Long-Lived Superheavy Particles
in Dynamical Supersymmetry-Breaking Models
in Supergravity

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Abstract

Superheavy particles of masses \( \simeq 10^{13} - 10^{14} \) GeV with lifetimes \( \simeq 10^{10} - \) \( 10^{22} \) years are very interesting, since their decays may account for the ultra-high energy (UHE) cosmic rays discovered beyond the Greisen-Zatsepin-Kuzmin cut-off energy \( E \sim 5 \times 10^{10} \) GeV. We show that the presence of such long-lived superheavy particles is a generic prediction of QCD-like SU(\( N_c \)) gauge theories with \( N_f \) flavors of quarks and antiquarks and the large number of colors \( N_c \). We construct explicit models based on supersymmetric SU(\( N_c \)) gauge theories and show that if the dynamical scale \( \Lambda \simeq 10^{13} - 10^{14} \) GeV and \( N_c = 6 - 10 \) the lightest composite baryons have the desired masses and lifetimes to explain the UHE cosmic rays. Interesting is that in these models the gaugino condensation necessarily occurs and hence these models may play a role of so-called hidden sector for supersymmetry breaking in supergravity.
1 Introduction

In QCD-like SU($N_c$) gauge theories with $N_f$ flavors of quarks $Q$ and antiquarks $\bar{Q}$, any charges (like baryon number) associated with conserved vector currents are not spontaneously broken [1]. Thus, the lightest bound states of $Q$’s or $\bar{Q}$’s carrying nonvanishing baryon numbers are almost stable and they will decay into the ordinary quarks and leptons through some baryon-number violating nonrenormalizable operators suppressed by the gravitational scale $M_\ast \simeq 2.4 \times 10^{18}$ GeV. If the dynamical scale $\Lambda$ of the SU($N_c$) gauge interactions is well below the gravitational scale and $N_c$ is sufficiently large, the lifetimes of the superheavy baryons may be longer than the age of the present universe. Therefore, the presence of long-lived superheavy baryons is a generic prediction of QCD-like gauge theories for a certain parameter region of $N_c$ and $\Lambda$.

However, any (quasi-)stable particles much heavier than $O(1)$ TeV are cosmologically dangerous, since they would easily overclose the universe if they were once in the thermal equilibrium [2]. One may usually invoke some inflationary stage in the universe’s evolution to dilute the number density of such superheavy $X$ particles. If the reheating temperature after the inflation is much lower than the masses of $X$ particles one may completely neglect the thermal production of $X$ particles. It has been, however, suggested [3, 4] that if the $X$-particle masses $m_X$ are of order of the Hubble constant $H$ at the final epoch of inflation, gravitational interactions may give nonnegligible contributions to the $X$-particle production just after the end of the inflation. The numerical calculation in Ref. [3], in fact, shows that when $m_X \simeq (0.04 - 2) \times H$ a suitable amount of the $X$ particles is produced to form a part of the dark matter in the present universe. It is remarkable that the decays of such $X$ particles will generate significant effects on the spectrum of high energy cosmic ray if the lifetimes of the $X$ particles are of order of the age of the present universe.

Several events of the ultra-high energy (UHE) cosmic rays [5, 6, 7] have been recently observed beyond the Greisen-Zatsepin-Kuzmin (GZK) bound $E \sim 5 \times 10^{10}$ GeV [8]. These are naturally explained [9, 10] by the decay products of superheavy $X$ particle
of mass $m_X \simeq 10^{13} - 10^{14}$ GeV\(^1\) with lifetime $\tau_X \simeq 10^{10} - 10^{22}$ years\(^2\) if its energy density $\rho_X$ lies in the range

$$\rho_X / \rho_c \simeq 10^{-12} - 1.$$ \hspace{1cm} (1)

Here, $\rho_c \simeq 8.1 h^2 \times 10^{-47}$ GeV\(^4\) with $h \simeq 0.5 - 1.0$ is the critical density of the present universe. Since the required window of the energy density $\rho_X$ is very wide, this scenario seems very plausible and attractive.

