Application of novel analysis techniques to Cosmic Microwave Background astronomy

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A dissertation submitted for the degree of Doctor of Philosophy in the University of Cambridge.
Preface

This dissertation is the result of work undertaken at the Mullard Radio Astronomy Observatory, Cambridge between October 1994 and September 1997. The work described here is my own, unless specifically stated otherwise. To the best of my knowledge it has not, nor has any similar dissertation, been submitted for a degree, diploma or other qualification at this, or any other university. This dissertation does not exceed 60 000 words in length.

Aled Wynne Jones

To Isabel and Ffion
I am not sure how the universe was formed. But it knew how to do it, and that is the important thing.

Anon. (child)

It is enough just to hold a stone in your hand. The universe would have been equally incomprehensible if it had only consisted of that one stone the size of an orange. The question would be just as impenetrable: where did this stone come from?

Jostein Gaarder (in ‘Sophie’s World’)

Acknowledgements

Firstly I would like to thank the two people who have introduced me to the immense field of microwave background anisotropies, Anthony Lasenby and Stephen Hancock. Without them I would not have begun to uncover the beauty at the beginning of time. I would also like to thank Joss Bland-Hawthorn whose supervision and enthusiasm during my time in Australia has made me more inquisitive in my field. The many collaborations involved in this project have introduced me to many people without whom this thesis would not have been written; Graca Rocha and Mike Hobson at MRAO, Carlos Gutierrez, Bob Watson, Roger Hoyland and Rafael Rebolo at Tenerife, and Giovanna Giardino, Rod Davies and Simon Melhuish at Jodrell Bank.
I want to say a special thank you to the two women in my life that have kept me going for the last few years. Thanks to Ffion, my sister, whose insanity has kept me sane and thanks to Isabel whose support and encouragement I could not have done without and whose love has made it all worth while.

Martina Wiedner, Marcel Clemens and Dave St. Jacques deserve a special mention for making my time in the department a little more bearable. Anna Moore for putting up with me for three months in Australia. Cynthia Robinson, Martin Gunthorpe, Nicholas Harrison, Liam Cox and Dafydd Owen for putting up with me for the first years of my research.

I could not finish thanking people without mentioning Pam Hicks and David Titterington (special thanks for all the colour overhead transparencies) who have kept the department running smoothly.

I am also very grateful to PPARC for awarding me a research studentship. May they know better next time.

Yn olaf diolch yn fawr i fy rhieni sydd wedi rhoi i fyny efo fi am yr ugain mlynedd diwethaf.
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Chapter 1

Introduction

The Big Bang is the name given to the theory that describes how the Universe came into existence about 15 billion years ago at infinite density and temperature and then expanded to its present form. The very early Universe was opaque due to the constant interchange of energy between matter and radiation. About 300,000 years after the Big Bang, the Universe cooled to a temperature of \( \sim 3000^\circ \text{C} \) because of its expansion. At this stage the matter does not have sufficient energy to remain ionised. The electrons combine with the protons to form atoms and the cross section for Compton scattering with photons is dramatically reduced. The radiation from this point in time has been travelling towards us for 15 billion years and has now cooled to a blackbody temperature of 2.7 degrees Kelvin. At this temperature the Planck spectrum has its peak at microwave frequencies (\( \sim 1 – 1000 \text{ GHz} \)) and its study forms a branch of astronomy called Cosmic Microwave Background astronomy (hereafter CMB astronomy). In 1965 Arno Penzias and Robert Wilson (Penzias & Wilson 1965) were the first to detect this radiation. It is seen from all directions in the sky and is very uniform. This uniformity creates a problem. If the universe is so smooth then how did anything form? There must be some bumps in the early universe that could grow to create the structures we see today.

In 1992 the NASA Cosmic Microwave Background Explorer (COBE) satellite was the first experiment to detect the bumps. These initial measurements were in the form of a statistical detection and no physical features could be identified. Today, experiments all around the world are finding these bumps that eventually grew into galaxies and clusters of galaxies. The required sensitivities called for new techniques in astronomy. The main principle behind all of these experiments is that, instead of measuring the actual brightness, they measure the difference in brightness between different regions of the sky. The experiments at Tenerife produced the first detection of the real, individual CMB fluctuations.

There are many different theories of how the universe began its life and how it evolved into the structures seen today. Each of these theories make slightly different predictions of how the universe looked at the very early stages which up until now have been impossible to prove or disprove. Knowing the structure of the CMB, within a few years it should be possible for astronomers to tell us where the universe came from, how it developed and where it will end up.
CHAPTER 1. INTRODUCTION

1.0.1 Outline of thesis content

This thesis covers four main topics: the detection of anisotropies of the CMB with data taken by the Tenerife differencing experiments and Jodrell Bank interferometers, the analysis of this data to produce actual sky maps of the fluctuations, the potential of future CMB experiments and subsequent analysis of the sky maps.

Chapter 2 introduces the processes involved in the formation of the fluctuations and the resultant radiation we expect to see in our sky maps. This includes both the CMB and other emission processes that operate at the frequencies covered by CMB experiments. These other processes are the Sunyaev–Zel’dovich effect, point source emissions and various Galactic foregrounds (namely dust, bremsstrahlung and synchrotron emission).

Chapters 3 and 4 present the experiments discussed in the thesis and the data obtained from them. Preliminary analysis is done on the data to put constraints on Galactic emissions and point source contributions. The data discussed covers the 5 GHz to 33 GHz frequency range which is at the lower end of the useful CMB spectral window. Lower frequency surveys are used to put constraints on the spectral index of some of the Galactic foregrounds (the frequency range is not large enough to put any useful constraints on dust emission which is expected to dominate at higher frequencies). The Tenerife experiments are used to put constraints on the level of the CMB anisotropy.

Chapter 5 introduces the concepts of the Maximum Entropy Method (MEM), the Wiener filter, CLEAN and Singular Value Decomposition. These four methods offer different alternatives to find the best underlying cosmological signal within the noisy data. The usual approach of a positive–only Maximum entropy is enlarged to cover both positive and negative fluctuations, as well as multifrequency and multicomponent observations. Simulations performed (Chapter 6) have shown that MEM is the best method of the four tested when attempting a reconstruction of the CMB signal.

Chapter 7 presents the final sky maps of the CMB produced with the Maximum Entropy algorithm, as well as maps of the various contaminants that the experiments are also sensitive to. The maps from a range of different experiments can be used to put constraints on various cosmological parameters such as the density parameter, $\Omega_\sigma$, Hubble’s constant, $H_\sigma$, and the spectral index of the large scale CMB power spectrum, $n$.

Chapter 8 presents subsequent analysis performed on the sky maps. These include examining the topology using genus as well as looking at the power spectrum and correlation functions. The methods discussed are first applied to simulations to test their usefulness at distinguishing between the origins of the fluctuations and then applied to the reconstructed CMB sky maps.

New constraints on the power spectrum and some of the cosmological parameters will be given in the final chapter. Here, the data and analysis described will be brought together and the future of CMB experiments discussed.

Rhagarweiniad
O ble rydym ni i gyd yn dod? Pa brosesau wnaeth ddigwydd i greu popeth a welwn o’n cwmpas? I ateb y cwestiynau hyn byddai’n fanteisiol gallu teithio yn ôl mewn amser. Yn ôledd fìsseg mae hyn yn amhobisl, ond maent yn caniatáu rhywbeth sy’n ail orau i hynnyn. Gallwn edrych yn ôl mewn amser amser amser. Mae golau’n teithio ar gyfer ymdrechion gyntaf, felly os edrychwn yn ôl ddigon bell gallwn weld yn ôl i ddechreuad y bydysawd. Er enghraiff, pan edrychwn ar yr haul mae mor bell fel ein bod yn edrych arno fel ag yr oedd wyth munud yng Nghymru.

Heddiw mae seryddwyr yn gallu gweld cyn belled yn ôl mewn amser ag sy’n bosibl. Gymysgedd o fás ac ynni ymbelydrol yw’r bydysawd. Dros dymheredd arbennig (~4000°C), oherwydd ynni uchel yr ymbelydriad, mae’r rhynghweithiad rhwng más ac ynni yn gwneud y bydysawd yn dywyll. Yn nechrau’r bydysawd roedd popeth wedi ei wasgu’n belen fechan poeth a ddechreuodd ehangu ac oeri wedyn. Felly, os, edrychwn yn ôl mewn amser pryd y daeth y bydysawd yn glir. Mae’r ymbelydriad i’r pryd hwn mewn amser amser wedi bod yn teithio tuag atom am 15 bilion o flynnyddoedd ac mae wedi oeri i dymheredd o −270°C erbyn hyn. Mae’r tymheredd hwn yn cyfateb i ymbelydriad microdon. Arno Penzias a Robert Wilson oedd y rhai cyntaf i ddarganfod yr ymbelydriad hwn yn 1965. Mae’n ymddangos o bob cyferiad yn yr awyr ac mae’n unffurf iawn. Mae’r unffurfedd hwn yn achosi problem. Os yw’r bydysawd mor wastad sut y gwnaeth unrhyw beth fforio? Mae’n rhaid fod yna rai gwrymiau yn y bydysawd cynnar i greu’r adeileddau a welwn heddiw.

Lloern Cosmic Microwave Background Explorer (COBE) NASA yn 1992 oedd yr arbrawf cyntaf i ddarganfod y gwrymiau. Ni llwyddodd i dynnu lluniau o’r gwrymiau mewn gwirionedd oherwydd bod cymaint o swn yn gwneud hynny’n amhobisl. Daeth ar draws swn na’r arfer a’r unig eglurhad y gellid ei roi am hynny oedd presenoldeb gwrymiau yn y bydysawd. Roedd hyn yn ffodus i seryddwyr neu byddent wedi gorfod newid eu holl ddamcaniaethau. Erbyn Heddiw mae mapiau o’r bydysawd cynnar hyd yn oed yn cael eu cynyrru. Led led y byd mae arbrofion yn darganfod gwrymiau a dyfodd yn y diweddd yn alaethau a chlystyrau o alaethau.

Roedd yr gwaith hwn mor sensitif fel bod rhaid dyfeisio technegau newydd mewn seryddiaeth. Datblygydd dulliau newydd o dymnu lluniau o’r awyr gyda thelegopau newydd. Yr egwyddor sylfaenol y tu ôl i’r holl arbrofion hyn oedd, yn hytrach na’u bod yn mesur y disglerdeb gwirioneddol, eu bod yn mesur y gwahaniaeth mewn disglerdeb rhwng gwahanol ranbarthau o’r awyr. Cynhyrchodd yr arbrofion yn Tenerife y mapiau cyntaf o’r awyr. Mae Telesgop Anisotropy Caergrawnt hefyd yn cynyrru mapiau o ranbarthau llai o’r awyr, gan weld gwyriniau llai na’r rhai a welwyd o Tenerife.

