The visible effect of a very heavy magnetic monopole at colliders

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Abstract

If a heavy Dirac monopole exists, the light–to–light scattering below the monopole production threshold is enhanced due to strong coupling of monopoles to photons. At the next Linear Collider with electron beam energy 250 GeV this photon pair production could be observable at monopole masses less than 2.5–6.4 TeV in the $e^+e^-$ mode or 3.7–10 TeV in the $\gamma\gamma$ mode, depending on the monopole spin. At the upgraded Tevatron such an effect is expected to be visible at monopole masses below 1–2.5 TeV. The strong dependence on the initial photon polarizations allows to find the monopole spin in experiments at $e^+e^-$ and $\gamma\gamma$ colliders. We consider the $Z\gamma$ production and the $3\gamma$ production at $e^+e^-$ and $pp$ or $p\bar{p}$ colliders via the same monopole loop. The possibility to discover these processes is significantly lower than that of the $\gamma\gamma$ case.

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I. INTRODUCTION

The magnetic charge (monopole) was introduced by Dirac [1] to restore the symmetry among electricity and magnetism in particle theory. Dyons — particles with both magnetic and electric charge — were considered later by Schwinger [2] (see also [3]). All attempts to discover the monopole gave negative results, and there is no place for such a particle in modern theories of our world. This is the reason, why the members of the physics community do not believe in the existence of this particle. However, there are no precise reasons against its existence. Moreover, the idea of the monopole is very attractive: if it exists somewhere, the mysterious quantization of the electric charge of particles can be explained.

We discuss here only the point–like monopole, assuming that it was not observed yet only due to its very high mass. (This particle differs strongly from nonlocal monopoles discussed in the context of gauge theories after Polyakov and ’t Hooft papers [4].)

Our basic idea is simple: The existence of monopoles provides for a $\gamma \gamma$ elastic scattering at large angles (“via monopole loop”), which is sufficiently strong below the monopole production threshold. This effect is observable at colliders with energies smaller than the monopole mass. It can be used as a method to find a lower bound for the monopole mass from data. The idea was proposed for the first time in 1982 in Ref. [5] where first calculations for $e^+e^-$ and $\gamma\gamma$ colliders were presented. The same effect for hadron collider was considered recently [6]. The calculations of the last paper were the base to estimate a lower limit for the monopole mass from D0 data [7]. A resembling idea for the process $e^+e^- \rightarrow Z \rightarrow 3\gamma$ has been used in Refs. [8,9] and tested at LEP [10]. A similar monopole loop effect should influence the photon and $Z$ boson polarization operators. The last effect allows to obtain limitations for the monopole mass from the anomalous magnetic momentum of the muon [11] (> 120 GeV) and data at the $Z$ peak [8] (> 1 TeV).

In this paper we present more detailed calculations for the production of large angle high energy photons at colliders signaling the presence of virtual monopole loops and consider some new features of this observable effect. We compare the discovery potential at different future colliders and for several processes with monopole loops. We show how to define the spin of the monopole via well observed polarization effects at $e^+e^-$ or $\gamma\gamma$ colliders. The $3\gamma$ production at $e^+e^-$ and $p\bar{p}$ ($pp$) colliders via a monopole loop is found to be less suitable for observing the effect.

Besides, we consider the similar $Z\gamma$ production.

Throughout the paper we denote the monopole mass by $M$ and its spin by $J$ (we consider the opportunities $J = 0$, $J = 1/2$, or $J = 1$).

II. EFFECTIVE DYNAMICS FOR HEAVY MONOPOLES INTERACTING WITH PHOTONS AND $Z$ BOSONS MUCH BELOW THE PRODUCTION THRESHOLD

A. Basic assumptions

A theory with two point–like charges, electric and magnetic, cannot be standard QED. This theory needs to describe the electromagnetic field with a vector potential having the Dirac string [1,2] or some its surrogate. To have unambiguous results, the elementary electric and magnetic charges $e$ and $g$ ought to be quantified so that
\[ g = \frac{2\pi n}{e} \quad n = \pm 1, \pm 2, \ldots \] (2.1)

with \( \alpha \equiv e^2/(4\pi) = 1/137 \) which leads to \( \alpha_g \equiv g^2/(4\pi) = n^2/(4\alpha) \approx 34n^2 \).

The explicit form of such a theory is unknown (see Refs. [3,8] and Appendix B). Its construction needs really new approaches. In particular, in the standard approach this theory should even violate \( U(1) \) local gauge invariance of QED [12].

We consider below mainly the effective 4\( \gamma \) vertex, describing in particular the light–to–light scattering. It is assumed that the theory of photon–monopole interactions restores the electro–magnetic duality at least for an energy region below the threshold of monopole production (the region of our interest)

\[ \omega \ll M. \] (2.2)

Here \( \omega \) is the characteristic photon energy in the discussed process. The main features of a typical matrix element with external photons in this region are:

- gauge invariance provides a factor \( \omega \) for each photon leg;
- to make this factor dimensionless it should be written as \( \omega/M \);
- a factor \( g \) belongs to each vertex.

Therefore, the amplitude with \( n \) external photons is at least proportional to \( (g\omega/M)^n \). (In fact, for \( n > 4 \) photons an additional factor \( (\omega/M)^{n-4} \) arises — cf. Ref. [13].)

We assume that at low enough photon energies the photon–monopole interaction can be described effectively as a QED–like theory which is valid in tree and 1–loop approximation with the expansion parameter

\[ g_{\text{eff}} = c(J) \frac{g}{4\pi} \frac{\omega}{M} = \frac{c(J) \omega n}{4\sqrt{\pi \alpha} M} < 1. \] (2.3)

Here \( c(J) \) is some numerical factor of order unity depending on the monopole spin. Such a theory is gauge invariant and has the correct low energy behavior. Besides, at \( e/(4\pi) \ll g_{\text{eff}} < 1 \) effects of ordinary particle production can be neglected.

