Self-gravitating Line Sources of Weak Hypercharge

T. Dereli
Department of Physics, Middle East Technical University
06531 Ankara, Turkey
E.mail: tekin@dereli.physics.metu.edu.tr

Robin W Tucker
Department of Physics, University of Lancaster
Bailrigg, Lancs. LA1 4YB, UK
E.mail: r.tucker@lancaster.ac.uk
(15 August 1998)

We explore the role of the Cremmer-Scherk mechanism in the context of low energy effective string theory by coupling the antisymmetric 3-form gauge field to an Abelian gauge potential carrying weak hypercharge. The theory admits a class of exact self-gravitating solutions in the spontaneously broken phase in which dual fields acquire massive perturbative modes. Despite the massive nature of these fields they admit non-perturbative progressive longitudinal modes that together with pp-type gravitational waves travel in a direction of a line source at the speed of light.

Considerable effort has been devoted to the search for classical string-like solutions in relativistic field theories. Such solutions range from the pioneering work on vortices as models for dual strings [1] to more recent investigations on the properties of global and superconducting cosmic strings [2]-[4]. A common feature in recent work has been the role played by singular sources as models for the strings themselves. Such source descriptions often lend themselves to a formulation in terms of de Rham periods. Thus in the Higgs vacuum of the global Abelian Higgs model [3], the phase \( \theta \) of a complex scalar field satisfies the massless field equations \( d * d \theta = 0 \) in a regular space-time domain. In such a domain one may introduce a 2-form potential \( \hat{B} \) by \( d \hat{B} = *d\theta \). Classical sources enter the theory as solutions with \( \int_C d\theta = 2\pi \) for some closed space-like curve \( C = \partial \Sigma_2 \) bounding a space-like disc \( \Sigma_2 \). For such solutions, \( \hat{B} \) can be promoted to a distribution on space-time satisfying \( d * d\hat{B} = 2\pi \delta \) with \( \int_{\Sigma_2} \delta = 1 \). One then identifies the solution as a line source threading \( C \) at each instant and such “axionic” strings have interesting cosmological implications [5].

In a recent note we have suggested that fields arising in low energy effective string actions may have consequences for the standard model of the electroweak interactions [7]. By coupling the antisymmetric 3-form gauge field \( H \) to an Abelian gauge potential \( A \) carrying weak hypercharge via a gauge covariant derivative of the standard Higgs weak isospinor, we showed explicitly how the masses of the \( W^\pm, Z^0 \) could depend on this coupling. A salient feature of this generalised Cremmer-Scherk mechanism [6] was the manner in which the \( H \) field became assimilated into the physical degrees of freedom of the vector bosons via a spontaneous breakdown of a local gauge symmetry. Since the \( H \) field along with the dilaton \( \phi \) is thought to have implications for cosmology, it is of interest to explore the gravitational sector of the low energy effective string action in the presence of the Cremmer-Scherk interaction. Although the model discussed in Ref.[7] involves the full non-Abelian \( SU(2) \times U(1) \) gauge theory of the electroweak standard model, we shall here restrict attention to a single local Abelian internal symmetry gauge group for simplicity but retain the hypercharge interpretation. The Cremmer-Scherk mechanism is controlled by a coupling constant \( \lambda \) and we are interested in the phase with \( \phi \neq 0 \). Thus to lowest order in string fields we investigate the dynamics derived from the action density 4-form

\[
\Lambda[g, \phi, A, B] = \kappa R \ast 1 - \frac{(2\alpha - 3)}{4} d\phi \wedge *d\phi + \frac{1}{2} e^{-2\phi} dB \wedge *dB + \frac{1}{2} e^{-2\phi} dA \wedge *dA + \lambda A \wedge dB \tag{1}
\]

where \( A \) is a 1-form, \( B \) a 2-form, \( \phi \) the dilaton 0-form on spacetime \( M \) with a metric \( g = \eta_{ab} e^a \otimes e^b \), curvature scalar \( R \) and associated Hodge map \( * \). The field equations derived from (1) by varying \( A, B, \phi, g \), respectively, are

\[
d(e^{-2\phi} dA) + \lambda dB = 0, \tag{2}
\]

\[
d(e^{-2\phi} dB) - \lambda dA = 0, \tag{3}
\]

\[
d * d\phi = \frac{2}{(2\alpha - 3)} e^{-2\phi} (dB \wedge *dB + dA \wedge *dA), \tag{4}
\]