Bydd yr holl arbrofion hyn yn rhoi prawf ar ddamcaniaethau’r seryddwyr. Ceir canmoedd o wahanol ddamcaniaethau ynglŷn â’r modd y ddechreuodd y bydysawd a sut y dathlygddod i’r adeileddau a welir heddiw. Mae pob un o’r damcaniaethau hyn yn cynnwys sy’n ymgasglu ychydig yn wahanol ynglŷn â’r modd yr edrychais’r bydysawd yn ynnar ym ei hanes a hwyd yma bu’n amhobisl eu profi neu eu gwrthbrofi.

O fewn ychydig flynyddoedd dylai fod yn bosibl i seryddwyr dweud wrthym o ble y daeth y bydysawd, sut y dathlygddod ac ym mhlwyddod y byddyn gorffen. Mae’n gyfnod cyffrous i seryddwyr gyda holl gwyriniau’r bydysawd yn disgwyl i gael eu
CHAPTER 1. INTRODUCTION

darganfod.
Chapter 2

The Universe and its evolution: the origin of the CMB and foreground emissions

In this chapter I summarise the various processes that go into forming the power spectrum of fluctuations in the Cosmic Microwave Background. This includes primordial effects as well as foreground effects like the Sunyaev-Zel’dovich effect. I also describe the various foregrounds that have significant emissions at the frequencies of interest to Microwave Background experiments.

The theory of the Big Bang stems from Edwin Hubble’s observations that every galaxy is moving away from every other galaxy (providing they are not in gravitational orbit about each other). Hubble’s law tells us that the velocity of recession away from a point in the Universe is proportional to the distance to that point. Today that constant of proportionality is called Hubble’s constant, $H_0$. If we extrapolate this law back in time, there comes a point where everything in the Universe is very close together. To get everything we see today into a very small region requires an enormous amount of energy and this is where the Big Bang theory is born. This hot, dense ‘soup’ expanded, cooled and eventually formed all the structures that we see today.

2.1 Symmetry breaking and inflation

In physics, as things get hotter they generally get simpler. At a relatively low temperature ($\sim 10^{15}$ K), compared to the Big Bang, the electromagnetic force and the weak force, which binds the nucleus together, combine to form the electro–weak force. At higher temperatures ($\sim 10^{28}$ K) the strong force (described by Quantum Chromo-Dynamics), which keeps the proton from splitting into its quarks, joins the electro–weak force to become one force. This theory is called Grand Unification (GUT). It is hypothesised that the last force, the force of gravity, joins the other forces in a theory of quantum gravity, at even higher temperatures (corresponding to the very earliest times in the Universe). At this stage everything in the Universe
is indistinguishable from everything else. Matter and energy do not exist as separate entities and the Universe is very isotropic. To create structure in such a Universe there are two main theoretical models (or a combination of the two).

Starting from Newtonian physics and considering the effect of gravity on a unit mass at the edge of a sphere of radius \( R \), then

\[
\ddot{R}(t) = -\frac{GM}{R^2(t)}, \tag{2.1}
\]

where \( t \) is the time since the Big Bang and \( M \) is the mass inside the sphere, given by

\[
M = \frac{4}{3} \pi R^3(t) \rho(t). \tag{2.2}
\]

In an expanding Universe, with no spontaneous particle creation, the amount of matter present does not change and so \( M \) is constant if we follow the motion of the edge of the sphere. Thus, if we write the present density of the Universe as \( \rho_0 \) then we have

\[
\rho(t) = \frac{R^3(t_0)}{R^3(t)} \rho_0, \tag{2.3}
\]

where \( t_0 \) corresponds to now. The gravitational force per unit mass in Equation 2.1 is therefore given by

\[
\ddot{R}(t) = -\frac{4}{3} \pi G \rho_0 R^{-2}(t) \tag{2.4}
\]

where \( R(t_0) \), the radius of the sphere today, is taken as unity. Integrating Equation 2.4 gives

\[
\dot{R}^2(t) = \frac{8}{3} \pi G \rho_0 R^{-1}(t) - k c^2. \tag{2.5}
\]

The constant of integration is found by including General Relativistic considerations where \( k \) is a measure of the curvature of space. So the equation of evolution of the Universe is

\[
\left( \frac{\dot{R}}{R} \right)^2 - \frac{8 \pi G \rho}{3} = -\frac{k c^2}{R^2}. \tag{2.6}
\]

We define Hubble’s constant, the rate of expansion of the Universe, as \( H = \frac{\dot{R}}{R} \). In General Relativity the Universe is said to be closed if the density is high enough to prevent it from expanding forever. If the density is too low then the Universe will continue to expand forever and is called open. The point at which the density becomes critical (in between a closed and open Universe) corresponds to a flat space, or \( k = 0 \). This critical density is found from Equation 2.6 to be

\[
\rho_{\text{crit}} = \frac{3 H^2}{8 \pi G} \tag{2.7}
\]
2.1. SYMMETRY BREAKING AND INFLATION

Figure 2.1: The expansion of the Universe. The red line shows the Universe whose size expands at the speed of light whereas the blue line includes an early inflationary period.

and we can define a new parameter called the closure parameter, \( \Omega = \frac{\rho}{\rho_{\text{crit}}} \). Using this definition, if \( \Omega > 1 \) then the Universe is heavier than the critical density (closed) and if \( \Omega < 1 \) then it is lighter (open).

If another constant is included in Equation 2.1 then an effective zero energy (the scalar field) can be added to the General Relativistic description of the Universe. This can be thought of as a ‘vacuum energy’, the lowest possible state in the Universe. This alters Equation 2.5 so that it includes the Cosmological constant

\[
\dot{R}^2(t) = \frac{8}{3} \pi G \rho_s R^{-1}(t) + \frac{\Lambda}{3} R^2 - k c^2
\]

(2.8)

If this cosmological constant is the dominant term (as the scalar field is expected to be at very high temperatures), and the other two terms in Equation 2.8 become negligible, it is possible to solve and find

\[
R(t) \propto \exp \left[ \left( \frac{\Lambda}{3} \right)^{\frac{1}{2}} t \right]
\]

(2.9)

which represents an exponential expansion. This expansion, in which the Universe expands at speeds faster than light, is dubbed inflation (Guth 1981 and Linde 1982). Figure 2.1 shows the effect of inflation on the size of the Universe.

With such an expansion, small quantum fluctuations (produced by Heisenberg’s uncertainty principle) would expand up into large inhomogeneities in the Universe. These inhomogeneities are the density fluctuations that then go on to form the structure that is present in the Universe today. Exponential expansion stops when the \( \Lambda \) term becomes less dominant.

Another possible way to create fluctuations is through phase transitions in the early Universe. The theory of phase transitions does not require inflation but it does not rule it out either. When one of the fundamental forces becomes separated from the rest, the Universe is said to undergo a phase transition. If, during an early phase transition, some of the energy of the Universe is trapped between two regions undergoing the transition in a slightly different way and is frozen out, then a topological defect (Coulson et al. 1994) is formed. Depending on the original GUT the Universe is described by we get different defects. There are four possible defects, corresponding to zero, one, two and three dimensions, called monopoles, strings (see Brandenberger 1989), domain walls and textures (Turok 1991) respectively. A string, for example, can be thought of as a frozen one–dimensional region of ‘early’ Universe. It separates regions that went through the phase transition at slightly different times so that the geometry around the string is different from normal space–time geometry. In particular the angle surrounding a string is less than 360°. These defects can act as seeds for structure formation through their gravitational interaction.

After inflation the Universe was still very hot and radiation dominated so that no atoms could be formed. A thermal equilibrium between matter and radiation was set
up by continual scattering. As the Universe cooled, processes not fully understood as yet caused an excess of matter over anti-matter which started to form basic nuclei (deuterium, helium and lithium). This ‘soup’ of interacting particles and radiation has a very high optical depth and so the radiation could not escape.

### 2.2 Dark matter

We have already defined the closure parameter of the Universe, $\Omega$, as the ratio of the actual density of the Universe to the critical density. For $\Omega > 1$ (a closed Universe) gravity will dominate and the Universe will collapse in on itself in a finite time. For $\Omega < 1$ (an open Universe) the expansion will dominate and the Universe will continue growing forever. We can weigh the Universe by making observations of the stars and galaxies and estimating how heavy the objects are that we can see. In 1978 observations were first reported of the rotation curve of galaxies and it was calculated that there must be a lot more mass, unaccounted for by light (see Rubin, Ford & Thonnard, 1978). Later, observations were made of velocities of galaxies in clusters and it was found that even more unseen mass was required to give the galaxies their observed peculiar velocities. The Milky Way is in orbit around the Virgo cluster with a peculiar velocity of $\sim 600 \text{ km s}^{-1}$ (see Gorenstein & Smoot 1981 for the first measurement of this peculiar velocity, determined from the dipole in the CMB). The luminous mass in the Universe can account for $\Omega_{\text{lum}} = 0.003h^{-1}$, where $h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and so, taking into account the non–luminous mass, or dark matter, this is a lower limit on $\Omega$ (see White 1989 for a review on dark matter).

It remains to be seen whether this dark matter can make $\Omega = 1$ as simple inflation predicts. The obvious question that comes to mind is – “In what form does the dark matter come?” The most obvious candidate for dark matter is non–luminous baryon matter. This can not exist as free hydrogen or dust clouds, otherwise we would expect to see large black objects across the sky blocking out the starlight. If baryonic, the dark matter must exist as gravitationally bound matter, either in the form of brown dwarfs (Carr 1990), planets, neutron stars or black holes. These may exist in an extra–galactic halo around our galaxy and are called Massive Compact Halo Objects or MACHO’s for short (Alcock et al 1993). However, the amount of baryonic matter present in the Universe is constrained by the relative proportions of hydrogen, helium, deuterium, lithium and beryllium that are observed as these were formed together in the early Universe. This gives us $0.009 \leq \Omega_b h^2 \leq 0.02$ (Copi et al 1995) and taking $h = 0.5$ then $\Omega_b < 0.1$ which means that 90% of the Universe is made up of non–baryonic matter if it is spatially flat and closed ($\Omega = 1$).