We expect that with increasing \( \omega \) (when our interaction becomes formally strong) the real interaction strength of such a theory becomes saturated. So, multiloop effects of QED, including essential contributions from regions \( |q_i^2| \gtrsim M^2 \), with \( q_i \) being Euclidean loop momenta, are smoothed. (From experience at strong coupling is is known that cross sections are usually less than those of low order perturbation theory.)

The validity range of the used approach is given by the following limitations:

1. multiloop radiative corrections should be small;
2. production cross sections for larger numbers of photons should be small;
3. the unitarity for the elastic \( \gamma\gamma \) scattering should not be violated.

Let us discuss the status of these limitations in more detail.

1) In standard QED the integration over loop momentum \( q \) is convergent for the \( \gamma\gamma \rightarrow \gamma\gamma \) process due to gauge invariance. Therefore, for loop integrals we can perform a Wick rotation into the Euclidean region. In this case the integration region is limited by virtualities \( |q^2| \lesssim \omega^2 \), where the effective expansion parameter is small, and our QED–like approach is valid. If some convergent QFT–like monopole dynamics exists, these arguments show that radiative corrections should be \( \sim g_{\text{eff}}^2 \) and multiloop effects are negligible.
At first glance, the experience with radiative corrections in QED contradicts the above statement. The 2–loop calculations in standard QED [15] give corrections \( \sim \alpha/\pi \) which would transform to meaningless large values by simply replacing \( \alpha \rightarrow \alpha g \). However, these corrections have no relation to our problem. Indeed, these \( \omega \)–independent corrections in QED arise from integration regions with virtualities \(|q_i^2| \sim m_e^2\) (i.e. for our problem the inequality (2.2) with \( m_e \rightarrow M \) is strongly violated). Here the monopole dynamics should be quite different from QED. As it was noted above, this interaction is expected to be more smooth than that of QED.

2) Based on the general Heisenberg–Euler effective Lagrangian for heavy spin 1/2 and spin 0 particles in the loop [13,14], it is easy to see that the production of additional photons is suppressed by a factor \((\omega/M)^{4 g_{\text{eff}}}/4\) per photon. This factor is very small within the region (2.3) (see e.g. [8]).

3) To respect the unitarity limit \( \sigma_{\text{uni}}(\omega) \sim 4\pi(2J + 1)/\omega^2 \) (the result for scalar particles is used, for simplicity), the cross section of the considered \( \gamma \gamma \rightarrow \gamma \gamma \) process has to be less than the sum of \( S \) and \( D \) partial waves, describing the process at \( \omega \ll M \)

\[
\sigma_{\gamma \gamma \rightarrow \gamma \gamma}(\omega) < \frac{24\pi}{\omega^2}. \tag{2.4}
\]

In other words, our theory can be valid only at small enough energy

\[
\omega < \omega_m \left[ \sigma_{\gamma \gamma \rightarrow \gamma \gamma}(\omega_m) = \frac{24\pi}{\omega_m^2} \right]. \tag{2.5}
\]

Using the cross section (3.4) given below at \( \omega = \omega_m \), we obtain at \( g_{\text{eff}} \approx 1 \) estimates for \( c(J) \). Therefore inequalities (2.3) and (2.5) are satisfied simultaneously if \( c(J = 1) \sim 2.41, c(J = 1/2) \sim 1.33, c(J = 0) \sim 0.94 \). These estimates well agree with the experience in usual theories where we have for the effective coupling constant \( c(J = 1) \approx 4 \) (QCD), \( c(J = 1/2) \approx 2 \) (QED), \( c(J = 0) \approx 1 \).

We conclude that using one–loop results of QED without radiative corrections within the region (2.5) is a self–consistent approach, which has chances to be the correct description of nature.

### B. The 4–photon effective Lagrangian

Taking into account the above stated arguments we describe the photon–monopole interaction by a Heisenberg–Euler effective Lagrangian with coefficients given by standard QED. For the 4–photon interaction via a monopole loop (Fig. 1) we have

\[
\mathcal{L}_{\text{eff}} = -\frac{F_{\mu\nu}F^{\mu\nu}}{4} + \mathcal{L}_{4\gamma} + \ldots \tag{2.6}
\]

with

\[
\mathcal{L}_{4\gamma} = \frac{1}{36} \left( \frac{g}{\sqrt{4\pi M}} \right)^4 \left[ \beta_+ + \beta_- (F^{\mu\nu}F_{\mu\nu})^2 + \frac{\beta_+ - \beta_-}{2} (F^{\mu\nu} \tilde{F}_{\mu\nu})^2 \right]. \tag{2.7}
\]

Here \( F_{\mu\nu} \) is the electromagnetic field strength tensor and \( \tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}/2 \). (We assume also an additional gauge–fixing term to invert the photon propagator.) The constants \( \beta_\pm \) depend
on the monopole spin. The numerical coefficient is introduced to simplify the numbers $\beta_{\pm}$. We use also the combinations

$$P = \beta_+^2 + 2\beta_-^2, \quad B = \frac{\beta_+^2 - 2\beta_-^2}{\beta_+^2 + 2\beta_-^2}.$$  \hspace{1cm} (2.8)

The numbers for these coefficients are collected in Table I.

The effective Lagrangian (2.6) is valid to describe processes with virtual photons having virtualities $|q^2| \lesssim \omega^2$. The relevant scale for the virtuality dependence of the $\gamma\gamma \to \gamma\gamma$ cross section (possible form factor) is given by the unique inner parameter of the monopole loop — the monopole mass $M$. This dependence appears as ratios $|q^2|/M^2$ which are very small in the region (2.5). Therefore, the light–to–light cross section can be used on-shell.

