SPINNING PARTICLE AS A NON-TRIVIAL ROTATING
SUPER BLACK HOLE WITH BROKEN N=2
SUPERSYMMETRY

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Abstract

A non-trivial super black hole solution is considered as representing a combined model of the Kerr spinning particle and superparticle.

Treatment is based on the broken N=2 supersymmetry in supergravity in analogue with Deser-Zumino model of broken supersymmetry in N=1 supergravity. There appears a non-linear realization of broken supersymmetry, which is very specific for the Kerr geometry and which leads to a family of the exact non-trivial rotating and charged super black hole solutions (super-Kerr-Newman solutions).

Peculiarities of the super-Kerr-Newman solutions and in particular the appearance of the short ranging traveling waves of torsion and other fields, build of the nilpotent Grassmann variables, on the pure bosonic Kerr-Newman background are discussed.

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1 Introduction

Since 1968 it has been mentioned that the Kerr-Newman solution possesses some remarkable properties which allow to consider it as a model of spinning particle [1, 2]. This point of view was also suggested by superstring theory [3, 4]. In previous papers [5, 6] we considered supergeneralization of the Kerr solution as a model of spinning particle. Such super black hole class of solutions contains fermionic degrees of freedom and possesses the combined properties of the Kerr spinning particle and superparticle models.

In the present paper we consider such non-trivial super black holes as solutions with broken supersymmetry in N=2 supergravity. We show that some of supersolutions have fermionic traveling waves and traveling waves of torsion on the Kerr-Newman background geometry.

2 Non-trivial solutions from trivial ones

The problem of non-trivial supergravity solutions is connected with the fact that any solution of Einstein gravity is indeed a trivial solution of supergravity field equations with a zero spin-3/2 field [7, 8, 9, 10]. By using a supergauge freedom of supergravity (supertranslations) one can turn the gravity solutions into a form containing spin-3/2 field. However, since this field can be gauged away by the reverse transformation, such supersolutions are indeed TRIVIAL. There existed even an opinion that all the super black hole solutions are trivial. However, some examples of the non-trivial super black hole solutions were given by Aichelburg and Güven [11, 12], and also in two dimensional dilaton supergravity by Knutt-Wehau and Mann [13].

In previous papers [5, 6] we showed that NON-TRIVIAL examples of the super-Kerr geometry can be obtained by a TRIVIAL supershift of the Kerr solution taking into account some non-linear B-slice constraints. Indeed, the complex structure of the Kerr geometry prompts how to avoid this triviality problem.

The Kerr-Schild form of the Kerr geometry [14] \( g_{ik} = \eta_{ik} + 2h_{ik}k_k \) allows to give a complex representation of the Kerr solution as a geometry generated by a complex source propagating along a complex world line \( x^l_0(\tau) \) in auxiliary Minkowski space \( \eta = \text{diag}(-, +, +, +) \). This representation shows that from complex point of view the Schwarzschild and Kerr geometries are equivalent.
and connected by a TRIVIAL complex shift.

The non-trivial twisting structure of the Kerr geometry arises as a result of the shifted real slice of the complex retarded-time construction [4]. If the real slice is passing via ‘center’ of the solution $x_0$ there appears a usual spherical symmetry of the Schwarzschild geometry. The specific twisting structure results from the complex shift of the real slice regarding the source.

Similarly, it is possible to turn a TRIVIAL super black hole solution into a NON-TRIVIAL if one finds an analogue to the real slice in superspace.

The trivial supershift can be represented as a replacement of the complex world line by a superworldline

$$X_0^i(\tau) = x_0^i(\tau) - i\theta \sigma^i \bar{\zeta} + i\zeta \sigma^i \bar{\theta}, \quad (1)$$

parametrized by Grassmann coordinates $\zeta, \bar{\zeta},$ or as a corresponding coordinate replacement in the Kerr solution

$$x^i = x^i + i\theta \sigma^i \bar{\zeta} - i\zeta \sigma^i \bar{\theta}; \quad \theta' = \theta + \zeta, \quad \bar{\theta}' = \bar{\theta} + \bar{\zeta}, \quad (2)$$

Assuming that coordinates $x^i$ before the supershift are the usual c-number coordinates one sees that coordinates acquire nilpotent Grassmann contributions after supertranslations. Therefore, there appears a natural splitting of the space-time coordinates on the c-number ‘body’-part and a nilpotent part - the so called ‘soul’. The ‘body’ subspace of superspace, or B-slice, is a submanifold where the nilpotent part is equal to zero, and it is a natural analogue to the real slice in complex case.

It has been shown in [5, 6] that reproducing the real slice procedure of the Kerr geometry in superspace we have to consider super light cone constraints

$$s^2 = [x_i - X_0^i(\tau)][x^i - X_0^i(\tau)] = 0, \quad (3)$$

and B-slice, where coordinates $x^i$ do not contain nilpotent contributions. Selecting the body and nilpotent parts of this equation we obtain three equations. The first one is the real slice condition of the complex Kerr geometry claiming that complex light cones, described by set

$$x = x_0(\tau) + \Psi \sigma \bar{\Psi}, \quad (4)$$

can reach the real slice. Here $\Psi$ is the commuting two-component spinor determining the principal null congruence of the Kerr geometry

$$k_i = P^{-1} \Psi \sigma_i \bar{\Psi}, \quad (5)$$
The nilpotent part yields two B-slice conditions

\[ [x^i - x_0^i(\tau)](\theta \sigma_i \bar{\zeta} - \zeta \sigma_i \bar{\theta}) = 0; \tag{6} \]

\[ (\theta \sigma \bar{\zeta} - \zeta \sigma \bar{\theta})^2 = 0. \tag{7} \]

Equation (6) may be rewritten using (4) in the form

\[ (\theta^\alpha \sigma_{\alpha \dot{\alpha}} \bar{\zeta^{\dot{\alpha}}} - \zeta^\alpha \sigma_{\alpha \dot{\alpha}} \bar{\theta^{\dot{\alpha}}}) \Psi^\beta \sigma_{\beta \dot{\beta}} \bar{\Psi}^{\dot{\beta}} = 0 \tag{8} \]

which yields

\[ \bar{\Psi} \bar{\theta} = 0, \quad \bar{\Psi} \bar{\zeta} = 0, \tag{9} \]

which in turn is a condition of proportionality of the commuting spinors \( \bar{\Psi}(x) \) and anticommuting spinors \( \bar{\theta} \) and \( \bar{\zeta} \), this condition providing the left null superplanes of the supercones to reach B-slice.