The non–baryonic matter must take the form of Weakly Interacting Massive Particles (WIMPS; Turner 1991) which only interact with baryonic matter through gravity (otherwise they would have been detected already). The theory of WIMPS can be subdivided into two categories; hot dark matter (HDM) and cold dark matter (CDM). Hot dark matter has large thermal velocities (for example, heavy neutrinos) and will wipe out structure on galactic scales in the early Universe due to streaming in the last scattering surface. This is a ‘top–down’ scenario. Cold dark matter
2.3. COMING OF AGE

has low thermal velocities (for example, axions and supersymmetric partners to the baryonic matter) and will enhance the gravitational collapse of galactic size structures. This is a ‘bottom–up’ scenario. The two theories are not mutually exclusive and the combination of CDM and HDM is called mixed dark matter (MDM).

Constraints are already possible on some of the dark matter candidates. For example, in HDM if the dominant form of matter consists of heavy neutrinos, then it can be shown (see Efstathiou 1989) that for a critical Universe (so $\Omega = 1$) the neutrinos need to be about 30 eV in mass. With better measurements of the power spectrum of the fluctuations seen in the CMB it will be possible to rule out or confirm the existence of such particles. No candidate dark matter has yet been detected.

2.3 Coming of age

At a redshift of $z \sim 1100$, the Universe had cooled to a temperature of $\sim 3000$ K. At this temperature electrons become coupled with protons and form atoms. This essentially increases the photon mean free path from close to zero to infinity in a very short time ($\Delta z \sim 80$). So the furthest we can look back is to this last scattering surface. This period is called the recombination era and is the origin of the microwave background radiation studied in this thesis. By observing this radiation the imprints of the fluctuations from the early part of the Universe can be studied and hence cosmological models can be tested. The temperature of the microwave background has now cooled down through the effects of cosmic expansion and has been measured to a very high degree of accuracy. It is found (see Mather, J.C. et al 1994) to be at

$$T_0 = 2.726 \pm 0.010\text{K} \ (95\% \text{ confidence}). \quad (2.10)$$

The pattern of fluctuations in the radiation from the last scattering surface will tell us a lot about the early Universe. There are two models for how the matter fluctuations couple to the radiation fluctuations. These are adiabatic and isocurvature fluctuations. Inflation naturally produces the former but in special conditions can produce the latter. Adiabatic fluctuations are perturbations in the density field which conserve the photon entropy of each particle species (the number in a comoving volume is conserved). Isocurvature fluctuations are fluctuations in the matter field with equal and opposite fluctuations in the photon field, keeping the overall energy constant and therefore a constant curvature of space–time.

Due to the early coupling between matter and radiation, prior to the last scattering surface, an almost perfect blackbody would exist throughout the Universe at last scattering. For blackbody emission, the spectrum (i.e. the CMB) is given by the differential Planck spectrum

$$\Delta T_A = \frac{\Delta T x^2 e^x}{(e^x - 1)^2}. \quad (2.11)$$

Therefore the change in intensity is
\[ \Delta I(\nu) = \frac{\Delta T x^4 e^x}{(e^x - 1)^2} \]

where \( x = \frac{h\nu}{kT} \). The Universe progressively became more transparent and so the fluctuations seen at the last scattering surface are a superposition of fluctuations within a last scattering volume (comprising of the region between a totally opaque and a totally transparent Universe). This can be expressed in terms of the optical depth, \( \tau \), as

\[ \frac{\Delta T}{T} = \int_0^z \frac{\delta T(z)}{T} e^{-\tau(z)} \frac{dT}{dz} dz, \]

where \( g(z) = e^{-\tau(z)} \frac{dT}{dz} \) is a Gaussian centred on \( z \sim 1100 \) with \( \Delta z \sim 80 \), the width of the last scattering surface (see, for example, Jones & Wyse 1985). Radiation from fluctuations smaller than the width of last scattering will add incoherently and therefore the radiation pattern of fluctuations will be erased on these small scales. This corresponds to an angular size of \( \theta = 3.8' \Omega^\circ_{1/2} \) on the sky today so all anisotropies smaller than this will be heavily suppressed.

### 2.3.1 The dipole

The main source of anisotropy in the CMB is not intrinsic to the Universe. It is produced by the peculiar velocity of the observer. Moving towards an object causes emitted light to appear blueshifted. As the Earth is not stationary with respect to the CMB (it is moving around the sun, the sun around the galaxy, the galaxy around the Virgo cluster etc.) there will be a part of the CMB that the Earth moves towards and a part that it moves away from. Therefore, we expect to see a large dipole created by this Doppler effect. This dipole was clearly detected by the COBE satellite (see Figure 2.2). It is necessary to remove this before attempting to study the intrinsic fluctuations in the CMB.

### 2.3.2 Sachs-Wolfe effect

As the Universe grows older the observable Universe gets bigger, due to the finite speed of light. The particle horizon of an observer is the distance to the furthest object that could have affected that observer. Any objects further than this point are not, and never have been, in causal contact with the observer. At the last scattering surface the particle horizon corresponds to \( \theta \sim 2^\circ \) as seen from Earth today. No physical processes will act on scales larger than this. Therefore, at the epoch of recombination fluctuations must have been produced by matter perturbations already present at this time. Inflation gives us a mechanism for the creation of these fluctuations. These matter perturbations give rise to perturbations in the gravitational potential. Radiation will experience different redshifts depending on
the potential and hence produce large angular scale anisotropies in the CMB. This process is called the Sachs–Wolfe effect (Sachs & Wolfe 1967).

It can be shown (see for example Padmanabhan 1993) that the angular dependence of the Sachs–Wolfe temperature fluctuations on scales greater than the horizon size is given by

$$\frac{\Delta T}{T} \propto \theta^{(1-n)/2},$$

(2.14)

where $n$ is the spectral index of the initial power spectrum of fluctuations ($P(k) = A_k^n$). In inflation the natural outcome is a spectral index $n = 1$ because the fluctuations originate from quantum fluctuations that have no preferred scale (although recently it has been shown that inflation does allow other possible values of $n$). This special case is called the Harrison–Zel’dovich (Harrison 1970 and Zel’dovich 1972) spectrum and leads to the $\Delta T/T$ fluctuations being constant on all angular scales larger than the horizon size. These fluctuations have been observed by the COBE satellite at an angular scale of $7^\circ$. The combined maps from the three observing frequencies after two years of COBE measurements at this angular scale are shown in Figure 2.3.

### 2.3.3 Doppler peaks

An overdensity in the early Universe does not collapse under the effect of self-gravity until it enters its own particle horizon when every point within it is in causal contact with every other point. The perturbation will continue to collapse until it reaches the Jean’s length, at which time radiation pressure will oppose gravity and set up acoustic oscillations. Since overdensities of the same size will pass the horizon size at the same time they will be oscillating in phase. These acoustic oscillations occur in both the matter field and the photon field and so will induce ‘Doppler peaks’ in the photon spectrum.

The level of the Doppler peaks in the power spectrum depend on the number of acoustic oscillations that have taken place since entering the horizon. For overdensities that have undergone half an oscillation there will be a large Doppler peak (corresponding to an angular size of $\sim 1^\circ$). Other peaks occur at harmonics of this. As the amplitude and position of the primary and secondary peaks are intrinsically determined by the number of electron scatterers and by the geometry of the Universe, they can be used as a test of the density parameter of baryons and dark matter, as well as other cosmological constants.

### 2.3.4 Defect anisotropies

The anisotropies produced by the various forms of defects arise from the effect that the defect has on the surrounding space–time (for example see Coulson et al 1994). Not only do they leave imprints on the CMB at the time of last scattering but a
large proportion of the fluctuations due to defect anisotropies would be produced at latter times. As an example, consider the effect of cosmic strings (see Kaiser & Stebbins 1984). A string moving with velocity $v$ will leave behind it a ‘gap’ in space–time. The angle around the string is not $360^\circ$ but is reduced by $8\pi G \mu$, where $\mu$ is the energy density per unit length in the string. Therefore, photons travelling through space behind the string will experience a Doppler boost, with respect to photons in front of the string, as they have less space to travel through. The value of this boosting is

$$\frac{\Delta T}{T} = 8\pi G \mu \frac{v}{c}$$  \hspace{1cm} (2.15)

and is called the Kaiser–Stebbins effect. This results in a linear discontinuity in the CMB when the string passes in front and is easily discernible from the Gaussian anisotropies produced by the other processes. The higher dimensional defects will produce more complicated discontinuities. Recently however, Magueijo et al 1996 and Albrecht et al 1996 have shown that this discontinuity effect may be masked by the defect interaction with the CMB prior to recombination. On large angular scales the discontinuities will also add together and mimic a Gaussian field (the central limit theorem). Therefore, only a high resolution (on the arcmin scale), high sensitivity, experiment will be able to distinguish between defect and inflationary signatures on the CMB.

### 2.3.5 Silk damping and free streaming

Prior to the last scattering surface the photons and matter interact on scales smaller than the horizon size. Through diffusion the photons will travel from high density regions to low density regions ‘dragging’ the electrons with them via Compton interaction. The electrons are coupled to the protons through Coulomb interactions and so the matter will move from high density regions to low density regions. This diffusion has the effect of damping out the fluctuations and is more marked as the size of the fluctuation decreases. Therefore, we expect the Doppler peaks to vanish at very small angular scales. This effect is known as Silk damping (Silk 1968).

Another possible diffusion process is free streaming. It occurs when collisionless particles (e.g. neutrinos) move from high density to low density regions. If these particles have a small mass then free streaming causes a damping of the fluctuations. The exact scale this occurs on depends on the mass and velocity of the particles involved. Slow moving particles will have little effect on the spectrum of fluctuations as Silk damping already wipes out the fluctuations on these scales, but fast moving, heavy particles (e.g. a neutrino with 30 eV mass), can wipe out fluctuations on larger scales corresponding to 20 Mpc today (Efstathiou 1989).

### 2.3.6 Reionisation

Another process that will alter the power spectrum of the CMB is reionisation. If, for some reason, the Universe reheated to a temperature at which electrons and
protons became ionised after recombination, then the interaction with the photons would wipe out any small scale anisotropies expected. Today, there is reionisation around quasars and high energy sources but this occurred too late in the history of the Universe to have any large effect on the CMB. Little is known about the period between the last scattering surface and the furthest known quasar ($z \sim 4$) so reionisation cannot be ruled out.