C. Monopoles interacting with the $Z$ boson. Effective Lagrangian

With a heavy monopole the standard $SU(2) \times U(1)$ theory has to be considered as low energy limit of some unknown theory. New hypotheses are necessary to describe the interaction of monopoles with $Z$ and $W$ bosons. We consider the simplest one [8,9]. For heavy monopoles ($M \gg M_Z$) it is naturally to assume that the monopole interacts with the fundamental field of the $SU(2) \times U(1)$ theory before symmetry breaking, namely, the isoscalar field $B$. Since the physical photon and the $Z$ boson are obtained by rotation in the $(B, W^0)$ plane, the monopole interactions with $Z$ and $\gamma$ are assumed to be identical except of an additional factor $\tan \theta_W$ in the monopole coupling to $Z$. In this approach the additional term for the $Z3\gamma$ coupling in the effective Lagrangian has the same form as in Eq. (2.7) with the natural change of one field strength $F_{\mu\nu} \to Z_{\mu\nu}$:

$$\mathcal{L}_{Z3\gamma} = \frac{1}{9} \tan \theta_W \left( \frac{g}{\sqrt{4\pi M}} \right)^4 \left[ \frac{\beta_+ + \beta_-}{2} (F_{\mu\nu} F_{\mu\nu}) (Z_{\mu\nu} F_{\mu\nu}) + \frac{\beta_+ - \beta_-}{2} (F_{\mu\nu} \tilde{F}_{\mu\nu}) (Z_{\mu\nu} \tilde{F}_{\mu\nu}) \right].$$  \hspace{1cm} (2.9)

The factor of 4 in the definition of the $Z3\gamma$ coupling as compared with Eq. (2.7) is chosen in such a way that the two Lagrangians lead to the same strength, and the expression in momentum space, for the $Z3\gamma$ and $4\gamma$ couplings modulo a factor $\tan \theta_W$. For scalar or spinor monopoles the coefficients $\beta_{\pm}$ in $\mathcal{L}_{Z3\gamma}$ are the same as those in $\mathcal{L}_{4\gamma}$ (2.7), a possible difference in $\beta_{\pm}$ for vector monopoles [9] is neglected in the following.

III. HELICITY AMPLITUDES AND CROSS SECTIONS

A. Light to light scattering

The helicity amplitudes corresponding to the effective Lagrangian (2.7) can be written in compact form ($\lambda_{1,2} —$ helicities of initial photons, $\lambda_{3,4} —$ helicities of final photons)

$$\mathcal{M}(\lambda_1, \lambda_2; \lambda_3, \lambda_4) = \frac{8}{9} \left( \frac{g\omega}{\sqrt{4\pi M}} \right)^4 \mathcal{N}(\lambda_1, \lambda_2; \lambda_3, \lambda_4).$$  \hspace{1cm} (3.1)
with the only nonvanishing amplitudes
\[ \mathcal{N}(\lambda, \lambda; \lambda, \lambda) = 4\beta_+, \]
\[ \mathcal{N}(\lambda, \lambda; -\lambda, -\lambda) = 2\beta_-(3 + \cos^2 \theta), \] (3.2)
\[ \mathcal{N}(\lambda, -\lambda; \pm \lambda, \mp \lambda) = \beta_+(1 \pm \cos \theta)^2. \]
The corresponding total cross section of the $\gamma\gamma$ elastic scattering is given by
\[ \sigma(\gamma_1\gamma_2 \rightarrow \gamma\gamma) = \sigma^{\text{tot}}_{\text{unp}} \left[ 1 + \frac{5 - 2B}{7} \lambda_1\lambda_2 \right], \] (3.3)
\[ \sigma^{\text{tot}}_{\text{unp}} = R\omega^6, \] (3.4)
\[ R = \frac{28P}{405\pi} \left( \frac{g\sqrt{4\pi M}}{\sqrt{\rho}} \right)^8 \equiv \frac{28P}{405\pi} \left( \frac{n}{2\sqrt{\alpha M}} \right)^8. \] (3.5)

Let $s$, $t$ and $u$ be the standard Mandelstam variables $(s + t + u = 0)$, $\theta$ is the scattering angle in the c.m.s. (with $0 < \theta < \pi/2$). In our case $s = 4\omega^2$, $t = -2\omega^2(1 - \cos \theta)$. The angular distribution is roughly isotropic, but the details of the distribution are nontrivial:
\[ d\sigma(\gamma_1\gamma_2 \rightarrow \gamma\gamma) = \frac{5}{56}\sigma^{\text{tot}}_{\text{unp}} \left[ \left( 3 + \cos^2 \theta \right)^2 (1 - B\lambda_1\lambda_2) + 8(1 + B)\lambda_1\lambda_2 \right] d\cos \theta \]
\[ = \frac{5}{7}\sigma^{\text{tot}}_{\text{unp}} \left[ \left( \frac{s^2 + t^2 + u^2}{s^2} \right)^2 (1 - B\lambda_1\lambda_2) + 2(1 + B)\lambda_1\lambda_2 \right] \frac{dt}{s}. \] (3.6)

**B. $Z\gamma$ production**

The helicity amplitudes for the process $\gamma\gamma \rightarrow Z\gamma$ are obtained from the effective Lagrangian (2.9):
\[ \mathcal{M}(\lambda_1, \lambda_2; \lambda_3, \lambda_Z) = \frac{8}{9} \tan \theta_W \left( \frac{g\omega}{\sqrt{4\pi M}} \right)^4 \sqrt{\rho} \times \mathcal{N}(\lambda_1, \lambda_2; \lambda_3, \lambda_Z) \] (3.7)
with $\rho = 1 - M_Z^2/s$, $\lambda_Z$ is the helicity of the $Z$. In addition to the helicity amplitudes (3.2), the further nonvanishing amplitudes are
\[ \mathcal{N}(\lambda, \lambda; -\lambda, \lambda) = 2\beta_+ \sin^2 \theta, \]
\[ \mathcal{N}(\lambda, \lambda; -\lambda, 0) = 2\sqrt{2\rho} \beta_- \lambda \sin \theta \cos \theta, \] (3.8)
\[ N(\lambda, -\lambda; \pm \lambda, \pm \lambda) = \rho \beta_+ \sin^2 \theta, \]

\[ N(\lambda, -\lambda; \pm \lambda, 0) = \sqrt{2\rho} \beta_+ \lambda (1 \pm \cos \theta) \sin \theta. \]