In this paper, we construct explicit models based on supersymmetric (SUSY) SU($N_c$) gauge theories with $N_f$ flavors of quarks $Q$ and antiquarks $\bar{Q}$, in which the lightest baryons $B$ and antibaryons $\bar{B}$ have the desired mass and lifetime, that is, $m_B = m_{\bar{B}} \simeq 10^{13} - 10^{14}$ GeV and $\tau_B = \tau_{\bar{B}} \simeq 10^{10} - 10^{22}$ years. In these models the gaugino condensation necessarily occurs and hence the models may play a role of so-called hidden sector for SUSY breaking in supergravity \cite{13}. Thus, the long-lived superheavy $B$ and $\bar{B}$ are regarded as byproducts\(^3\) of the hidden sector gauge theories for dynamical SUSY breaking.\(^4\)

It should be noted here that in contrast to the non-SUSY case, the SUSY QCD-like gauge theories may yield baryon-number violating vacua due to the presence of scalar quarks. In these vacua we have no longer quasi-stable baryons. However, if quarks $Q$ and antiquarks $\bar{Q}$ have SUSY-invariant masses, the unwanted baryon-number violating vacua disappear as shown in Ref. \cite{16}. Thus, we consider the SUSY QCD-like gauge theories with massive quarks throughout this paper.

In section 2, we briefly discuss vacua of SUSY SU($N_c$) gauge theories with $N_f$ pairs of massive quarks and antiquarks. We restrict our discussion to the case of $N_f = N_c + 1$ and show that there is a unique SUSY-invariant vacuum preserving the baryon-number conservation. Thus, we always have stable baryons and antibaryons in this theory. In

\(^1\)There is an analysis which suggests $m_X \simeq 10^{12}$ GeV \cite{11}.

\(^2\)The required lifetime may be accounted for by imposing discrete gauge symmetries even if the $X$ are elementary particles \cite{12}.

\(^3\)A similar idea has been considered in connection with string theory \cite{14}.

\(^4\)We may consider non-SUSY SU($N_c$) gauge theories which cause dynamical breaking of the Peccei-Quinn symmetry at $\Lambda \simeq 10^{13}$ GeV \cite{15}. In these models we may have naturally quasi-stable baryons of masses $\sim 10^{13} - 10^{14}$ GeV, which have the required lifetimes $\tau_B \simeq 10^{10} - 10^{22}$ years for $N_c \simeq 5, 6$. The main decay mode of such baryons will be $B \rightarrow l+\text{Higgs boson or } 2\times\text{Higgs bosons}$.
section 3, we extend the above model to the supergravity, in which we introduce non-renormalizable operators. We show that possible baryon-number violating operators induce spontaneous breakdown of the baryon-number conservation and the baryon-meson mixings occur. Owing to the baryon-number violating effects even the lightest baryons are no longer stable. However, we see that the lifetimes of the baryons can be chosen as in the required range to account for the UHE cosmic rays by taking \( N_c = 8, 9 \) and 10. We also show that the gaugino condensation necessarily occurs and the SUSY may be broken in the dilaton stabilized vacua. In section 4, we argue that the lifetimes of the baryons become longer if the baryons carry nonvanishing charges of some extra symmetries. As for such symmetries we adopt the matter parity \( Z_2 \) or the discrete baryon parity \( Z_3 \), since they are often used to guarantee the stability of usual proton [17]. In these cases we find the desired lifetimes are obtained for somewhat smaller \( N_c = 6 - 9 \). The last section 5 is devoted to discussion and conclusions.

2 Supersymmetric QCD with Massive Quarks

Let us consider SUSY SU(\( N_c \)) gauge theories with \( N_f \) flavors of quarks \( Q^i_a \) and antiquarks \( \bar{Q}^a_i \), where \( a = 1, \ldots, N_c \) and \( i, \bar{i} = 1, \ldots, N_f \). We omit the color index \( a \), hereafter. We neglect the mass term for \( Q^i \) and \( \bar{Q}^a \) for the time being. Then, we have a global SU(\( N_f \)) × SU(\( N_f \)) × U(1)_V × U(1)_R symmetry. We restrict our consideration to the case \( N_f = N_c + 1 \), since the dynamics is the clearest in this case.

