Limits on Sparticle Dark Matter

John Ellis

Abstract. Arguments are given that the lightest supersymmetric particle should be a neutralino $\chi$. Minimizing the fine tuning of the gauge hierarchy favours $\Omega_\chi h^2 \sim 0.1$. There are important constraints on the parameter space of the MSSM from the stability of the electroweak vacuum. Co-annihilation with the next-to-lightest supersymmetric particle is potentially significant. Incorporating the latest accelerator constraints from LEP and elsewhere, we find that $50 \text{ GeV} < m_\chi < 600 \text{ GeV}$ and $\tan \beta > 2.5$, if soft supersymmetry breaking parameters are assumed to be universal.

THE LIGHTEST SUPERSYMMETRIC PARTICLE IN THE MSSM

The motivation for supersymmetry at an accessible energy is provided by the gauge hierarchy problem [1], namely that of understanding why $m_W \ll m_P$, the only candidate for a fundamental mass scale in physics. Alternatively and equivalently, one may ask why $G_F \sim g^2/m_W^2 \gg G_N = 1/m_P^2$, where $M_P$ is the Planck mass, expected to be the fundamental gravitational mass scale. Or one may ask why the Coulomb potential inside an atom is so much larger than the Newton potential, which is equivalent to why $e^2 = \mathcal{O}(1) \gg m_p m_e/m_P^2$, where $m_{p,e}$ are the proton and electron masses.

One might think it would be sufficient to choose the bare mass parameters: $m_W \ll m_P$. However, one must then contend with quantum corrections, which are quadratically divergent:
\[ \delta m_{H,W}^2 = \mathcal{O} \left( \frac{\alpha}{\pi} \right) \Lambda^2 \]  

which is much larger than \( m_W \), if the cutoff \( \Lambda \) representing the appearance of new physics is taken to be \( \mathcal{O}(m_P) \). This means that one must fine-tune the bare mass parameter so that it is almost exactly cancelled by the quantum correction (1) in order to obtain a small physical value of \( m_W \). This seems unnatural, and the alternative is to introduce new physics at the TeV scale, so that the correction (1) is naturally small.

At one stage, it was proposed that this new physics might correspond to the Higgs boson being composite [2]. However, calculable scenarios of this type are inconsistent with the precision electroweak data from LEP and elsewhere. The alternative is to postulate approximate supersymmetry [3], whose pairs of bosons and fermions produce naturally cancelling quantum corrections:

\[ \delta m_W^2 = \mathcal{O} \left( \frac{\alpha}{\pi} \right) |m_B^2 - m_F^2| \]  

that are naturally small: \( \delta m_W^2 \lesssim m_W^2 \) if

\[ |m_B^2 - m_F^2| \lesssim 1 \text{TeV}^2. \]  

There are many other possible motivations for supersymmetry, but this is the only one that gives reason to expect that it might be accessible to the current generation of accelerators and in the range expected for a cold dark matter particle.

The minimal supersymmetric extension of the Standard Model (MSSM) has the same gauge interactions as the Standard Model, and the Yukawa interactions are very similar:

\[ \lambda_d Q D^c + \lambda_L E^c H + \lambda_u Q U^c \bar{H} + \mu \bar{H} H \]  

where the capital letters denote supermultiplets with the same quantum numbers as the left-handed fermions of the Standard Model. The couplings \( \lambda_{d,f,u} \) give masses to down quarks, leptons and up quarks respectively, via distinct Higgs fields \( H \) and \( \bar{H} \), which are required in order to cancel triangle anomalies. The new parameter in (4) is the bilinear coupling \( \mu \) between these Higgs fields, that plays a significant role in the description of the lightest supersymmetric particle, as we see below. The gauge quantum numbers do not forbid the appearance of additional couplings

\[ \lambda L L E^c + \lambda' L Q D^c + \lambda U^c D^c D^c \]  