At large energies \((s \gg M_Z^2)\) we obtain the angular distribution including the polarizations of initial photons:

\[
\begin{align*}
    d\sigma(\gamma_1 \gamma_2 \rightarrow Z\gamma) &= \frac{2}{81\pi} \tan^2 \theta_W \frac{1}{\omega^2} \left( \frac{g\omega}{\sqrt{4\pi M}} \right)^8 \left\{ \beta_+^2 (3 + \cos^2 \theta) + \beta_-^2 (5 + 3 \cos^2 \theta) \\
    &+ \lambda_1 \lambda_2 \left[ \beta_+^2 (1 - \cos^2 \theta) + \beta_-^2 (5 + 3 \cos^2 \theta) \right] \right\} d\cos \theta.
\end{align*}
\]

Using the notations of Eqs. (2.8),(3.4) the corresponding total cross section can be written as

\[
\sigma(\gamma_1 \gamma_2 \rightarrow Z\gamma) = \frac{5}{42} \tan^2 \theta_W [(19 + B) + \lambda_1 \lambda_2 (11 - 7B)] \sigma_{\text{tot}}^{\text{unp}}(\gamma\gamma \rightarrow \gamma\gamma). \tag{3.10}
\]

This cross section is less than that of the \(\gamma\gamma \rightarrow \gamma\gamma\) process (3.4). With the lower detection efficiency for the \(Z\) boson compared with the high energy photon, the process \(\gamma\gamma \rightarrow Z\gamma\) is less suitable than the light–to–light scattering to discover the monopole loop effect.

**C. 3\(\gamma\) production. Total cross section**

We consider the 3\(\gamma\) production in \(e^+e^-\) or \(q\bar{q}\) collisions. Let us denote the total cross section of the process \(e^+e^- \rightarrow 3\gamma\) via a monopole loop (Fig. 2) with a s–channel photon by \(\sigma_i (3\gamma)\). In the total cross section both \(Z\) and \(\gamma\) exchange have to be taken into account. The monopole loop factors are identical for both exchanges up to \(\tan \theta_W\) in the amplitude (coefficients \(\beta_\pm\) for \(J = 1\) monopoles in \(4\gamma\) and \(3\gamma\) couplings are set to be equal). With these changes we have at \(s \gg M_Z^2\) for \(e^+e^-\) or \(q\bar{q}\) collisions

\[
\sigma(f^i \bar{f}^i \rightarrow 3\gamma) = K_i \sigma_i (3\gamma), \tag{3.11}
\]

where

\[
K_i = \left( q_i + \frac{g_V^i}{2 \cos^2 \theta_W} \right)^2 + \left( \frac{g_A^i}{2 \cos^2 \theta_W} \right)^2.
\]

Here \(g_V^i\) and \(g_A^i\) are the vector and axial coupling of fermion \(f^i\), \(q_i\) is the charge of \(f_i\) in units of \(e\). For electrons we obtain \(K_e = 1.174\), for \(u\)–quarks and \(d\)–quarks \(K_u = 0.739\) and \(K_d = 0.416\), respectively. Using the results of Ref. [9] for the process \(e^+e^- \rightarrow Z \rightarrow 3\gamma\) via a monopole loop, we write the cross section \(\sigma_i (3\gamma)\) for the \(e^+\) and \(e^-\) energy \(E\) in the notations for the \(\gamma\gamma \rightarrow \gamma\gamma\) process given in Eq. (3.5) as

\[
\sigma_i (3\gamma) = \frac{\alpha}{252\pi} RE^6 \frac{3\beta_+^2 + 5\beta_-^2}{3\beta_+^2 + 6\beta_-^2}. \tag{3.12}
\]

Differential distributions for the 3\(\gamma\) production in \(e^+e^-\) collisions via \(Z\) exchange were analyzed in Ref. [9]. The result at the \(Z\) peak was given in Ref. [8].
IV. DISCOVERING THE MONOPOLE EFFECT AT COLLIDERS

Let us discuss the opportunity to see the effect described by the effective Lagrangian (2.7) at \( \gamma \gamma \), \( e^+e^- \) and proton colliders. We focus our interest to a first observation where the cross section is not very large and the collider energy is \( E \ll M \). In this region the cross section of the process raises rapidly with the energy of the initial (real or virtual) photons. So, the effect will be seen as the production of high energy photons at large angles.

A. Photon Colliders

The constructed linear colliders will be used both in \( e^+e^- \) mode and in \( \gamma \gamma \) or \( e\gamma \) modes (Photon Colliders) with the following typical parameters (obtained without a special optimization for the photon mode) [18,19].

- **Characteristic photon energy** \( \omega \approx 0.8E \). (\( E = 0.25 \div 1 \text{ TeV} \) is the electron energy in the basic \( e^+e^- \) collider).
- **Annual luminosity** \( \mathcal{L}_b \approx 100 \text{ fb}^{-1} \) (or larger).
- **Mean energy spread** \( \langle \Delta \omega \rangle \approx 0.07 \omega \).
- **Mean photon helicity** \( \langle \lambda_\gamma \rangle \approx 0.95 \), the sign of which can be changed easily [18].

The considered process will be seen as elastic large angle light–to–light scattering. Since the cross section raises quickly with energy, our effect will be most easily observed for photons with highest energy. The scattering at \( \theta = \pi/2 \) is almost twice as large as the forward scattering (see Eq. 3.6). Both total and differential cross sections depend strongly on the sign of the initial photon helicities. This dependence is very different for different values of the monopole spin.

The background is given by the light–to–light scattering via a W–boson loop with a cross section of about 20 fb at the discussed energies [17]. In contrast to the effect of interest, the background angular distribution of the produced photons is shifted to the beam collision axis, with transverse momenta \( \mathcal{O}(M_W) \). Besides, this background varies slowly with energy.

Neglecting the difference in the angular distributions for signal and background at 0.25–1 TeV, we conclude: To see our effect unambiguously with a good Signal/Background ratio, a production cross section above 1 fb is necessary.

B. \( e^+e^- \) Colliders

To search for the monopole effect in \( e^+e^- \) collisions, the emission of high energy photons with large transverse momenta via photon fusion should be studied (\( e^+e^- \rightarrow e^+e^- \gamma \gamma \)). To describe this process we use the leading log equivalent–photon approximation (cf. e.g. [20]) having an accuracy \( \sim 1/\ln(E^2/m_e^2) \approx 0.05 \).