For \( N_f = N_c + 1 \), the low energy physics is described by canonically-normalized gauge invariant composite fields, mesons \( M^i_{\bar{j}} \simeq Q^i Q_{\bar{j}} / \Lambda \), baryons \( B_i \simeq \epsilon^{ijk \cdots} Q^j Q^k \cdots Q^l / \Lambda^{N_c - 1} \) and antibaryons \( \bar{B}^i \simeq \epsilon^{ijk \cdots} \bar{Q}_{\bar{j}} \bar{Q}_{\bar{k}} \cdots \bar{Q}_{\bar{l}} / \Lambda^{N_c - 1} \) [18]. The dynamically generated superpotential is given by

\[
W_{\text{dyn}} = B_i M^i_{\bar{j}} \bar{B}^j - \frac{1}{\Lambda^{N_c - 2}} \det M. \tag{2}
\]

The mesons \( M^i_{\bar{j}} \), baryons \( B_i \) and antibaryons \( \bar{B}^i \) are all massless to satisfy the 't Hooft anomaly matching conditions [18].

Now we introduce the mass term for quarks \( Q^i \) and antiquarks \( \bar{Q}_{\bar{i}} \). Then, the total
effective superpotential is given by
\[ W = B_i M_j^i \bar{B}^j - \frac{1}{\Lambda^{N_c-2}} \det M + m_i^j \Lambda M_j^i. \] (3)

It is a straightforward task to see a SUSY invariant vacuum:
\[ \langle M_j^i \rangle = \Lambda^{\frac{N_c-1}{N_c}} \frac{1}{\sqrt{c}} (m^{-1})^i_j, \] (4)
\[ \langle B_i \rangle = \langle \bar{B}^i \rangle = 0. \] (5)

In this vacuum the mesons \( M_j^i \), baryons \( B_i \) and antibaryons \( \bar{B}^i \) have the following mass terms
\[ W_{\text{mass}} = \Lambda^{\frac{N_c-1}{N_c}} \frac{1}{\sqrt{c}} (m^{-1})^i_j B_i \bar{B}^j - \frac{1}{2} \Lambda^{\frac{1}{N_c}} (m_i^j m_i^k - m_i^j m_i^l) M_j^i M_j^k. \] (6)

Thus, we obtain the masses for these composite fields as
\[ m_B \simeq (m \Lambda^{N_c-1})^{\frac{1}{N_c}}, \] (7)
\[ m_M \simeq (m^{N_c-1} \Lambda)^{\frac{1}{N_c}}, \] (8)
where we have assumed a common mass \( m_i^j = m \delta_i^j \), for simplicity. We will identify these composite baryons \( B_i \) and antibaryons \( \bar{B}^i \) with the long-lived superheavy \( X \) particle introduced to explain the UHE cosmic rays discovered beyond the GZK bound. Therefore, we take
\[ m_B \simeq (m \Lambda^{N_c-1})^{\frac{1}{N_c}} \simeq 10^{13} - 10^{14} \text{ GeV}. \] (9)

Using the Konishi anomaly relation [19] we determine the gaugino condensation as
\[ \langle \lambda \bar{\lambda} \rangle = \Lambda^{\frac{2N_c-1}{N_c}} (\det m)^{\frac{1}{N_c}} \] (10)
\[ \simeq (m^{N_c+1} \Lambda^{2N_c-1})^{\frac{1}{N_c}}. \]

This condensation may give a dominant contribution to the SUSY breaking in supergravity.
3 Extension to the Supergravity