### 2.3.7 The power spectrum

The usual approach to presenting CMB observations is through spherical harmonics. The expansion of the fluctuations over the sky can be written as

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

(2.16)

where $\theta$ and $\phi$ are polar coordinates. Here $a_{\ell m}$ represent the coefficients of the expansion. For a random Gaussian field all of the statistical information can be obtained by considering the two–point correlation function, given by

$$C(\beta) = \left\langle \frac{\Delta T}{T}(n_1) \frac{\Delta T}{T}(n_2) \right\rangle$$

(2.17)

for the unit vectors $n_1$ and $n_2$ that define the directions such that $n_1 \cdot n_2 = \cos(\beta)$. Substituting Equation 2.16 into Equation 2.17 gives

$$C(\beta) = \sum_{\ell m} \sum_{\ell' m'} <a_{\ell m} a_{\ell' m'}^* > Y_{\ell m}(\theta, \phi) Y_{\ell m'}^*(\theta', \phi').$$

(2.18)

If the CMB has no preferred direction, so that it is statistically rotationally symmetric, then

$$C(\beta) = \frac{1}{4\pi} \sum_{\ell} (2\ell + 1) C_\ell P_\ell(\cos \beta)$$

(2.19)

defining $<a_{\ell m} a_{\ell' m'}^*> = C_\ell \delta_{\ell \ell'} \delta_{m m'}$ and the multiplication of spherical harmonics give the Legendre polynomials $P_\ell(\cos \beta)$. If this is taken as a true indicator of the CMB then the $C_\ell$ values can be used to give a complete statistical description of the fluctuations. These $C_\ell$ values can be predicted from theory (normalised to an arbitrary value) and constitute the power spectrum of the CMB. For example, if a standard power law ($P(k) = A k^n$) can be used to describe the fluctuations, as in the case of the Sachs Wolfe effect, then $C_\ell$ is given by (see Bond & Efstathiou, 1987)

$$C_\ell = C_2 \frac{\Gamma \left[ \ell + (n - 1)/2 \right] \Gamma \left[ (9 - n)/2 \right]}{\Gamma \left[ \ell + (5 - n)/2 \right] \Gamma \left[ (3 + n)/2 \right]}$$

(2.20)

where $C_\ell$ is now normalised to the quadrupole term $C_2$ and $\Gamma[x]$ are the Gamma functions.

The power spectrum of the CMB is made up of a combination of all the competing processes already described. At large angular scales (corresponding to small $\ell$ values in the Fourier plane) the level of fluctuations (and hence $\ell(\ell + 1)C_\ell$) will be constant.
due to the Sachs–Wolfe effect. At intermediate angular scales ($\sim 2^\circ$) the level will start to rise when the acoustic oscillations begin to act. At the smallest angular scales the level will approach zero as the dissipative processes take place. The actual shape of the power spectrum can be calculated for all inflationary scenarios but this is much more difficult for defects. In all simulations of the CMB discussed in this thesis the power spectrum used is that for a standard CDM inflationary model with $H_0 = 50$ km s$^{-1}$, $\Omega = 1$ and a baryon density parameter, $\Omega_b = 0.05$. Figure 2.4 shows the predicted spectrum for a standard CDM realisation of the Universe.

2.4 The middle ages

With the seeds of fluctuations sown, gravity started to enhance the differences. Over–dense regions grew at the expense of under–dense regions and new structures formed. Over the next few billion years galaxies and clusters of galaxies would decouple from the Hubble flow with the help of gravity. These structures (both large and small) are the structures that are seen in the night sky. Unfortunately for a CMB astronomer, these structures are also part of what they see when they point their telescopes at the sky. The CMB emits in the microwave region of the spectrum but so do extra–galactic sources and sources within the galaxy. This section describes some of the foreground processes that have significant emission at frequencies of interest to CMB astronomers, like extra–Galactic point sources, as well as processes that interact with the CMB photons to alter their spectra, like the Sunyaev–Zel’’dovich effect.

2.4.1 Sunyaev–Zel’dovich effect

Hot ionised gas interacts with the CMB photons to alter their power spectrum. Such a hot region is found around clusters of galaxies. To first order, the Doppler scattering of the photons from the electrons in the hot gas averages to zero. However, to second order, the inverse Compton effect will distort the power spectrum. A relatively cold photon passing through a hot gas will gain a boost in its energy, moving its temperature up slightly leaving a hole in the CMB. Therefore, there will be less CMB photons at lower frequencies while at higher frequencies there will be an excess of CMB photons. This results in a frequency dependence of the spectrum given by (for derivation see Rephaeli & Lahav 1991)

$$
\Delta I = \frac{2(kT)^3}{hc^2}yg(x),
$$

(2.21)

where $T$ is the temperature of the CMB, $\nu$ is the frequency, $y$ is the Comptonisation parameter which is dependent on the electron interaction with the photon, and $g(x)$ is given by
2.4. THE MIDDLE AGES

Figure 2.5: The functional form of the thermal (solid line) and kinetic (dashed line) SZ effect.

\[
g(x) = \frac{x^4 e^x}{(e^x - 1)^2} \left[ x \coth \left( \frac{x}{2} \right) - 4 \right]
\]  
(2.22)

with \( x = \frac{\hbar \nu}{kT} \).

If the cluster is moving with its own peculiar velocity (i.e., it is not moving solely with the Hubble flow), then there will be an extra Doppler shift in the spectrum. This boosts the spectrum up slightly from the normal CMB spectrum but still preserves its blackbody nature. The frequency dependence of this effect is

\[
\Delta I = -\frac{2(kT)^3 v_r}{(hc)^2} \frac{\tau}{c} h(x),
\]  
(2.23)

where \( v_r \) is the peculiar velocity of the cluster along the line of sight, \( \tau \) is the optical depth of the cluster and \( h(x) \) is given by

\[
h(x) = \frac{x^4 e^x}{(e^x - 1)^2}.
\]  
(2.24)

These two effects combine together to form the Sunyaev-Zel’dovich (SZ) effect. They occur on the angular scale of clusters of galaxies, which is generally below the scale where Silk damping has wiped out the fluctuations intrinsic to the CMB. Figure 2.5 shows \( g(x) \) and \( h(x) \) as a function of frequency. As can be seen the thermal SZ effect (arising from the inverse Compton scattering) has a very characteristic spectrum which makes it easy to identify (the zero point is at 217 GHz) whereas the kinetic SZ effect (arising from the Doppler boost) has the same spectrum as the differential CMB blackbody and is therefore harder to distinguish using statistical techniques.

Since the SZ effect arises from the CMB interacting with cluster gas, the spatial power spectrum of the anisotropies will closely follow that of the cluster gas. The cluster gas is gravitationally tied to galaxy clusters, which are distributed in a Poissonian manner (white noise) across the sky. Therefore, the power spectrum of the SZ effect \( (C_\ell) \), like that of the extra–galactic point sources, is expected to be constant with \( \ell \).

2.4.2 Extra–galactic sources

One of the main foregrounds that is seen in CMB data originates from extra-galactic sources. These are usually unresolved point sources such as quasars and radio–loud galaxies. A study of the contribution by unresolved point sources to CMB experiments has been produced by Franceschini et al (1989). They used numerous surveys, including VLA and IRAS data, to put limits on the contribution to single beam CMB experiments by a random distribution of point sources. This analysis assumes that there are no unknown sources that only emit radiation in a frequency range between \( \sim 30 \) GHz and \( \sim 200 \) GHz. This range of frequency has not been properly surveyed
CHAPTER 2. THE UNIVERSE AND ITS EVOLUTION

Figure 2.6: Curves of constant $\log(\Delta T_A/T_A)$ for a random distribution of point sources with a detection limit of $5\sigma$. Taken from Franceschini et al 1989.

and therefore there is still a cause of concern in the CMB community. In spite of this, the analysis by Franceschini et al will be used to put constraints on the contribution from unresolved point source to the data discussed in this thesis. Figure 2.6 shows the expected fluctuation levels for a random distribution of unresolved sources (it was assumed that all sources above $5\sigma$ of the root mean square ($rms$) point source contribution could be resolved and subtracted effectively) as found in Franceschini et al. This analysis assumed that the point sources exhibit a Poissonian flux distribution with no clustering and so this estimate is likely to be an underestimate of the total $rms$ signal expected (clustering enhances the level of fluctuations). More recent studies of the contribution by point sources to CMB experiments (e.g. De Zotti et al 1997) show similar results.

In CMB data it is often not easy to distinguish between a resolved point source convolved with the beam and CMB fluctuations. Therefore, an estimate for these resolved sources, as well as the expected level of unresolved sources, is needed. The survey carried out with the 300 ft Green Bank telescope in 1987 (Condon, Broderick & Seielstad, 1989) is used for the estimates. This survey consists of data between $0^\circ$ to $+75^\circ$ in declination at 1400 MHz and 4.85 GHz. At 1400 MHz the survey has a resolution of $12'$ and is complete to $\sim 30$ mJy, and at 4.85 GHz the resolution is $4'$ and is complete to $\sim 8$ mJy. To predict the point source levels for the various experiments considered in this thesis the surveys must be converted to a common resolution and gridding.

All fluxes in the two frequency surveys, above the sensitivity levels, are then compared, pixel by pixel, to find the spectral index of each pixel. The spectral index is then used to extrapolate the flux up to the frequency of the experiment being considered. In this way a spatially varying spectral index for the point sources is obtained. This has obvious disadvantages as it does not take into account steepening spectral indices or the variability of sources but it is the best simple estimate and will provide good constraints on the data. The maps are then converted from flux, $S$, into antenna temperature, $T_A$, using the Rayleigh–Jeans approximation to the differential of the Planck spectrum given by

$$T_A = \left(\frac{\lambda^2}{2k\Omega_b}\right) S,$$  \hspace{1cm} (2.25)

where $\lambda$ is the wavelength of the experiment, $\Omega_b$ is the area of the beam ($\Omega_b = 2\pi\sigma^2$ where $\sigma$ is the beam dispersion) and $k$ is the Boltzmann constant. At $\sim 60$ GHz $\frac{h\nu}{kT}$ becomes 1 and Equation 2.25 is no longer a good approximation and the full spectrum should be used (e.g. for the Planck Surveyor satellite). For the 5 GHz Jodrell Bank interferometer (see next chapter for description of the experiments) the conversion using the Rayleigh–Jeans approximation is

$$\frac{T_A}{S} = 64\mu K/Jy$$ \hspace{1cm} (2.26)
2.4. THE MIDDLE AGES

Figure 2.7: The variability of 3C345 as a function of time at various frequencies. Taken from the Michigan and Metsahovi monitoring program.

and for the 10 GHz, 8.3° FWHM beam switching experiment at Tenerife

\[ \frac{T_A}{S} = 14 \mu K / Jy. \]  (2.27)

The final maps of the expected point source temperatures are then convolved with the experimental beam and compared with the data from that particular experiment.