\[
\frac{\kappa}{2} R_{bc} \wedge *(e_a \wedge e^b \wedge e^c) = \tau_a[\phi] + \tau_a[A] + \tau_a[B], \tag{5}
\]
where
\[
\tau_a[\phi] = \frac{(2\alpha - 3)}{4}(\iota_a d\phi * d\phi + d\phi \iota_a * d\phi)
\]
\[
\tau_a[A] = \frac{1}{2} \epsilon^{-2\phi}(\iota_a dA \wedge *dA - dA \wedge \iota_a * dA)
\]
\[
\tau_a[B] = \frac{1}{2} \epsilon^{-2\phi}(\iota_a dB \wedge *dB + dB \wedge \iota_a * dB)
\]
in terms of the interior operator with \(\iota_a(e^h) = \delta^b_a\). In a regular source-free domain of space-time (2) and (3) imply
\[
d\tilde{A} = \lambda e^{2\phi} \tilde{B},
\]
\[
d\tilde{B} = \lambda e^{2\phi} \tilde{A},
\]
in terms of the variables \(\tilde{A} = A - \frac{1}{\lambda} df_0\), \(\tilde{B} = B - \frac{1}{\lambda} df_1\) in the gauge equivalence classes \([A]\) and \([B]\), respectively. One may fix gauges by taking solutions with particular \(f_0\) and \(f_1\). Using (7) or (8) the entire theory can be recast in terms of either the fields \(\{g, \phi, \tilde{A}\}\) or the fields \(\{g, \phi, \tilde{B}\}\), and the two descriptions refer to dual sectors of the same theory. Moreover, in terms of the \(\{g, \phi, \tilde{A}\}\) description the theory admits vector fields satisfying a generalised Einstein-dilaton-Proca system:
\[
d(e^{-2\phi} * d\tilde{A}) + \lambda^2 e^{2\phi} * \tilde{A} = 0,
\]
\[
d * d\phi = -\frac{2\lambda^2}{(2\alpha - 3)} e^{2\phi} \tilde{A} \wedge *\tilde{A} + \frac{2}{(2\alpha - 3)} e^{-2\phi} d\tilde{A} \wedge *d\tilde{A}.
\]
\[
\kappa R_{bc} \wedge *(e_a \wedge e^b \wedge e^c) = \tau_a[\phi] + \tau_a[\tilde{A}] + \frac{\lambda^2}{2} e^{2\phi}(\iota_a * \tilde{A} \wedge \tilde{A} + *\tilde{A} \wedge \iota_a \tilde{A}).
\]
It is clear how in the absence of gravitation the vector field \(\tilde{A}\) acquires massive propagating modes. In a similar manner the theory admits a description in terms of \(\{g, \phi, \tilde{B}\}\) satisfying the generalised Einstein-dilaton-massive-Kalb-Ramond system:
\[
d(e^{-2\phi} * d\tilde{B}) - \lambda^2 e^{2\phi} \tilde{B} = 0,
\]
\[
d * d\phi = -\frac{2\lambda^2}{(2\alpha - 3)} e^{2\phi} \tilde{B} \wedge *\tilde{B} + \frac{2}{(2\alpha - 3)} e^{-2\phi} d\tilde{B} \wedge *d\tilde{B}.
\]
\[
\kappa R_{bc} \wedge *(e_a \wedge e^b \wedge e^c) = \tau_a[\phi] + \tau_a[\tilde{B}] + \frac{\lambda^2}{2} e^{2\phi}(\iota_a * \tilde{B} \wedge \iota_a \tilde{A} - \iota_a * \tilde{B} \wedge \tilde{B}).
\]

Working with the fields \(\{g, \phi, \tilde{A}\}\), and restricting to cylindrical symmetry we seek solutions for \(\tilde{A}\) and \(\phi\) with the metric
\[
g = du \otimes dv + dv \otimes du + d\rho \otimes d\rho + \rho^2 d\psi \otimes d\psi + 2\mathcal{H}(u, \rho) du \otimes du,
\]
in a coordinate system \((u, v, \rho, \psi)\). We take \(\mathcal{H}\) to have the form
\[
\mathcal{H} = f(u)^2 h(\rho)
\]
and
\[
\tilde{A} = f(u) \beta(\rho) du
\]
with the dilaton constant,
\[ \phi = \phi_0. \]  