We now extend the above model to the supergravity. So far we have considered only renormalizable interactions, but in the framework of supergravity it is quite natural to introduce nonrenormalizable interactions. Namely, we introduce in general

\[
W_{\text{tree}} = m_i^j Q^i \bar{Q}_j + \frac{b^i_{\cdot \cdot}}{M_s^{N_c - 3}} \epsilon_{\cdot \cdot k} Q^i Q^j \cdots Q^l + \frac{\bar{b}^i_{\cdot \cdot \cdot}}{M_s^{N_c - 3}} \epsilon^{\cdot \cdot j} \bar{Q}_j \bar{Q}_k \cdots \bar{Q}_l, \tag{11}
\]

with \( b_i, \bar{b}_i = O(1) \). We have omitted the other nonrenormalizable terms which are irrelevant for our purposes here. Then, the total effective superpotential is given by

\[
W = W_{\text{dyn}} + W_{\text{tree}} = B_i M_j^i \bar{B}_j - \frac{1}{M_s^{N_c - 2}} \text{det} M
\]

\[
+ m_i^j \Lambda M_j^i + b^i \left( \frac{\Lambda}{M_s} \right)^{N_c - 3} \Lambda^2 B_i + \bar{b}_i \left( \frac{\Lambda}{M_s} \right)^{N_c - 3} \Lambda^2 \bar{B}_i. \tag{12}
\]

It is a straightforward task to see a SUSY invariant vacuum:

\[
\langle M_j^i \rangle = \Lambda^{\frac{N_c - 1}{N_c}} (\text{det} m)^{\frac{1}{N_c}} (m^{-1})_{j}^{i}, \tag{13}
\]

\[
\langle B_i \rangle = -\bar{b}_i \left( \frac{\Lambda}{M_s} \right)^{N_c - 3} \Lambda^2 \bar{B}_i, \tag{14}
\]

\[
\langle \bar{B}_i \rangle = -b^i \left( \frac{\Lambda}{M_s} \right)^{N_c - 3} \Lambda^2 B_i, \tag{15}
\]

up to the leading order in \( \Lambda/M_s \). In this vacuum the mesons \( M_j^i \), baryons \( B_i \) and antibaryons \( \bar{B}_i \) have the following mass terms

\[
W_{\text{mass}} = \Lambda^{\frac{N_c - 1}{N_c}} (\text{det} m)^{\frac{1}{N_c}} (m^{-1})_{j}^{i} B_i \bar{B}_j
\]

\[
- \frac{1}{2} \Lambda^{\frac{1}{N_c}} (\text{det} m)^{\frac{1}{N_c}} (m^j_{\cdot \cdot k} m^l_{\cdot \cdot i} - m^l_{\cdot \cdot k} m^j_{\cdot \cdot i}) M_j^i M_l^k
\]

\[
- b_k \left( \frac{\Lambda}{M_s} \right)^{N_c - 3} \Lambda^{\frac{N_c + 1}{N_c}} (\text{det} m)^{\frac{1}{N_c}} m^j_{\cdot \cdot k} B_i M_j^i
\]

\[
- \bar{b}_k \left( \frac{\Lambda}{M_s} \right)^{N_c - 3} \Lambda^{\frac{N_c + 1}{N_c}} (\text{det} m)^{\frac{1}{N_c}} m^j_{\cdot \cdot k} \bar{B}_l M_j^l. \tag{16}
\]

\(^5\)There exist also baryon-number violating nonrenormalizable operators in the Kähler potential. However, they are negligible compared with the baryon-number violating operators in the superpotential.
Thus, we obtain mixing masses between the composite meson and baryon fields as

\[ m_{BM} \simeq \left( \frac{\Lambda}{M_*} \right)^{N_c-3} (m^{-1} \Lambda^{N_c+1})^{\frac{1}{N_c}}. \tag{17} \]

The diagonal masses \( m_B \) for baryons and \( m_M \) for mesons are the same as in Eq. (8) up to the leading order in \( \Lambda/M_* \). Notice that as long as \( \Lambda \ll M_* \) and \( N_c \gg 3 \) the mixings between mesons and baryons are very small.\(^6\)

Let us now discuss the decay of these \( B_i \) and \( \bar{B}^i \). When the masses \( m_M \) for the mesons are all larger than the half of those of \( B_i \) and \( \bar{B}^i \), these composite baryons should decay directly into ordinary light particles including quarks and leptons through the mixing terms in Eq. (16) together with the following nonrenormalizable operator:

\[ W = \frac{f}{M_*} Q \bar{Q} H \bar{H}. \tag{18} \]

The lifetimes of \( B_i \) and \( \bar{B}^i \) are determined as

\[ \tau_B \simeq \frac{1}{f^2} \left( \frac{M_*}{\Lambda} \right)^{2(N_c-2)} (m^{-3} \Lambda^{N_c+3})^{\frac{1}{N_c}}, \tag{19} \]

which should be taken \( \simeq 10^{10} - 10^{22} \) years to account for the UHE cosmic rays.