This prediction for the level of point source contamination is a good first approximation but for accurate subtraction from the data more is required. Many point sources are highly variable and so will contribute to each data set differently. Without simultaneous observations of each point source in the data this is very difficult to account for. For example, one of the main contaminants to the data sets, discussed in this thesis, is 3C345. The variability of this source at various frequencies is shown as a function of time in Figure 2.7. It is seen that, over the period that the data discussed in this thesis was taken, 3C345 varied in flux by more than a factor of two. This would have a large effect on the subtraction of this source from the individual data scans and making a prediction by averaging the data over ten years of data collection will give incorrect results. It is noted that this is likely to be the most variable point source in the region that is of interest in this thesis. The data for the variability of the point sources has only recently become available and so only the first approximation for fitting point sources was used in the analysis presented in this thesis. Therefore, care was taken to exclude any highly variable point sources from regions that were used for CMB analysis.

The distribution of extragalactic point sources across the sky is Poissonian. This is just a simple white noise power spectrum and, therefore, it is flat with varying angular scale. In contrast to the varying spectrum of the CMB \( (C_\ell \propto (\ell(\ell + 1))^{-1}) \) we expect the power spectrum \( (C_\ell) \) to be constant for all values of \( \ell \). This constant value is determined through observational constraints.

2.4.3 Galactic foregrounds

Galactic emission processes, such as bremsstrahlung (free-free), synchrotron and dust emission, are all important foregrounds in CMB experiments. The following is a brief description of the processes involved in the foreground emissions and gives the best estimate of the spectral dependencies of each.

Dust emission

At the higher frequency range of the microwave background experiments, dust emission starts to become dominant. This is the hardest galactic foreground to estimate as it depends on the properties of the individual dust grains and their environment.

The emission from an ensemble of dust grains follows an opacity law. For this process the intensity is given by
\[ I(\nu) = \int \epsilon(\nu)dl, \]  

(2.28)

where \( \epsilon(\nu) \) is the emissivity at frequency \( \nu \), and the integral is along the line of sight. The brightness temperature is found from the black body equation and is therefore a solution of

\[ I(\nu) = \frac{2h\nu^3}{c^2} \frac{1}{(e^{h\nu/kT} - 1)}. \]  

(2.29)

In the Rayleigh-Jeans approximation (where \( h\nu \ll kT \)),

\[ T_b = \frac{c^2 I(\nu)}{2\nu^2 k}. \]  

(2.30)

This equation is used to convert between temperature and flux for all of the foreground emissions. Considering a constant line-of-sight density of dust, it is possible to combine Equations 2.30 and 2.28 to give

\[ T_b \propto \epsilon(\nu)\nu^{-2}. \]  

(2.31)

When modelling dust emission, it is therefore necessary to find the emissivity as a function of frequency, as well as the flux level at a particular frequency.

From surveys of the dust emission (for example the COBE FIRAS results and the IRAS survey) it can be shown that low galactic latitude dust (dust in the galactic plane) is modelled well by a blackbody temperature of 21.3 K and an emissivity proportional to \( \nu^{1.4} \), while at high galactic latitudes it is well modelled by a blackbody temperature of 18 K and an emissivity proportional to \( \nu^2 \) (Bersanelli et al. 1996). Since the observations discussed in this thesis are all at high galactic latitudes the dust models used have an assumed blackbody temperature of 18 K and a spectra which follows

\[ \Delta I(\nu) \propto \Delta T x^6 e^x \left( e^x - 1 \right)^2, \]  

(2.32)

where \( x = \frac{h\nu}{kT} \). This equation is obtained by the differentiation of Equation 2.29 with respect to \( T \), multiplied by the dust emissivity.

As the dust emission comes from regions of warm interstellar clouds it is very likely that there will also be ionised clouds associated with the neutral clouds (perhaps embedded within the neutral clouds or surrounding them, see McKee & Ostriker 1977 for an example of correlated features) and so we should expect bremsstrahlung from the same region (see below for description of bremsstrahlung). This results in an expected correlation between the dust and bremsstrahlung. This correlation has been detected (see for example Kogut et al. 1996a or Oliveira-Costa et al. 1997 who find a cross-correlation between the DIRBE dust maps and the low frequency Saskatoon data which is contaminated by bremsstrahlung). In a full analysis of any data this correlation should be taken into account.

From IRAS observations of dust emission (Gautier et al. 1992), it was found that the dust fluctuations have a power law that decreases as the third power of \( \ell \). This
Figure 2.8: Electron with charge $e$ passing through the Coulomb field of an ion with charge $Ze$. 

has also been confirmed at larger angular scales by the COBE DIRBE satellite. This means that at small angular scales (large $\ell$) there is less power in the dust emission.

**Bremsstrahlung**

When a charged particle is accelerated in a Coulomb field it will emit radiation to oppose this acceleration; a braking radiation or *Bremsstrahlung* (also known as free–free emission). In ionised clouds of gas with no magnetic field this process will be the dominant source of radiation. The expected spectrum of this emission can be derived by considering the classical non-relativistic case. Also one can make the simplification that only the electron in an electron-ion interaction will emit the radiation, as the acceleration is inversely proportional to the mass of the particle and so the ion, being much heavier than the electron, can be effectively thought of as stationary and, therefore, does not emit. Since the ion is stationary the electron moves in a fixed Coulomb field.

First consider the radiation from one electron. To derive the functional form of the radiation we assume that the electron does not deviate a great deal from its original path while interacting with the Coulomb field (this is a good approximation if the electron is moving very fast so that the main change in its momentum will be normal to the path and any change parallel is negligible). Figure 2.8 shows the path of the electron as it passes the ion. The parameter $b$ represents the electron’s closest approach to the ion.

The dipole moment of the electron is given by

$$d = -eR$$

and its second derivative with respect to time, in terms of the velocity, $v$, is

$$\ddot{d} = -e\dot{v}.$$  \hspace{1cm} (2.34)

The electric field from a dipole in the non-relativistic case is given by

$$E_{rad} = \frac{q}{4\pi\varepsilon_0 r c^2} n \times (n \times \dot{v}),$$

where $n$ is the line of sight from the observer to the particle and $r$ is the distance. In the case when the electron is not deviated a great deal from its original path, so that $\ddot{d}$ is along the normal to $v$, the electric field at a point $i$, distance $r$ away from the dipole, is given by

$$E(t) = \frac{1}{4\pi\varepsilon_0} |\ddot{d}(t)| \frac{\sin \theta}{rc^2},$$

\hspace{1cm} (2.36)
where \(E(t)\) and \(|\mathbf{d}(t)|\) represent the magnitudes of \(E(t)\) and \(\mathbf{d}(t)\), and \(\theta\) is the angle between the direction of \(\mathbf{d}\) and the point \(i\). From this electric field the radiation energy per unit area per unit frequency is given by

\[
\frac{dW}{dAd\omega} = \frac{\epsilon_o}{\pi} |\hat{E}(\omega)|^2,
\]

(2.37)

where \(\hat{E}(\omega)\) is the Fourier transform of \(E(t)\). This follows from Parseval’s theorem for Fourier transforms. When integrated over \(dA\), after substitution for the Fourier transform of Equation 2.36, it follows that

\[
\frac{dW}{d\omega} = \frac{2\mu_o \omega^4}{3c} |\hat{d}(\omega)|^2.
\]

(2.38)

The Fourier transform of Equation 2.34 is given by

\[
-\omega^2 \hat{d}(\omega) = -\frac{e}{2\pi} \int_{-\infty}^{\infty} \dot{v} e^{i\omega t} dt,
\]

(2.39)

which will integrate to zero, because it oscillates, over long integration times. For short interaction times, however, the exponential is essentially unity and we have

\[
\hat{d}(\omega) \sim \frac{e}{2\pi \omega^2} \Delta \mathbf{v},
\]

(2.40)

where \(\Delta \mathbf{v}\) is the change in electron velocity during the collision. With the assumption that the electron does not deviate from its path so that the change in velocity \((\Delta \mathbf{v})\) is normal to the path,

\[
\Delta \mathbf{v} = \frac{Ze^2}{4\pi \epsilon_o m} \int_{-\infty}^{\infty} \frac{b}{(b^2 + v^2 t^2)^{3/2}} dt = \frac{Ze^2}{2\pi \epsilon_o b m v}.
\]

(2.41)

Using Equations 2.38, 2.40 and 2.41 it follows that

\[
\frac{dW}{d\omega} = \frac{Z^2 \mu_o e^6}{6\pi^4 \epsilon_o^2 m^2 b^2 v^2};
\]

(2.42)

remembering that this result is only valid for the short interaction times, or equivalently, interactions that are within a certain distance.

Now expand this to include \(n_e\) electrons per unit volume, interacting with \(n_i\) ions per unit volume. Integrating over all interactions results in

\[
\frac{dW}{d\omega dV dt} = 2\pi v n_e n_i \int_{b_{min}}^{b_{max}} \frac{dW}{d\omega} b db,
\]

(2.43)

where we have taken the velocity of each electron to be \(v\) so that the flux of electrons incident on an ion is \(n_e v\) (the element of area around an ion is given by \(2\pi b db\)). By substituting from Equation 2.42 and integrating the final result it is found that

\[
\frac{dW}{d\omega dV dt} = \frac{Z^2 \mu_o e^6 n_e n_i}{3\pi^2 \epsilon_o^2 m^2 v} \sqrt{3} \gamma_ff(v, \omega),
\]

(2.44)
where the $b_{\text{max}}$ and $b_{\text{min}}$ parameters have been absorbed into the $g_{ff}(v,\omega)$ Gaunt factor. This factor depends on the energy of the interaction and includes quantum corrections when the full quantum analysis is considered. For the final part of the derivation consider a thermal ensemble of interacting pairs. Averaging over a Maxwellian distribution gives

$$
\frac{dW(T,\omega)}{d\omega dV dt} = \frac{\int_{v_{\text{min}}}^{\infty} dW}{\int_{0}^{\infty} v^2 e^x \exp \left( \frac{-m v^2}{2 kT} \right) dv},
$$

(2.45)

where $v_{\text{min}}$ is taking into account the photon discreteness as the electron’s kinetic energy must be at least as big as the photon energy that it is creating. The final result for the emissivity of bremsstrahlung is therefore

$$
\epsilon(\nu) \propto n_i n_e T^{-\frac{1}{2}} e^{-\frac{h \nu}{kT}} g_{ff}(T,\nu),
$$

(2.46)

where $\nu = \frac{c^2}{2\pi}$. This result has been tabulated on numerous occasions (see review article by Bressaard & van de Hulst, 1962). At the GHz frequency range of interest in this thesis, it is shown that $\epsilon(\nu) \propto \nu^{-0.1}$ which, from Equation 2.31, in terms of the temperature fluctuations, gives $T_b \propto \nu^{-2.1}$.

Kogut et al (1995) have made fits to bremsstrahlung and determined that its power spectrum decreases as the third power of $\ell$. This agrees with the assumed correlation of bremsstrahlung and dust, as the dust is found to have the same $\ell$ dependence. To model this emission the IRAS templates, normalised to the appropriate $rms$, were used as described in the dust emission section.