but these violate lepton or baryon number, and we assume they are absent. One significant aspect of the MSSM is that the quartic scalar interactions are determined, leading to important constraints on the Higgs mass, as we also see below.
Supersymmetry must be broken, since supersymmetric partner particles do not have identical masses, and this is usually parametrized by scalar mass parameters $m_{0i}^2 |\phi_i|^2$, gaugino masses $\frac{1}{2} M_a \tilde{V}_a \cdot \tilde{V}_a$ and trilinear scalar couplings $A_{ijk} \lambda_{ijk} \phi_i \phi_j \phi_k$. These are commonly supposed to be inputs from some high-energy physics such as supergravity or string theory. It is often hypothesized that these inputs are universal: $m_{0i} \equiv m_{0i}, M_a \equiv M_{1/2i}, A_{ijk} \equiv A$, but these assumptions are not strongly motivated by any fundamental theory. The physical sparticle mass parameters are then renormalized in a calculable way:

$$m_{0i}^2 = m_{0i}^2 + C_i m_{1/2}^2, \quad M_a = \left( \frac{\alpha_a}{\alpha_{GUT}} \right) m_{1/2}$$

where the $C_i$ are calculable coefficients [4] and MSSM phenomenology is then parametrized by $\mu, m_0, m_{1/2}, A$ and $\tan \beta$ (the ratio of Higgs v.e.v.'s).

Precision electroweak data from LEP and elsewhere provide two qualitative indications in favour of supersymmetry. One is that the inferred magnitude of quantum corrections favour a relatively light Higgs boson [5]

$$m_h = 66^{+74}_{-39} \pm 10 \text{ GeV}$$

which is highly consistent with the value predicted in the MSSM: $m_h \lesssim 150 \text{ GeV}$ [6] as a result of the constrained quartic couplings. (On the other hand, composite Higgs models predicted an effective Higgs mass $\gtrsim 1 \text{ TeV}$ and other unseen quantum corrections.) The other indication in favour of low-energy supersymmetry is provided by measurements of the gauge couplings at LEP, that correspond to $\sin^2 \theta_W \simeq 0.231$ in agreement with the predictions of supersymmetric GUTs with sparticles weighing about 1 TeV, but in disagreement with non-supersymmetric GUTs that predict $\sin^2 \theta_W \sim 0.21$ to 0.22 [7]. Neither of these arguments provides an accurate estimate of the sparticle mass scales, however, since they are both only logarithmically sensitive to $m_0$ and/or $m_{1/2}$.

The lightest supersymmetric particle (LSP) is expected to be stable in the MSSM, and hence should be present in the Universe today as a cosmological relic from the Big Bang [8]. This is a consequence of a multiplicatively-conserved quantum number called $R$ parity, which is related to baryon number, lepton number and spin:

$$R = (-1)^{3B+L+2S}$$

It is easy to check that $R = +1$ for all Standard Model particles and $R = -1$ for all their supersymmetric partners. The interactions (5) would violate $R$, but not a Majorana neutrino mass term or the other interactions in $SU(5)$ or $SO(10)$ GUTs. There are three important consequences of $R$ conservation: (i) sparticles are always
produced in pairs, e.g., $pp \rightarrow q\bar{q}X$, $e^+e^- \rightarrow \mu^+\mu^-$, (ii) heavier sparticles decay into lighter sparticles, e.g., $\tilde{q} \rightarrow q\tilde{g}$, $\tilde{\mu} \rightarrow \mu\tilde{\gamma}$, and (iii) the LSP is stable because it has no legal decay mode.

If such a supersymmetric relic particle had either electric charge or strong interactions, it would have condensed along with ordinary baryonic matter during the formation of astrophysical structures, and should be present in the Universe today in anomalous heavy isotopes. These have not been seen in studies of $H$, $He$, $Be$, $Li$, $O$, $Na$, $B$ and $F$ isotopes at levels ranging from $10^{-11}$ to $10^{-29}$ [9], which are far below the calculated relic abundances from the Big Bang:

$$\frac{n_{\text{relic}}}{n_p} \gtrsim 10^{-6} \text{ to } 10^{-10} \quad (9)$$

for relics with electromagnetic or strong interactions. Except possibly for very heavy relics, one would expect these primordial relic particles to condense into galaxies, stars and planets, along with ordinary bayonic material, and hence show up as an anomalous heavy isotope of one or more of the elements studied. There would also be a ‘cosmic rain’ of such relics [10], but this would presumably not be the dominant source of such particles on earth. The conflict with (9) is sufficiently acute that the lightest supersymmetric relic must presumably be electromagnetically neutral and weakly interacting [8]. In particular, I believe that the possibility of a stable gluino can be excluded. This leaves as candidates for cold dark matter a sneutrino $\tilde{\nu}$ with spin 0, some neutralino mixture of $\tilde{\gamma}/H^0/\tilde{Z}$ with spin 1/2, and the gravitino $\tilde{G}$ with spin 3/2.

LEP searches for invisible $Z^0$ decays require $m_{\tilde{g}} \gtrsim 43$ GeV [11], and searches for the interactions of relic particles with nuclei then enforce $m_{\tilde{g}} \gtrsim$ few TeV [12], so we exclude this possibility for the LSP. The possibility of a gravitino $\tilde{G}$ LSP has attracted renewed interest recently with the revival of gauge-mediated models of supersymmetry breaking [13], and could constitute warm dark matter if $m_{\tilde{G}} \simeq 1$ keV. In this talk, however, I concentrate on the $\tilde{\gamma}/H^0/\tilde{Z}$ neutralino combination $\chi$, which is the best supersymmetric candidate for cold dark matter.

The neutralinos and charginos may be characterized at the tree level by three parameters: $m_{1/2}$, $\mu$ and $\tan\beta$. The lightest neutralino $\chi$ simplifies in the limit $m_{1/2} \rightarrow 0$ where it becomes essentially a pure photino $\tilde{\gamma}$, or $\mu \rightarrow 0$ where it becomes essentially a pure higgsino $\tilde{H}$. These possibilities are excluded, however, by LEP and the FNAL Tevatron collider [11]. From the point of view of astrophysics and cosmology, it is encouraging that there are generic domains of the remaining parameter space where $\Omega_\chi h^2 \simeq 0.1$ to 1, in particular in regions where $\chi$ is approximately a $U(1)$ gaugino $\tilde{B}$, as seen in Fig. 1 [14].

Purely experimental searches at LEP enforce $m_\chi \gtrsim 30$ GeV, as seen in Fig. 2 [15]. This bound can be strengthened by making various theoretical assumptions, such as
FIGURE 1. Regions of the \((\mu, M_2)\) plane in which the supersymmetric relic density may lie within the interesting range \(0.1 \leq \Omega h^2 \leq 0.3\) [14].

the universality of scalar masses \(m_{\tilde{Q}_i}\), including in the Higgs sector, the cosmological dark matter requirement that \(\Omega_{\chi} h^2 \leq 0.3\) and the astrophysical preference that \(\Omega_{\chi} h^2 \geq 0.1\). Taken together as in Fig. 3, we see that they enforce

\[
m_{\chi} \gtrsim 42 \text{ GeV}
\]  

and LEP should eventually be able to establish or exclude \(m_{\chi}\) up to about 50 GeV. As seen in Fig. 4, LEP has already explored almost all the parameter space available for a Higgsino-like LSP, and this possibility will also be thoroughly explored by LEP [15].

**WHAT IS THE “NATURAL” RELIC LSP DENSITY?**

Should one be concerned that no sparticles have yet been seen by either LEP or the FNAL Tevatron collider? One way to quantify this is via the amount of fine-tuning of the input parameters required to obtain the physical value of \(m_W\) [16]:

\[
\Delta_o = Max_i \left| \frac{a_i}{m_W} \frac{\partial m_W}{\partial a_i} \right|
\]  

where \(a_i\) is a generic supergravity input parameter. As seen in Fig. 5, the LEP exclusions impose [17]
FIGURE 2. Experimental lower limit on the lightest neutralino mass, inferred from unsuccessful chargino and neutralino searches at LEP [15].

\[ \Delta_o \gtrsim 8 \] \hspace{1cm} (12)

Although fine-tuning is a matter of taste, this is perhaps not large enough to be alarming, and could in any case be reduced significantly if a suitable theoretical relation between some input parameters is postulated [17].