When \( 2m_M < m_B \), we have new decay channels \( B_i (\bar{B}^i) \rightarrow 2M_j \). Since the mesons \( M_j \) decay into ordinary light particles very quickly through the interactions Eq. (18), the lifetimes of \( B_i \) and \( \bar{B}^i \) decaying into the ordinary light particles are determined by the \( 2M_j \) decay channels which are given by

\[ \tau_B \simeq \left( \frac{M_*}{\Lambda} \right)^{2(N_c-3)} (m^{2N_c-7} \Lambda^{-N_c+7})^{\frac{1}{N_c}}. \tag{20} \]

These are somewhat shorter than the previous lifetimes Eq. (19). Thus, we conclude

\[
\begin{cases}
\tau_B^{-1} \simeq f^2 \left( \frac{\Lambda}{M_*} \right)^{2(N_c-2)} (m^{-3} \Lambda^{N_c+3})^{\frac{1}{N_c}} & \text{for } 2m_M > m_B, \\
\tau_B^{-1} \simeq \left( \frac{\Lambda}{M_*} \right)^{2(N_c-3)} (m^{2N_c-7} \Lambda^{-N_c+7})^{\frac{1}{N_c}} & \text{for } 2m_M < m_B,
\end{cases}
\tag{21}
\]

\(^6\)The baryon-number condensations in Eqs. (14, 15) induce kinetic mixings between the meson and baryon fields in the Kähler potential, which give the same-order effects as those discussed in the text. We neglect them, for simplicity, since they do not affect our main conclusions.
to have the required lifetime $\tau_B \simeq 10^{10} - 10^{22}$ years.

Now we are at the point to discuss the gaugino condensation Eq. (10). It is well known that this gaugino condensation may induce SUSY breaking together with the dilaton field stabilization in supergravity [13]. Assuming the gravitino mass $m_{3/2} \simeq 1 \text{ TeV}$ we determine the gaugino condensation scale $\langle \lambda \lambda \rangle^{1/3} \simeq 10^{13} \text{ GeV}$. From the constraints Eqs. (9, 10, 21) we obtain the following relations:

\[
\begin{align*}
\frac{1}{2} M_s^{2(N_c-2)} \langle \lambda \lambda \rangle^{\frac{2N_c}{N_c-2}} \tau_B^{-1} &= m_B^{\frac{2N_c^2-N_c+6}{N_c-2}} (\text{for } 2m_M > m_B), \\
M_s^{2(N_c-3)} \langle \lambda \lambda \rangle^{\frac{2}{N_c-2}} \tau_B^{-1} &= m_B^{\frac{2N_c^2-9N_c+16}{N_c-2}} (\text{for } 2m_M < m_B).
\end{align*}
\]

These relations are consistent with the required values of $m_B$, $\tau_B$ and $\langle \lambda \lambda \rangle$ only when the numbers of colors $N_c$ are

\[
\begin{align*}
N_c &= 8, 9, 10 \quad (\text{for } 2m_M > m_B), \\
N_c &= 9, 10 \quad (\text{for } 2m_M < m_B).
\end{align*}
\]

Here, we have assumed the coupling constant $f \simeq 1$. Then, $\Lambda$ and $m$ are determined as

\[
\begin{align*}
\Lambda &\simeq 10^{13.0} - 10^{14.5} \text{ GeV}, \\
m &\simeq 10^{10.5} - 10^{13.0} \text{ GeV}.
\end{align*}
\]

Note that these numerical values yield too large a mass term for the Higgs doublets in Eq. (18). We postpone the discussion on this point to the final section.

