The problem of nonlinear Landau damping in quark-gluon plasma

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Abstract

On the basis of the semiclassical kinetic equations for quark-gluon plasma (QGP) and Yang-Mills equation, the generalized kinetic equation for waves with regard to its interaction is obtained. The physical mechanisms defining nonlinear scattering of a plasmon by QGP particles are analyzed. The problem on a connection of nonlinear Landau damping rate of longitudinal oscillation with damping rate, obtained on the basis of hard thermal loops approximation, is considered.

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1. INTRODUCTION

In recent 15-20 years, a theoretical investigations of properties of quark-gluon plasma has been of great interest. It is connected with intensive looking for a QGP in the experiments with collision of ultrarelativistic heavy ions.

Two methods to study of the nonequilibrium phenomena in a QGP are used: method of temperature Green functions and kinetic approach. Significant progress has been achieved in the development of the first method. The effective perturbative theory was constructed in the papers by Pisarski and Braaten, Frenkel and Taylor [1] on the basis of the resummation of so-called hard-thermal loops (HTL’s), and the problem on the sign and gauge dependence of the damping rate of the long wavelength excitations of QGP was solved. The progress of the thermal QCD makes possible a new view at the existing of kinetic theory of QGP, the basis of which was laid in the papers [2] by Heinz, Elze, Vasak and Gyullasy, Mrowczynski and others, and it has given impetus to its further development.

In spite of the fact that the language and methods appear very different, there are close similarities between HTL approach and transport theory. Originally the kinetic theory was used by Silin to derive HTL in the photon’s self-energy [3]. HTL’s in the quark and gluon self-energies can be computed similarly. Moreover, Kelly, Liu, Lucchesi and Manuel [4] have shown that the generating functional of HTL’s (with an arbitrary number of soft external bosonic legs) can be derived from the classical kinetic theory of QGP. This points clearly to the classical nature of the hard thermal effects.

A further step in development of kinetic theory was made by Blaizot and Iansu [5]. In contrast to the early papers on transport theory of QGP [2], these authors use from outset the ideas developed in thermal QCD in deriving of the kinetic equations. The equations obtained by them isolate consistently the dominant terms in the coupling constant $g$ in hierarchy of equations which describe the response of plasma to weak and slowly varying disturbances, and encompass all HTL’s. However, here it should be noted that if the influence of the average fermionic field is neglected, then the expression for current induced by soft gauge fields, obtained in [6] (and the nonlinear equation of motion, connected with it) fully coincide with the corresponding expression obtained in [4] from usual classical kinetic theory on the basis of consistent expansion of distribution function in powers of the coupling constant. In spite of the fact that intermediate approximation schemes in which these equations were derived, mix leading and nonleading contributions with respect to the powers of $g$ and so, are not entirely consistent, this, somewhat justifies use of the classical and semiclassical kinetic equation found in [2].

Such close interlacing of two methods of investigation of nonequilibrium phenomena in QGP leads to the question: can one calculate whether the damping rate of bosonic modes corresponding to the hard thermal result [7], remaining with the framework of classical (semiclassical) kinetic theory only? Xiaofei and Jiarong [8] were the first to put this question. Because of obtained results, they have given a positive answer.

As was shown by Heinz and Siemens [9], in linear approximation the Landau damping
is absent in QGP. In fact, the only mechanism, with which one can associate the damping following from the kinetic theory with damping from HTL approach, is so-called non-linear Landau damping. It bound up with the nonlinear effects of interaction of waves and particles in QGP. The multiple time-scale method which has proved successful in studying the nonlinear properties of electromagnetic plasma (EMP) [10], was used in [8] for determination of this association. By means of this method the nonlinear shift of an eigenfrequency of longitudinal oscillations in the temporal gauge, the imaginary part of which defines required nonlinear Landau damping rate, had been obtained by Xiaofei and Jiarong. Futher, the limiting expression of the derived damping rate for $k = 0$-mode was obtained, and numerical computations for approximate estimate were performed. The value derived by this means is in close agreement with similar numerical one obtained by Braaten and Pisarski [7] on the basis of effective perturbative theory.

However, under close examination of above-mentioned paper we found certain inaccuracies in computations, which were of both principle and nonprinciple character. The elimination of these inaccuracies finally leads not only to a numerical modification of the limiting value of nonlinear Landau damping rate obtained in [8], but what is more important, it changes the sign of obtained expression (this subject will be considered in detail in Sec.10 below). This points to some prematurity of the statements in [8] on obtained connection between nonlinear Landau damping rate and damping rate from the HTL-approach.

Moreover, the physical mechanisms lying in the basis of the process of nonlinear scattering of longitudinal waves by QGP particles has not been revealed by Xiafei and Jiarong. The contribution to this process from the effect of self-action of the gauge field was dropped, although in a "soft" region of excitations it is of the same order as the contributions, bound up with the effects of the medium. An important problem of the gauge dependence for the obtained expression of nonlinear Landau damping rate was not considered. In connection with above remarks it is evident that this problem requires further investigation.

In this paper, we consider the above problem, using the approach based on obtaining of generalized kinetic equation for waves in quark-gluon plasma developed by Kadomtsev, Silin, Tsytovich and others [11] in connection with EMP as a basic method of investigation of nonlinear processes in QGP.

We have shown that within the limits of nonlinear theory, developed for EMP and immediately (i.e. without any crucial changes) applied to investigation of nonlinear processes in QGP, nonlinear Landau damping rate $\gamma_l(k)$ defines not an inverse time of damping at the expense of absorption (dissipation), but the inverse time of spectral pumping of the energy of waves in the direction of a small wave numbers. The inequality $\gamma_l(0) < 0$ is a consequence of this fact, i.e. $k = 0$-mode, in contrast to [8], in this approximation increases.

The outline of the paper is as follows. In Sec.2, we derive a system of self-consistent
equations in the covariant gauge for regular and random parts of the distribution functions of QGP particles and gauge field is obtained. In Sec.3 the first order approximation of the color current is considered and the correlation function of random oscillations is introduced. In Secs.4 and 5 the second and the third orders approximation of the color current are studied, and the terms leading in the coupling constant are separated. In Sec.6 the generalized kinetic equation for waves in QGP is derived. From there the kinetic equation describing the nonlinear interaction of longitudinal oscillations is extracted in Sec.7. In Sec.8 the physical mechanisms defining the nonlinear scattering of waves by plasma particles are considered. In Sec.9 the estimate of a value of the nonlinear Landau damping rate for longitudinal eigenwaves is made. In Sec.10 the inaccuracies, which were made in [8] in the process of computation of nonlinear Landau damping rate in the temporal gauge, are indicated, and its correct expression is written out. Comparison of this expression with similar one, obtained in the covariant gauge is performed. In Conclusion possible ways of further development of the scrutinized theory are discussed.

2. THE INITIAL EQUATIONS. THE METHOD OF AVERAGING OVER STATISTICAL ENSEMBLE

We use metric $g^{\mu\nu} = \text{diag}(1,-1,-1,-1)$ and choose units such that $c = k_B = 1$. The gauge field potentials are $N_c \times N_c$-matrices in a color space defined by $A_\mu = A_\mu^a t^a$ with $N_c^2 - 1$ hermitian generators of $SU(N_c)$ group in the fundamental representation. The field strength tensor $F^{\mu\nu} = F^a_{\mu\nu} t^a$ with

$$F^a_{\mu\nu} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

obeys the Yang-Mills (YM) equation in a covariant gauge

$$\partial_\mu F^{\mu\nu}(x) - ig[A_\mu(x), F^{\mu\nu}(x)] - \xi^{-1}\partial^\nu \partial^\mu A_\mu(x) = -j^\nu(x),$$

where $\xi$ is a gauge parameter. $j^\nu$ is the color current

$$j^\nu = gt^a \int d^4 p \{p^\nu [f_q - f_{\bar{q}}] + \text{Tr} (T^a f_g)\},$$

where $T^a$ are hermitian generators of $SU(N_c)$ in adjoint representation ($(T^a)^{bc} = -i f^{abc}, \text{Tr}(T^a T^b) = N_c \delta^{ab}$). We denote the trace over color indices in adjoint representation as $\text{Tr}$. Distribution functions of quarks $f_q$, antiquarks $f_{\bar{q}}$, and gluons $f_g$ satisfy the semiclassical kinetic equations (neglecting spin effects)

$$p^\nu D_\mu f_{q,\bar{q}} + \frac{1}{2} gp^\rho \{F_{\mu\rho}, \frac{\partial f_{q,\bar{q}}}{\partial p_\nu}\} = 0,$$

$$p^\nu \tilde{D}_\mu f_g + \frac{1}{2} gp^\rho \{F_{\mu\rho}, \frac{\partial f_g}{\partial p_\nu}\} = 0,$$
where $\mathcal{D}_\mu$ and $\tilde{\mathcal{D}}_\mu$ are covariant derivatives which act as 
\[
\mathcal{D}_\mu = \partial_\mu - ig[A_\mu(x), \cdot],
\]
\[
\tilde{\mathcal{D}}_\mu = \partial_\mu - ig[A_\mu(x), \cdot],
\]
$[\cdot, \cdot]$ denotes commutator, $\{\cdot, \cdot\}$ denotes the anticommutator, and $A_\mu$, $F_{\mu\nu}$ are defined as $A_\mu = A^a_\mu T^a$, $F_{\mu\nu} = F^a_{\mu\nu} T^a$. Upper sign in the first equation (4) refers to quarks and lower one - to antiquarks.

We begin with consideration of kinetic equations (4). The distribution functions $f_{q,\bar{q}}$ and $f_g$ can be decomposed into two parts: regular and random (turbulent) ones 
\[
f_s = f_s^R + f_s^T, \quad s = q, \bar{q}, g,
\]
so that 
\[
\langle f_s \rangle = f_s^R, \quad \langle f_s^T \rangle = 0,
\]
where angular brackets $\langle \cdot \rangle$ indicate a statistical ensemble of averaging. The initial values of parameters which characterize the collective degree of a plasma freedom is such statistical ensemble. For almost linear collective motion to be considered below it may be initial values of oscillation phases.

Further we set 
\[
A_\mu = A_\mu^R + A_\mu^T, \quad \langle A_\mu^T \rangle = 0,
\]
by definition. For simplicity the regular part of the field $A_\mu^R$ will be considered equal to zero.

Averaging the equation (4) over statistical ensemble, in view of (5)-(7), we obtain the equations for the regular part of the distribution functions $f_{q,\bar{q}}^R$ and $f_g^R$ 
\[
p^\mu \partial_\mu f_{q,\bar{q}}^R = igp^\mu \langle [A_\mu^T, f_{q,\bar{q}}^T] \rangle \mp \frac{1}{2} gp^\mu \{\langle (F_{\mu\nu}^T)_L \rangle, \frac{\partial f_{q,\bar{q}}^R}{\partial p_\nu} \} \mp 
\mp \frac{1}{2} gp^\mu \{\langle (F_{\mu\nu}^T)_NL \rangle, \frac{\partial f_{q,\bar{q}}^R}{\partial p_\nu} \},
\]
\[
p^\mu \partial_\mu f_g^R = igp^\mu \langle [A_\mu^T, f_g^T] \rangle - \frac{1}{2} gp^\mu \{\langle (F_{\mu\nu}^T)_L \rangle, \frac{\partial f_g^T}{\partial p_\nu} \} - \frac{1}{2} gp^\mu \{\langle (F_{\mu\nu}^T)_NL \rangle, \frac{\partial f_g^R}{\partial p_\nu} \} - 
\mp \frac{1}{2} gp^\mu \{\langle (F_{\mu\nu}^T)_NL \rangle, \frac{\partial f_g^T}{\partial p_\nu} \}.
\]
Here, indices ”$L$” and ”$NL$” denote the linear and nonlinear parts with respect to field $A_\mu^a$ of the strength tensor (1).