Finally, by introducing the Kerr projective spinor coordinate \( Y(x) \) we have \( \bar{\Psi}^2 = Y(x), \quad \bar{\Psi}^1 = 1 \), and we obtain

\[ \bar{\theta}^\dot{\alpha} = \begin{pmatrix} \bar{\theta}^i \\ Y(x) \bar{\theta}^i \end{pmatrix} \tag{10} \]

\[ \bar{\zeta}^{\dot{\alpha}} = \begin{pmatrix} \bar{\zeta}^i \\ Y(x) \bar{\zeta}^i \end{pmatrix} \tag{11} \]

It also leads to \( \bar{\theta} \bar{\theta} = \bar{\zeta} \bar{\zeta} = 0 \), and equation (7) is satisfied automatically.

Thus, as a consequence of the B-slice and superlightcone constraints we obtain a non-linear submanifold of superspace \( \theta = \theta(x), \quad \bar{\theta} = \bar{\theta}(x) \). The original four-dimensional supersymmetry is broken, and the initial supergauge freedom which allowed to turn the super geometry into trivial one is lost. Nevertheless, there is a residual supersymmetry based on free Grassmann parameters \( \theta^1, \quad \bar{\theta}^1 \).

It was mentioned that the above B-slice constraints yield in fact the non-linear realization of broken supersymmetry introduced by Volkov and Akulov [15, 16] and considered in N=1 supergravity by Deser and Zumino [17]. In terminology of broken supersymmetry the Grassmann parameters \( \zeta^\alpha(x), \quad \bar{\zeta}^{\dot{\alpha}}(x) \) represent some fermion Goldstone field on space-time which has to be eaten by spin-3/2 Rarita-Schwinger field. As a result supergauge will be fixed.

In this paper we consider the application of this approach to obtaining the superversions of the Kerr-Newman solution to N=2 supergravity formulated in [18].
3 Broken Supersymmetry in N=1 Supergravity

Here we use spinor notations of the book [16]. The indices \( i, j, k, l \ldots \) are related to coordinate, and \( a, b, c, d \ldots \) are reserved for tetrad.

The Volkov-Akulov model of non-linear realization of supersymmetry [15, 16] is based on selecting a submanifold of superspace setting the correspondence of the Grassmann coordinates

\[
\Theta = \left( \begin{array}{c} \theta^a \\ \bar{\theta}^\dot{a} \end{array} \right)
\]

(12)

to a Goldstone field \( \lambda(x) \) which is a Majorana fermion. This yields the submanifold \( \Theta(x) = b\lambda(x) \) which is non-linear in general case. The non-linear residual supertransformations are

\[
\delta_\epsilon \lambda = b^{-1}\epsilon + ib(\bar{\epsilon}\gamma^i\lambda)\partial_i\lambda(x),
\]

(13)

and contain inhomogeneous term \( b^{-1}\epsilon = \left( \begin{array}{c} \zeta^a \\ \bar{\zeta}^\dot{a} \end{array} \right) \)

Considered by Deser and Zumino case of broken N=1 supergravity is based on this model [17], and it is proposed that \( \epsilon \) admits local transformations \( \epsilon(x) \).

The Lagrangian is given by

\[
\mathcal{L} = -(i/2)\bar{\lambda}\gamma^i D_i \lambda - (i/2b)\bar{\lambda}\gamma^i\psi_i + \mathcal{L}_{sg}.
\]

(14)

where the supergravity Lagrangian is

\[
\mathcal{L}_{sg} = -e R/2k^2 - i/2\epsilon^{ijkl}\bar{\psi}_i\gamma^5\gamma_j D_k\psi_l,
\]

(15)

and

\[
D_i = \partial_i - \frac{1}{2}\omega_{ab}\Sigma^{ab}; \quad \Sigma^{ab} = \frac{1}{4}[\gamma^a, \gamma^b].
\]

(16)

\footnote{Further we use the Dirac four-component spinor notations.}

\footnote{We have omitted here cosmological term since here we shall consider only the region of massless fields.}
The Lagrangian is invariant by the above non-linear supertransformations, and tetrad $e^a$ and Rarita-Schwinger fields $\psi_i$ are transformed as follows

$$\delta_\epsilon \lambda = b^{-1}\epsilon + ib(\bar{\epsilon}\gamma^i\lambda)\partial_i\lambda,$$
$$\delta_\epsilon e^a_i = -ik\bar{\epsilon}\gamma^a\psi_i,$$
$$\delta_\epsilon \psi_i = -2kD_i\epsilon. \quad (17) (18) (19)$$

It is assumed that this construction is similar to the Higgs mechanism of the usual gauge theories. The Goldstone fermion $\lambda(x)$ can be eaten by appropriate local transformation $\epsilon(x)$ with a corresponding redefinition of the tetrad and spin-3/2 field. It means that starting from the gravity solution with zero spin-3/2 field and some Goldstone fermion field $\lambda$ one can obtain in such a way a non-trivial supergravity solution with non-linear realization of broken supersymmetry.