### 4 Models with Discrete Gauge Symmetries

In the previous section we find that the desired lifetimes of the baryons are obtained if $N_c = 8, 9$ and 10 without invoking any extra symmetries. In this section, we impose the matter parity $Z_2$ or the baryon parity $Z_3$ on our model, since these discrete gauge symmetries are often used to suppress very rapid decays of the usual proton. The charges for the minimal SUSY standard-model (MSSM) particles under the matter parity $Z_2$ and the baryon parity $Z_3$ are given in Table 1. If the composite baryons do

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7 The SUSY-breaking effects do not change the order of magnitude of $\langle \lambda \lambda \rangle$ in Eq. (10).

8 In the case that neutrinos acquire Majorana masses through operators $W = (1/M_B)\bar{l}lHH$, the anomaly-free discrete gauge symmetries are only the matter parity $Z_2$ and the baryon parity $Z_3$ [12].
not carry nonvanishing charges of the $Z_2$ and $Z_3$, the analyses are the same as in the previous section. If these composite baryons have nontrivial $Z_2$ or $Z_3$ charges, however, there are no mixing mass terms between mesons and baryons, since the linear terms of $B_i$ and $\bar{B}^i$ in Eq. (12) are forbidden (i.e. $b^i = \bar{b}_i = 0$) and hence $\langle B_i \rangle = \langle \bar{B}^i \rangle = 0$. Thus, the decay channels of $B_i$ and $\bar{B}^i$ are different from those in the previous section.

In this section, we discuss the lifetimes of the baryons with the discrete $Z_2$ or $Z_3$ [12].

First, we consider the case of the matter parity $Z_2$. The charges for the $B_i$ and $\bar{B}^i$ must be opposite for quarks $Q^i$ and antiquarks $\bar{Q}^i$ to have invariant masses. Thus, we suppose that both $B_i$ and $\bar{B}^i$ are odd under the $Z_2$. Then, the lowest dimensional operators which cause decays of the composite baryons are

$$W = \frac{1}{M^{N_c-1}_s} QQ \cdots HQ + \frac{1}{M^{N_c-1}_s} \bar{Q} \bar{Q} \cdots \bar{Q} \bar{l} H,$$  

(26)

where $Q$’s and $\bar{Q}$’s, for example, carry $Z_2$ charges $(2r+1)/N_c$ $(r \in \mathbb{Z})$ and $-(2r+1)/N_c$, respectively.\(^9\) Then, the lifetimes of $B_i$ and $\bar{B}^i$ are determined as

$$\tau_B \simeq \left( \frac{M_s}{\Lambda} \right)^{2(N_c-1)} (m\Lambda^{N_c-1})^{-\frac{1}{N_c}}.$$  

(27)

Next, let us turn to the case of the baryon parity $Z_3$. We suppose that the $B_i$ carry +1 and the $\bar{B}^i$ carry −1 of the $Z_3$ charges. Then, the lightest baryons decay into the MSSM particles through the following operator:

$$W = \frac{1}{M^{N_c}_s} \bar{Q} \bar{Q} \cdots \bar{Q} \bar{u} \bar{d} \bar{d},$$  

(28)

\(^9\)The $Z_2$ is anomaly free.

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Table 1: Charges for the MSSM particles under the matter parity $Z_2$ and the baryon parity $Z_3$. $q, \bar{u}, d, l$ and $\bar{e}$ denote $SU(2)_L$-doublet quark, up-type antiquark, down-type antiquark, $SU(2)_L$-doublet lepton and charged antilepton chiral multiplets. $H$ and $\bar{H}$ are chiral multiplets for Higgs doublets.