Subtracting (8) from (4), we define the equations for $f_{q,\bar{q}}^T$ and $f_g^T$ 
\[
p^\mu \partial_\mu f_{q,\bar{q}}^T = igp^\mu \langle [A_\mu^T, f_{q,\bar{q}}^T] - \langle [A_\mu^T, f_{q,\bar{q}}] \rangle \rangle \mp \frac{1}{2} gp^\mu \{\langle (F_{\mu\nu}^T)_L \rangle, \frac{\partial f_{q,\bar{q}}^R}{\partial p_\nu} \} \mp 
\]

5
\[ \pm \frac{1}{2} gp^\mu \left\{ (F^{T}_{\mu\nu})_L, \frac{\partial f_{q,q}}{\partial p_\nu} \right\} - \langle \{(F^{T}_{\mu\nu})_L, \frac{\partial f_{q,q}}{\partial p_\nu} \} \rangle \pm \frac{1}{2} gp^\mu \left\{ (F^{T}_{\mu\nu})_{NL}, \frac{\partial f_{q,q}}{\partial p_\nu} \right\} \]

\[ \pm \frac{1}{2} gp^\mu \left\{ (F^{T}_{\mu\nu})_{NL}, \frac{\partial f_{q,q}}{\partial p_\nu} \right\} - \langle \{(F^{T}_{\mu\nu})_{NL}, \frac{\partial f_{q,q}}{\partial p_\nu} \} \rangle, \quad (9) \]

\[ p^\mu \partial_\mu f_g^T = ig p^\mu ([A^T, f_g^T] - \langle [A^T, f_g^{T(1)}] \rangle) - \frac{1}{2} g p^\mu \left\{ (F_{\mu\nu})_L, \frac{\partial f_{g}}{\partial p_\nu} \right\} - \]
\[ - \frac{1}{2} g p^\mu \left\{ (F_{\mu\nu})_{NL}, \frac{\partial f_{g}}{\partial p_\nu} \right\} - \langle \{(F_{\mu\nu})_{NL}, \frac{\partial f_{g}}{\partial p_\nu} \} \rangle. \]

The system of equations (8) and (9) is suitable for investigation of nonequilibrium processes in QGP such that the excitation energy of waves is a small quantity in relation to the total energy of particles. In this case it can be used expansion in series in power of oscillations amplitude of the random functions \( f_s^T \)

\[ f_s^T = \sum_{n=1}^{\infty} f_s^{T(n)}, \quad s = q, \bar{q}, g, \quad (10) \]

where index \( n \) depicts that \( f_s^{T(n)} \) is proportional to the \( n \)-th power of \( A^T_\mu \). Substituting expansions (10) into (9), and collecting terms of the same order with respect to \( A^T_\mu \), we derive the system of equations

\[ p^\mu \partial_\mu f_{q,q}^{T(1)} = \pm \frac{1}{2} g p^\mu \left\{ (F_{\mu\nu})_L, \frac{\partial f_{q,q}^{R}}{\partial p_\nu} \right\}, \quad (11) \]

\[ p^\mu \partial_\mu f_{q,q}^{T(2)} = ig p^\mu ([A^T, f_{q,q}^{T(2)}] - \langle [A^T, f_{q,q}^{T(2)}] \rangle) \]

\[ \pm \frac{1}{2} g p^\mu \left\{ (F_{\mu\nu})_L, \frac{\partial f_{q,q}^{T(1)}}{\partial p_\nu} \right\} - \langle \{(F_{\mu\nu})_L, \frac{\partial f_{q,q}^{T(1)}}{\partial p_\nu} \} \rangle \pm \frac{1}{2} g p^\mu \left\{ (F_{\mu\nu})_{NL}, \frac{\partial f_{q,q}^{T(1)}}{\partial p_\nu} \right\}, \quad (12) \]

\[ p^\mu \partial_\mu f_{q,q}^{T(3)} = ig p^\mu ([A^T, f_{q,q}^{T(3)}] - \langle [A^T, f_{q,q}^{T(3)}] \rangle) \pm \frac{1}{2} g p^\mu \left\{ (F_{\mu\nu})_L, \frac{\partial f_{q,q}^{T(2)}}{\partial p_\nu} \right\} - \]
\[ - \langle \{(F_{\mu\nu})_{NL}, \frac{\partial f_{q,q}^{T(2)}}{\partial p_\nu} \} \rangle \pm \frac{1}{2} g p^\mu \left\{ (F_{\mu\nu})_{NL}, \frac{\partial f_{q,q}^{T(1)}}{\partial p_\nu} \right\} \) etc.. \quad (13) \]

Similar equations are obtained for \( f_g^{T(n)} , \quad n = 1, 2, 3, \dots \)

The nonlinear color current is expressed as

\[ j_\mu = j_\mu^R + f_\mu^T , \quad \langle j_\mu \rangle = j_\mu^R , \quad j_\mu^T = \sum_{n=1}^{\infty} j_\mu^{T(n)}, \quad (14) \]
where
\[
{j^T_\mu}^{(n)} = g t^a \int d^4 p \, p_\mu [\text{Sp} \, t^a (f_\bar{q}^{T(n)} - f_q^{T(n)}) + \text{Tr} (T^a f_g^{T(n)})].
\] (15)

Now we turn to the Yang-Mills equation (2), connecting the gauge field with the color current. Averaging Eq. (2) and subtracting the averaged equation from (2) in view of Eqs. (17) and (14), we find (for \(A^\tau_\mu = 0\))

\[
\partial_\mu (F^{T\mu\nu})_L - \xi^{-1} \partial^\rho \partial_\mu A^\tau_\mu + j^{T(1)\nu} = - \xi^{-1} J^{NL} + i g \partial_\mu (\langle [A^{T\mu}, A^{T\nu}] \rangle - \langle [A^{T\mu}, A^{T\nu}] \rangle) + g \langle [A^\tau_\mu, (F^{T\mu\nu})_L] \rangle + g^2 \langle [A^\tau_\mu, [A^{T\mu}, A^{T\nu}]] \rangle + g^2 \langle [A^\tau_\mu, [A^{T\mu}, A^{T\nu}]] \rangle.
\] (16)

Here, in the left-hand side we collect all linear terms with respect to \(A^\tau_\mu\) and we denote:\n\[
j^{T\nu}_{NL} = j^{T(2)\nu} + j^{T(3)\nu} + \ldots.
\]

To account for nonlinear interaction between waves and particles in QGP (in first non-vanishing approximation over the energy of waves), it is sufficiently to restrict the consideration to the cubic terms with respect to oscillation amplitude in expansion (10).

We introduce the following assumption. Eqs. (8) represent the transport equations for averaged distribution functions. The correlation functions in the right-hand side of these equations have meaning of the collision terms of QGP particles with waves and describe the influence of plasma waves to a background state.

We suppose that a characteristic time of nonlinear relaxation of the oscillations is small quantity as compared with a time of relaxation of the distribution particles \(f_\bar{q}^R\). Therefore we neglect by variation of regular part of the distribution functions with space and time, assuming that these functions are specified and describe the global equilibrium in QGP

\[
f^{R}_{\bar{q} q} = f^{0}_{\bar{q} q} + 2 N_f \theta(p_0) \frac{1}{(2\pi)^3} \delta(p^2) e^{(pu - \mu)/T} + 1,
\]

\[
f^{R}_{\bar{q} g} = f^{0}_{\bar{g} g} + 2 N_f \theta(p_0) \frac{1}{(2\pi)^3} \delta(p^2) e^{(pu + \mu)/T} - 1.
\] (17)

where \(N_f\) - being the number of flavours for massless quarks, \(u_\mu\) is the four-velocity of the plasma at temperature \(T\), and \(\mu\) is the quark chemical potential.

3. THE LINEAR APPROXIMATION. THE CORRELATION FUNCTION OF THE RANDOM OSCILLATIONS

Let us turn our attention to obtaining of kinetic equation for waves. The initial equation is Eq. (16). The left-hand side of Eq. (16) contains a linear approximation of the color current, explicit form of which is easily defined from Eq. (11). We prefer to work in momentum space; the corresponding equations are obtained by using

\[
A_\mu(x) = \int d^4 k A_\mu(k) e^{-ikx},
\]
and similar translations for \( f^T_{q,q} \), \( f^T_g \). The result of Fourier transformation for Eq. (11) is

\[
 f^{T(1)}_{q}(k,p) = \mp g \frac{\chi^{\mu\lambda}(k,p)}{pk + ip_0\epsilon} \frac{\partial f^0_{q}}{\partial p^\lambda} A_{\nu}(k),
 f^{T(1)}_{g}(k,p) = -g \frac{\chi^{\mu\lambda}(k,p)}{pk + ip_0\epsilon} \frac{\partial f^0_{g}}{\partial p^\lambda} A_{\nu}(k),
\]

(18)

Here, \( \chi^{\mu\lambda}(k,p) = (pk)g^{\mu\lambda} - p^{\nu}k^{\lambda} \), and the suffix ”\( T \)” for a gauge-field is omitted. Substituting (18) into (15) (more precisely, in Fourier transformation of (15)) we define a well-known form [2, 12] of linear over field approximation of the current

\[
 j^{T(1)\mu}(k) = \Pi^{\mu\nu}(k) A_{\nu}(k),
\]

(19)

where

\[
 \Pi^{\mu\nu}(k) = g^2 \int d^4p \frac{p^\mu(p^\nu(k\partial_\mu) - (kp)\partial^\nu_p)\nu_{eq}}{pk + ip_0\epsilon}
\]

is the high temperature polarization tensor, and \( \nu_{eq} = \frac{1}{2}(f^0_q + f^0_g) + \nu_c f^0_q \).

Further we rewrite Eq. (16) in the momentum space. Taking into account (19), we obtain

\[
 [k^2 g^{\mu\nu} - (1 + \xi^{-1})k^\mu k^\nu - \Pi^{\mu\nu}(k)] A^b_\nu(k) = j^{T_{NL}}_{b\mu}(k) +
 f^{bcd} \int S^{(1)\mu\nu\lambda\sigma}_{k,k_1,k_2}(A^c_\nu(k_1)A^d_\lambda(k_2) - \langle A^c_\nu(k_1)A^d_\lambda(k_2) \rangle) \delta(k - k_1 - k_2) dk_1 dk_2 +
 f^{bce} f^{fde} \int \Sigma^{\mu\nu\rho\lambda\sigma}_{k,k_1,k_2,k_3}(A^c_\nu(k_1)A^d_\lambda(k_2)A^f_\rho(k_3) - \langle A^c_\nu(k_1)A^d_\lambda(k_2)A^f_\rho(k_3) \rangle) \delta(k - k_1 - k_2 - k_3) dk_1 dk_2 dk_3,
\]

(20)

where

\[
 S^{(1)\mu\nu\lambda\sigma}_{k,k_1,k_2} = -i g(k^\nu g^{\mu\lambda} + k_2^\nu g^{\mu\lambda} - k_2^\mu g^{\nu\lambda}), \Sigma^{\mu\nu\rho\lambda\sigma}_{k,k_1,k_2,k_3} = g^2 g^{\nu\lambda} g^{\rho\sigma}.
\]

(21)

Let us multiply Eq. (20) by the complex conjugate amplitude \( A^a_{\mu}(k') \) and average it

\[
 [k^2 g^{\mu\nu} - (1 + \xi^{-1})k^\mu k^\nu - \Pi^{\mu\nu}(k)] \langle A^a_{\mu}(k')A^b_\nu(k) \rangle = \langle A^a_{\mu}(k')j^{T_{NL}}_{b\mu}(k) \rangle +
 f^{bcd} \int S^{(1)\mu\nu\lambda\sigma}_{k,k_1,k_2}(A^c_\nu(k_1)A^d_\lambda(k_2) - \langle A^c_\nu(k_1)A^d_\lambda(k_2) \rangle) \delta(k - k_1 - k_2) dk_1 dk_2 +
 f^{bce} f^{fde} \int \Sigma^{\mu\nu\rho\lambda\sigma}_{k,k_1,k_2,k_3}(A^c_\nu(k_1)A^d_\lambda(k_2)A^f_\rho(k_3) - \langle A^c_\nu(k_1)A^d_\lambda(k_2)A^f_\rho(k_3) \rangle) \delta(k - k_1 - k_2 - k_3) dk_1 dk_2 dk_3.
\]

(22)

We introduce the correlation function of the random oscillations

\[
 I^{ab}_{\mu\nu}(k',k) = \langle A^a_{\mu}(k')A^b_{\nu}(k) \rangle.
\]

(23)

In conditions of the stationary and homogeneous of QGP, i.e. when the correlation function (23) in the coordinate representation depends on the difference of coordinates and time \( \Delta x = x' - x \) only, we have

\[
 I^{ab}_{\mu\nu}(k',k) = I^{ab}_{\mu\nu}(k') \delta(k' - k).
\]

(24)
By the effects of the nonlinear interaction of waves and particles, the state of QGP becomes weakly inhomogeneous and weakly nonstationary. The dependence of $I_{\mu \nu}^{ab}$ on $k' - k = \Delta k$ is no longer $\delta$-shaped, i.e. $k' \neq k$ and it "smeared" on $\Delta k$, with $|\Delta k/k| \ll 1$.

Let us introduce $I_{\mu \nu}^{ab}(k', k) = I_{\mu \nu}^{ab}(k, \Delta k)$, and insert the correlation function in the form

$$I_{\mu \nu}^{ab}(k, x) = \int I_{\mu \nu}^{ab}(k, \Delta k) e^{-i\Delta k x} d\Delta k,$$

slowly depending on $x$. In Eq. (22) we make the change $k \leftrightarrow k'$, $a \leftrightarrow b$, complex conjugate and subtract obtained equation from Eq. (22), beforehand expanding of the polarization tensor into Hermitian and anti-Hermitian parts

$$\Pi^\sigma(0) = \Pi^{H\sigma}(k) + \Pi^{A\sigma}(k), \quad \Pi^{H\sigma}(k) = \Pi^{+H\sigma}(k), \quad \Pi^{A\sigma}(k) = -\Pi^{*A\sigma}(k).$$

By using the definition (23), we obtain the result

$$[(k^2 g^{\mu \nu} - (1 + \xi^{-1})k^\mu k^\nu) - (k'^2 g^{\mu \nu} - (1 + \xi^{-1})k'^\mu k'^\nu) - (\Pi^{H\mu \nu}(k) - \Pi^{H\mu \nu}(k'))] I_{\mu \nu}^{ab}(k', k) -$$

$$- [\Pi^{A\mu \nu}(k) + \Pi^{A\mu \nu}(k')] I_{\mu \nu}^{ab}(k', k) = \langle A^{* a}_\mu(k') J_{NL}^{ab}(k) \rangle - \langle A^{b}_\mu(k) J_{NL}^{*ab}(k') \rangle +$$

$$+ f^{bcf} \int S_{k, k_1, k_2}^{(I) \mu \nu \lambda} \langle A^{a}_\mu(k') A^{\nu}_\nu(k_1) A^{b}_\lambda(k_2) \rangle \delta(k' - k_1 - k_2) d k_1 d k_2 -$$

$$- f^{acd} \int S_{k', k_1, k_2}^{(I) \mu \nu \lambda} \langle A^{b}_\mu(k) A^{\nu}_\nu(k_1) A^{c}_\lambda(k_2) \rangle \delta(k' - k_1 - k_2) d k_1 d k_2 -$$

$$+ f^{bcf} f^{fde} \int \sum_{k, k_1, k_2, k_3} \langle A^{a}_\mu(k') A^{\nu}_\nu(k_1) A^{d}_\lambda(k_2) A^{* a}_\lambda(k_3) \rangle \delta(k' - k_1 - k_2 - k_3) d k_1 d k_2 d k_3 -$$

$$- f^{acf} f^{fde} \int \sum_{k', k_1, k_2, k_3} \langle A^{b}_\mu(k) A^{\nu}_\nu(k_1) A^{* c}_\lambda(k_2) A^{a}_\lambda(k_3) \rangle \delta(k' - k_1 - k_2 - k_3) d k_1 d k_2 d k_3.$$