There are two obstacles for indirect application of this scheme to the Kerr-Newman case. First one is the electromagnetic charge which demands to change the expression for supercovariant derivative that leads to non-Majorana values for spin-3/2 field. The second one is the complex character of supertranslations in the Kerr case that also yields the non-Majorana supershifts. Thus, this scheme has to be extended to $N=2$ supergravity.

## 4 Broken supersymmetry in $N=2$ supergravity

The consistent $N=2$ supergravity described by Ferrara and Nieuwenhuizen [18] is based on the complex (non-Majorana) supergauge field $\epsilon = (\epsilon_1 + i\epsilon_2)/\sqrt{2}$ and contains a complex spin-3/2 field $\chi = (\psi + i\phi)/\sqrt{2}$ and the vector potential of electromagnetic field $A_i$.

The expression for supercovariant derivative (see Appendix B) is extended by electromagnetic contribution $F_{ab}$ and torsion terms $D_{Ni} = (k^2/2)\Sigma^{ab}k_{iab}$,

$$D_{Ci} = D_i + D_{Ni} - (ik/2\sqrt{2})F_{ab}\Sigma^{ab}\gamma_i. \quad (20)$$

The action is invariant under the following complex local supersymmetry transformations

$$\delta_\epsilon e^a_i = k(\bar{\epsilon}\gamma^a\chi_i - \bar{\chi}_i\gamma^a\epsilon) \quad (21)$$
\[ \delta_{\epsilon} \chi_i = \left( \frac{2}{k} \right) D_{Ci} \epsilon. \]  
(22)

\[ \delta_{\epsilon} A_i = -i \sqrt{2} (\bar{\epsilon} \chi_i - \bar{\chi}_i \epsilon). \]  
(23)

The extension of the expression (17) on the complex non-Majorana case is given by

\[ \delta_{\epsilon} \lambda = b^{-1} \epsilon + i b (\bar{\epsilon} \gamma^i \lambda) \partial_i \lambda, \]  
(24)

Now we have to use the above considered complex supershift and superlight-cone constraints of the Kerr geometry (10, 11) in N=2 supergravity. One should note, that the above superlightcone constraints restrict values of supershift parameters \( \bar{\zeta} \), leaving the values of \( \zeta \) free, which leads in general case to a non-Majorana spinor

\[ \epsilon = \begin{pmatrix} \zeta_{\alpha} \\ \bar{\zeta}^\alpha \end{pmatrix} \]  
(25)

corresponding to a complex supershift. Further, we should note that supercovariant derivative contains the nonlinear in \( \epsilon \) terms from torsion \( D_N \), which hinders the indirect use of finite supertranslations. At the other hand the Kerr constraints on \( \bar{\zeta} \), (11), select a submanifold displaying a remarkable nilpotency \( \bar{\zeta}^2 = 0 \). It means, that the values of \( \bar{\zeta} \) on this submanifold lie in a degenerate subalgebra of the Grassmann algebra [8, 19].

To avoid the non-linear terms from torsion we will extend this property on the four-component spinor \( \epsilon \), and will restrict supershift by the form

\[ \epsilon = 2^{-1/2} \begin{pmatrix} s \eta_{\alpha} \\ s \bar{\eta}^\alpha \end{pmatrix}, \]  
(26)

where

\[ s^2 = s^2 = 0; \quad \bar{s}s + s\bar{s} = 0, \]  
(27)

This form possesses the nilpotency \( \epsilon^2 = 0 \) that leads to cancelling the non-linear term \( D_N \), and gives rise to validity (21, 22, 23) under finite supertranslations. The formulated in N=1 case problem of the absorption of the

\[ \text{It means that the complex expansion } \zeta_{\alpha} = \Sigma \zeta_{(1)}^{\alpha} s_q + \Sigma \zeta_{(2)}^{(2)qr} s_q s_r \ldots, \text{ where } s_p s_q + s_q s_p = 0, \text{ contains there only the first non-zero term } s^2 = \bar{s}^2 = 0; \quad \bar{s}s + s\bar{s} = 0. \text{ This degeneracy take place only on the considered subspace of supershifts, however, even in a very narrow neighborhood of this subspace the full algebra has to be taken into account. In particular, by application spinor supercovariant derivatives when the Grassmann variables have to be considered as independent.} \]
Goldstone fermion $\lambda$ is reduced to a choice of supergauge field

$$\epsilon(x) = -b\lambda(x)$$

that leads to satisfying (24) and yields $\lambda' = \lambda + \delta\lambda \to 0$, if we assume that $\lambda$ has corresponding nilpotency.

Below we present few examples of supershift functions and will see that the most interesting case is when the $\lambda$ satisfies the massless Dirac equation.

5 Examples of N=2 solutions

We first represent the results in the case of arbitrary nilpotent supershift satisfying the constraints (11). It is convenient to perform all calculations in the local Lorentz frame of the Kerr geometry where $\sigma$-matrices take the form [9, 12, 20]

$$\sigma_i = 2^{1/2} \begin{pmatrix} e^3_i & e^2_i \\ e^1_i & -e^4_i \end{pmatrix} \hspace{1cm} \bar{\sigma}_i = 2^{1/2} \begin{pmatrix} -e^4_i & -e^2_i \\ -e^1_i & e^3_i \end{pmatrix}$$

(28)

The corresponding matrices with tetrad indices $a, b, c, d...$ will be given by $\sigma^a = \sigma_i e^{ai}$ and take the form

$$\sigma^a = 2^{1/2} \begin{pmatrix} \delta^a_3 & \delta^a_1 \\ \delta^a_2 & -\delta^a_3 \end{pmatrix}$$

(29)

