas to those in Fig. 4. A full sensitivity analysis which includes all the $\Delta_a$ is beyond the scope of this paper.

By contrast, a Higgsino-like lightest neutralino is more plausible in the absence of gaugino mass unification. For starters, breaking the gaugino mass unification condition alters the RGE evolution of the squarks, and hence of $m_{H_D}$ and $m_{H_U}$, and thus admits smaller $\mu$. As recently emphasized [18], since the dominant terms in (2) driving $m_{H_U}^2$ negative scale with the squark masses$^2$, which in turn receive large contributions $\sim M_3^2$, a reduction of $M_3$ below its unification value can significantly reduce the value of $|\mu|$ inferred from (1) (and hence the tuning associated with the $\mu$ parameter), particularly at low $\tan \beta$. Alternatively, putting $M_1$ above its unification value increases the size of the Higgsino region in the $\{\mu, M_2\}$ plane by making the bino heavier vis-a-vis the Higgsinos. Taking $M_1 \gtrsim M_2$ at the electroweak scale roughly doubles the size of the Higgsino regions in Fig. 3 and more than doubles the allowed areas in Fig. 4.

We have shown that while non-universal Higgs mass parameters allow for a Higgsino-type neutralino in models with gaugino mass unification, it is at the price of living in a narrow strip in the $\{m_{H_U}, m_{H_D}\}$ parameter space. This fact is often obscured by scatter plots which tend to emphasize the maximum extent to which the low energy parameters can be affected by GUT scale non-universalities, rather than the likelihood of the resulting parameters. In this work we have considered only non-universalities in the Higgs masses. Extending scalar mass non-universality to the sfermions can have a significant impact on the running of $m_{H_U}$ via the D-term contribution $S$. However, it is likely that more tuning would be required to cancel the $S$ term contribution to (2) with the Yukawa terms and permit a small $|\mu|$. Thus a gaugino-type neutralino is still preferred in SUSY models with gaugino mass unification,
even in the absence of scalar, in particular Higgs, mass unification.

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