We assume that anti-Hermitian part of $\Pi^A$ is small in comparison with $\Pi^H$ and it is a value of the same smallness order, as the nonlinear terms in the right-hand side of Eq. (25). Therefore it can be suggested that $\Pi^{A\sigma}(k) \simeq \Pi^{A\sigma}(k')$, and the term with $\Pi^A$ can be rearranged to the right-hand side of Eq. (25). The remaining terms in the left-hand side of Eq. (25) we expanded in a series in powers of $\Delta k$ to first smallness order. Multiplying obtained equation by $e^{-i\Delta k x}$ and integrating over $\Delta k$ with regard to

$$\int \Delta k \delta(k') I_{\mu \nu}^{bc}(k, \Delta k) e^{-i\Delta k x} d\Delta k = i \frac{\partial I_{\mu \nu}^{bc}(k, x)}{\partial x^{^\lambda}};$$

we obtain finally

$$\frac{\partial}{\partial k^\lambda} [k^2 g^{\mu \nu} - (1 + \xi^{-1})k^\mu k^\nu - \Pi^{H\mu \nu}(k)] \frac{\partial I_{\mu \nu}^{ab}}{\partial x^{^\lambda}} = 2i \Pi^{A\mu \nu} I_{\mu \nu}^{ab} -$$

$$- i \int d k' \{ \langle A^{* a}_\mu(k') J_{NL}^{bc}(k) \rangle - \langle A^{b}_\mu(k) J_{NL}^{*bc}(k') \rangle \}.$$
\[ -i \{ f^{|abcd} \int dk' dk_1 dk_2 S^{(l) \mu \nu \lambda}_k, k_1, k_2, k_3 (A^a_{\mu}(k') A^c_{\nu}(k_1) A^d_{\lambda}(k_2)) \delta(k - k_1 - k_2) - \]
\[ - f^{|acdf} \int dk' dk_1 dk_2 S^{(q) \mu \lambda}_k, k_1, k_2, k_3 (A^b_{\mu}(k) A^{ac}_{\nu}(k_1) A^{bd}_{\lambda}(k_2)) \delta(k' - k_1 - k_2) \} - \\
\[ -i \{ f^{|bcf} f^{|de} \int \sum_{k', k_1, k_2, k_3} \langle A^a_{\mu}(k') A^c_{\nu}(k_1) A^d_{\lambda}(k_2) A^e_{\sigma}(k_3) \rangle \delta(k - k_1 - k_2 - k_3) dk_1 dk_2 dk_3 - \]
\[ - f^{|acf} f^{|de} \int \sum_{k', k_1, k_2, k_3} \langle A^b_{\mu}(k) A^{ac}_{\nu}(k_1) A^{bd}_{\lambda}(k_2) A^{de}_{\sigma}(k_3) \rangle \delta(k' - k_1 - k_2 - k_3) dk_1 dk_2 dk_3, \]

where \[ J^{\alpha \nu}_{NL}(k) = J^{(2)\alpha \nu}(k) + J^{(3)\alpha \nu}(k). \]

We made several remarks relative to obtained Eq. (26). The term with \[ \Pi^A \] introducing in the right-hand side of Eq. (26) corresponds to linear Landau damping. However, as shown by Heinz and Siemens [9], linear Landau damping for waves in QGP is absent and hence this term vanishes.

As it will be shown below, the terms with \[ \Sigma \] make no contribute to processes of the nonlinear interaction of waves with particles of QGP, and hereafter they will be dropped.

4. THE SECOND APPROXIMATION OF THE COLOR CURRENT

Now we concerned with computation of the nonlinear corrections to the current in the right-hand side of basic Eq. (26). In this section we consider the second order of \[ J^{(2)\alpha \nu}(k). \]

At first we define \[ f^{T(2)}_{q, \bar{q}}. \] We carry out the Fourier transformation of Eq. (12)

\[ -i(pk) f^{T(2)}_{q, \bar{q}}(k, p) = gp^\mu \int \{ i([A_\mu(k_1), f^{T(1)}_{q, \bar{q}}(k_2, p)] - \langle [A_\mu(k_1), f^{T(1)}_{q, \bar{q}}(k_2, p)] \rangle) + \]
\[ \frac{1}{2} \{ \langle (F_{\mu \nu})_L(k_1), \frac{\partial f^{T(1)}_{q, \bar{q}}(k_2, p)}{\partial p_\nu} \rangle - \langle \langle (F_{\mu \nu})_L(k_1), \frac{\partial f^{T(1)}_{q, \bar{q}}(k_2, p)}{\partial p_\nu} \rangle \rangle \} \pm \]
\[ \pm ig([A_\mu(k_1), A_\nu(k_2)] - \langle [A_\mu(k_1), A_\nu(k_2)] \rangle) \frac{\partial f^{0}_{q, \bar{q}}}{\partial p_\nu} \} \delta(k - k_1 - k_2) dk_1 dk_2. \]

Substituting the obtained \[ f^{T(1)}_{q, \bar{q}} \] from (18) into the last equation and collecting similar terms, we obtain

\[ f^{T(2)}_{q, \bar{q}} = \mp g^2 \frac{[t^b, t^c] p^\mu p^\lambda}{pk + ip_0} \int \frac{(k_2 \partial_\mu f^{0}_{q, \bar{q}})}{pk_2 + ip_0} (A^b_{\nu}(k_1) A^c_{\lambda}(k_2) - \langle A^b_{\nu}(k_1) A^c_{\lambda}(k_2) \rangle) \delta(k - k_1 - k_2) dk_1 dk_2 + \]
\[ + g^2 \frac{[t^b, t^c]}{2} \int \chi^{\nu \lambda}(k_1, p) \frac{\partial}{\partial p^\mu} \left( \frac{\chi^{\sigma \rho}(k_2, p)}{pk_2 + ip_0} \right) (A^b_{\nu}(k_1) A^c_{\sigma}(k_2) - \langle A^b_{\nu}(k_1) A^c_{\sigma}(k_2) \rangle) \]
\[ \delta(k - k_1 - k_2) dk_1 dk_2. \]
The expression for $j_g^{T(2)}(k, p)$ is obtained from (27) by choosing upper sign and replacements $f_0^a \to f_0^b$, $t^a \to T^a$. Substituting obtained expressions $f_s^{T(2)}$, $s = q, \bar{q}, g$ into (15) (for $n = 2$), we find required current correction

$$ j_g^{T(2)\mu}(k) = -ig^3 f^{abc} \int d^4p \frac{p^\mu p^\rho p^\lambda}{pk + ip_0\epsilon} (k_2 \partial_\mu N_{eq}) (A_b^\nu(k_1) A_c^\nu(k_2) - (A_b^\nu(k_1) A_c^\nu(k_2))) (28) $$

$$ + \frac{g^3}{4} f^{abc} \int d^4p \frac{p^\mu \chi^\lambda(k_1, p)}{pk + ip_0\epsilon} \frac{\partial}{\partial p^\lambda} \left( \chi^\nu(k_2, p) \frac{\partial (f_q^0 - f_q^0)}{\partial p^\nu} \right) (A_b^\nu(k_1) A_c^\nu(k_2) - (A_b^\nu(k_1) A_c^\nu(k_2))) \delta(k - k_1 - k_2)dk_1 dk_2, $$

The contribution of gluons to the expression with symmetric structure constant $d^{abc}$ here drops out. This is connected with the fact that in calculation of trace of anti-commutators we have: $\text{Sp} t^b t^c = \frac{1}{2} d^{abc}$ - for quarks and antiquarks, and $\text{Tr} T^a \{ T^b, T^c \} = 0$ - for gluons. The symmetry of contributions can be restored if we note that besides usual gluon current $j_g^\mu(x) = gt^a \int d^4p p^\mu \text{Tr}(T^a f_g(x, p))$, the kinetic equation for gluons admits a covariant conserving quantity

$$ \lambda g t^a \int d^4p \text{Tr} (f^a f_g(x, p)), (29) $$

where $(f^a)^{bc} = d^{abc}$ and $\lambda$ is a certain arbitrary constant. The covariant continuity of (29) is evident from the identity: $[P^a, T^b] = i f^{abc} P^c$. On addition of (29) to (3) we have contributions to the nonlinear current corrections only. Adding (29) to the second current iteration (15) and taking into account the equality

$$ \text{Sp} P^a \{ T^b, T^c \} = N_c d^{abc}, $$

instead of (28) we derive more general expression for $j_g^{T(2)}$

$$ j_g^{T(2)\mu}(k) = \int S_{k,k_1,k_2} \frac{f^{abc} S_{k,k_1,k_2} + d^{abc} S_{k,k_1,k_2}}{S_{k,k_1,k_2}^{(III)\mu\nu\lambda}(k_1, k_2, k_3)}, $$

where

$$ S_{(III)\mu\nu\lambda}^{k,k_1,k_2} = -i g^3 \int d^4p \frac{p^\mu p^\rho p^\lambda}{pk + ip_0\epsilon} \frac{(k_2 \partial_\mu N_{eq})}{pk + ip_0\epsilon}, $$

$$ S_{k,k_1,k_2}^{(III)\mu\nu\lambda} = \frac{g^3}{2} \int d^4p \frac{p^\mu \chi^\nu(k_1, p)}{pk + ip_0\epsilon} \frac{\partial}{\partial p^\sigma} \left( \chi^\rho(k_2, p) \frac{\partial M_{eq}}{\partial p^\sigma} \right), $$

$$ M_{eq} = \frac{1}{2} (f_q^0 - f_{q\bar{q}}^0) + \lambda N_c f_g^0. $$

The tensor structure of $S_{k,k_1,k_2}^{(III)\mu\nu\lambda}$ exactly coincides with appropriate expression obtained in calculation of $j_g^{T(2)\mu}$ in electromagnetic plasmas [11], and hence part of current
with \( d^{abc} \) has a meaning of Abelian part of the color current \( j^{T(2)\mu} \). The term with \( S_{k,k_1,k_2}^{(I)} \) is purely non-Abelian, i.e. it has no Abelian counterpart.

Let us estimate orders of \( S^{(I)} \) and \( S^{(III)} \). Following usual terminology [1], we call an energy or a momentum ”soft" when it is of order \( gT \), and ”hard" when it is of order \( T \). We will be considered, as in Ref. [5], that collective excitations carrying soft momenta, i.e. \( k \sim gT \), and plasma particles have the typical hard energies: \( p \sim T \). On this basis, we have the following estimate for \( S^{(I)} \)

\[
S_{k,k_1,k_2}^{(II)\mu\nu\lambda} \sim g^2 T. \tag{33}
\]

Here, we considered that by virtue of the definitions (17): \( N_{eq} \sim 1/T^2 \).

In expression (32) the integral of energy with gluon distribution function is infrared divergent. In a similar manner [8], we regulate it by introducing an electric mass cut-off of order \( gT \), and only take the leading term in \( g \). Then it can be found that in (32), the part related to the gluon distribution function is of order \( g^2T(g \ln g) \) and the other part related to the quark and antiquark distribution functions is of order \( g^3T \), i.e.

\[
S_{k,k_1,k_2}^{(II)\mu\nu\lambda} \sim g^2 T(g \ln g) + g^3 T.
\]

Hence \( S^{(II)} \), which is purely non-Abelian, is of lower order in the coupling constant than \( S^{(III)} \), which has an Abelian counterpart. This fact was first seen in Ref. [8].

5. THE THIRD APPROXIMATION OF THE COLOR CURRENT

Now we calculate the nonlinear correction of \( j^{T(3)\mu} \). We carry out the Fourier transformation of Eq. (13) and taking into account (18), (27), after cumbersome transformations, we define

\[
f_{k,q}^{T(3)}(k,p) = \int \left[ \mp g^3 \int f^{bcf} f^{cde} t^f \frac{p^\nu p^\lambda p^\sigma}{pk + ip_0 \epsilon} \right. \frac{(k_2 \partial_p f_{q,q}^0)}{pk_2 + ip_0 \epsilon} -
\]

\[
- \frac{ig^3}{2} \left\{ t^b, \{ t^d, t^e \} \right\} p^\nu \chi^{\lambda\alpha}(k_1, p) \frac{\partial}{\partial p^\rho}(\chi^{\sigma \rho}(k_2, p) \partial f^0_{q,q}) +
\]

\[
+ \frac{ig^3}{2} \left\{ t^b, \{ t^d, t^e \} \right\} \chi^{\nu\tau}(k_3, p) \frac{\partial}{\partial p^\sigma}\left(\frac{p^\lambda p^\rho}{p(k_1 + k_2) + ip_0 \epsilon pk_2 + ip_0 \epsilon} \right)
\]

\[
\mp \frac{g^3}{4} t^b, \{ t^d, t^e \} \chi^{\nu\tau}(k_3, p) \frac{\partial}{\partial p^\sigma}\left(\frac{\chi^{\lambda\alpha}(k_1, p)}{p(k_1 + k_2) + ip_0 \epsilon pk_2 + ip_0 \epsilon} \right)
\]

\[
\left( A^{(\nu)}_\alpha(k_3) A^{(\lambda)}_\beta(k_1) A^{(\tau)}_\sigma(k_2) - A^{(\nu)}_\alpha(k_3) A^{(\tau)}_\beta(k_1) A^{(\lambda)}_\sigma(k_2) - A^{(\nu)}_\alpha(k_3) A^{(\lambda)}_\beta(k_1) A^{(\sigma)}_\tau(k_2) - A^{(\nu)}_\alpha(k_3) A^{(\sigma)}_\beta(k_1) A^{(\lambda)}_\tau(k_2) \right). \tag{34}
\]
\[ \delta(k - k_1 - k_2 - k_3) \, dk_1 dk_2 dk_3 + \]
\[ + \frac{ig^3}{2} \int \frac{f^{cde} \{ t^b, t^c \}}{pk + ip_0} \, p^\lambda \frac{\partial}{\partial p^\sigma} \left( \chi^{\mu \nu(k_3, p)} \frac{\partial f\{ q, k \}}{\partial p^\mu} \right) (A^b_\mu(k_3) A^d_\nu(k_1) A^c_\sigma(k_2) - (A^b_\mu(k_3) A^d_\nu(k_1) A^c_\sigma(k_2))) \]
\[ \delta(k - k_1 - k_2 - k_3) \, dk_1 dk_2 dk_3. \]

The gluon part of \( f_q^{T(3)}(k, p) \) is obtained from (34) by replacements \( f_q^0 \to f_q^0 \) and \( t^a \to T^a \).