**Synchrotron emission**

When a relativistic particle interacts with a magnetic field $\mathbf{B}$, it will radiate. The equations of motion describing the motion of a charged particle are

$$
\frac{d}{dt} (\gamma m \mathbf{v}) = \frac{q}{c} \mathbf{v} \times \mathbf{B}
$$

(2.47)

and

$$
\frac{d}{dt} (\gamma m c^2) = q \mathbf{v} \cdot \mathbf{E} = 0.
$$

(2.48)

Equation 2.48 implies that $\gamma$, the relativistic correction factor, is constant (the magnitude of the velocity is constant) and only the particles direction is altered. The velocity perpendicular to the field is therefore given by

$$
\frac{d\mathbf{v}}{dt} = \frac{q}{\gamma mc} \mathbf{v} \times \mathbf{B}
$$

(2.49)

and its magnitude is constant. The velocity (magnitude and direction) parallel to the field must be constant. In the non-relativistic case the power from a charged particle is given by the surface integral of the Poynting flux.
CHAPTER 2. THE UNIVERSE AND ITS EVOLUTION

Figure 2.9: The emission cones of synchrotron radiation.

\[ P = \int \int \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} dS, \] (2.50)

where \( \mu_0 \) is the permeability of free space. Using Equation 2.35 and \( \mathbf{B} = \frac{1}{c} [\mathbf{n} \times \mathbf{E}] \) it is easily seen that

\[ P = \int \int \frac{\mu_0 q^2 \dot{v}^2}{6\pi c} \sin \theta dS, \] (2.51)

where \( \dot{v} \) is the acceleration of the particle and \( \theta \) is the angle between the line-of-sight from the observer to the particle and the acceleration. Transforming to the relativistic particle \( (\dot{v}'' = \gamma^2 \dot{v}||) \) which is zero here and \( \dot{v}''_\perp = \gamma^2 \dot{v}_\perp \) the power emitted in synchrotron radiation is given by

\[ P = \left\langle \frac{\mu_0 q^4 \gamma^2 B^2 \dot{v}_\perp^2}{6\pi c m^2} \right\rangle. \] (2.52)

Due to the relativistic speed at which the electron spirals through the \( \mathbf{B} \) field, the radiation will be beamed into a small cone. The electron emits radiation in one direction over the angle \( \Delta \theta \) as seen in Figure 2.9. The frequency of rotation of the electron is given by

\[ \omega_B = \frac{qB}{\gamma mc} \] (2.53)

(from Equation 2.48). The radius of the circle shown in Figure 2.9 is

\[ a = \frac{v}{\omega_B \sin \alpha}, \] (2.54)

where the \( \sin \alpha \) term is the projection of the circle into a plane normal to the field and the angle \( \Delta \theta \) is \( \frac{\pi}{2} \). The distance that the particle has travelled between the start and finish of the pulse is

\[ \Delta S = a \Delta \theta = \frac{2v}{\gamma \omega_B \sin \alpha} \] (2.55)

and the duration of the pulse is

\[ \Delta t = \frac{\Delta S}{v} = \frac{2}{\gamma \omega_B \sin \alpha}. \] (2.56)

The time between the start and the finish of the pulse as seen by an observer is less than \( \Delta t \) by a factor \( \frac{\Delta S}{c} \), which is the time taken for the radiation to travel across \( \Delta S \). So the observer sees a time difference between pulses of

\[ \Delta t_o = \frac{2}{\gamma \omega_B \sin \alpha} \left(1 - \frac{v}{c}\right) \] (2.57)
and as

\[ 1 - \frac{v}{c} \sim \frac{1}{2\gamma^2}, \tag{2.58} \]

the time interval is proportional to the inverse third power of \( \gamma \). Let the inverse of the pulse delay be defined as the frequency

\[ \omega_c = \frac{3}{2} \gamma^3 \omega_B \sin \alpha. \tag{2.59} \]

The synchrotron emission is dominated by this frequency. If the full relativistic calculation is performed, then it can be shown that the total power (from Equation 2.52) is proportional to \( \omega \omega_c \).

For an ensemble of electrons emitting synchrotron radiation, assuming an energy distribution of the form

\[ N(E) = N_0 E^{-p} dE, \tag{2.60} \]

the total radiation power is given by

\[ P_{\text{tot}}(\omega) = N_0 \int_{E_{\text{min}}}^{E_{\text{max}}} P \left( \frac{\omega}{\omega_c} \right) E^{-p} dE. \tag{2.61} \]

\( \omega_c \) depends on energy (through \( \gamma \) in Equation 2.59) and it is possible to substitute in Equation 2.61 to give

\[ \omega_c = \frac{3E^2 qB \sin \alpha}{2m^3 c^4}. \tag{2.62} \]

Substituting for \( x = \frac{\omega}{\omega_c} \) in Equation 2.61 the functional form of the power law is found to be

\[ P_{\text{tot}}(\omega) \propto \omega^{-\left(\frac{p-1}{2}\right)} \int_{x_1}^{x_2} P(x) x^{\left(\frac{p-3}{2}\right)} dx. \tag{2.63} \]

If the energy limits are sufficiently wide then the integral is a constant and a power law in \( \omega \) with spectral index \( \alpha = \frac{p-1}{2} \) is obtained, so that \( P_{\text{tot}}(\omega) \propto \omega^{-\alpha} \). As the electrons radiate they lose energy and so in the full treatment the energy distribution of the electron should be a function of time \((N(E,T))\). The electrons with the highest energy radiate faster than the ones with the lowest energy and so the spectrum is expected to steepen at higher frequencies as the source grows older. In the galactic plane, where fairly young sources are found, the spectral index for a typical synchrotron source is \( \alpha \sim 0.75 \) resulting in the temperature fluctuations having a power law \((T_b \propto \nu^{-2.75})\). At higher galactic latitudes the spectral index of the synchrotron steepens with energy (Lawson et al 1987) but due to the lack of full sky surveys at the GHz frequency range it is very difficult to estimate the frequency dependence of this steepening.

To estimate the effect of synchrotron radiation on experiments in the GHz range low frequency maps are extrapolated using this form of power law. However, there are inherent problems with this. Normally, the 408 MHz (Haslam et al 1982) and
1420 MHz (Reich & Reich 1986) surveys are used to compute synchrotron radiation. The 408 MHz survey has a FWHM of 0.85° and a 10% error in scale. The 1420 MHz survey has a FWHM of 0.58° and a 5% error in scale. However, there are errors in the zero level (which would not affect differencing experiments except if the maps are consequently calibrated incorrectly) and there is also atmospheric noise present in the final maps. Figure 2.10 shows the high galactic latitude region of the 1420 MHz survey and it is obvious from the stripes that there are a large amount of artefacts left in the survey, as well as a number of point sources, that will cause errors in any extrapolation. The 408 MHz survey is slightly better but still contains some artefacts. Other than the errors inherent in the maps themselves, there is also the extrapolation problem discussed above: we expect the spectral index to steepen as we increase in frequency but we do not know by how much. Therefore, using the low frequency surveys to estimate the synchrotron emission at higher frequencies will lead to errors.

For the high Galactic latitude region, Bersanelli et al (1996) have computed the power spectrum of low frequency surveys (the 408 MHz and the 1420 MHz). With the assumption that the major source of radiation at these low frequencies is synchrotron, this power spectrum should closely follow that of synchrotron. For \( \ell > 100 \) the power spectrum falls off roughly as the third power of \( \ell \), similar to both the dust emission and bremsstrahlung.

### 2.5 Growing old

What will happen to the Universe in the future? There are three main possibilities. If the density of the Universe is large enough, so that \( \Omega > 1 \), then it is gravitationally bound and will recollapse. The end of this collapse is commonly referred to as the Big Crunch, but there is a lot of debate on whether there really will be a Big Crunch or just a whimper. If there is not enough mass to keep the Universe bound (\( \Omega < 1 \)), then it will continue expanding forever and become more and more sparse. If \( \Omega = 1 \), then there is just enough mass to keep the Universe bound so that it will neither expand forever nor will it collapse. At present, observations suggest that \( \Omega \) is very close to 1. It would be surprising to find \( \Omega \) very close to 1 but not equal to 1 as expansion has the effect of moving \( \Omega \) away from 1. For example, if \( \Omega = 0.1 \) today, then at the very early stages of the Universe it had to be \( 1 - 10^{-60} \). This constitutes the most accurately determined number in physics and hence causes a problem of why \( \Omega \) was, originally, so close to unity. Inflation gives us a solution to this problem as it naturally predicts that \( \Omega = 1 \) (if the cosmological constant is zero).
Chapter 3

Microwave Background experiments

In this chapter I describe the various considerations that go into designing a Microwave Background experiment. The experiments used to produce the data discussed in this thesis will also be summarised.

When making measurements of the CMB fluctuations there are many technical problems that need to be addressed. A basic CMB experiment must be able to make high sensitivity observations while minimising both foreground and atmospheric emissions (see Section 3.1 for description of the atmospheric emission). An estimate of the level of Galactic free-free and synchrotron emissions can be made by using experiments at lower frequencies (100 MHz to 10 GHz), where these emissions are expected to be dominant, and then extrapolating to higher frequencies. The Jodrell Bank 5 GHz experiment is used for this purpose. Dust emission becomes important at frequencies higher than \( \sim 200 \) GHz and so is not considered as a contaminant to the low frequency experiments in this thesis. Between 10 GHz and 200 GHz the CMB is expected to dominate over the Galactic foregrounds, although the contamination from the atmosphere increases with frequency. Therefore, a ground based experiment operating at frequencies between 10 GHz and 100 GHz, or a space based experiment operating at frequencies between 10 GHz and 200 GHz, should be used as a measure on the CMB. The ground based experiments chosen for this purpose are the Tenerife experiments (10 GHz to 33 GHz). The results from these are compared to the COBE satellite results (30 GHz to 90 GHz) to check the consistency of the two results (they both operate at similar angular scales) and the primordial nature of the signal detected. As examples of the possible future CMB experiments both the Planck Surveyor and MAP satellites are discussed.

When designing any experiment systematic errors need to be well understood so that useful constraints can be made on the data. Today there are two types of receivers that can reach high sensitivity and have well understood systematic errors at the frequencies of interest to a CMB astronomer. At lower frequencies (< 100 GHz) High Electron Mobility Transistor (HEMT) devices are used, whereas at higher frequencies Bolometer devices are used. The HEMT devices work with an antenna receiver, the signal from which is then amplified with transistors. The bolometers
are solid-state devices that increase in temperature with incoming radiation. Both of these receiver systems need to be cooled to lower the noise signal.