The four component spinor

$$\phi = \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix}$$

(30)

will be called as aligned to $e^3$ if $e^3_\gamma \phi(3) = 0$ that has the consequence $A = D = 0$. Similarly, spinor $\phi^{(4)}$ is aligned to $e^4$ if $e^4_\gamma \phi^{(4)} = 0$, which yields $B = C = 0$. Therefore, any four component spinor can be split on the sum of two aligned spinors $\phi = \phi^{(3)} + \phi^{(4)}$. Physically, the $e^3$ ($e^4$) aligned solution describes the waves in the $e^3$ ($e^4$) null direction. In many cases (especially massless) solutions of these aligned parts are independent. In the algebraically special Kerr-Newman geometry all the tensor fields are aligned.
to the principal null direction \(e^3\). The supershift constraints (11) represent in fact a condition for the two-component spinor \(\bar{\zeta}\) to be aligned to \(e^3\). In the local Lorentz frame it takes the form \(\bar{\zeta}^2 = 0\), the second component \(\bar{\zeta}^1 = C\) is free. Therefore, the general form of supershift satisfying the Kerr’s superconstraints and nilpotency condition will be

\[
\epsilon = s2^{-1/2} \begin{pmatrix} \eta_\alpha \\ \bar{\zeta}_\alpha \end{pmatrix},
\]

(31)

where

\[
\eta_\alpha = \begin{pmatrix} A \\ B \end{pmatrix},
\]

(32)

and

\[
\bar{\zeta}_\alpha = \begin{pmatrix} C \\ 0 \end{pmatrix}.
\]

(33)

For the sake of convenience we also give contravariant components of \(\eta\)

\[
\eta^a = \begin{pmatrix} B \\ -A \end{pmatrix},
\]

(34)

The resulting complex Rarita-Schwinger field has contributions from metric

\[
\chi_g = \chi_{g\alpha} e^\alpha = (s\sqrt{2}/k) \begin{pmatrix} dA + AH - Bd\bar{Y} \\ dB - AG - BH \\ dC - C\bar{H} \\ CdY \end{pmatrix},
\]

(35)

and from electromagnetic field

\[
\chi_F = -i(s/\sqrt{2}) \begin{pmatrix} C\bar{N}e^3 \\ C(\bar{S}e^3 - \bar{N}e^1) \\ A(Ne^4 + Se^1) + B(Ne^2 - Se^3) \\ -ANe^1 + BNe^3 \end{pmatrix},
\]

(36)

where we have introduced the notations for the geometry parameters

\[
H = [(\bar{Z} - Z)h - h_{+4}]e^3/2; \quad G = h\bar{Z}e^1 - (h_{+2} - hY_{+3})e^3,
\]

(37)

and for the combinations of the tetrad components of electromagnetic field

\[
N = F_{12} + F_{34}, \quad S = 2F_{31}.
\]

(38)
The function $Y(x)$ is the main function determining the principal null congruence (5) and the Kerr tetrad. It can be expressed as a projective spinor coordinate $Y = \bar{\psi}^0/\bar{\psi}^1$, and satisfies the equation $dY = Z[e^1 - (P_Y/P)e^3]$, where $P = 2^{-1/2}(1 + YY)$ for the stationary Kerr-Newman background. The corresponding Kerr tetrad $e^a$ is given in the Appendix C. The resulting nilpotent contribution to tetrad contains two terms, contributions from metric and from electromagnetic field $\delta e^a = \delta g e^a + \delta F e^a$. The metric part will be

\[
\begin{align*}
\delta g e^1 &= \bar{s}s\sqrt{2}[(\bar{C}C - \bar{B}B)dY - \bar{A}d\bar{B} + Bd\bar{A} + \bar{A}\bar{A}G + \bar{A}B(H + \bar{H})] \\
\delta g e^2 &= -\delta e^1 \\
\delta g e^3 &= \bar{s}s\sqrt{2}[(\bar{A}B)dY - A\bar{B}dY + Ad\bar{A} - \bar{A}dA + A\bar{A}(\bar{H} - H)] \\
\delta g e^4 &= \bar{s}s\sqrt{2}[(\bar{C}d\bar{C} - C\bar{dC} + \bar{B}dB - Bd\bar{B} + \bar{A}\bar{B}G - \bar{A}BG + (B\bar{B} - C\bar{C})(\bar{H} - H)]
\end{align*}
\]

(39)

Electromagnetic contribution to tetrad is

\[
\begin{align*}
\delta F e^1 &= -ik\bar{s}s/2[e^1(C\bar{A}\bar{N} - \bar{C}AN) - C\bar{A}\bar{S}e^3] \\
\delta F e^2 &= -\delta F e^1 \\
\delta F e^3 &= ik\bar{s}s\sqrt{2}(C\bar{A}\bar{N} + \bar{C}AN)/2 \\
\delta F e^4 &= ik\bar{s}s(\bar{N}e^4 + Se^1)(C\bar{A} + \bar{C}A)/2
\end{align*}
\]

(40)

Nilpotent contribution to vector potential (23) also contains two terms $\delta A = \delta_D A_a e^a + \delta_F A_a e^a$,

\[
\delta g A = -i(\bar{s}s\sqrt{2}/k)(\bar{C}dA + \bar{A}d\bar{C} - Ad\bar{C} + C\bar{dC} + 2A\bar{C}H - 2\bar{A}CH + 2\bar{B}CdY - 2B\bar{C}d\bar{Y}),
\]

(41)

and

\[
\begin{align*}
\delta_F A &= \bar{s}s/2[e^1[S\bar{A}\bar{A} + (\bar{N} - \bar{N})\bar{A}\bar{B}] + e^2[S\bar{A}\bar{A} + (\bar{N} - \bar{N})\bar{A}\bar{B}] + e^3[(B\bar{B} + C\bar{C})(N + \bar{N}) - A\bar{B}\bar{S} - \bar{A}\bar{B}\bar{S}] + e^4A\bar{A}(N + \bar{N})]
\end{align*}
\]

(42)

We have the free spinor components

\[
\eta_a = \left( \begin{array}{c} A(x) \\ B(x) \end{array} \right),
\]

(43)

determining the supershift. Now we consider some particular cases leading to simplification of general expressions.