Substituting obtained expressions for \( f_q^{T(3)} \) and \( f_{\tilde{q}}^{T(3)} \) into (15) (with regard to additional current (29)) and taking into account the equalities

\[ \operatorname{Tr} (\{ T^a, T^b \} \{ T^d, T^e \}) = N_c d^{abc} d^{cde} + 4 \delta^{ab} \delta^{cd} + 2 \delta^{ad} \delta^{eb} + 2 \delta^{ae} \delta^{bd}, \]

we find the required form of \( j^{T(3)\mu} \)

\[ j^{T(3)\mu}(k) = \int \Sigma_{k, k_1, k_2, k_3}^{abcd\mu\lambda\sigma} (A^b_\mu(k_3) A^d_\nu(k_1) A^c_\sigma(k_2) - A^b_\mu(k_3) A^d_\nu(k_1) A^c_\sigma(k_2)) - \]
\[ - (A^b_\mu(k_3) A^d_\nu(k_1) A^c_\sigma(k_2)) \, \delta(k - k_1 - k_2 - k_3) \, dk_1 dk_2 dk_3 + \]
\[ + d^{abc} f^{cde} \int R_{k, k_1, k_2, k_3}^{\mu\nu\lambda\sigma} (A^b_\mu(k_3) A^d_\nu(k_1) A^c_\sigma(k_2) - A^b_\mu(k_3) A^d_\nu(k_1) A^c_\sigma(k_2)) \]
\[ \delta(k - k_1 - k_2 - k_3) \, dk_1 dk_2 dk_3. \]

Here,

\[ \Sigma_{k, k_1, k_2, k_3}^{abcd\mu\lambda\sigma} = f^{abc} f^{cde} \Sigma_{k, k_1, k_2, k_3}^{(I)\mu\nu\lambda\sigma} + f^{abc} d^{cde} \Sigma_{k, k_1, k_2, k_3}^{(I)\mu\nu\lambda\sigma} + \delta^{ab} \delta^{cd} \Sigma_{k, k_1, k_2, k_3}^{(III)\mu\nu\lambda\sigma} + d^{abc} f^{cde} \Sigma_{k, k_1, k_2, k_3}^{(IV)\mu\nu\lambda\sigma} + \]
\[ + d^{abc} d^{cde} \Sigma_{k, k_1, k_2, k_3}^{(V)\mu\nu\lambda\sigma} + (\delta^{ab} \delta^{cd} + \delta^{ad} \delta^{bc} + \delta^{ae} \delta^{bd}) \Sigma_{k, k_1, k_2, k_3}^{(VI)\mu\nu\lambda\sigma}, \]

\[ \Sigma_{k, k_1, k_2, k_3}^{(I)\mu\nu\lambda\sigma} = - g^4 \int d^4 p \, p^\mu p^\nu p^\lambda p^\sigma \frac{1}{pk + ip_0} \frac{(k_2 \partial_\rho N_{eq})}{p(k_1 + k_2) + ip_0}, \]
\[ \Sigma_{k, k_1, k_2, k_3}^{(II)\mu\nu\lambda\sigma} = - g^4 \frac{N_c}{2} \int d^4 p \, p^\mu \chi^{\nu\lambda\sigma} \frac{\partial}{\partial p^\rho} \left( \frac{\chi^{\nu\lambda\sigma}(k_1, p)}{p(k_1 + k_2) + ip_0} \frac{\partial}{\partial p^\rho} \left( \frac{\chi^{\nu\lambda\sigma}(k_2, p)}{p(k_1 + k_2) + ip_0} \frac{\partial}{\partial p^\rho} N_{eq} \right) \right), \]
\[ \Sigma_{k, k_1, k_2, k_3}^{(V)\mu\nu\lambda\sigma} = \frac{N_c}{2} \Sigma_{k, k_1, k_2, k_3}^{(III)\mu\nu\lambda\sigma}. \]

The expression for \( \Sigma^{(VI)} \) is obtained from (38) by exception of quark and antiquark contributions. The availability of the term with \( \Sigma^{(VI)} \) is reflection of more complicated color structure of the gluon kinetic equation in comparison with quark and antiquark equations.

The terms with \( \Sigma^{(I)}, \Sigma^{(IV)} \) and \( R \) are defined as the interference of Abelian and non-Abelian contributions. In the case of isotropic, homogeneous and colorless plasma, the correlation function (23) is proportional to unit matrix in color space, i.e. plasma oscillations are degenerate in color association. This leads to the fact that the coefficients
standing before these interference terms, supporting the color indicies, are vanish in kinetic
equation for waves and therefore their explicit form is not given here. With the availability,
e.g., of external color field only, when degeneration is removed (the correlation function
(48) becomes nontrivial matrix in color space), the interference of Abelian and non-
Abelian contributions will be presented.

At the end of this section we estimate the order of $\Sigma^{(I)}$ and $\Sigma^{(III)}$. It follows from the
expression (37) that
\[\Sigma^{(I)\mu\nu\lambda\sigma}_{k,k_1,k_2,k_3} \sim g^2.\]
(39)
Cutting off, as in the previous section, integration limit for the gluon distribution function,
we find
\[\Sigma^{(III)} \sim \Sigma^{(V)} \sim g^3 + g^4, \Sigma^{(VI)} \sim g^3.\]
By this means, purely non-Abelian contribution of $\Sigma^{(I)}$ is of lower order in the coupling
constant than Abelian - $\Sigma^{(III)}, \Sigma^{(V)}$ and $\Sigma^{(VI)}$.

6. THE GENERALIZED KINETIC EQUATION FOR WAVES

Let us consider the initial equation for waves (26). Substituting obtained nonlinear
corrections of the induced current by field (30) and (36) into this equation, and considering
the terms of leading order in $g$ only, we obtain
\[
\frac{\partial}{\partial k_\lambda}[k^2 g^{\mu\nu} - (1 + \xi^{-1}) k^\mu k^\nu - \Pi^{H\mu\nu}(k)] \frac{\partial I^{ab}_{\mu\nu}}{\partial k^\lambda} =
\]
\[= -i \int dk' \{ f^{bced} S_{k,k_1,k_2,k_3}^{(I)\mu\nu\lambda\sigma}(A_{\mu}^{a}(k') A_{\nu}^{c}(k_1) A_{\lambda}^{d}(k_2)) dk_1 dk_2 \delta(k - k_1 - k_2) -
\]
\[-f^{abcde} S_{k',k_1,k_2,k_3}^{(I)\mu\nu\lambda\sigma}(A_{\mu}^{a}(k) A_{\nu}^{b}(k') A_{\lambda}^{c}(k_1) A_{\sigma}^{d}(k_2)) dk_1 dk_2 \delta(k' - k_1 - k_2) +
\]
\[+ f^{bef} f^{de} \Sigma_{k,k_1,k_2,k_3}^{(I)\mu\nu\lambda\sigma}(\langle A_{\mu}^{a}(k') A_{\nu}^{e}(k_3) A_{\lambda}^{d}(k_1) A_{\sigma}^{c}(k_2) \rangle - \langle A_{\mu}^{a}(k') A_{\nu}^{c}(k_3) A_{\lambda}^{d}(k_1) A_{\sigma}^{e}(k_2) \rangle)
\]
\[dk_1 dk_2 dk_3 \delta(k - k_1 - k_2 - k_3) -
\]
\[-f^{acg} f^{def} \Sigma_{k',k_1,k_2,k_3}^{(I)\mu\nu\lambda\sigma}(\langle A_{\mu}^{a}(k) A_{\nu}^{c}(k_3) A_{\lambda}^{d}(k_1) A_{\sigma}^{e}(k_2) \rangle - \langle A_{\mu}^{a}(k') A_{\nu}^{e}(k_3) A_{\lambda}^{d}(k_1) A_{\sigma}^{c}(k_2) \rangle)
\]
\[dk_1 dk_2 dk_3 \delta(k' - k_1 - k_2 - k_3)\}.\]
(40)
Here, $S^{(I)\mu\nu\lambda\sigma}_{k,k_1,k_2} \equiv S^{(I)\mu\nu\lambda\sigma}_{k,k_1,k_2} + S^{(III)\mu\nu\lambda\sigma}_{k,k_1,k_2}$.

By virtue of weak nonlinearity, oscillations phases of a field weakly correlate among
themselves. Therefore mean value of four random quantities can be approximately divided
into product of mean values of two fields. For mean value of three fields this decomposition
vanishes, and it should be considered a weak correlation of fields. For this purpose we use
the nonlinear equation of a field (20), taking into account in the right-hand side of (20)
the terms of the second order in $A$
\[
[k^2 g^{\mu\nu} - (1 + \xi^{-1}) k^\mu k^\nu - \Pi^{H\mu\nu}(k)] A_{\nu}^{a}(k) =
\]

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\[ f^{abc} \int S_{k,k_1,k_2}^{\mu\nu\lambda}(A^b_\nu(k_1)A^c_\lambda(k_2) - \langle A^b_\nu(k_1)A^c_\lambda(k_2) \rangle) \delta(k - k_1 - k_2) dk_1 dk_2. \] (41)

The approximate solution of this equation is in the form

\[ A^{(0)}_\mu(k) = A^{(0)a}_\mu(k) - D_{\mu\nu}(k) f^{abc} \int S_{k,k_1,k_2}^{\nu\lambda\sigma}(A^{(0)b}_\nu(k_1)A^{(0)c}_\lambda(k_2)) - \langle A^{(0)b}_\nu(k_1)A^{(0)c}_\lambda(k_2) \rangle \delta(k - k_1 - k_2) dk_1 dk_2, \] (42)

where \( A^{(0)a}_\mu(k) \) is a solution of homogeneous Eq. (41) corresponding noninteracting fields, and

\[ D_{\mu\nu}(k) = -[k^2 g_{\mu\nu} - (1 + \xi^{-1}) k_\mu k_\nu - \Pi_{\mu\nu}(k)]^{-1} \] (43)

represents the medium modified (retarded) gluon propagator.

Now we substitute (42) into correlators of three fields introducing in Eq. (40). Because \( A^{(0)} \) represents amplitudes of entirely uncorrelated waves, the correlation function with three \( A^{(0)} \) drops out. In this case it should be defined more exactly each of the terms in \( \langle A^{(0)a}_\mu(k')A^b_\nu(k_1)A^c_\lambda(k_2) \rangle \) and \( \langle A^b_\nu(k_1)A^{(0)c}_\nu(k_1)A^{(0)d}_\sigma(k_2) \rangle \). In the correlation functions of four amplitudes, within the accepted accuracy, it can be done distinction between the fields \( A \) and \( A^{(0)} \).