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1The numerical results of that paper should be reduced by a factor \( 0.76 = (128/137)^4 \) since the photons here are real, and the fine structure constant is \( \alpha = 1/137 \), but not 1/128.
Let us denote the energies of the separate virtual photons by \( \omega_i = x_i E \). Then the effective mass of produced \( \gamma \gamma \) system is \( \hat{s} = 4x_1x_2E^2 \). The result for the collision of unpolarized electrons is written with high accuracy via the spectra of the equivalent photons \( dn_i (Q^2 = -q^2 > 0) \):

\[
\sigma(ee \rightarrow ee\gamma\gamma) = RE^6 \int \frac{dx}{x} \frac{dQ^2}{Q^2} \left[ 1 - x + \frac{1}{2} x^2 - (1 - x) \frac{Q^2_{\text{min}}}{Q^2} \right],
\]

(4.1)

\[
dn(x) = \frac{\alpha}{\pi} x \int_{Q^2_{\text{min}}}^{Q^2_{\text{max}}} \frac{dQ^2}{Q^2} \left[ 1 - x + \frac{1}{2} x^2 - (1 - x) \frac{Q^2_{\text{min}}}{Q^2} \right],
\]

(4.2)

\[
Q^2_{\text{min}} = \frac{m_e^2 x^2}{1 - x}.
\]

Since the virtuality dependence for the subprocess cross section is negligible, the integration over the virtuality is spread over the whole kinematical region, till \( Q^2_{\text{max}} \approx \hat{s} \), where the product of the photon fluxes in Eq. (4.1) become much higher than the precise QED expression. Within the considered accuracy we obtain from Eq. (4.2) the photon spectra

\[
dn(x_i) = \frac{\alpha}{\pi} L f(x_i),
\]

(4.3)

\[
f(x_i) = 1 - x_i + \frac{1}{2} x_i^2, \quad L = \ln \frac{4E^2}{m_e^2}.
\]

To take into account polarization effects of initial leptons in Eq. (4.1) we note that the doubled longitudinal electron and positron helicities \( \xi_i \) are transferred to the mean helicities of their equivalent (virtual) photons \( \lambda_i^v \) via [21]

\[
\lambda^v = A(x) \xi, \quad A(x) = \frac{x - x^2/2}{1 - x + x^2/2},
\]

(4.4)

Therefore, the integrand in the total cross section (4.1) acquires the additional factor

\[
\left[ 1 + 5 - \frac{2}{7} A(x_1)A(x_2) \xi_1 \xi_2 \right].
\]

(4.5)

Combining Eqs. (4.1)-(4.5), the energy spectrum for the initial equivalent photons takes the form

\[
d\sigma(e_1 e_2 \rightarrow ee\gamma\gamma) = \frac{\alpha^2}{\pi} RE^6 L^2 \left[ 1 + \frac{5 - 2B}{7} A(x_1)A(x_2) \xi_1 \xi_2 \right] f(x_1)x_1^2 dx_1 f(x_2)x_2^2 dx_2,
\]

(4.6)

and we calculate the total cross section, the characteristic photon energy \( \langle \omega \rangle \) and the mean spread of the virtual photon energy \( \langle \Delta \omega \rangle^2 \):

\[\text{Eqs. (4.9), (4.10) were obtained in Ref. [5].}\]
\[
\sigma(e_1 e_2 \to ee\gamma\gamma) = R\alpha^2 E^6 D^2 \left[ 1 + \frac{81}{847} (5 - 2B) \xi_1 \xi_2 \right],
\]
(4.7)

\[
D = \frac{1}{\alpha E^3} \int_0^E \omega^3 dn \approx \frac{11}{60\pi} L,
\]
(4.8)

\[
\langle \omega \rangle = \frac{\int \omega \times \omega^3 dn}{\int \omega^3 dn} = \frac{8}{11} E \approx 0.73E,
\]
(4.9)

\[
\langle \Delta \omega \rangle = \sqrt{\langle \omega^2 \rangle - \langle \omega \rangle^2} = \sqrt{\frac{36}{847} E} = 0.283 \langle \omega \rangle.
\]
(4.10)

So, the effect looks like the production of a resonance with \( M \approx 1.4E \) and \( \Gamma/M \approx 0.283 \). The “resonance” will be produced almost at rest. For the considered energies \( E = 100-1000 \) GeV we have \( D = 1.50 - 1.77 \). Differential cross sections for the produced photons are given in Appendix A.

A different possibility for observing the influence of the monopole loop effect seems to be the production, through annihilation, of three photons (see Fig. 2). In the discussed energy range the ratio of total cross sections for three to two photon production (Eqs. (3.11) and (4.7)) is roughly \( K_e/(252\pi\alpha D^2) \approx 1/12 \). Therefore, the three photon production is unsuitable for our goal.

**Background.** A characteristic feature of the discussed process is the radiation of two noncollinear high energy photons with \( \omega \sim E \). There are only two processes of standard QED with radiation of two high energy photons having large \( p_\perp \) and with not too small cross sections. The first of them is the process \( e^+ e^- \to \gamma\gamma \) with a cross section for large angle production \( \sim \alpha^2/E^2 \). This process differs from that considered above since here the photons are collinear and their energies \( \omega = E \). The second QED process is \( e^+ e^- \to 3\gamma \) with a very low cross section for the radiation of large angle high energy photons \( \sim \alpha^2/E^2 \).

**C. Numerical estimates for \( e^+ e^- \) and \( \gamma\gamma \) colliders**

The effects discussed have a very good signature and a small background. Therefore, a relatively low cross section can be used to demonstrate the observability of the effect.

Let us first summarize our estimates for the characteristic photon energy \( \langle \omega \rangle \) and the ratio of cross sections with initial antiparallel and parallel helicities of photons or electrons \( \sigma_-/\sigma_+ \) for the electron beam energy \( E \) in Table II. These ratios show the very high potential to determine the monopole spin from measurements with polarized beams.