\footnote{This function is determined by the Kerr theorem \cite{4,14}. A fixation of $Y$ selects the null planes in $CM^4$ and null rays of the principal null congruence.}
5.1 Case I

The simplest spinor shift $A(x) \neq 0, \quad B = 0$, considered in the basis having the $\sigma$-matrices adapted to the auxiliary Minkowski space $\eta_{\mu\nu}$. The peculiarity of this shift is that while transformed to the aligned basis $^6$ this spinor takes the same simple form that allows to simplify expressions

$$\eta_{\alpha} = \begin{pmatrix} A(x) \\ 0 \end{pmatrix},$$

(44)

We will also assume that $C = 1$, and obtain for $\delta_g$ contribution to tetrad

$$\delta_g e^1 = \bar{s}s\sqrt{2}[dY + \bar{A}AG]$$
$$\delta_g e^2 = -\delta e^1$$
$$\delta_g e^3 = \bar{s}s\sqrt{2}[Ad\bar{A} - \bar{A}dA + A\bar{A}(\bar{H} - H)]$$
$$\delta_g e^4 = \bar{s}s\sqrt{2}(\bar{H} - H),$$

(45)

and $\delta_F$ contribution

$$\delta_F e^1 = -ik\bar{s}s/2[e^1(\bar{A}N - AN) - \bar{A}\bar{S}e^3]$$
$$\delta_F e^2 = -\delta_F e^1$$
$$\delta_F e^3 = ik\bar{s}se^3(\bar{A}N + AN)/2$$
$$\delta_F e^4 = ik\bar{s}s(Ne^4 + Se^1)(\bar{A} + A)/2.$$  (46)

Contributions to vector potential $A$ will be

$$\delta_g A = -i(\bar{s}s\sqrt{2}/k)(dA - d\bar{A} + 2AH - 2\bar{A}\bar{H}),$$
$$\delta_F A = \bar{s}s/2[(e^1 + e^2)A\bar{A} + (e^3 + e^4 A\bar{A})(N + \bar{N})].$$

(47)
(48)

One can see that the wave oscillations of the shift, $A = e^{ipix}$, can lead to the wave oscillations of the terms $\delta_F e^a$ in tetrad and the term $\delta_g A$ in the vector potential. Since in the Kerr-Newman geometry function $N$ has the behavior $N \sim Z^2$, the tetrad oscillations are growing near the Kerr singular ring and take the form of waves travelling along the singularity. However, in this case we do not have clear proposals concerning mechanisms or the possible origin of such oscillations in supershift. Some suggestion to this case follows from observation that such oscillating spinor $\eta$ resembles oscillating solutions of the Dirac equation.

$^6$Transformations of the Dirac spinors from the $e^3$-aligned basis to the basis of auxiliary Minkowski space are given in Appendix D.
5.2 Case II

The spinor shift has parameters $A = 0$, $B \neq 0$, considered in the basis where $\sigma$-matrices are adapted to the auxiliary Minkowski space $\eta_{ik}$. Being transformed to the aligned basis this spinor takes the form $A = BY$; $B \neq 0$. This case does not lead to essential simplifications in respect to the general case, and we will not write down the expressions for this case, but we should note that appearance of traveling waves for oscillating $B$ can be observed in this case, too.

5.3 Case III

The case of supershifts aligned to $e^3$: $A = 0; B \neq 0$, considered in the aligned $e^3$ basis. This case yields maximal simplifications of the expressions and is motivated by super-QED model of broken supersymmetry for the region of matter fields.

There is no electromagnetic contributions to tetrad in this case and we have

$$\delta e^1 = \bar{s}s\sqrt{2}(C\bar{C} - B\bar{B})dY$$

$$\delta e^2 = -\delta e^1$$

$$\delta e^3 = 0$$

$$\delta e^4 = \bar{s}s\sqrt{2}[(C\bar{C} - B\bar{B})(\bar{Z} - Z) + \bar{C}dC - Cd\bar{C} + B\bar{d}B - Bd\bar{B}]$$

The contribution to the Kerr-Newman vector potential will be

$$\delta_g A = -i(\bar{s}s2\sqrt{2}/k)(\bar{B}CdY - B\bar{C}d\bar{Y})$$

$$\delta_F A = -\bar{s}s/2e^3[(B\bar{B} + CC)(N + \bar{N})]$$

The case $C = \bar{B}$ could be called pseudo-Majorana. In this case there are no nilpotent contributions to tetrad at all, $\delta e^a = 0$, and metric has pure bosonic form similar to the case of extreme black holes without an angular momentum. However, the Kerr-Newman vector potential has nilpotent contributions, and moreover, it can have traveling waves from the term

$$\delta_g A = -i(\bar{s}s2\sqrt{2}/k)(\bar{B}^2dY - B^2d\bar{Y})$$

\[7\] If $\sigma$-matrices were adapted to auxiliary Minkowski frame this shift is given by parameters $A = -Y$; $B = 1$. See matrices of transformation in Appendix D.
In the case of independent $B$ and $C$ there also appears tetrad traveling waves. The interpretation of $\lambda$ as a Goldstone fermion which has to be massless support the proposal that supershift $\epsilon(x)$ has to satisfy the massless Dirac equation. In the Appendix B we have given the solutions of the massless Dirac equation on the Kerr-Newman background in the aligned to $e^3$ case. The corresponding functions $B$ and $C$ have the form

$$B = \bar{Z} f_B(\bar{Y}, \bar{\tau})/P, \quad C = Z f_C(Y, \tau)/P,$$

where $f_B$ and $f_C$ are arbitrary analytic functions, and $\tau_2 = \tau_4 = 0$, and $Y_2 = Y_4 = 0$.