Finally Eq. (40) becomes

\[ \frac{\partial}{\partial k_\lambda} [k^2 g^{\mu\nu} - (1 + \xi^{-1}) k^\mu k^\nu - \Pi^{\mu\nu}(k)] \frac{\partial f^{ab}}{\partial x^\nu} = -i \int dk' dk_1 dk_2 dk_3 \{ f^{bcf} f^{fde} \delta(k - k_1 - k_2 - k_3) \tilde{S}_{k,k_1,k_2,k_3}^{\mu\nu\lambda\sigma} \langle \{ A^a_\mu(k')A^b_\nu(k_1)A^c_\lambda(k_3)A^d_\sigma(k_2) \} - \langle A^a_\mu(k')A^b_\nu(k_1)A^c_\lambda(k_3)A^d_\sigma(k_2) \rangle \rangle - f^{u\sigma c} f^{fde} \delta(k' - k_1 - k_2 - k_3) \tilde{S}_{k',k_1,k_2,k_3}^{\nu\mu\lambda\sigma} \langle \{ A_{\mu}(k)A^c_\lambda(k_3)A^{\sigma d}(k_1)A^e_\sigma(k_2) \} - \langle A_{\mu}(k)A^c_\lambda(k_3)A^{\sigma d}(k_1)A^e_\sigma(k_2) \rangle \rangle - f^{a\sigma c} f^{fde} \delta(k_1 - k_2 - k_3) \tilde{S}_{k,k_1,k_2,k_3}^{\nu\mu\lambda\sigma} \langle \{ A_{\mu}(k_1)A^c_\lambda(k_3)A^{\sigma d}(k_1)A^e_\sigma(k_2) \} - \langle A_{\mu}(k_1)A^c_\lambda(k_3)A^{\sigma d}(k_1)A^e_\sigma(k_2) \rangle \rangle \} \] (44)

\[ + i f^{bcd} f^{ace} \int dk' \int dk_1 dk_2 dk_3 \langle D_{\mu\nu}(k') - D_{\nu\mu}(k) \rangle S_{k,k_1,k_2,k_3}^{\mu\nu\lambda\sigma} \langle \{ A^{(c)}_{\mu}(k_1)A^{d}_{\nu}(k_2) \} \rangle \langle A_{\lambda}^{(e)}(k_1)A_{\sigma}^{(e)}(k_2) \rangle \rangle \delta(k - k_1 - k_2) \delta(k' - k'_1 - k'_2). \]

Here,

\[ \tilde{S}_{k,k_1,k_2,k_3}^{\mu\nu\lambda\sigma} = \Sigma_{k,k_1,k_2,k_3}^{(I)\mu\nu\lambda\sigma} - (S_{k,k_1,k_2}^{\mu\rho\sigma} + S_{k,k_1,k_2}^{\mu\nu\rho} - S_{k,k_2,k_1}^{\nu\mu\rho}) D_{\rho\sigma}(k_1 + k_2) S_{k_1,k_2,k_1,k_2}^{\mu\nu\lambda\sigma}. \] (45)

It follows from the definition (43) that the propagator is of order \( 1/g^2 T^2 \). Taking into account (33) and (39) we see that all terms in the right-hand side of (44) are of the same order. This explains why in the expansion of the current (14) the following term \( j^{T(3)} \) should be retained in addition to the first nonlinear correction \( j^{T(2)} \); it leads to the effects of the same quantity order.
Let us divide the mean of four fields in Eq. (44) into three pairwise products of two-point correlations by the rule

\[
\langle A(k_1)A(k_2)A(k_3)A(k_4) \rangle = \langle A(k_1)A(k_2) \rangle \langle A(k_3)A(k_4) \rangle + \langle A(k_1)A(k_3) \rangle \langle A(k_2)A(k_4) \rangle + \langle A(k_1)A(k_4) \rangle \langle A(k_2)A(k_3) \rangle.
\]

Taking into account that the spectral densities in the right-hand side of Eq. (44) can be considered as stationary and homogeneous those, i.e. having the form (24), and setting \(I_{\mu\nu}^{ab} = \delta^{ab}I_{\mu\nu}\), we find instead of Eq. (44)

\[
\frac{\partial}{\partial k_\lambda} \left[ k^2 g^{\mu\nu} - (1 + \xi^{-1})k^\mu k^\nu - \Pi^{\mu\nu}(k) \right] \frac{\partial I_{\mu\nu}}{\partial x^\lambda} = 2N_c \int dk_1 \text{Im} (\tilde{\Sigma}_{k,k_1,\mu}^{\lambda} - \tilde{\Sigma}_{k,k_1,\nu}^{\lambda}) I_{\mu\lambda}(k) I_{\nu\sigma}(k_1) + \sum_{\mu\nu} \text{Im}(D_{\mu\nu}(k)) \int dk_1 dk_2 (S_{k,k_1,k_2}^{\mu\nu} - S_{k,k_2,k_1}^{\mu\nu}) (S_{k,k_1,k_2}^{\nu\sigma} - S_{k,k_2,k_1}^{\nu\sigma}) I_{\mu\lambda}(k_1) I_{\nu\sigma}(k_2) \delta(k - k_1 - k_2).
\]

As it is known [13, 14], in global equilibrium QGP the oscillations of three types can be extended: the longitudinal, transverse and nonphysical oscillations. In this connection we define the spectral density \(I_{\mu\nu}(k, x) = I_{\mu\nu}^{l}\) in the form of expansion

\[
I_{\mu\nu}^{l} = P_{\mu\nu} I_{k}^{l} + Q_{\mu\nu} I_{k}^{l} + \xi D_{\mu\nu} I_{k}^{l(n)} \equiv I_{k}^{l(n)}(k, x),
\]

where the transverse and the longitudinal projectors [14] are \(P_{\mu\nu} = g_{\mu\nu} - k_\mu k_\nu/k^2 - Q_{\mu\nu}\), \(Q_{\mu\nu} = \bar{u}_\mu \bar{u}_\nu / \bar{u}^2\), and \(\bar{u}_\mu = k^2 u_\mu - k_\mu (ku)\); \(D_{\mu\nu} = k_\mu k_\nu/k^2\). By using these projectors the propagator (43) can be written in more convenient form

\[
D_{\mu\nu}(k) = -\frac{P_{\mu\nu}}{k^2 - \Pi'} - \frac{Q_{\mu\nu}}{k^2 - \Pi'} + \xi \frac{D_{\mu\nu}}{k^2 + i\epsilon}.
\]

Here, \(\Pi' = \frac{1}{2} \Pi^{\mu\nu} P_{\mu\nu}\), \(\Pi' = \Pi^{\mu\nu} Q_{\mu\nu}\). At finite temperature, the velocity of plasma introduces a preferred direction in space-time which breaks manifest Lorentz invariance. Let us assume that we are in the rest frame of the heat bath, so that \(u_\mu = (1, 0, 0, 0)\).

Eq. (46) with the expansions (47) and (48) enables us to investigate various nonlinear processes in QGP: the nonlinear scattering of longitudinal waves in longitudinal or transverse waves; the scattering of transverse waves in longitudinal or transverse waves; the merger of two longitudinal waves in one transverse wave etc.. In this work we restrict our consideration to investigation of most simple process - the nonlinear scattering of longitudinal waves by particles of QGP in longitudinal those.
Further derivation of kinetic equation for longitudinal oscillations is similar to corresponding derivation in the theory of electromagnetic plasma, therefore we restrict our consideration to its schematic description.

Now we omit nonlinear terms and anti-Hermitian part of the polarization tensor in Eq. (22). Further substituting the function $\delta^{ab}Q_{\mu\nu}(k)I_k^l\delta(k' - k)$ instead of $I_{\mu\nu}^{ab}(k', k)$, we lead to the equation

$$\text{Re} (\varepsilon^l(k)) I_k^l = 0.$$  

Here, we use relation: $1 - \Pi^l(k)/k^2 = \varepsilon^l(k)$. The solution of this equation has the structure

$$I_k^l = I_k^l\delta(\omega - \omega_k^l) + I_{-k}^l\delta(\omega + \omega_k^l), \quad \omega_k^l > 0,$$

where $I_k^l$ is a certain function of a wave vector $k$ and $\omega_k^l \equiv \omega^l(k)$ is a frequency of the longitudinal eigenwaves in QGP.

The equation describing the variation of spectral density of longitudinal oscillations is obtained from Eq. (46) by replacement: $I_{\mu\nu} \to Q_{\mu\nu}(k)I_k^l$, where $I_k^l$ is defined by (49). $\delta$-functions in (49) enable us to remove integration over frequency and thus we have instead of Eq. (46)

$$\frac{k^2}{\omega = \omega_k^l} \frac{\partial I_k^l}{\partial x^\lambda} = 2N_c I_k^l \int d\mathbf{k}_1 I_{\mathbf{k}_1}^l (\text{Im} [\Sigma_{k,k,k_1,-k_1} - \Sigma_{k,k,k_1,k_1} - \Sigma_{k_k}])Q_{\mu\nu}(k)Q_{\nu\sigma}(k_1)\omega_k^l = \omega_{k_1}^l = \omega_{k_2}^l + \omega_{k_2}^l$$

(50)

where

$$G_{k,k_1,k_2} = \text{Im}(\mathcal{D}_{\mu\sigma}(k))(S^{\mu\nu}_{k,k_1,k_2} - S^{\mu\nu}_{k,k_2,k_1})(S^{\alpha\sigma}_{k,k_1,k_2} - S^{\alpha\sigma}_{k,k_2,k_1})Q_{\mu\nu}(k_1)Q_{\nu\sigma}(k_2)\delta(k - k_1 - k_2).$$

Let us consider the terms entering in the right-hand side of Eq. (50). The integral with the function $G_{k,k_1,k_2}$ is different from zero if the conservation laws are obeyed

$$k = k_1 + k_2,$$

$$\omega_k^l = \omega_{k_1}^l + \omega_{k_2}^l.$$  

(51)

These conservation laws describe a decay of one longitudinal wave in two longitudinal waves. However for a spectrum of the longitudinal oscillations in QGP, the equalities (51) do not hold simultaneously, no matter what the values of the wave vectors $k, k_1$ and $k_2$ may be, i.e. this nonlinear process is forbidden. Therefore the integral with $G_{k,k_1,k_2}$ vanishes. Remaining integrals with $G$-functions differ from (51) in that some of the interacting waves are not radiated but absorbed. They also vanish.
The expression
\[
\left( \sum_{k,k_1} C_{\mu \nu \sigma \lambda} \right) Q_{\mu \lambda}(k) Q_{\nu \sigma}(k_1) \bigg|_{\omega = \omega_k, \omega_1 = \omega_{k_1}}, \tag{52}
\]
contains the factors
\[
\frac{1}{pk + ip_0 \epsilon}, \frac{1}{pk_1 + ip_0 \epsilon}, \frac{1}{p(k - k_1) + ip_0 \epsilon},
\]
by the definitions of functions entering in it. Imaginary parts of first two factors should be setting equal to zero, because they are connected with linear Landau damping of longitudinal waves (which is absent in QGP), and therefore the imaginary part of the expression (52) properly introducing in Eq. (50) will be defined as
\[
\text{Im} \left( \frac{1}{p(k - k_1) + ip_0 \epsilon} \right)_{\omega = \omega_k, \omega_1 = \omega_{k_1}} = -\frac{i\pi}{p_0} \delta(\omega_k - \omega_{k_1} - \mathbf{v} \cdot (\mathbf{k} - \mathbf{k}_1)).
\]
It follows that nonlinear term in the right-hand side of (50) with the function (52) is different from zero if the conservation law is obeyed
\[
\omega_k - \omega_{k_1} - \mathbf{v} \cdot (\mathbf{k} - \mathbf{k}_1) = 0.
\]
This conservation law describes the process of scattering of longitudinal wave (plasmon) in longitudinal one by the particles in QGP.

Let us consider in more detail the term in (52) (see definition (45)) with propagator \( D_{\rho \alpha}(k - k_1) \). By expansion (48) this propagator represents the nonlinear interaction of longitudinal waves with longitudinal ones through three types of intermediate oscillations: the transverse, longitudinal and nonphysical oscillations depending on a gauge parameter. The term with
\[
\left( \frac{P_{\rho \alpha}(k_2)}{k_2^2 - \Pi^I(k_2)} \right)_{\omega = \omega_k, \omega_1 = \omega_{k_1}}
\]
(hereafter \( k_2 \equiv k - k_1 \)) in general, describes two fundamentally different nonlinear processes:

1. if \( k_2 = (\omega_k - \omega_{k_1}, \mathbf{k} - \mathbf{k}_1) \) is a solution of the dispersion equation \( k_2^2 - \Pi^I(k_2) = 0 \), then this term describes the process of merger of two longitudinal oscillations in transverse eigenwave;

2. otherwise, it defines the process of nonlinear scattering of longitudinal waves in longitudinal those through the transverse virtual oscillation (for a virtual wave in distinction to the eigenwave, a frequency \( \omega \) and a wave vector \( \mathbf{k} \) are not connected with each other by the dispersion dependence: \( \omega \neq \omega(k) \)).

The equality \( k_2^2 - \Pi^I(k_2) = 0 \) does not hold for longitudinal oscillations, as we see above and therefore, the term
\[
\left( \frac{Q_{\rho \alpha}(k_2)}{k_2^2 - \Pi^I(k_2)} \right)_{\omega = \omega_k, \omega_1 = \omega_{k_1}}
\]
defines only the process of scattering of longitudinal waves in longitudinal those through the longitudinal virtual oscillation.

Let us consider the contribution of nonphysical intermediate oscillations

\[ \left( \xi \frac{D_{\rho\alpha}(k_2)}{k_2^2 + i\epsilon} \right)_{\omega = \omega'_l, \omega'_1 = \omega'_k}. \]

By direct calculation one can show, that in contraction with tensor \( D_{\rho\alpha}(k_2) \) the complex factor \( 1/p(k - k_1) + ip_\eta \epsilon \) is reduced in the expressions with \( S \)-functions. In particular it follows that contribution from the process of nonlinear scattering of longitudinal waves, connected with nonphysical intermediate oscillations drops out. Therefore the gauge parameter \( \xi \) is absent in the equation for longitudinal wave.

The remaining terms with \( \tilde{\Sigma} \) are distinguished from above considered terms by a sign of \( k_1 \), and describe the processes of simultaneous radiation or absorption by particles of two waves. The contribution of these processes is exponentially small in relation to the scattering process and therefore these terms are omitted.

Summing the preceding and going from the function \( I^l_k \) to the function

\[ W^l_k = -\left( \omega k^2 \frac{\partial \text{Re} \epsilon^l(k)}{\partial \omega} \right)_{\omega = \omega'_l} I^l_k, \]

having the physical meaning of spectral density of energy of longitudinal oscillations, we find from (50) the required kinetic equation for longitudinal waves in QGP

\[ \frac{\partial W^l_k}{\partial t} + V^l_k \frac{\partial W^l_k}{\partial x} = -\tilde{\gamma}\{ \left( \frac{W^l_k}{\omega^l_k} \right) \} W^l_k, \quad (53) \]

where

\[ V^l_k = \frac{\partial \omega^l_k}{\partial k} = -\left[ \frac{\partial \text{Re} \epsilon^l(k)}{\partial k} \right]_{\omega = \omega'_l} \]

is the group velocity of longitudinal oscillations and

\[ \tilde{\gamma}\{ \left( \frac{W^l_k}{\omega^l_k} \right) \} \equiv \gamma^l(k) = 2N_c \int d\mathbf{k}_1 \left( \frac{W^l_{k_1}}{\omega^l_{k_1}} \right) \left[ \frac{1}{k^2 k_1^2} \left( \frac{\partial \text{Re} \epsilon^l(k)}{\partial \omega} \right)^{-1} \frac{\partial \text{Re} \epsilon^l(k_1)}{\partial \omega_1} \right] \]

represents the damping rate caused by nonlinear effects and being the linear functional of spectral density of energy.