It is interesting to note that the amount of fine-tuning \( \Delta_o \) is minimized when \( \Omega_\chi h^2 \sim 0.1 \) as preferred astrophysically, as seen in Fig. 6 [18]. This means that solving the gauge hierarchy problem naturally leads to a relic neutralino density in the range of interest to astrophysics and cosmology. I am unaware of any analogous argument for the neutrino or the axion.

**IS OUR ELECTROWEAK VACUUM STABLE?**

For certain ranges of the MSSM parameters, our present electroweak vacuum is unstable against the development of vev’s for \( \tilde{q} \) and \( \tilde{l} \) fields, leading to vacua that would break charge and colour conservation. Among the dangerous possibilities are flat directions of the effective potential in which combinations such as \( L_i Q_3 D_3 \), \( H_2 L_i \), \( LLE \), \( H_2 L \) acquire vev’s. Avoiding these vacua imposes con-
FIGURE 3. Theoretical lower limits on the lightest neutralino mass, obtained by using the unsuccessful Higgs searches (H), the cosmological upper limit on the relic density (C), the assumption that all input scalar masses are universal, including those of the Higgs multiplets (UHM), and combining this with the cosmological upper (cosmo) and astrophysical lower (DM) limits on the cold dark matter density [11].

Constraints that depend on the soft supersymmetry breaking parameters: they are weakest for $A \simeq m_{1/2}$. Figure 7 illustrates some of the resulting constraints in the $(m_{1/2}, m_0)$ plane, for different values of $\tan \beta$ and signs of $\mu$ [19]. We see that they cut out large parts of the plane, particularly for low $m_0$. In combination with cosmology, they tend to rule out large values of $m_{1/2}$, but this aspect needs to be considered in conjunction with the effects of co-annihilation, that are discussed in the next section.

CO-ANNIHILATION EFFECTS ON THE RELIC DENSITY

As $m_\chi$ increases, the LSP annihilation cross-section decreases and hence its relic number and mass density increase. How heavy could the LSP be? Until recently, the limit given was $m_\chi \lesssim 300$ GeV [20]. However, it has now been pointed out that there are regions of the MSSM parameter space where co-annihilations of the $\chi$ with the stau slepton $\tilde{\tau}$ could be important, as seen in Fig.8 [21]. These co-annihilations would suppress $\Omega_\chi$, allowing a heavier neutralino mass, and we now find that [21]

$$m_\chi \lesssim 600 \text{ GeV}$$

is possible if we require $\Omega_\chi h^2 \leq 0.3$. In the past, it was thought that all the cosmologically-preferred region of MSSM parameter space could be explored by
FIGURE 4. The regions of the $(\mu, M_2)$ plane where the lightest supersymmetric particle may still be a Higgsino, taking into account the indicated LEP constraints [14]. The Higgsino purity is indicated by $p^2$.

the LHC [22], as seen in Fig. 9, but it now seems possible that there may be a delicate region close to the upper bound (13). This point requires further study.

CURRENT LEP CONSTRAINTS

The LEP constraints on MSSM particles have recently been updated [15], constraining the parameter space and hence the LSP. The large luminosity accumulated during 1998 has enabled the lower limit on the chargino mass to be increased essentially to the beam energy: $m_{\chi^\pm} \gtrsim 95$ GeV, except in the deep Higgsino region, where the limit decreases to about 90 GeV because of the small mass difference between the chargino and the LSP, which reduces the efficiency for detecting the $\chi^\pm$ decay products. There are also useful limits on associated neutralino production $e^+e^- \to \chi\chi'$, which further constrain the LSP. Without further theoretical assumptions, the purely experimental lower limit on the neutralino mass has become

$$m_{\chi} \gtrsim 32 \text{ GeV}$$  \hspace{1cm} (14)

for large values of $m_0$ whatever the value of $\tan \beta$, decreasing to a minimum of 28 GeV for small $m_0$.