3.1 Atmospheric effect

Another foreground that is seen with experiments looking at the microwave background is closer to Earth than those already discussed. This is the atmosphere. Fluctuations in the atmosphere are hard to distinguish from actual extra-terrestrial fluctuations when limited frequency coverage is available. There are three ways to overcome this problem. The first method is to eliminate the atmospheric effect completely. Space missions are the best way to do this but their main problem is cost. High altitude sites (either at the top of a mountain or in a balloon) can reduce the atmospheric contribution, as can moving the experiment to a region with a stable atmosphere. A cheaper alternative to physically moving the experiment is to observe with the experiment for a long time. As the atmospheric effects occur on a short time-scale, compared with the life-time of the experiment (typically of order a few months for each data set taken with ground based CMB experiments), and the extra-terrestrial fluctuations are essentially constant, by integrating over a long time the contribution from the extra-terrestrial fluctuations are increased with respect to the atmospheric effects. Stacking together $n$ data points (taken from $n$ separate observations) will reduce the variable atmospheric signal with respect to the constant galactic or cosmological one by a factor of $\sqrt{n}$ (providing that they are independent with respect to the atmospheric signal and any atmospheric effects on scales larger than the beam which affect the gain have been removed). The third way, which can also be combined with both the first and second way, is to design the experiment to be as insensitive as possible to atmospheric variations.

An obvious design consideration is to make the telescope sensitive to frequencies at which the atmospheric contribution is a minimum. By avoiding various bands in the spectrum, where much emission is expected (for example water lines), the atmosphere becomes less of a problem. Above a frequency of about 100 GHz the atmospheric effect is too large to allow useful observations from a ground based telescope. Taken with the increasing foreground contamination from the Galaxy at low frequencies (it is expected that the Galaxy dominates over the CMB signal at frequencies below 10 GHz) this reduces the observable frequencies for ground based CMB experiments to between 10 and 100 GHz. This narrow observable range results in the need for balloon or satellite experiments so that a larger frequency coverage can be made to check the consistency of the results and to check the contamination from the various foregrounds that are expected.

The largest atmospheric variations occur mainly on longer time scales than the integration time of telescopes (typically of order a few minutes), as the variations are produced by pockets of air moving over the telescope. If an experiment could be insensitive to these ‘long’ term variations then it should effectively see through the atmosphere. It is noted that these ‘long’ term variations are still on short time scales compared to the lifetime of the experiment. An interferometer extracts a small
range of Fourier coefficients from the sky, reducing any incoherent signal (short time scale variations) or any signal that is coherent on large angular scales (long time scale variations), and so should see through the atmosphere very well. Similarly, an experiment that switches between two positions on the sky relatively quickly will also reduce the long term atmospheric variations. This technique is called beam switching. Church (1995) modelled the atmosphere to predict the contribution that atmospheric emission would make to interferometer and beam switching experiments operating at GHz frequencies. Church found that the level of atmospheric ‘snapshot’ fluctuations expected was below 1 mK in favourable conditions for an interferometer operating at sea level. After averaging over a relatively short time scale (much shorter than the average lifetime of an experiment), the atmospheric noise was well below the system noise and so negligible. A beam switching experiment is less well able to eliminate the atmospheric emission but operating at high altitudes, where the atmosphere is drier, should allow good observations to be made with this type of set up.

3.2 General Observations

Once the measurements of the CMB have been taken it is then necessary to present data in a way that is consistent between all experiments. In this section I will attempt to summarise the way in which most CMB data are presented.

3.2.1 Sky decomposition

The usual method of presenting the results from a CMB experiment is through the power spectrum of the spherical harmonic expansion discussed in the previous chapter. Another value often quoted is related to this analysis. The COBE group published their data in terms of $Q_{\text{rms-ps}}$ which is given by

$$Q_{\text{rms-ps}} = T \sqrt{\frac{5C_2}{4\pi}}$$  \hspace{1cm} (3.1)

where $C_2$ is related to the $C_\ell$ values in the lower $\ell$ range (where the Sachs Wolfe effect dominates) through Equation 2.20. The relative values of the $C_\ell$s throughout the $\ell$ range depend mainly on the spectral index, $n$, Hubble’s constant, $H_0$, the density parameter, $\Omega_0$, and, to a lesser extent, the other cosmological constants. For example, in the cold dark matter model of the Universe the height of the Doppler peak depends mainly on Hubble’s constant, whereas its position depends mainly on the density parameter.

3.2.2 The effect of a beam

One important thing to note is that the observations from a particular experiment will not generally measure the $C_\ell$s directly. This is due to the effect of the beam on the data. A different experiment will be sensitive to different angular scales (and so
CHAPTER 3. MICROWAVE BACKGROUND EXPERIMENTS

a different range of $C_\ell$s. If an experiment has a Gaussian beam (most experiments
are not perfectly Gaussian but can be approximated by one) then it will measure

$$C_m(\beta) = \frac{1}{4\pi} \sum_\ell (2\ell + 1)C_\ell P_\ell(\cos \beta) \exp[-(\ell + \frac{1}{2})^2 \sigma^2]$$

(3.2)

where $\sigma$ is the dispersion of the Gaussian (see Scaramella & Vittorio, 1988). This
exponential term follows through into the equation for temperature fluctuations
and must be taken into account when analysing the data if the results are to be
represented in this form.

3.2.3 Sample and cosmic variance

With the data presented like this and all systematic errors taken into account there
are still two errors which must be considered. In some cases these errors will be
larger than those caused by the systematic or instrumental noise. The sample vari-
ance of the data arises from an experiment that only measures a fraction of the sky.
This is due to the uncertainty that the part of sky measured was a ‘special’ part.
As the distribution of the CMB is expected to follow a Gaussian pattern there is a
probability that the level of fluctuations in the fraction of sky that one experiment
measured is different to that in the fraction of sky measured by another experiment.
This is sample variance and is inversely proportional to the sky area covered by
the experiment. The other error is sample variance on a cosmological scale. The
observable Universe is just one realisation of the parameters (for example the $C_\ell$s
in the case of a Gaussian field) taken from a Gaussian distribution with the ensem-
ble average described by the underlying theory. Therefore, at large angular scales,
where there are less degrees of freedom (given by $2\ell + 1$ for the Gaussian field), the
uncertainty caused by having just one realisation is greatest. There is no way to
reduce cosmic variance as this would require the study of another observable Uni-
verse. Cosmic variance will dominate at large angular scales while sample variance,
if present, dominates at small angular scales.

3.2.4 The likelihood function

If it is assumed that the CMB is described by a two–dimensional, random Gaussian
field then the properties of the fluctuations can be described completely by their
auto–correlation function $C(\beta)$ (see Equation 2.17). The data can then be used
to find the most likely variables that describe the auto–correlation function (for
example $n$ or $C_2$, and hence $Q_{RMS-PS}$, in Equation 2.19 and 2.20).

In the case of the Tenerife experiments care must be taken to account for the
switch beam and it is possible to write the covariance matrix for two points $i$ and $j
with coordinates $(\alpha_i, \delta_i)$ and $(\alpha_j, \delta_j)$ as

$$M_{ij} = <\left\{\Delta T(\alpha_i, \delta_i) - \frac{1}{2} [\Delta T(\alpha_i + \beta/\cos(\delta_i), \delta_i) + \Delta T(\alpha_i - \beta/\cos(\delta_i), \delta_i)]\right\}>$$
\[
\left\{ \Delta T(\alpha_j, \delta_j) - \frac{1}{2} [\Delta T(\alpha_j + \beta/\cos(\delta_j), \delta_j) + \Delta T(\alpha_j - \beta/\cos(\delta_j), \delta_j)] \right\} > (3.3)
\]

where \(\Delta T(\alpha_i, \delta_i)\) is the fluctuation in temperature at point \((\alpha_i, \delta_i)\) after convolution with the Gaussian beam pattern for a single antenna and \(\beta\) is the switching angle. With the noise \(\epsilon_i\) on point \(i\) included the total covariance matrix is given by

\[
V_{ij} = M_{ij} + \langle \epsilon_i \epsilon_j \rangle (3.4)
\]

where the noise term is non–zero only when \(i = j\) if it is uncorrelated from point to point.

The likelihood function of this covariance matrix is defined as

\[
L(\Delta T | p_i) \propto \frac{1}{(\det V)^{\frac{1}{2}}} \exp \left( -\frac{1}{2} \Delta T^T V^{-1} \Delta T \right) (3.5)
\]

where \(p_i\) are the parameters to be fitted in the covariance matrix and \(\Delta T\) are the data. The maximum value of this function corresponds to the most probable values of the parameters \(p_i\) if we interpret the likelihood curves in a Bayesian sense with a uniform a priori probability distribution. As the likelihood function calculates how probable a set of parameters are given a data set, rather than trying to predict the parameters directly, both sample and cosmic variance are taken into account in the analysis.

### 3.3 The Jodrell Bank 5 GHz interferometer

The CMB is dominant over the Galactic foreground emissions at frequencies higher than \(\sim 10\) GHz (and below \(\sim 200\) GHz). Therefore, to obtain a good estimate of these foregrounds it is necessary to make observations at lower frequencies. These observations can then be used to put constraints on the foreground contribution to other CMB experiments.

The 5 GHz interferometer located at Jodrell Bank, Manchester is a twin horn, broad–band, multiplying interferometer (see Figure 3.1). The horns are corrugated and have a well defined beam with low side lobes to minimise ground spill–over. They have an aperture diameter of 0.56 m and a half power beam width of 8°. The principle of interferometry ensures that uncorrelated signals from the atmosphere are averaged down, whilst the astronomical signals, which are correlated between antennae, add coherently. Thus despite the relatively poor atmospheric conditions prevalent at Jodrell Bank, an antenna temperature sensitivity of \(\sim 100\) µK per beam can be attained in one good day of observing.

The antennae are arranged in an East–West configuration with a central frequency of 4.94 GHz and a variable baseline (the two baselines to be discussed in this
thesis are 1.79 m and 0.702 m). The half power receiver bandwidth is 337 MHz. The horns are mounted horizontally and view the sky reflected through a plane mirror. The mirrors can be tilted so that the centre of the beam is at a specific declination. The rotation of the Earth sweeps the beam across the full right ascension range every sidereal day (see Davies et al 1996a). Repeated 24 hour drift scans were taken at 2.5° intervals in declination spanning the range 30° to 55° inclusive. Since the beam full width half maximum (FWHM) is ∼ 8° in declination, this provides a fully sampled map of the sky. The receivers are HEMTs cooled to ∼ 15 K by a closed cycle helium refrigerator. The receiver noise temperature is 20 ± 2 K.