One can write (53) in the form which is more close to usual representation if the spectral density of number of longitudinal oscillations is entered

\[ N^l_k = \frac{W^l_k}{\omega^l_k}. \]
It fulfills the role of distribution function of a number of plasmons. Then instead of (53) we have
\[
\frac{dN^l_k}{dt} = \frac{\partial N^l_k}{\partial t} + \mathbf{V}_k \frac{\partial N^l_k}{\partial \mathbf{x}} = -\dot{\gamma} \{ N^l_k \} N^l_k. \tag{55}
\]

8. THE PHYSICAL MECHANISM OF THE NONLINEAR SCATTERING OF WAVES

Now we transform \( \gamma^l(k) \) to the form allowing more neatly explain the physical meaning of the terms entering in the nonlinear damping rate. The first transformation of this type was proposed by Tsytovich for electromagnetic plasma [11].

By the definition \( \Sigma \) (45) we have
\[
(\Sigma^{(I)}_{\mu\nu\lambda\sigma})_{k,k,-k_1,k_1} = (\Sigma^{(I)}_{\mu\nu\lambda\sigma})_{k,k,-k_1,k_1} + \Sigma^{(I)}_{\mu\nu\lambda\sigma} - \Sigma^{(I)}_{\mu\nu\lambda\sigma} + Q_{\mu\lambda}(k)Q_{\nu\sigma}(k_1) +
\]
\[
\frac{1}{k^2 - \Pi'(k_2)} (S^{\mu\nu}_{k,k,k_1} - S^{\mu\nu}_{k,k,k_1}) Q_{\rho\alpha}(k_2) Q_{\mu\lambda}(k) Q_{\nu\sigma}(k_1). \tag{56}
\]
Here, for simplicity in the last term in the right-hand side of Eq. (56) the effects connected with existence of transverse intermediate oscillations are neglected. At first we consider the expression with \( \Sigma^{(I)} \). By virtue of definition (37) we have
\[
\operatorname{Im} (\Sigma^{(I)}_{k,k,-k_1,k_1} - \Sigma^{(I)}_{k,k,-k_1,k_1}) Q_{\mu\lambda}(k) Q_{\nu\sigma}(k_1) =
\]
\[
= - \frac{\pi g^4}{k^2 k_1^2 k_2^4} \int d^4 p \frac{(p\mu(k))^2 (p\mu(k_1))^2}{(pk)^2} \delta(pk_2)(k_2 \partial_p N_{eq}).
\]

The contribution to \( \gamma^l(k) \) from this term may be introduce in the form
\[
- \int w_p^{\Sigma}(k,k_1) N^l_k \rho_0 (k_2 \partial_p N_{eq}) d^4 p d\mathbf{k}_1, \tag{57}
\]
where
\[
w_p^{\Sigma}(k,k_1) =
\]
\[
= \frac{2 \pi N_e}{(k^2 k_1^2)^2} \left( \frac{\partial \operatorname{Re} \varphi^l(k)}{\partial \omega} \right)_{\omega = \omega_k}^{-1} \left( \frac{\partial \operatorname{Re} \varphi^l(k_1)}{\partial \omega_1} \right)_{\omega_1 = \omega_{k_1}} \delta(k_1 - \mathbf{k}_1 - \mathbf{v}(k - k_1)) |\Lambda^\Sigma(k,k_1)|^2. \tag{58}
\]
\[
\Lambda^\Sigma(k,k_1) = \frac{g^2 \left| \omega_k^l (k \mathbf{v}) - \mathbf{k}^2 \right|}{|k||k_1| \omega_k^l - (k \mathbf{v})}. \tag{59}
\]

To clear up the physical meaning of contribution (57), it is convinient to compare it with appropriate contribution in the theory of electromagnetic plasma. In this case as shown in [11] this contribution describes the Thomson scattering of a wave \( \omega_k^l \) by particles: a wave \( \omega_k^l \) sets particles of plasma into oscillation and oscillating particles radiate a wave \( \omega_{k_1}^l \). The corresponding function \( w_p^{\Sigma}(k,k_1) \) presents the probability of
this scattering. As it was shown above in quark-gluon plasma for a soft long-wavelength excitations all Abelian contributions is at most $g \ln g$ times the non-Abelian ones and the basic scattering mechanism here, is essentially another (next we consider it briefly, the details will be published elsewhere).

For revealing this mechanism we use the classical pattern of QGP description [2], in which the particles states are characterized besides coordinate and momentum by the color vector $Q = (Q^a)$, $a = 1, \ldots, N_c^2 - 1$ also. As was shown by Heinz [2] there is an intimate connection between the classical kinetic equations and semiclassical ones (4). Therefore in this case use of classical notions is justified.

Let the field acting on a color particle in QGP represents a set of longitudinal plane waves

$$\tilde{A}_\mu^a(x) = \int [Q_{\mu\nu}(k)A_{k\nu}^a]_{\omega = \omega_k} e^{ikx - i\omega_k t} dk.$$  

(60)

The particle motion in this wave field is described by the system of equations

$$m \frac{d^2x^\mu}{d\tau^2} = gQ^a \tilde{F}_{a\mu\nu} \frac{dx^\nu}{d\tau},$$  

(61)

$$\frac{dQ^a}{d\tau} = -gf^{abc} \frac{dx^\mu}{d\tau} \tilde{A}_\mu^b Q^c.$$  

(62)

Here, $\tau$ is a proper time of a particle. The system (61), (62) is solved by the method of successive approximations - expansion in the field amplitude. A zeroth approximation describes uniform restlinear motion, and the next one - constrained charge oscillations in the field (60). With a knowledge of the motion law of a charge, the intensity of radiating by it longitudinal waves can be defined. In this case Eq. (61) defines the Abelian contribution to radiation, whereas (62) - non-Abelian one and interference of these two contributions equals zero. The scattering probability calculated by this means, based on Eq. (62) is coincident with obtained above (58).

In this manner the contribution (57) to $\gamma^l(k)$ are caused by not the spatially oscillations of a color particle, as it occurs in electromagnetic plasma, but the initiation of a precession of a color vector $Q$ of a particle in field of a longitudinal wave (60) (Eq. (62) conserves the length of a color vector).

Let us consider now more complicated term in (56) connected with $S$-functions. By exact calculation, using the definitions (21) and (31), it is not difficult to see that the following equality is obeyed

$$(S_{k,k_1,k_2}^{\mu\nu\rho} - S_{k,k_2,k_1}^{\mu\nu\rho}) \bar{u}_\mu(k)\bar{u}_\rho(k_2)\bar{u}_\nu(k_1) = - (S_{k_2,-k_1,k}^{\alpha\lambda\sigma} - S_{k_2,k_1,-k}^{\alpha\lambda\sigma}) \bar{u}_\alpha(k_2)\bar{u}_\lambda(k)\bar{u}_\sigma(k_1) \equiv$$

$$\equiv S_{k,k_1}^{(I)} + S_{k,k_1}^{(II)}.$$  

(63)

Then the contribution to $\gamma^l(k)$ from $S$-functions can be represented as

$$2N_c \int dk_1 N_{k_1} \frac{1}{k_1^2 k_2^2 k_3^2} \left( \frac{\partial \Re \varepsilon^l(k)}{\partial \omega} \right)^{-1} \omega = \omega_k \left( \frac{\partial \Re \varepsilon^l(k_1)}{\partial \omega_1} \right)^{-1} \omega_1 = \omega_{k_1}.$$
in electromagnetic plasma. The second contribution determining by the function $S$ damping rate in the form

$$\text{Im} \left( \frac{1}{\varepsilon^2(k_2)} \right) \frac{1}{\varepsilon(k_2)} (S_{k,k_1})^2 \right)_{\omega=\omega'_k, \omega_1=\omega'_k}. $$

Next we use the relation

$$\text{Im} \left( \frac{1}{\varepsilon^2(k_2)} (S_{k,k_1})^2 \right) = \text{Im} \varepsilon'(k_2) \frac{|S_{k,k_1}|^2}{|\varepsilon'(k_2)|^2} - 2 \text{Im}(-iS_{k,k_1}) \text{Re} \left( \frac{\varepsilon'(k_2)}{\varepsilon'(k_2)} \right). $$

Taking into consideration the equality

$$\text{Im} \varepsilon'(k_2) = -\frac{\pi g^2}{k_2^2} \int d^4 p (p_0)^2 \delta(p k_2)(k_2 \partial_p N_{eq}) ,$$

one can write contribution from the first term in the right-hand side of (64) to the nonlinear precession of color vectors with particles forming this cloud in the wave field as a result of interaction with impinging wave, but as a consequence of the initiation of through the oscillation of a coherent polarization cloud which surrounds the quark is associated with the scattering of wave by the Debye screening shell of the particle. However, in contrast to the electromagnetic plasma here, the scattering is accounted for not through the oscillation of a coherent polarization cloud which surrounds the quark as a result of interaction with impinging wave, but as a consequence of the initiation of precession of color vectors with particles forming this cloud in the wave field $\omega'_k$. This process of scattering represents a pure collective effect. For calculation of its probability it is necessary to solve the kinetic equation describing a charge motion in a polarization cloud in the field which is equal to the sum of fields of impinging wave (60) and a charge producing a screening cloud.

The remaining term in the right-hand side of (64) describes the interference of the above-mentioned scattering mechanisms. It is easily to see this having used

$$\text{Im} (-iS_{k,k_1}) = \pi g^3 k_2^2 \int d^4 p \frac{p_0}{p k} (p \bar{u}(k))(p \bar{u}(k_1)) \delta(p k_2)(k_2 \partial_p N_{eq}).$$

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Thus, summing the preceding, instead of (54) we have

\[
\gamma'(k) = -\int (\omega_k' - \omega_{k1}') Q_p(k, k_1) \left( \frac{W_{k1}'}{\omega_{k1}'} \right) p_0 \frac{dN_{eq}(p_0)}{dp_0} d^4p dk_1, \tag{68}
\]

where

\[
Q_p(k, k_1) = \frac{2\pi N_c}{[(\omega_k')^2 - k^2]^2[(\omega_{k1}')^2 - k_1^2]^2} \left( \frac{\partial \text{Re} \varepsilon'(k)}{\partial \omega} \right)^{-1}_{\omega = \omega_k'} \left( \frac{\partial \text{Re} \varepsilon'(k_1)}{\partial \omega_1} \right)^{-1}_{\omega_1 = \omega_{k1}'}, \tag{69}
\]

\[
\delta(\omega_k' - \omega_{k1}' - v(k - k_1))|\Lambda^\Sigma(k, k_1) + \Lambda^S(k, k_1)|^2,
\]

and the expressions for \(\Lambda^\Sigma\) and \(\Lambda^S\) are given by (59) and (67), respectively.

Now we note that it is convenient to interpret the terms entering in the \(\Lambda \equiv \Lambda^\Sigma + \Lambda^{S(I)} + \Lambda^{S(II)}\) by using a quantum language. In this case the term \(\Lambda^\Sigma\) connected with the Thomson scattering can be represented as the Compton scattering of the oscillation quantum (plasmon) by QGP particle. \(\Lambda^{S(I)}\) defines the scattering of a plasmon through a virtual wave with the propagator \(1/(k_2^2 - \Pi'(k_2))\), where a vertex of a three-wave interaction is induced by a self-action of a field. The term \(\Lambda^{S(II)}\) defines the plasmon scattering by a particle through a virtual wave with the same propagator and effective vertex of three-wave interaction connected with the medium effects. In this case \(\Lambda\) fulfills role of the scattering amplitude.

The kernel \(Q_p(k, k_1)\) possesses two main properties. The following inequality results from definition (69)

\[
Q_p(k, k_1) \geq 0. \tag{70}
\]

Next, from (59) it follows that \(\Lambda^\Sigma(k, k_1) = \Lambda^{\ast \Sigma}(k_1, k)\). The correctness of this equality results from the conservation law: \(\omega_k' - \omega_{k1}' - v(k - k_1) = 0\). For \(\Lambda^S\) we have the similar relation: \(\Lambda^S(k, k_1) = \Lambda^{\ast S}(k_1, k)\). Its proof trivially follows from the definitions of \(S^{(I)}_{k,k_1}, S^{(II)}_{k,k_1}\) and \(\varepsilon'(k)\):

\[
-iS^{(I)}_{k,k_1} = g \{(k + k_1) \bar{u}(k_2))(\bar{u}(k_2)\bar{u}(k_1)) - (k_1 \bar{u}(k))(\bar{u}(k_2))\bar{u}(k_1)\bar{u}(k_2)) - 2(k\bar{u}(k_2))(\bar{u}(k_2)\bar{u}(k_2))\}, \tag{71}
\]

\[
-iS^{(II)}_{k,k_1} = -g^3 \int d^4p' \left( \frac{p'\bar{u}(k_2))(p'\bar{u}(k))(p'\bar{u}(k_1))\left( \frac{(k\partial_{p'}N_{eq})}{p'k} - (k_1\partial_{p'}N_{eq})}{p'k_1} \right), \tag{72}
\]

\[
\varepsilon'(k) = 1 + \frac{3\omega_p^2}{k^2}[1 - F(\frac{\omega}{|k|})], \quad F(x) = \frac{x}{2} \left[ \ln \left( \frac{1+x}{1-x} \right) - i\pi \theta(1 - |x|) \right]. \tag{73}
\]

The consequence of these equalities is a main property of a symmetry kernel \(Q_p(k, k_1)\) with respect to permutation of a wave vectors \(k\) and \(k_1\)

\[
Q_p(k, k_1) = Q_p(k_1, k). \tag{74}
\]
Now let us consider consequence of the properties (70) and (74). Integrating (55) over 
\( dK/(2\pi)^3 \), and using (74), we find

\[
\frac{dN^l}{dt} \equiv \frac{d}{dt} \left( \int N_k^l \frac{dK}{(2\pi)^3} \right) = 0,
\]

i.e. the general number of plasmons \( N^l \) in the process of the nonlinear scattering conserves exactly.