The solutions are singular on the Kerr singular ring, $Z^{-1} \equiv P^{-1}(r + ia \cos \theta) = 0$, and contain traveling waves if there is an oscillating dependence on complex time parameter $\tau$. The angular dependence of these solutions on $\phi$ is determined by the degree of function $Y = e^{i\phi} \tan \theta/2$. One sees that any non-trivial analytic dependence on $Y$ will lead to a singularity in $\theta$. Thus, besides the singular ring the solution contain an extra axial singularity which is coupled topologically with the Kerr singular ring threading it. One should note that singular Rarita-Schwinger ‘hair’ was mentioned earlier by Aichelburg and Güven [12, 20] as an obstacle to form a super black hole in the case of the Kerr geometry. However, when considering this solution as a model of spinning particle we have to treat a region of parameters $a \gg m$ leading to a naked disk-like super-source [5] without horizons that allows to avoid the objections relating to rotating black holes.

One should also note that this ‘axial’ singularity can change the position in time, so that the phase speed of the position can exceed the speed of light. One simple example of such function is

$$B = Z/[f_1(\tau) - Y f_2(\tau)]^{-1},$$

where the position of the ‘axial’ singularity is determined by the root of equation $Y = f_1/f_2$ depending on the retarded-time parameter $\tau$.

\textsuperscript{8}Also see solutions in [21]

\textsuperscript{9}In some sense this singularity can scan the space time. At the other hand the topological coupling with the Kerr ring provides its guiding role for the ring that allows to consider this couple of singularities as a wave-pilot construction in the spirit of the old de Broglie ideas.
We should note, that a non-zero torsion in the super-Kerr geometry $Q^a = \tilde{\psi}_b e^b \wedge \gamma^a \psi e^c$ gives rise to corresponding traveling waves of torsion.  \(^1\)

In general case $Q^a$ contains contributions from metric $Q^a_g$, electromagnetic field $Q^a_F$, and terms of interplay $Q^a_I$. We write down here the torsion terms only for the simplest and the most interesting case III, when $A = 0$. Contributions from metric are

\[ Q^1_g = i\tilde{s}s\sqrt{2}/k^2 (Cd\bar{C} - C\bar{C}H - \bar{B}d\bar{B} + \bar{B}BH) \wedge dY \]
\[ Q^2_g = \bar{Q}^1_g \]
\[ Q^3_g = i\tilde{s}s\sqrt{2}/k^2 (C\bar{C} - \bar{B}\bar{B})dY \wedge d\bar{Y} \]
\[ Q^4_g = i\tilde{s}s\sqrt{2}/k^2 [(d\bar{C} - \bar{C}H) \wedge (dC - C\bar{H}) + (d\bar{B} - \bar{B}\bar{H}) \wedge (d\bar{B} - \bar{B}\bar{H})] \]

Contributions from electromagnetic field are

\[ Q^1_F = i\tilde{s}s/(2\sqrt{2})N\bar{N}(B\bar{B} - C\bar{C})e^1 \wedge e^3 \]
\[ Q^2_F = \bar{Q}^1_F \]
\[ Q^3_F = 0 \]
\[ Q^4_F = i\tilde{s}s/(2\sqrt{2})(B\bar{B} - C\bar{C})(\bar{N}e^1 - \bar{S}e^3) \wedge (Ne^2 - Se^3). \] \(^{(58)}\)

Terms of interplay are

\[ Q^1_I = \tilde{s}s/(k\sqrt{2})N(Bd\bar{C} - \bar{C}dB) \wedge e^3 \]
\[ Q^2_I = \bar{Q}^1_I \]
\[ Q^3_I = 0 \]
\[ Q^4_I = \tilde{s}s/(k\sqrt{2})[B(d\bar{C} - \bar{C}H) \wedge (Ne^2 - Se^3) - C(d\bar{B} - \bar{B}\bar{H}) \wedge (\bar{N}e^1 - \bar{S}e^3) + \text{d(59)}] \]

In the case $\bar{B} = C$ all the torsion terms disappear.

6 Conclusion

We have represented three families of the rotating and charged super black hole solutions which are supergeneralizations of the Kerr-Newman solution.\(^\text{10}\)

\(^{10}\)The Kerr-Newman solution with torsion based on the Poincaré gauge theory has been considered in [22].
and represent a combined model of the Kerr spinning particle and superparticle [5].

Treatment is based on the partially broken supersymmetry in N=2 supergravity with a non-linear realization of broken supersymmetry, which is very specific for the Kerr geometry, and which leads to a family of the exact non-trivial rotating and charged super black hole solutions.

Main peculiarity of this super-Kerr-Newman geometry is the appearance of the traveling waves, build of the nilpotent Grassmann fields, on the bosonic Kerr-Newman background. In contrast to traveling waves in the Einstein–Maxwell theory these fields are short range and do not lead to radiative catastrophes. However, the connected with these fields axial singular line needs a more detailed investigation.

The considered families of solutions are depending on the Goldstone fermion field $\lambda$. At present stage of investigation our knowledge regarding the origin of Goldstone fermions is very restricted. Usually, their origin is connected with the breaking of supersymmetry in the region of matter fields. For example, spontaneously broken supersymmetry in the Wess-Zumino model of super-QED [16] leads to massless Goldstone fermions, and their sources are connected with massive Dirac fields.

When considering the super black hole solution we are interested mainly in the region of massless fields. However, as it was mentioned in [5, 6], for the known parameters of spinning particles the angular momentum is very high, regarding the mass parameter. In this case the black hole horizons disappear and a ‘hard core’ region of superconducting disk-like source has to be taken into account. Treatment of this region is going out of the frame of this paper. However, we should note that investigations of such super-sources should be connected with Seiberg-Witten theory [23] and with some other interesting models of broken supersymmetry together with gauge symmetry breaking [24, 25, 26, 27].