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<th>$q$</th>
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<td>$Z_2$</td>
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<tr>
<td>$Z_3$</td>
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where $Q$'s and $\bar{Q}$'s carry $\mathbb{Z}_3$ charges $\frac{(3r+1)}{N_c}$ and $-\frac{(3r+1)}{N_c}$, respectively. The lifetimes of $B_i$ and $\bar{B}^i$ are given by

$$\tau_B \simeq \left( \frac{M_s}{\Lambda} \right)^{2N_c} \left( m^3 A^{N_c-3} \right)^{\frac{-1}{N_c}}.$$  \hspace{1cm} (29)$$

Note that even if we assign the $\mathbb{Z}_3$ charge $-1$ for $B_i$ and $+1$ for $\bar{B}^i$, the lightest baryons decay through the operator

$$W = \frac{1}{M_s N_c} QQ \cdots \bar{u} \bar{d} \bar{d},$$ \hspace{1cm} (30)$$

so that the lifetimes are the same as in Eq. (29).

From the constraints Eqs. (9, 10, 27, 29) we obtain the following relations:

$$\begin{align*}
M_s^{2(N_c-1)} \langle \lambda \lambda \rangle - \frac{2m^2 N_c - 4}{N_c^2} \tau_B^{-1} = m_B^{N_c - 2} & \quad \text{(for } \mathbb{Z}_2), \\
M_s^{2N_c} \langle \lambda \lambda \rangle - \frac{2m^2 N_c - 4}{N_c^2} \tau_B^{-1} = m_B^{N_c - 2} & \quad \text{(for } \mathbb{Z}_3),
\end{align*}$$ \hspace{1cm} (31)$$

which are consistent with the required values of $m_B$, $\tau_B$ and $\langle \lambda \lambda \rangle$ only when the numbers of colors $N_c$ are

$$\begin{align*}
N_c &= 7, 8, 9, \quad \text{(for } \mathbb{Z}_2), \\
N_c &= 6, 7, 8 \quad \text{(for } \mathbb{Z}_3).
\end{align*}$$ \hspace{1cm} (32)$$

Therefore, when the $B_i$ and $\bar{B}^i$ carry nonvanishing charges of the matter parity $\mathbb{Z}_2$ or the baryon parity $\mathbb{Z}_3$, the required numbers of colors $N_c$ to obtain the desired lifetimes are smaller than those in the case without $\mathbb{Z}_2$ or $\mathbb{Z}_3$. Notice that $\Lambda$ and $m$ are almost the same as in Eqs. (24, 25) despite the change of $N_c$.

5 Discussion and Conclusions

In this paper we have constructed explicit models based on SUSY SU($N_c$) gauge theories with $N_f$ flavors of quarks and antiquarks which naturally accommodate the superheavy composite baryons $B$ and $\bar{B}$ introduced to account for the UHE cosmic rays beyond the GZK bound. The models contain three crucial parameters, the quark mass $m$, the dynamical scale $\Lambda$ and the number of colors $N_c$. The number of flavors $N_f$ is fixed as $N_f = N_c + 1$, for simplicity. In these models the gaugino condensation

\footnote{The $\mathbb{Z}_3$ is anomaly free.}
always occurs, which may cause the SUSY breaking in supergravity. Assuming 
\langle \lambda \lambda \rangle^{1/3} \approx 10^{13} \text{ GeV} \ (\text{for } m_{3/2} \approx 1 \text{ TeV}) \text{ and the desired properties for the composite baryons } (m_B = m_{\bar{B}} \approx 10^{13} - 10^{14} \text{ GeV and } \tau_B = \tau_{\bar{B}} \approx 10^{10} - 10^{22} \text{ years}), \text{ we have obtained } \Lambda \approx 10^{13} - 10^{14.5} \text{GeV and } N_c = 6 - 10. \text{ Namely, we have found that the required long lifetimes of superheavy } B \text{ and } \bar{B} \text{ are naturally explained in SUSY QCD-like gauge theories with large number } N_c \text{ of color degrees of freedom. Although we have restricted our analysis only to the case of } N_f = N_c + 1, \text{ it is possible to consider other cases.}