From (68) it follows that in the case of a global equilibrium plasma, when

\[
\frac{dN_{eq}(p)}{dp} < 0,
\]

and by inequality (70), waves of a high frequencies are damped out, and a smaller ones are increased. Thus, due to the nonlinear interaction of a plasma oscillations a pumping-over from short to long waves occurs in the spectrum, practically leaving its total energy fixed. Therefore, the nonlinear decrement \( \gamma^l(k) \) defines actually the inverse time of a spectral pumping.

In the limit of \( |k| \to 0 \) from (68) we obtain

\[
\gamma^l(0) = -\int (\omega_{pl} - \omega^l_{k_1})Q_p(0, k_1) \left( \frac{W^l_{k_1}}{\omega^l_{k_1}} \right) p_0 \frac{dN_{eq}(p_0)}{dp_0} d^4p d^4k_1.
\]

The frequency \( \omega^l_{k_1} \) is monotonically increasing function of a wave number, therefore \( \omega_{pl} - \omega^l_{k_1} \leq 0 \). From this following inequality folows

\[
\gamma^l(0) < 0,
\]

i.e. \( |k| = 0 \)-mode is not damped, in contrast to Ref. [8]. From the physical point of view this result is clear. As it was shown above, nonlinear interaction of waves leads to the spectral pumping from short to long waves. The mode \( |k| = 0 \) is a limiting in this series, and therefore \( \gamma^l(0) < 0 \). It is clear that in the region \( |k| \approx 0 \) the other nonlinear mechanisms switching off instability are to come into effect. One of such mechanism, connected with regard to the terms of a higher-degree of nonlinearity in expansion of the color current (14) will be discussed in Conclusion.

9. THE ESTIMATE OF \( \gamma^l(0) \)

Now we consider the quantity \( \gamma^l(0) \). As in the paper [8] it is convenient to introduce the cylindrical coordinate system in which the direction of polar axis is selected to be the same as the direction \( k_1 \). Then the coordinates for \( k, p, \) and \( p' \) are \( k = (|k|, \alpha, \beta), \ p = (|p|, \theta, \varphi), \) and \( p' = (|p'|, \theta', \varphi') \) correspondly. By \( \phi \) and \( \phi' \) we denote the corresponding angles between \( p \) and \( k, \ p' \) and \( k \). They can be expressed as

\[
\cos \phi = \sin \theta \sin \alpha \cos(\varphi - \beta) + \cos \theta \cos \alpha.
\]
The expression for \( \cos \phi' \) is similar to above equation.

In the limit of \( |k| \to 0 \) the kernel (69) transforms to

\[
Q_p(0, k_1) = \frac{\pi N_c}{\omega_{pl}^2 |k_1| |(\omega_{k_1}^l)^2 - k_1^2|} \left( \frac{\partial \text{Re} \varepsilon^l(k_1)}{\partial \omega_1} \right)^{-1} \left. \delta(\cos \theta - \rho_{k_1}^l) |\Lambda^\Sigma(0, k_1) + \Lambda^S(0, k_1)|^2 \right|_{\omega_1 = \omega_{k_1}^l},
\]

where

\[
\rho_{k_1}^l = (\omega_{k_1} - \omega_{pl})/|k_1| \geq 0.
\]

Using definitions (59) and (67), we find the expressions \( \Lambda^\Sigma(0, k_1) \) and \( \Lambda^S(0, k_1) \).

Turning \( |k| \) to zero in (59), we have

\[
\Lambda^\Sigma(0, k_1) = g^2 |k_1| \cos \phi (v_{k_1}^l \cos \theta - 1).
\]

Here, we denote the phase velocity of longitudinal oscillations by \( v_{k_1}^l = \omega_{k_1}^l / |k_1| \).

Calculation of \( \Lambda^S(0, k_1) \) is more complicated. From (67) we obtain

\[
\Lambda^S(0, k_1) = -\frac{g}{|k_1|^3} \frac{1}{1 - \rho_{k_1}^l} \left. \frac{1}{k_1^2} \frac{1}{\omega_{pl}^2 [1 - F(-\rho_{k_1}^l)]} \right|_{\omega = \omega_{k_1}^l, \omega_1 = \omega_{k_1}^l}.
\]

Here, we use definition of the function \( \varepsilon^l(k) \) (73). Dividing (71) by \( |k| \) and going to the limit \( |k| \to 0 \), after the simple, but slightly cumbersome computations, we define

\[
\lim_{|k| \to 0} \left. \frac{-i S_{k,k_1}^{(I)}}{|k|} \right|_{\omega = \omega_{k_1}^l, \omega_1 = \omega_{k_1}^l} = 2g \cos \alpha \omega_{pl} |k_1|^3 \{ \omega^2_{pl} - k_1^2 (1 - v_{k_1}^l \rho_{k_1}^l) \}.
\]

For definition of the limit of \( (-i S_{k,k_1}^{(II)})/|k| \), instead of (72) we use expression, which is defined from it, if the following identity is accounted for

\[
\frac{1}{p'k_2 + ip'\epsilon} \frac{1}{p'k_1} = \left( \frac{1}{p'k_2 + ip'\epsilon} + \frac{1}{p'k_1} \right) \frac{1}{p'k_1}
\]

This replacement leads to more simple limiting expression

\[
\lim_{|k| \to 0} \left. \frac{-i S_{k,k_1}^{(II)}}{|k|} \right|_{\omega = \omega_{k_1}^l, \omega_1 = \omega_{k_1}^l} = \frac{3g}{4\pi} \omega_{pl}^2 \int_0^{2\pi} d\phi' \int_{-1}^1 d(\cos \theta') \cos \phi' k_1^l (v_{k_1}^l \cos \theta' - 1)(\rho_{k_1}^l \cos \theta' - 1)
\]

\[
\left( \frac{\rho_{k_1}^l}{\rho_{k_1}^l - \cos \theta' - i\epsilon} - \frac{v_{k_1}^l}{v_{k_1}^l - \cos \theta'} \right).
\]

\[81]
Here, we take into account that in view of definition of the equilibrium function $N_{eq}$ and (17) (for $\mu = 0$):

$$\int_{-\infty}^{+\infty} |p'|^2 dp' \int_{-\infty}^{+\infty} p_0 dp_0 \frac{dN_{eq}(p_0)}{dp_0} = \frac{3}{4 \pi} \frac{\omega_{pl}^2}{g^2}.$$  

Now we substitute instead of $\cos \phi'$ the expression similar (76) in (81). Next integrating over $\theta'$ we obtain

$$\lim_{|k| \to 0} \frac{-i S_{k,k_1}^{(II)}}{|k|} \bigg|_{\omega = \omega_{k}, \omega_{l} = \omega_{k_1}} = 3g\omega_{pl}^2 \cos \alpha k_1^l v_{k_1}^l \rho_{k_1}^l \left\{ \frac{\omega_{pl}}{3|k|} - \frac{1 - v_{k_1}^l \rho_{k_1}^l}{v_{k_1}^l \rho_{k_1}^l} [\rho_{k_1}^l (1 - (\rho_{k_1}^l)^2)] (1 - F(-\rho_{k_1}^l)) + \frac{k_1^l}{3\omega_{pl}^2} v_{k_1}^l (1 - (v_{k_1}^l)^2) \right\}.$$  

Substituting (80) and (82) in (79), we find unknown expression for $\Lambda^S(0, k_1)$.

Now we explicitly select the angular dependence on square of the scattering amplitude entering in $Q_p(0, k_1)$

$$Q(0, k_1) = \frac{\omega_{pl} - \omega_{k_1}}{\omega_{k_1}} \int d^4p Q_p(0, k_1) p_0 \frac{dN_{eq}(p_0)}{dp_0} = \frac{3\omega_{pl}^2}{4\pi g^2} \rho_{k_1}^l \int_0^{2\pi} d\varphi \int_0^1 d(\cos \theta) Q_p(0, k_1).$$  

Let us consider the integrals over angles $\varphi$ and $\theta$ in the kernel $Q(0, k_1)$ (84). These angles enter into $\Lambda^S(0, k_1)$ and $\delta$-function in (77), and the element $\Lambda^S(0, k_1)$ is independent from those at all.

Now we explicitly select the angular dependence on square of the scattering amplitude entering in $Q_p(0, k_1)$

$$|\Lambda^S(0, k_1) + \Lambda^S(0, k_1)|^2 = g^4 \cos^2 \phi k_1^l (v_{k_1}^l \cos \theta - 1)^2 +$$

$$+ 2g^2 \cos \phi |k_1| (v_{k_1}^l \cos \theta - 1) \text{Re} \Lambda^S(0, k_1) + |\Lambda^S(0, k_1)|^2.$$  

Using the formula of the angles connection (76), and taking into account relations

$$\int_0^{2\pi} \cos^2 \phi d\varphi = \pi [(3 \cos^2 \theta - 1) \cos^2 \alpha + 1 - \cos^2 \theta], \int_0^{2\pi} \cos \phi d\varphi = 2\pi \cos \theta \cos \alpha,$$

we integrate over $\varphi$ in (84). The remaining integral over $\theta$ by the $\delta$-function is computed elementary. Summing preceding, we find

$$Q(0, k_1) = \frac{\pi N_{eq} g^2 k_1^l \rho_{k_1}^l}{\omega_{pl}} \left( 1 - v_{k_1}^l \rho_{k_1}^l \right)^2 \left( \frac{\partial \text{Re} \varepsilon(k_1)}{\partial \omega_1} \right)^{-1} \left[ \frac{3}{4} [(3(\rho_{k_1}^l)^2 - 1) \cos^2 \alpha + \right]$$
\[ +1 - \left(\rho_{k_1}^l\right)^2 - 3\rho_{k_1}^l \cos^2 \alpha \Re \tilde{\Lambda}^S(|k_1|) + \frac{3}{2} \cos^2 \alpha |\tilde{\Lambda}^S(|k_1|)|^2 \theta(1 - \rho_{k_1}^l). \] (85)

Here, instead of \( \Lambda^S(0, k_1) \) we introduce a new function depending only on \(|k_1|\) by means of relation

\[
\Lambda^S(0, k_1) = g^2 |k_1| \cos \alpha (1 - v_{k_1}^l \rho_{k_1}^l) \tilde{\Lambda}^S(|k_1|). 
\]

The peculiarity of obtained expression (85) is the absence of the angle dependence on \( \beta \). This enables us to represent \( \gamma^l(0) \) in the form

\[
\gamma^l(0) = -2\pi \int_0^\infty |k_1|^2 d|k_1| \int_{-1}^1 d(\cos \alpha) Q(k_1) W^l_{k_1}.
\] (86)

If we consider that the isotropy of the oscillations directions takes place at the time interval which is much less than the characteristic that of the nonlinear interaction, then the spectral density \( W^l_{k_1} \) can be considered isotropic with respect to \( k_1 \) directions. This enables us to integrate over angle in (86) to completion. Now we introduce the spectral function

\[
W^l_{|k_1|} = 4\pi |k_1|^2 W^l_{k_1},
\]

such that the integral \( \int_0^\infty W^l_{|k_1|} d|k_1| = W^l \) is total energy of longitudinal oscillations in QGP. Substituting (85) into (86) and integrating over \( d(\cos \alpha) \) we find more suitable form for estimate \( \gamma^l(0) \)

\[
\gamma^l(0) = - \int_0^\infty Q(|k_1|) W^l_{|k_1|} d|k_1|,
\] (87)

where

\[
Q(|k_1|) = \frac{\pi N_c g^2 k_1^2 \rho_{k_1}^l (1 - v_{k_1}^l \rho_{k_1}^l)^2}{2\omega_{pl}^2 \left( (\omega_{k_1}^2 - k_1^2)(3\omega_{pl}^2 - (\omega_{k_1}^2 - k_1^2)^2) \right) \left( 1 - 2\rho_{k_1}^l \Re \tilde{\Lambda}^S(|k_1|) + |\tilde{\Lambda}^S(|k_1|)|^2 \right) },
\]

\[
\tilde{\Lambda}^S(|k_1|) = \frac{1}{k_1^2 + 3\omega_{pl}^2(1 - F(-\rho_{k_1}^l))} \left[ 3\omega_{pl}^2 \rho_{k_1}^l (1 - F(-\rho_{k_1}^l)) + \frac{\omega_{pl}^2}{2(1 - (\rho_{k_1}^l)^2)} \frac{k_1^2}{\omega_{pl}^2} \frac{1 - (v_{k_1}^l)^2}{\rho_{k_1}^l} - \frac{\omega_{pl}^2}{|k_1|} \left( 1 - v_{k_1}^l \rho_{k_1}^l \right) \right] + \frac{2\omega_{pl}^2}{3(1 - (\rho_{k_1}^l)^2)} \left( |k_1| - \frac{\omega_{pl}^2}{|k_1|} \right). \] (88)

Here, we set \( \theta \)-function entering in (85) equals to unit, since the function \( \rho_{k_1}^l \) by its definition, for any \(|k_1|\) satisfies the inequality

\[
\rho_{k_1}^l \leq 1.
\]

For estimate of \( \gamma^l(0) \) order in (87) we cutt-off upper integration limit on the characteristic value \(|k_1| \approx \omega_{pl} \). As was shown in [13] in the region \(|k_1| \geq \omega_{pl} \) there is the strong damping of longitudinal oscillations, therefore here, this cutting has meaning. Besides, the realized above analysis of separation of leading in \( g \) terms is obeyed only in
the long-wavelength spectrum region. Let us approximate the kernel $Q(|k_1|)$, using the approximations

$$\omega'_{k_1} \approx \omega_{pl}, \, \rho'_{k_1} \approx \frac{3|k_1|}{10\omega_{pl}}, \, \upsilon'_{k_1} \approx \frac{\omega_{pl}}{|k_1|} \text{ etc.}$$

Leaving the leading in $|k_1|$ term in the expansion (88), we obtain

$$Q(|k_1|) \approx \frac{3\pi}{40} N_c g^2 \frac{|k_1|}{(\omega_{pl})^4}. \quad (89)$$

Note that here, the contribution to $Q(|k_1|)$ from the Compton scattering process proves to be negligible by comparison with two other contributions. The function $W_{k_1}$ is approximated by its equilibrium value:

$$W'_{k_1} \approx \frac{4\pi}{T}$$

and therefore

$$W_{|k_1|} \approx 16\pi^2 |k_1|^2 T. \quad (90)$$

Substituting (89) and (90) in (87), we finally define

$$\gamma^l(0) \approx -9 N_c g^2 T. \quad (91)$$

10. THE GAUGE DEPENDENCE

In this section we consider the problem on the gauge dependence of nonlinear Landau damping rate of longitudinal oscillations. Let us compare derived expression for $\gamma^l(k)$ (68) with the kernel (69) in covariant gauge, with similar expression computed in temporal gauge $A_0^a = 0$.