To estimate the possible monopole mass bound, we demand to have 10 events in the \( e^+ e^- \) mode of TESLA (at \( E = 250 \) GeV and \( E = 1 \) TeV with luminosity integral 500 fb\(^{-1}\)) and a 1 fb cross section in the \( \gamma\gamma \) mode of that collider (with 100 fb\(^{-1}\) integrated luminosity). Table III shows the ultimate values of the monopole mass (in TeV) depending on its spin at different initial energies of the electron bunch \( E \) (in TeV) and the corresponding values
of $\langle \omega \rangle / \omega_m$. For a possible experiment at LEP2 we use the luminosity integral $0.5 \text{ fb}^{-1}$ and assume to have 10 events. Besides, we include in Table III “universal estimates” assuming that the process with a monopole loop is observable for a cross section $\sigma \geq \sigma_0 = 0.1\alpha^2/(3E^2)$ for both $e^+e^-$ and $\gamma\gamma$ collisions. This leads to $\sigma_0 = 0.7 \text{ fb}$ at $E = 1 \text{ TeV}$ and $\sigma_0 = 11 \text{ fb}$ at $E = 0.25 \text{ TeV}$.

Table III shows that we are far from the boundary (2.5). Therefore, we expect no new phenomena with increasing energy except a fast rising cross section (for example, the cross section becomes twice as large increasing the collider energy by 12%).

D. Proton colliders

The proton colliders are those with highest beam energies. The main observable effect here should be $pp \rightarrow \gamma\gamma + \ldots$ in the collision of two virtual photons, created by the protons (or antiprotons) in both elastic and inelastic processes ($p \rightarrow p\gamma^*, p \rightarrow \gamma^* + \ldots$) with the $\gamma^*\gamma^* \rightarrow \gamma\gamma$ subprocess. These calculations were performed for the Tevatron and the LHC in Ref. [6], where total and differential cross sections for the produced photons were obtained. The total cross section (obtained numerically using known approximations for the proton structure functions [22] and form factors) can be written in the form

$$
\sigma_{pp \rightarrow \gamma\gamma X} = 108 P \left( \frac{nE}{M} \right)^8 \left( \frac{N(E)}{N(1 \text{ TeV})} \right)^2 \left( \frac{1 \text{ TeV}}{E} \right)^2 \text{ fb}.
$$

(4.11)

Here $N(E)$ is the normalized flux of the effective photons, it varies within 10% in the energy interval 1–7 TeV.

This relatively low (in comparison with the $e^+e^-$ case) cross section is due to the soft structure of the proton with respect to the photons. This fact results also in a relatively low fraction of the total energy transferred to the photons.

With a good accuracy for the 1–7 TeV energy interval the average energy of the colliding (virtual) photons and their energy spread are

$$
\langle \omega \rangle = 0.314 E, \quad \langle \Delta \omega \rangle = 0.149 E.
$$

(4.12)

The energies of produced photons are close to this value.

Luminosity integrals of 2 fb$^{-1}$ at $E = 0.9 \text{ TeV}$ (upgraded Tevatron) and 100 fb$^{-1}$ at $E = 7 \text{ TeV}$ (LHC) have been considered. Supposing 10 events to record the effect, the following ultimate values for the monopole mass (in TeV) and corresponding values of the ratio $\langle \omega \rangle / \omega_m$ were obtained [6], see Table IV. The third line in this Table represents the results of data processing at the Tevatron [7] based on equations from Ref. [6]. Here a much smaller luminosity integral 70 pb$^{-1}$ was used, and the limitations for the monopole mass are much less restrictive.

To complete the study for proton colliders one has to estimate the process $pp \rightarrow 3\gamma + \ldots$ via a monopole loop arising from the $q\bar{q}$ fusion into a single photon similar to Fig. 2. We calculate the corresponding cross section, using Eqs. (3.11), (3.12) and the GRV parametrizations for the structure functions [22]. It is useful to write the result in the form

11
\[
\sigma_{pp \rightarrow \gamma \gamma \gamma X} = D(E) P \left( \frac{nE}{M} \right)^8 \left( \frac{3 \beta_1^2 + 5 \beta_2^2}{3 \beta_1^2 + 6 \beta_2^2} \right) \times \left( \frac{1 \text{ TeV}}{E} \right)^2 \text{ fb},
\]

(4.13)

\[
D(\text{Tevatron}) = 4.2, \quad D(\text{LHC}) = 0.17.
\]

The large difference in the cross sections for the Tevatron and the LHC is due to the fact that the number of antiquarks in protons is much lower than that in antiprotons at relatively large values of the (anti)quark momentum fractions \(x_i\).

These cross sections are much smaller than those for the two–photon case (4.11). Therefore, this process is unsuitable to discover the monopole effect.

V. FINAL COMMENTS

We have studied different processes suitable to discover the point–like Dirac magnetic monopole in experiments at colliders before its direct observation. We have found that the study of the two–photon final state produced by two initial photons either real or virtual has the highest discovery potential. Among different colliders the most promising ones are Next Linear Colliders in the Photon Collider mode. For both the Photon (\(\gamma \gamma\)) and \(e^+e^-\) mode of these colliders the ratio \(\langle \omega \rangle/\omega_m\) is small enough to neglect unwanted corrections in the theory. With the lower detection efficiency for \(Z\) bosons compared with high energy photon, studies of the \(Z\gamma\) final state are less favorable.

For \(e^+e^-\) and proton colliders the cross sections for the \(3\gamma\) final state via an intermediate photon or \(Z\) state (Fig. 2) is much smaller than that of the \(\gamma \gamma\) production, they are unsuitable for a first observation of effects caused by monopoles.

The same approach provides the opportunity to study processes of higher order \(\gamma \gamma \rightarrow 4\gamma, \ldots, e^+e^- \rightarrow 5\gamma, \ldots\). Their cross sections are relatively small within the region of interest. The increasing photon multiplicity due to these processes can be observed only for an energy much larger than that necessary for a first observation of the effect.