7 Acknowledgments

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Appendix A. Some relations of DKS-formalism [14]

The rotation coefficients can be obtained from the following independent forms

$$\Gamma_{42} = -Ze^1 - Y; 3 e^3;$$
$$\Gamma_{12} + \Gamma_{34} = [h_{4} + (\bar{Z} - Z)\hbar]e^3;$$
$$\Gamma_{31} = hZ e^2 + (h_{4} - h\bar{Y}; 3 e^3),$$

(60)

taking into account property of skew-symmetry $\Gamma_{ab} = -\Gamma_{ba}$ and relations to the complex conjugate forms, for example: $\Gamma_{12} = \bar{\Gamma}_{21}$; $\Gamma_{42} = \bar{\Gamma}_{41}$; $\Gamma_{32} = \bar{\Gamma}_{31}$.

Basic relations:

$$P = 2^{-1/2}(1 + Y\bar{Y}); \quad Y = e^{i\phi} \tan \frac{\theta}{2}$$

(61)

$$Z_{+4} = -Z^2; \quad \bar{Z}_{-4} = -\bar{Z}^2; \quad Z_{+2} = (Z - \bar{Z})Y_{3};$$

(62)

$$Y_{+1} = Z; \quad Y_{-2} = Y_{+4} = 0; \quad \bar{Y}_{-2} = \bar{Z}; \quad \bar{Y}_{+1} = \bar{Y}_{-4} = 0;$$

(63)

$$Y_{-3} = -ZP_{Y}/P = -Z\bar{Z}^{-1}ZP_{2}/P;$$

(64)

Some other useful relations:

$$P_{Y}P_{Y}/P^2 = \frac{1}{4} \sin^2 \theta$$

(65)

$$P_{Y} - r_{1} = -ia(\cos \theta),_{1} = -iaZ^{1/2}P_{Y}/P^2$$

(66)

$$(r_{+1} - P_{Y})e^1 + (r_{-2} - P_{Y})e^2 = ia2^{1/2}/P^2[P_{Y}d\bar{Y} - P_{Y}dY] =$$

$$4aP_{Y}P_{Y}/P^2d\phi = a \sin^2 \theta d\phi$$

(67)

$$(r_{+1} - P_{Y})(r_{-2} - P_{Y}) = 2a^{2}P_{Y}P_{Y}Z\bar{Z}/P^{4}$$

$$= a^{2} \sin^2 \theta/(r^{2} + a^{2} \cos^2 \theta)$$

(68)

The complex radial coordinate $\tilde{r} = r + ia \cos \theta = PZ^{-1}$.

Some tetrad derivatives

$$Z_{+1} = (Z^3/P)F_{YY}'' + 2Z^2P_{Y}/P$$

(69)

$$\bar{Z}_{-2} = (\bar{Z}^3/P)\bar{F}_{YY}'' + 2\bar{Z}^2P_{Y}/P$$

(70)

$$(\ln \bar{Z}/P)_{-2} = \bar{Z}^22A/P - Y_{3}\bar{Z}/Z$$

(71)

$$Z_{-3}/Z = -(Z_{+1}/Z)P_{Y}/P + hZ - ZP_{Y}/P + (Z - \bar{Z})P_{Y}P_{Y}/P^{2}$$

(72)
Appendix B. Aligned solutions of the Dirac equation

The Dirac spinor in represented in general form

\[ \Psi_D = \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \tag{73} \]

The supercovariant derivative in complex N=2 case has the form

\[ \mathcal{D}C_i = \mathcal{D}_i + \mathcal{D}_{N_i} - (ik/2\sqrt{2})F_{ab}\Sigma^{ab}\gamma_i. \tag{74} \]

The metric part of the supercovariant derivative is

\[ \mathcal{D}_i = \partial_i - \frac{1}{2}\Gamma_{abi}\Sigma^{ab}. \]

For the Ricci rotation coefficients we use notations of DKS-paper [14] \( \omega_{iab} = -\Gamma_{abi} \), see Appendix A.

\[ \frac{1}{2}\Gamma_{k\ell}\Sigma^{\ell\ell}\Psi_D = \Gamma_{12}\Sigma^{12}\Psi_D + \Gamma_{31}\Sigma^{31}\Psi_D + \Gamma_{41}\Sigma^{41}\Psi_D + \Gamma_{42}\Sigma^{42}\Psi_D + \Gamma_{34}\Sigma^{34}\Psi_D + \Gamma_{32}\Sigma^{32}\Psi_D. \tag{75} \]

The Dirac equation \( \gamma^i\mathcal{D}C_i\Psi_D = 0 \).

A special class of \( e^3 \)-aligned solutions satisfying the constraint \( \gamma^i e_i^3 \Psi_D = 0 \), has the form

\[ \Psi_D = \begin{pmatrix} 0 \\ B \\ C \\ 0 \end{pmatrix}. \tag{76} \]

For the \( e^3 \)-aligned solutions the Kerr-Newman electromagnetic field drops out from the Dirac equation. In the nilpotent case (31) the nonlinear term drops out too. The spinor-valued 1-form expression \( \mathcal{D}\Psi_D \) takes the form

\[ \mathcal{D}\Psi_D = \begin{pmatrix} -B(\bar{Z}e^2 + \bar{Y}e^3) \\ dB + B[h_{+4} + (Z - \bar{Z})h/2]e^3 \\ dC + C[h_{+4} - (Z - \bar{Z})h/2]e^3 \\ C(Ze^1 + Ye^3) \end{pmatrix}. \tag{77} \]
As a result, the Dirac equation $\gamma^a D_a \Psi_D = 0$ yields the four equations:

$$ C_{,4} + ZC = 0; \quad C_{,2} - CY_{,3} = 0, \quad (78) $$

and

$$ B_{,4} + \bar{Z}B = 0; \quad B_{,1} - B\bar{Y}_{,3} = 0, \quad (79) $$

These equations can be easily solved by using the known basis relations of the Kerr-Schild formalism (See Appendix A).