As we have mentioned in Introduction, the first nonlinear Landau damping rate in temporal gauge was calculated in [8]. The inaccuracies in calculations were made in obtaining of equation for the second correction of a gauge field $A^{(2)}(k)$ (formula (3.19) in [8]). The elimination of these inaccuracies leads to the expression

$$(\omega^{(0)})^2 e(\omega^{(0)}, k) A^{(2)}(k) = g \sum_{k_1+k_2=k} \frac{k \cdot k_1 k \cdot k_2}{K_1 K_2} \frac{1}{K} [A^{(1)}(k_1), A^{(1)}(k_2)] -$$

$$-g^3 \int \frac{d^3 p}{(2\pi)^3 E_p} \left[ N_f \left( \frac{df^{(0)}(p)}{dE_p} + \frac{dJ^{(0)}(p)}{dE_p} \right) + 2N_c \frac{dG^{(0)}(p)}{dE_p} \right] \frac{1}{p \cdot k^{(0)} + ip^{(0)+}} \sum_{k_1+k_2=k} \left\{ \frac{1}{2p^0} \frac{p \cdot k_2}{p \cdot k_2^{(0)} + ip^{(0)+}} \right\} \frac{p \cdot k}{K_1} \frac{p \cdot k_1}{K_1} \frac{p \cdot k_2}{K_2} [A^{(1)}(k_1), A^{(1)}(k_2)]. \quad (92)$$

Here, we use notations accepted in [8]. The distinction (92) from (3.19) in [8] is as follows. Firstly - this is the availability of the first term in the right-hand side of (92), that is connected with self-action of a gauge field, and is of the same order in a soft region of excitations as the second term, connected with medium effects. As shown above this term also contributes to process of the nonlinear scattering of plasmons by QGP particles.
Second distinction lies in fact that instead of expression in braces in the second term in the right-hand side of Eq. (92) in [8] the following expression

\[
\frac{\omega^{(0)}_2}{p \cdot k^{(0)}_2 + ip^0} + \omega^{(0)}_1
\]

is used. The principal point is presence of the factor 1/2 in our case. This factor enables us finally to lead the expression for nonlinear Landau damping rate in the temporal gauge to the form that is similar to (88), (89). Further we obtain the expression for \(\gamma^l(k)\) in this gauge following our reasoning, do not repeating calculations in [8].

In temporal gauge the Yang-Mills equation (2) has the form

\[
\partial_\mu F^{\mu\nu}(x) = ig [A_\mu(x), F^\mu\nu(x)] - \xi^{-1} u^\nu u^\mu A_\mu(x) = -j^\nu(x).
\]

Here, as the fixed four-vector in the gauge condition \(n_\mu A^\mu = 0\) we choose the four-velocity of plasma: \(n_\mu \equiv u_\mu\).

In this gauge instead of the propagator (48) we have

\[
D_{\mu\nu}(k) = -[k^2 g_{\mu\nu} - k_\mu k_\nu + \xi^{-1} u_\mu u_\nu - \Pi_{\mu\nu}(k)]^{-1} =\]

\[
= - \frac{P_{\mu\nu}}{k^2 - \Pi^l(k)} - \frac{\tilde{Q}_{\mu\nu}}{k^2 - \Pi^t(k)} - \xi \frac{k^2}{(ku)^2} D_{\mu\nu}.
\]

In obtaining of the last equality in (93) we use relation [15]

\[
u_\mu u_\nu = \frac{k^2}{k^4} \tilde{Q}_{\mu\nu} - \frac{(ku)}{k^4} \sqrt{-2k^2 \tilde{u}^2} C_{\mu\nu} + \frac{(ku)^2}{k^2} D_{\mu\nu}
\]

and introduce notation

\[
\tilde{Q}_{\mu\nu} \equiv Q_{\mu\nu} + \sqrt{-2k^2 \tilde{u}^2} \frac{k^2}{k^4} \sqrt{-2k^2 \tilde{u}^2} C_{\mu\nu} + \frac{\tilde{u}^2}{k^2 (ku)^2} D_{\mu\nu},
\]

where \(C_{\mu\nu} = -(\bar{u}_\mu k_\nu + \bar{u}_\nu k_\mu)/\sqrt{-2k^2 \tilde{u}^2}\). In the rest frame of a plasma, tensor \(\tilde{Q}_{\mu\nu}\) has the structure

\[
\tilde{Q}_{\mu\nu} = \frac{k^2}{\omega^2} \begin{pmatrix}
0 & 0 \\
0 & k^2/k^2
\end{pmatrix}.
\]

Further we take, \(\xi = 0\), i.e. \(A_0 = 0\) is imposed strictly. By virtue of expansion of the polarization tensor \(\Pi_{\mu\nu} = P_{\mu\nu} \Pi^t + Q_{\mu\nu} \Pi^l\) and the properties of tensor structures \(P, Q, C\) and \(D\) [15], the expression \(\Pi^l\) remains unchanged, i.e.

\[
\Pi^l = Q_{\mu\nu} \Pi_{\mu\nu} \equiv \tilde{Q}_{\mu\nu} \Pi_{\mu\nu}.
\]

Instead of expansion of the spectral density (47) now we have

\[
I_{\mu\nu} = I^l_{\mu\nu} P_{\mu\nu} + I^l_{\mu\nu} \tilde{Q}_{\mu\nu}.
\]
Connection between the spectral density $I^l_k$ and the density of the energy of longitudinal oscillations is unchanged.

From the above it is easy to appreciate that for obtaining of nonlinear Landau damping rate in the temporal gauge it will sufficiently in the expression (7.9) to replace the projector $Q$ by $\tilde{Q}$, and in the expression for $\Sigma$ (45) use propagator (93) instead of (48). Finally we obtain the same expression (68), where now in the kernel (69) instead of the scattering amplitudes $\Lambda^\Sigma(k, k_1), \Lambda^{s(I)}(k, k_1)$ and $\Lambda^{s(II)}(k, k_1)$ it is necessary to use expressions

$$
\Lambda_0^\Sigma(k, k_1) = \frac{g^2}{|k||k_1|} \frac{(kv)(k_1v)}{\omega_k - (kv)} \left[ (\omega_k^l)^2 - k^2 \right] \left[ (\omega_{k_1}^l)^2 - k_1^2 \right],
$$

$$
\Lambda_0^{s(I, II)}(k, k_1) = \frac{g}{|k||k_1|} \frac{1}{k_2^2} \left( 1 - iS_0^{(I, II)}(k, k_1) \right)
$$

respectively. Here,

$$
-iS_0^{(I)}(k, k_1) = 0,
$$

$$
-iS_0^{(II)}(k, k_1) = -g^2 \frac{k_2^2 k_1^2}{\omega_1 \omega_2} \int \frac{d^4p'}{p' k_2 + ip_0} \left( \frac{(k \partial_{p'} N_{eq})}{p' k} - \frac{(k_1 \partial_{p'} N_{eq})}{p' k_1} \right).
$$

Now we compare the scattering amplitude $\Lambda$ in the covariant gauge with appropriate scattering amplitude $\Lambda_0$ in $A_0$-gauge. Using the above-mentioned explicit expressions for amplitudes, as the result of simple calculations we obtain

$$
\Lambda - \Lambda_0 = \frac{g^2}{|k||k_1|} \frac{k^2 k_1^2 - \omega_1^2 \omega_2^2}{2 \omega_1}(k + k_1)(v - v_{\parallel}) \neq 0,
$$

(94)

where $v_{\parallel} \equiv \frac{k_2(k_2v)}{k_2^2}$. Here the term with $v_{\parallel}$ in the right-hand side depends on the contribution to the scattering amplitudes the terms with longitudinal virtual oscillation. The last expression explicitly demonstrates the gauge-noninvariant character of obtained decrement of nonlinear Landau damping.

11. CONCLUSION

In our paper it was shown that the nonlinear interaction of longitudinal eigenwaves leads to effective pumping of energy across the spectrum sideways of small wave numbers. Consequence of this fact is the inequality: $\gamma^l(0) < 0$, i.e. $k = 0$ - mode is increased. The real nonlinear absorption of the energy of plasma waves by particles in QGP is the effect of higher-order.

It is clear that growth of $k = 0$ - mode is consequence of chosen approximation. In the region of a small $|k|$ effects, described by nonlinear terms in the expansion of the color current of higher-order in the field, come into play. One of such possible nonlinear effects is known from the theory of electromagnetic plasma [11]. This is as follows.
By the effect of pumping, all plasmons will be tend to concentrate near a small $|k| = |k_0| \rightarrow 0$. However, phase space, which the plasmons are occupied, proportionaled to $|k_0|^3$ will be also highly small. By virtue of this fact the intensive collision of plasmons is arised. This is the process

$$l + l_1 \leftrightarrow l' + l_1',$$

which is to lead to the scattering of plasmons from region of a small $|k|$ and thus to suppression of increase of $k = 0$-mode. The probability of this four-plasmon interaction is defined by the preceding method from the nonlinear current of fourth order $j^{T(4)}_\mu$ with regard to the process of interaction iteration of higher-order in field.

Let us consider in more detail approximations scheme, which we use in this paper. In fact, here two various levels of the approximations are used. The first of them is connected with the employment of usual approach, developed in EMP to QGP, i.e. the standard approximation of the current in terms of the oscillations amplitude and computation of interacting field in the form of a series of a perturbation theory in a free field. However, in contrast to EMP, in our case even if the first two nonlinear orders of the color current are taken into account, vastly more terms, defining the nonlinear scattering of waves are derived. Here we use second approximation level, connected with the notions going from the papers on hard thermal loops or more precisely, the set of the orders estimates of a various terms, developed by Blaizot and Iancu [5]. This set of estimates enables us to extract the leading terms in the coupling constant from the set of obtained terms. The surprising thing is that although this terms are purely non-Abelian, all basic conclusions, performed on the basis of these terms, coincide with the appropriate results in nonlinear theory of EMP in a qualitative sense.

The obtained expression of nonlinear Landau damping rate (68), (69) is not gauge invariant, that it was explicitly shown in Sec.10. Such gauge dependence of the nonlinear damping rate is a specific character of the non-Abelian theory. In the Abelian plasma a similar problem does not arise, since the theory of the nonlinear processes in EMP is stated in terms of the gauge-invariant electric and magnetic fields only. However in the case of QGP it is impossible to avoid the work with potentials, since ones explicitly appear in the kinetic equations (4) and YM Eq. (2). Therefore the initial function in our consideration is the two-point correlative function of the form (23) (more precisely, its longitudinal part, since in this paper we have restricted ourselves to the longitudinal excitations in QGP only), weakly inhomogeneous and weakly nonstationary, having no directly physical meaning and being the gauge-noncovariant value.

By virtue of fact that the process of the nonlinear scattering of longitudinal waves is the physical process, the scattering amplitude $\Lambda$, entering in the kernel (69) is bound to be gauge-invariant (within used in this paper approximation the spectral density of energy $W_{k_1}^l$ is gauge-invariant value). However, as it was shown in Sec.10 (Eq.(94)), the scattering amplitudes calculated in the covariant and $A_0$-gauges are not coincident. The simplest analysis of the right-hand side of (94) points to the fact that difference $\Lambda - \Lambda_0$ is equal to zero if in the last parentheses in the right-hand side besides longitudinal in relation
to the vector $k_2$ component of velocity $v_{||}$ the transverse component $v_\perp$: $(v_\perp k_2) = 0$ is present. Intuitively clear that such term is arised from discarded contribution to nonlinear scattering of term with transverse virtual oscillation. However if the longitudinal and transverse virtual oscillations taken into account, already it is impossible to write $\gamma^l(k)$ in compact and transparent form identical with those of (68), (69). Therefore in this case checking of gauge invariance becomes a very nontrivial problem and it is the subject of specific research.

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