We would like to mention that also a monopolium bound state \(R\) with mass \(M_R\) and spin \(J_R\) can exist, as has been discussed, for example, in Refs. [5,8]. To make a rough estimate, we assume \(2\omega \sim M_R\). If \(M_R \gtrsim 8\pi M/(gc(J)) \approx 1.32M/(nc(J))\), the monopolium resonance is invisible at the discussed energies (when inequality (2.5) is valid). However, there exists another possibility: \(M_R < 8\pi M/(gc(J)) \approx 1.32M/(nc(J))\) where the effective coupling \(g_{\text{eff}}\) is small near this resonance. In this case the two–photon decay of \(R\) is dominant, and (for a \(C\)-even resonance with \(J_R = 0, 2\)) the cross section for the transition \(\gamma \gamma \rightarrow R \rightarrow \gamma \gamma\) with \(2\omega \sim M_R\) is large. Then at \(2\omega > M_R\) a resonance peak could be expected in the \(\gamma \gamma\) elastic scattering. The \(C\)-odd resonance with \(J_R = 1\) should be observable as some new \(Z\)-like heavy vector boson decaying mainly into three photons.

ACKNOWLEDGMENTS

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APPENDIX A: THE ENERGY AND MOMENTUM DISTRIBUTION FOR THE PRODUCED PHOTONS AT $e^+e^-$ COLLIDER

Since the produced $\gamma\gamma$ system is almost at rest, we will have a roughly isotropic angular distribution in $e^+e^-$ collisions. More detailed distributions can be obtained, substituting Eq. (3.6) into the integrand (4.1) and performing a subsequent boost and a space rotation. In a first approximation we can neglect the transverse motion of the virtual photons (let us remind that in the main part of the integration region we have $Q_i^2 \approx q_{i\perp}^2/(1-x_i) \ll \hat{s}$ with the transverse momenta of the virtual photons $q_{i\perp}$). Therefore, the transverse momenta of the produced photons are balanced: $p_{3\perp} \approx -p_{4\perp} \equiv p_{\perp}$. Their 4–momenta in the c.m.s. of the protons are

$$p_{3,4} = p_{\perp}(\cosh \eta_{3,4}, \pm 1, 0, \sinh \eta_{3,4})$$

with the rapidities of the produced photons $\eta_{3,4}$. Using these notations we have $\sin \theta = p_{\perp}/(E\sqrt{x_1x_2})$ and $x_{1,2} = (p_{\perp}/2E)(\exp(\pm \eta_3) + \exp(\pm \eta_4))$. With the standard transformation

$$\frac{\partial^3}{E^2 \partial x_1 \partial x_2 \partial \cos \theta} \equiv 2 \frac{\partial^3}{\partial p_{\perp}^4 \partial \eta_3 \partial \eta_4}$$

the integrand of Eq. (4.1) with Eq. (4.3) represents the transverse momentum–rapidity distribution of the produced photons. The differential cross section of the $\gamma\gamma$ production (after integrating over one transverse momentum and azimuthal angle) can be written in the form

$$\frac{d^3\sigma}{d\eta_3 d\eta_4 dp_{\perp}^2} = \frac{5\alpha^2}{112\pi^2} RE^4 L^2 \Psi f(x_1)x_1^2 f(x_2)x_2^2,$$

(A2)

$$\Psi = \Phi [1 - BA(x_1)A(x_2)\xi_1\xi_2] + 8(1 + B)A(x_1)A(x_2)\xi_1\xi_2,$$

$$\Phi = \left\{ -\frac{p_{\perp}^2}{E^2 x_1 x_2} \right\}^2 \equiv \left\{ 4 - \frac{1}{\cosh^2 [(\eta_3 - \eta_4)/2]} \right\}^2.$$

Thus the measurement of this differential cross section allows to determine the monopole spin via the polarization dependence of the distribution.

$\gamma\gamma$ total transverse momentum distribution. The total transverse momentum of the produced photon pair $k_{\perp} = p_{\perp 3} + p_{\perp 4}$ is equal to the sum of transverse momenta of the virtual photons, $k_{\perp} = q_{\perp 1} + q_{\perp 2}$. This momentum distribution (written with logarithmic accuracy) can be found in Ref. [20] (we use Eq. (5.31) from that paper restoring omitted terms $\sim x_i^2$ in the photon spectra). In our case the mean value of $x_i$ is about 0.7. Therefore, with the used accuracy we also neglect these factors in the argument of the logarithm. Taking into account the electron polarization we have
\begin{equation}
\frac{d\sigma(e_1e_2 \to ee\gamma\gamma)}{d\xi_1d\xi_2} = \frac{2\alpha^2}{\pi^2} RE^6 \left[ 1 + \frac{5 - 2B}{7} A(x_1)A(x_2)\xi_1\xi_2 \right] \frac{d\mathbf{k}_1^2}{\mathbf{k}_1^2} \ln \frac{\mathbf{k}_1^2}{m_e^2} \int dx_1 f(x_1) dx_2 f(x_2) x_1^2 x_2^2 \quad (A3)
\end{equation}

and

\begin{equation}
\frac{d\sigma(e_1e_2 \to ee\gamma\gamma)}{d\xi_1d\xi_2} = \frac{121\alpha^2}{1800\pi^2} RE^6 \left[ 1 + \frac{81}{847} (5 - 2B) \xi_1\xi_2 \right] \frac{d\mathbf{k}_1^2}{\mathbf{k}_1^2} \ln \frac{\mathbf{k}_1^2}{m_e^2} \quad (A4)
\end{equation}

After integrating Eq. (A3) over the total transverse momentum, the energy spectrum coincides in leading logarithmic accuracy with Eq. (4.6).

**APPENDIX B: COMMENTS ON POSSIBLE CALCULATIONS USING STRING DEPENDENT MONOPOLE–PHOTON VERTEX**

The string–dependent monopole–photon coupling vertex can be written as (for spin 1/2 monopole)

\begin{equation}
\Gamma_{\mu}(q) = ig \varepsilon_{\mu\nu\sigma\tau} n^\nu q^\sigma \gamma^\tau. \quad (B1)
\end{equation}

Here \( q \) is the photon momentum and \( n \) is a space–like unit vector directed along the string (it is assumed to be a straight line for simplicity). The choice of the string is arbitrary: a reorientation of the string is a kind of gauge transformation.