There are other new LEP limits that come into play with supplementary theoretical assumptions. These include a lower limit on the slepton mass, assuming universality ($m_l \equiv m_\ell = m_\tilde{\mu} = m_\tilde{\tau}$):

$$m_l > 90 \text{ GeV}$$  \hspace{1cm} (15)

for $m_l - m_{\chi} \gtrsim 5$ GeV. There is also a new lower limit
FIGURE 5. The fine-tuning price $\Delta_0$ imposed by LEP for $\tan\beta = 10$, as a function of model parameters [17].

\begin{equation}
m_t > 85 \text{ GeV}
\end{equation}

assuming the dominance of $\tilde{t} \rightarrow c\chi$ decay, for $m_t - m_\chi \gtrsim 10 \text{ GeV}$. Most important, however, is the new lower limit on the mass of the lightest Higgs boson in the MSSM. The L3 collaboration reports

\begin{equation}
m_h > 95.5 \text{ GeV}
\end{equation}

for $\tan\beta \lesssim 3$. Combining all four LEP experiments, the lower limit (17) would probably be increased to 98 GeV, corresponding to the kinematic limit $\sqrt{s} (= 189 \text{ GeV}) - m_Z$.

The MSSM Higgs and other limits now appear to effectively exclude the possibility of Higgsino dark matter. Moreover, for $\mu < 0$, we now find $\tan\beta \gtrsim 3.0$, whereas a slightly smaller value is allowable if $\mu > 0$. For values of $\tan\beta$ close to these lower limits, the lower limit on $m_\chi$ increases sharply, qualitatively as in Fig. 6 but now shifted to the right. The valley in Fig. 6a for $\mu < 0$ is now filled in, so, pending a more complete evaluation, we estimate that

\begin{equation}
m_\chi \gtrsim 50 \text{ GeV}
\end{equation}

for either sign of $\mu$. 
FIGURE 6. The correlation between the fine-tuning price $\Delta_0$ and the relic density $\Omega h^2$, showing dependences on model parameters [18].

SUMMARY

We have seen that current experimental constraints impose $m_\chi \gtrsim 50$ GeV if universal soft supersymmetry breaking mass parameters are assumed, and that $m_\chi \lesssim 600$ GeV if we require $\Omega_\chi h^2 \leq 0.3$. Values of $m_\chi$ close to the lower limit may be explored by forthcoming runs of LEP in 1999 and 2000: the searches for the Higgs boson will be particularly interesting to follow. Thereafter, Run II of the Tevatron collider has the best accelerator chances to find supersymmetry, until the LHC comes along.

In the mean time, non-accelerator searches looking directly for LSP-nucleus scattering or indirectly at LSP annihilation products will be offering stiff competition. There is already one direct search that does not claim not to have observed LSP-nucleus scattering [23]. The possible signal would correspond to a domain of MSSSM parameter space close to the present limits. The LSP interpretation of the signal is not yet generally accepted, since a complete annual modulation cycle has not yet been reported. However, healthy scepticism should not obscure the fact that it is consistent with the limits on sparticle dark matter reported here. Time only will tell whether accelerator or non-accelerator experiments will win the race to discover supersymmetry.
FIGURE 7. The change in the domain of parameter space allowed by the requirements $0.1 \leq \Omega h^2 \leq 0.3$ after (shaded region) and before (dashed lines) including $\tilde{\tau}$ co-annihilation [20].

REFERENCES


FIGURE 8. The combined cosmological and experimental constraints on the constrained MSSM, for tan$\beta = 2,10$ and both $\mu < 0$ and $\mu > 0$. The dashed contours represent current and future LEP chargino bounds, dotted contours are slepton bounds, and dot-dashed contours are Higgs bounds. The light-shaded region gives $0.1 < \Omega_\chi h^2 < 0.3$. Below the solid contour, CCB minima are present in the LLE, LH$_2$ direction. The value $A_0 = -m_{1/2}$ has been chosen, so as to minimise the area containing CCB minima [19].

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FIGURE 9. The region of the \((m_0, m_{1/2})\) plane accessible to sparticle searches at the LHC [22].

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