Several comments are in order. We should mention first the so-called } \mu \text{ term problem. Owing to the } Q\bar{Q} \text{ condensation, the Higgs doublets seem to have an invariant mass term } \mu H\bar{H}, \text{ where } \mu \approx f \langle Q\bar{Q} \rangle / M_* \approx (10^8 - 10^{10}) \times f \text{ GeV. Thus, in order to induce a correct vacuum-expectation values for the Higgs doublets, a negative invariant mass of order } 10^8 - 10^{10} \text{ GeV must be introduced to cancell the unwanted large mass } f \langle Q\bar{Q} \rangle / M_. \text{ Alternatively, one can solve this problem by putting the coupling constant } f \text{ of the operator } (1/M_*)Q\bar{Q}H\bar{H} \text{ very small } f \approx 10^{-5} - 10^{-7}. \text{ We see that even if it is the case, the obtained } \Lambda \text{ and } m \text{ are almost the same as in Eqs. (24, 25), and our conclusion does not change much.}^{11} \text{ The small value of } f \text{ will be understood by some axial symmetries, which may also explain the small mass } m \text{ for } Q^i \text{ and } \bar{Q}_i.^{12}

Our model is also applicable to the gauge-mediated SUSY breaking scenario \cite{20}. \text{ If the gaugino condensation causes SUSY breaking partially, the induced gravitino mass } m_{3/2} \text{ must be smaller than 1 GeV in order to suppress dangerous flavor-changing neutral currents sufficiently. We find that the desired composite baryons are obtained if } m \approx 10^{2.0} - 10^{0.7} \text{ GeV, } \Lambda \approx 10^{13.3} - 10^{16.0} \text{ GeV and } N_c = 6 - 11 \text{ for } m_{3/2} \approx 10^{13} \text{ GeV (for } m_{3/2} \approx 1 \text{ TeV).}^{11} \text{ For the case of } 2m_M > m_B, \text{ if we take } f \approx 10^{-5} - 10^{-7} \text{ the dominant operators contributing to the baryon decays are not those in Eq. (18) but direct decay operators given by}

\begin{equation}
W = \frac{1}{M_*^{N_c-1}} QQ \cdots QH\bar{H} + \frac{1}{M_*^{N_c-1}} \bar{Q}\bar{\bar{Q}} \cdots \bar{\bar{Q}} H\bar{H},
\end{equation}

which give the desired number of colors } N_c = 7, 8, 9 \text{ instead of } N_c = 8, 9, 10 \text{ obtained in the text. On the other hand, the lifetimes are independent of the value of } f \text{ for the case of } 2m_M < m_B. \text{ }^{12} \text{ } R \text{ symmetry may be an example in which } H\bar{H} \text{ has } R \text{-charge zero. We naturally obtain the terms } mQ\bar{Q}(1 + f' H\bar{H}/M_*^2) \text{ in the superpotential with a coupling } f' \text{ of order one, which may yield an appropriate } \mu \text{ term. The small mass } m \text{ is regarded as a breaking term of the } R \text{ symmetry. To have unsuppressed baryon-number violating operators in Eq. (33), for example, we assume that both of } Q^i \text{ and } \bar{Q}_i \text{ carry } R \text{-charge } 2/N_c.
100 keV – 1 GeV.

Finally, we should comment on the gauge coupling constant $\alpha_c$ of the SUSY SU($N_c$) gauge theories considered in this paper. With $N_c = 6 – 10$ and $N_f = N_c + 1$ the solution to the one-loop renormalization group equation for the gauge coupling constant $\alpha_c$ is given by

$$\alpha_c^{-1}(\Lambda) = \alpha_c^{-1}(M_*) + \frac{2N_c - 1}{2\pi} \ln \left( \frac{\Lambda}{M_*} \right).$$

(34)

Using $\alpha_c(\Lambda) \simeq \infty$ we get $\alpha_c(M_*) \simeq 0.03 – 0.05$ at the gravitational scale $M_*$. It is interesting that the value of $\alpha_c(M_*)$ is roughly consistent with the hypothesis of unification with the standard-model gauge coupling constants. That is, the larger dynamical scale $\Lambda \simeq 10^{13} – 10^{14}$ GeV compared with the usual QCD scale $\Lambda_{\text{QCD}} \simeq 0.1$ GeV is a natural consequence of the large number of colors $N_c = 6 – 10$.

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References