First equation (78) gives $C = ZC_0$ where $C_{0,4} = 0$. Then, substituting $C$ and into second equation we obtain $C = \phi Z/P$, where $\phi$ must satisfy the conditions $\phi_{,2} = \phi_{,4} = 0$. Therefore, $\phi$ must be a function taking constant values on the null planes spanned by vectors $e^1$ and $e^3$. These null planes are "left" null planes of a foliation of space-time into complex null cones. In another terminology they represent a geometrical image of twistors and can be parametrized by twistor coordinates. All the twistor coordinates $\chi$ and $\mu = x^i \sigma_i \chi$ satisfy the relations $(...)_{,2} = (...)_{,4} = 0$, see [4]. Coordinate $Y$ is in fact one of the (projective) twistor coordinates. For our problem it will be convenient to use a retarded time coordinate $\tau$. The retarded time coordinate to a point $x$ is defined by a point of intersection of light cone emanated from $x$ with a world-line of source.

The light cone is split on the "left" and "right" null planes, therefore, the retarded time parameter takes constant values on the null cone. Besides, it satisfy the relations $\tau_{,2} = \tau_{,4} = 0$ on the "left" null planes. For the Kerr geometry one can use a known complex interpretation containing a complex world line for the source. In the rest frame the complex light cone equation $(t-\tau)^2 = \tilde{r}^2$ can be split with a selection of retarded fold $\tilde{r} = t - \tau$. Here $t$ is a real time coordinate, and $\tilde{r} = r + ia \cos \theta = P/Z$ is a complex radial distance from the real point $x$ to a point of source at the complex world line. It yields $\tau = t - \tilde{r}$. Therefore, the function $\phi$ can be an arbitrary analytic function of complex coordinates $Y$ and $\tau$, and we have solution $C = \phi(Y, \tau)Z/P$. For function $B$ one can obtain similarly $B = \bar{\phi}(\bar{Y}, \bar{\tau})\bar{Z}/P$, where coordinates $\bar{Y}$ and $\bar{\tau}$ satisfy the relations $(...)_{,1} = (...)_{,4} = 0$, and are constant on the "right" null planes of the light cone foliation.

\footnote{Details of this construction can be found in [4].}
Appendix C. The Kerr tetrad and some useful matrix expressions

The Kerr tetrad $e^a$ is determined by function $Y(x)$:

\[
\begin{align*}
e^1 &= d\zeta - Ydv, \\
e^2 &= d\bar{\zeta} - \bar{Y}dv, \\
e^3 &= du + \bar{Y}d\zeta + Yd\bar{\zeta} - Y\bar{Y}dv, \\
e^4 &= dv + h e^3,
\end{align*}
\]

where the null Cartesian coordinates are used $\sqrt{2}u = z + t$, $\sqrt{2}v = z - t$, $\sqrt{2}u = x + iy$, $\sqrt{2}v = x - iy$.

$\sigma$-matrices in the local Kerr geometry

\[
\sigma_i = 2^{1/2} \begin{pmatrix} e^3_i & e^2_i \\ e^1_i & -e^4_i \end{pmatrix} \quad \bar{\sigma}_i = 2^{1/2} \begin{pmatrix} -e^4_i & -e^2_i \\ -e^1_i & e^3_i \end{pmatrix}
\]

The corresponding matrices with tetrad indices $a, b, c, d...$ will be given by $\sigma^a = \sigma_i e^{ai}$ and take the form

\[
\sigma^a = 2^{1/2} \begin{pmatrix} \delta^a_1 & \delta^a_2 \\ \delta^a_2 & -\delta^a_3 \end{pmatrix}
\]

\[
\Sigma^{ab} = \frac{1}{4}(\gamma^a \gamma^b - \gamma^b \gamma^a) = \frac{1}{4} \begin{pmatrix} \sigma^a \sigma^b - \sigma^b \sigma^a & 0 \\ 0 & \bar{\sigma}^a \sigma^b - \bar{\sigma}^b \sigma^a \end{pmatrix}
\]

\[
\Sigma^{12} = \frac{1}{2} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \Sigma^{14} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\]

\[
\Sigma^{24} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \Sigma^{31} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}
\]

\[
\Sigma^{32} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \Sigma^{34} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\]
Appendix D. Transformations of the Dirac spinors from the $e^3$-aligned basis to the basis of auxiliary Minkowski space

Transformations of the Dirac spinors from the $e^3$-aligned basis to the basis of auxiliary Minkowski space are given by matrices

$$\left( \begin{array}{cc} M^{-1} & 0 \\ 0 & M^\dagger \end{array} \right) \left( \begin{array}{c} \phi_{al} \\ \bar{\chi}_{al} \end{array} \right) = \left( \begin{array}{c} \phi_{\text{Mink}} \\ \bar{\chi}_{\text{Mink}} \end{array} \right),$$

(85)

where

$$M = \left( \begin{array}{cc} 1 & \tilde{Y} \\ 0 & 1 \end{array} \right),$$

(86)

$$M^{-1} = \left( \begin{array}{cc} 1 & -\tilde{Y} \\ 0 & 1 \end{array} \right),$$

(87)

$$M^* = \left( \begin{array}{cc} 1 & Y \\ 0 & 1 \end{array} \right),$$

(88)

$$M^\dagger = \left( \begin{array}{cc} 1 & 0 \\ Y & 1 \end{array} \right),$$

(89)
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