Attempts to perform calculations using this vertex in the spirit of standard QED gave no satisfactory results. It seems that in the “good” theory some extra terms should be added to the interaction (B1). The known “string independent” result for dyon scattering is obtained using the unjustified extra prescription to consider only the most singular term \( 1/q^2 \) in the amplitude at \( q^2 \to 0 \) (omitting the string piece in the result) [23,24].

The authors of Ref. [25] pretend for a solution of this problem at least for the light–to–light scattering. Unfortunately, this proposal cannot be realized without strong additional ideas which seem unknown till now.

Indeed, using this singular coupling the result depends on the (nonphysical) direction of the string line \( n \). Such a result seems meaningless. In particular, the discussed cross section is infinite for photon momenta orthogonal to the string direction.

To avoid this difficulty one can try to average the reaction amplitude over the directions of \( n \) (using either different vectors for different vertices or a common vector for all vertices). To avoid technical difficulties, this averaging can be performed in a specific frame where \( n_0 = 0 \) (e.g., the c.m.s. of the light–to–light scattering) [26].

Unfortunately, this idea gives also no satisfactory results. Indeed, if a separate direction of \( n \) for each vertex is considered, the averaged vertex will be 0 (this can be concluded, e.g. from the calculations of paper [26] by inserting their Eq. (14) into Eq. (B1)). Considering a common vector \( n \) for all vertices, it is easy to see that the cross section is divergent (due to the singular nature of basic interaction (B1)). Therefore, both “natural” possibilities seem to be senseless.
There are no grounds for a difference between an electron and a monopole in the validity range of the duality approach. Introducing an additional factor $\omega/M$ for each photon leg as it has been proposed in Ref. [25] results in a strong difference between electricity and magnetism. It breaks down the basic idea of introducing a monopole. That is a new hypothesis which is one among a large variety of unjustified assumptions.
REFERENCES

### TABLES

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<th>$J$</th>
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<th>$\beta_-$</th>
<th>$P$</th>
<th>$B$</th>
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<tr>
<td>0</td>
<td>[14]</td>
<td>1/5</td>
<td>3/20</td>
<td>0.085</td>
<td>−0.059</td>
</tr>
<tr>
<td>1/2</td>
<td>[13]</td>
<td>11/10</td>
<td>−3/10</td>
<td>1.39</td>
<td>0.74</td>
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<tr>
<td>1</td>
<td>[16,17]</td>
<td>63/5</td>
<td>9/20</td>
<td>159.165</td>
<td>0.995</td>
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**TABLE I.** Coefficients $\beta_\pm$, $P$ and $B$ for monopoles of different spin

<table>
<thead>
<tr>
<th>collider</th>
<th>$\langle \omega \rangle$</th>
<th>$J=0$</th>
<th>$J=1/2$</th>
<th>$J=1$</th>
<th>$\langle \omega \rangle/\omega_m$</th>
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<tbody>
<tr>
<td>$\gamma\gamma$</td>
<td>$0.8E$</td>
<td>0.155</td>
<td>0.33</td>
<td>0.40</td>
<td>0.078</td>
</tr>
<tr>
<td>$e^+e^-$</td>
<td>0.73$E$</td>
<td>0.44</td>
<td>0.58</td>
<td>0.63</td>
<td>0.112</td>
</tr>
</tbody>
</table>

**TABLE II.** Mean photon energy and ratio of cross sections with initial antiparallel and parallel helicities of photons or electrons $\sigma_-/\sigma_+$.  

<table>
<thead>
<tr>
<th>$E$</th>
<th>collider</th>
<th>$J=0$</th>
<th>$J=1/2$</th>
<th>$J=1$</th>
<th>$\langle \omega \rangle/\omega_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>$\gamma\gamma$</td>
<td>3.75$n$</td>
<td>5.6$n$</td>
<td>10$n$</td>
<td>0.078</td>
</tr>
<tr>
<td>0.25</td>
<td>$e^+e^-$</td>
<td>2.5$n$</td>
<td>3.6$n$</td>
<td>6.4$n$</td>
<td>0.112</td>
</tr>
<tr>
<td>1.0</td>
<td>$\gamma\gamma$</td>
<td>11$n$</td>
<td>16$n$</td>
<td>28$n$</td>
<td>0.110</td>
</tr>
<tr>
<td>1.0</td>
<td>$e^+e^-$</td>
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<td>18$n$</td>
<td>0.154</td>
</tr>
<tr>
<td>0.1</td>
<td>LEP2</td>
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<td></td>
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<td>16.75$nE$</td>
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<td>6.5$nE$</td>
<td>11.7$nE$</td>
<td>0.25</td>
</tr>
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**TABLE III.** Lower limits for monopole masses in TeV for $e^+e^-$ and $\gamma\gamma$ colliders depending on the collider energy in TeV and the monopole spin and the ratio $\langle \omega \rangle/\omega_m$.  

<table>
<thead>
<tr>
<th>collider</th>
<th>$J=0$</th>
<th>$J=1/2$</th>
<th>$J=1$</th>
<th>$\langle \omega \rangle/\omega_m$</th>
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</thead>
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<td>Tevatron(2 fb$^{-1}$)</td>
<td>0.998$n$</td>
<td>1.42$n$</td>
<td>2.56$n$</td>
<td>0.44</td>
</tr>
<tr>
<td>LHC</td>
<td>7.40$n$</td>
<td>10.5$n$</td>
<td>19.0$n$</td>
<td>0.45</td>
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<tr>
<td>Tevatron [7]</td>
<td>0.61$n$</td>
<td>0.87$n$</td>
<td>1.58$n$</td>
<td>0.63</td>
</tr>
</tbody>
</table>

**TABLE IV.** Same as in Table III for proton colliders.
FIGURES

FIG. 1. $\gamma \gamma \rightarrow \gamma \gamma$ via monopole loop

FIG. 2. $e^+ e^- \rightarrow 3\gamma$ via monopole loop