It follows from (7) that the corresponding solution for \( \tilde{B} \) will have the form

\[ \tilde{B} = -\frac{e^{-2\phi_0}}{\lambda} f(u)\beta'(u) du \wedge \rho d\psi. \]  

The equations (9), (10) and (11) are satisfied provided the functions \( \beta(\rho) \) and \( h(\rho) \) solve

\[ \beta'' + \frac{1}{\rho} \beta' - \mu_0^2 \beta = 0, \]  

\[ e^{2\phi_0} \kappa h'' + \frac{1}{\rho} h' + (\beta')^2 + \mu_0^2 \beta^2 = 0 \]  

where \( \mu_0 = \lambda e^{2\phi_0} \). We seek smooth solutions to these equations for \( \rho > 0 \) such that \( d\tilde{A} \) tends to zero as \( \rho \rightarrow \infty \) and the gravitational field tends to that of a cylinder with arbitrary gravitational mass (which may be zero). We recall that \( \mathcal{H}(u, \rho) = 2\pi\sigma_0 \ln \rho \) yields a weak field Newtonian limit corresponding to a cylinder of mass density \( \sigma_0 \) per unit length. Therefore we require that \( h(\rho) \sim C \ln \rho \) as \( \rho \rightarrow \infty \). Such solutions exist for arbitrary \( f(u) \) in terms of modified Bessel functions:

\[ \beta(\rho) = K_0(\mu_0 \rho), \]  

\[ \frac{\kappa}{\mu_0^2} e^{2\phi_0} h(\rho) = \int_0^\rho \rho' \ln \rho' (K_1(\mu_0 \rho')^2 + K_0(\mu_0 \rho')^2) d\rho' \]  

\[ + \frac{1}{2} \rho^2 \ln \rho (K_0(\mu_0 \rho)K_2(\mu_0 \rho) - (K_0(\mu_0 \rho))^2) + C \ln \rho. \]  

where \( C \) is an arbitrary non-negative constant. The profiles \( \beta(\rho), h(\rho) - \mu_0^2 \frac{C}{\kappa} e^{-2\phi_0} \ln \rho \) are displayed in Figure 1 for \( \kappa = \mu_0 = 1, \phi_0 = 0 \).

The interpretation of these solutions depends on the form of \( f(u) \). When \( f \) is constant the solution is static. In terms of the coordinates \((t, x, y, z)\) where

\[ t = \frac{1}{\sqrt{2}} (u + v), \quad z = \frac{1}{\sqrt{2}} (v - u), \quad x = \rho \cos \psi, \quad y = \rho \sin \psi \]

we identify \( \rho = 0 \) as a line source for \( \{g, \tilde{A}\} \) along the \( z \)-axis at each instant. Writing the field strength \( d\tilde{A} \) in terms of hyper-electric \( e \) and hyper-magnetic \( b \) fields with respect to \( dt \), one finds that a radial \( e \) emanates from this source and it is everywhere transverse to \( b \). The fact that \( \int_{C_1} b \) for a closed space-like contour \( C_1 \) and the flux of \( e \) through a finite space-like cylinder depend on the extent of the integration regions is a reflection of the massive nature of the \( \tilde{A} \) field.

When \( f(u) \) is a non-constant bounded function, the solution describes a progressive gravitational wave with amplitude \( b(\rho) \) that propagates together with \( \tilde{A} \) with amplitude \( \beta(\rho) \) in the \( z \) direction at the speed of light. In the other spatial directions the \( \tilde{A} \) field falls off exponentially to zero at infinity while the behaviour of the gravitational field is determined by \( h(\rho) \). If one interprets the singular domain \( \rho = 0 \) as a straight wire, then it acts as a kind of gravitational optical fibre that guides a pp-type gravitational wave. Such an interpretation has potential astrophysical implications.

Given the surprising properties of these exterior self-gravitating Einstein-Proca solutions it may be of interest to explore the generalised Cremmer-Scherk mechanism [7] on the gravitational sector of low energy effective string theory. It may be of further interest to note that any Einstein-Proca solution can be used to generate a solution to non-Riemannian theories of gravity [8].

The authors are grateful to TUBITAK for the support of this research. RWT is also grateful to the Department of Physics, Middle East Technical University for hospitality.
   D. Förster, Nucl. Phys. B81(1975)84


   B. Carter, Phys. Lett. 228B(1989)466