The interferometer beam is made up of the convolution of two parts. Considering two infinitely thin horns separated by a distance $b$, as in Figure 3.2, the path difference between the signals arriving at the two horns can be shown to be $b \cos \theta$ by simple geometry. The number of extra waves that propagate in this path difference is given by

$$a = \frac{b \cos \theta}{\lambda}$$

(3.6)

where $\lambda$ is the wavelength of the incoming radiation. Therefore, the phase difference between the two horns is simply $2\pi a + \gamma$. Here $\gamma$ is an artificially added phase after the data has been collected by the horns. Note that the path difference compensation, shown in Figure 3.2, is an addition of phase that results in the two signals from the horns being coherent but the $\frac{\lambda}{4}$ added phase results in the two signals being 90° out of phase. Therefore, there are two output signals from the interferometer which are orthogonal to each other (they will be referred to as the cosine and sine channels).

The other part of the beam is a primary beam defined by the geometry of the horns. This is usually modelled by a Gaussian beam (providing the experiment has been well built). The beam response is multiplied with the infinitely thin horns' sinusoidal term, from the correlator output, to give the full response.

$$R(\alpha) = \exp \left[ -\frac{\theta^2}{2\sigma^2} \right] \cos(2\pi a + \gamma).$$

(3.7)

However, in this case $\theta$ is needed in terms of the Declination and Right Ascension (RA) of the source. If the beam centre is pointed towards $(\alpha_1, \delta_1)$, where $\delta_1$ is the declination and $\alpha_1$ is the RA, and the source is located at $(\alpha_2, \delta_2)$, then the angle $\theta$ between the source and beam axis in the Gaussian beam is given by

$$\theta = \cos^{-1}(\cos \delta_1 \cos \delta_2 \cos(\alpha_1 - \alpha_2) + \sin \delta_1 \sin \delta_2).$$

(3.8)

The interferometer baseline is changed due to projection effects and no longer depends solely on $\theta$. As the interferometer is East–West then the projection will only depend on the declination of the beam axis and not the source. The path difference is now given by

$$a = \frac{b \cos \theta}{\lambda}$$

(3.6)
3.4. THE TENERIFE EXPERIMENTS

Figure 3.3: Beam response for the 5 GHz interferometer cosine channel. The beam axis is at $40^\circ$ Declination and $0^\circ$ RA.

\[ a = \frac{b \cos \delta_1 \sin(\alpha_1 - \alpha_2)}{\lambda} \]  
\[ (3.9) \]

and the full beam response is given by

\[ R(\alpha_2, \delta_2) = \exp \left( -\frac{\theta^2}{2\sigma^2} \right) \cos(2\pi \frac{b}{\lambda} \cos \delta_1 \sin(\alpha_1 - \alpha_2) + \gamma) \]  
\[ (3.10) \]

for a beam pointed at $(\alpha_1, \delta_1)$ and $\theta$ is given in Equation 3.8. This beam response and its Fourier transform are plotted in Figure 3.3 for a beam pointed at $(\alpha = 0^\circ, \delta = 40^\circ)$.

The complex correlator in the interferometer produces two orthogonal sinusoidal outputs as already mentioned. In Equation 3.10 this corresponds to two different expressions, one with $\gamma = 0^\circ$ and one with $\gamma = 90^\circ$. The outputs in the two channels are binned into half degree pixels in Right Ascension. Variations in the output levels occurring on long time scales, to which the experiment is not sensitive, are removed from the data by smoothing it with a suitably large Gaussian and subtracting the result. These baselines originate from calibration errors (usually caused by atmospheric effects). Data collected over a period of time at the same declination is then stacked together to reduce the overall noise per pixel by a factor of $\sqrt{n}$, where $n$ is the number of days of data collected at that pixel. For a more detailed description of the experiment see Melhuish et al (1996).

3.4 The Tenerife experiments

From the ground the best frequency window for making CMB observations is between 10 GHz and 100 GHz. Python operates at 90 GHz and is located at the Antarctic plateau which is both high in altitude and has a very stable atmosphere, but most ground based experiments are confined to frequencies between 10 GHz and 40 GHz. This minimises both the Galactic foregrounds (free-free and synchrotron are dominant for frequencies less than 10 GHz) and the atmosphere (which becomes increasingly significant with higher frequency). The main data used in this thesis to put constraints on CMB emission are from the Tenerife experiments which operate in this window.

The Tenerife instruments consist of three radiometers, each with two independent channels, operating at frequencies of 10, 15 and 33 GHz. They are located at 2400 m altitude at the Teide Observatory in Tenerife. This area has a smooth airflow which reduces spatial fluctuations in the water vapour content. This ensures low atmospheric contamination of the experiments. Approximately 70% of the time there is less than 3 mm of precipitable water vapour above the site. Data taken during this period has very low atmospheric fluctuations and is regarded as ‘good’ data. As in the case of the 5 GHz interferometer they are drift scanning experiments.
so that the ground spill–over is constant and can be well accounted for. The instruments consist of two beams separated by an angle of $\theta_o = 8.1^\circ$ in the East-West direction. At 10 GHz there are two experiments, one with 8.3$^\circ$ FWHM and another with 4.9$^\circ$ FWHM, while at 15 GHz there is one with 5.2$^\circ$ FWHM and at 33 GHz there is one with 5.0$^\circ$ FWHM. In each experiment the difference between the two beams is calculated in real time. The beams are then ‘wagged’, by use of a tilting mirror (similar to the stationary tilted mirrors in the interferometer) by one beam separation (8.1$^\circ$) so that the East beam is now in the position of the old West beam. The new difference between the two beams is calculated in the ‘wagged’ position and the difference between the two beam–differences is then calculated. This double–difference between the two values gives the experiments a triple beam form which is given in Equation 3.11. The beam is shown in Figure 3.4 and the functional form is given by

$$R(\alpha, \delta) = \exp\left[-\frac{\theta^2}{2\sigma^2}\right] - \frac{1}{2} \left(\exp\left[-\frac{(\theta - \theta_o)^2}{2\sigma^2}\right] + \exp\left[-\frac{(\theta - \theta_o)^2}{2\sigma^2}\right]\right)$$

(3.11)

where $\sigma$ is the beam dispersion and $\alpha$ is the angular separation between the source and the beam centre. In terms of Declination and Right Ascension $\theta$ is given by Equation 3.8.

The mirror is tilted every $\sim$4 seconds and data is taken from each beam by use of a Dicke switch at 32 Hz. Over a period of 82 seconds (consisting of 8 difference pairs), the double–difference and its standard deviation, as well as a calibration signal, are recorded. The final data set consists of $1^\circ$ bins in Right Ascension with the average variance of the data taken from the 82 second cycles that contribute to that bin. The bandwidth of the receivers is 470 MHz at 10 GHz, 1.2 GHz at 15 GHz and 3 GHz at 33 GHz. The data is recorded over a continuous period of $4 \times 24$ hours and thus a single scan contains a maximum of four full coverages in Right Ascension, with data being taken over a period of up to 5 calendar days. The full data set, once collected (over a period of years), is then stacked together, as discussed in the next Chapter, after the removal of long period baseline drifts caused by slow variations in the atmosphere. This removal can be performed in a similar way to the 5 GHz interferometer but is actually done using the Maximum Entropy algorithm that will be described latter (see Chapter 5). The experiment is described in more detail in Davies et al (1992).

### 3.5 The COBE satellite

The first detection of fluctuations in the CMB was made by a NASA satellite in 1992 and the data from this experiment has always been central to CMB work.
Figure 3.5: The COBE satellite showing the location of the main experiments on board.

Therefore, a comparison between this data set and that from the Tenerife experiment will provide a very useful check on the consistency of the two data sets.

The NASA Cosmic Microwave Background Explorer (COBE) satellite, launched on November 18th 1989, had three experiments on board. The Diffuse Infrared Background Experiment (DIRBE) measured fluctuations in high frequency emission mainly produced by dust in our galaxy. The Far Infrared Absolute Spectrophotometer (FIRAS) measured the CMB spectrum between 1 cm and 100 µm. The results from FIRAS showed the CMB to have a black body spectrum that was correct to 1 part in $10^4$, the most accurate black body known to science. From this experiment the temperature of the CMB was shown to be $T_{CMB} = 2.726 \pm 0.010$ K at 95% confidence (see Mather et al 1994). The Differential Microwave Background Radiometer (DMR) maps the fluctuations in the CMB over the full sky. The satellite is shown in Figure 3.5.

The DMR experiment is made up of six radiometers, two at each frequency of 31.5 GHz, 53 GHz and 90 GHz. The frequencies were chosen to overlap the expected minimum in Galactic foreground emission. Each radiometer pair have two independent receivers (denoted by A and B) that measure the difference in the level of CMB in beams of FWHM 7° separated by 60° in the sky. After removal of the dipole effect (see Chapter 2, Section 2.3.1) the sum and difference between the A and B are calculated. The sum maps, which enhance any signal present in both A and B, give an estimate of the fluctuations present in the CMB while the difference maps, which remove any consistent signal, give a measure of the instrumental noise.

The data was calibrated by using a combination of an on–board calibration source, microwave emission from the Moon and the level of the dipole in the CMB (see Bennett et al 1992b). The first detection of fluctuations in the CMB was made by COBE using data from one year of flight (Smoot et al 1992). The dipole and the combined maps of the A+B channels from all three frequencies were shown in Chapter 2. The data used in this thesis are the publicly available processed data after four years of flight.

3.6 The Planck Surveyor satellite

The Planck Surveyor satellite is due to be launched by ESA in 2006. The goal of Planck is to make full–sky maps of the CMB fluctuations on all angular scales greater than 4 arc minutes with an accuracy set by astrophysical limits. The satellite and its proposed operation is described in detail in Bersanelli et al (1996). Since the publication of Bersanelli et al (1996) substantial improvements have been made to the telescope and the latest specifications (G. Efstathiou, private communication) will be used in the simulations performed in this thesis. The main improvements include a change in frequencies and a substantial improvement in noise sensitivities for the lower frequency channels and an additional 100 GHz channel which can be
CHAPTER 3. MICROWAVE BACKGROUND EXPERIMENTS

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>30</th>
<th>44</th>
<th>70</th>
<th>100</th>
<th>100</th>
<th>143</th>
<th>217</th>
<th>353</th>
<th>545</th>
<th>857</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of detectors</td>
<td>4</td>
<td>6</td>
<td>12</td>
<td>34</td>
<td>4</td>
<td>12</td>
<td>12</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Angular resolution (')</td>
<td>33'</td>
<td>23'</td>
<td>14'</td>
<td>10'</td>
<td>10.6'</td>
<td>7.4'</td>
<td>4.9'</td>
<td>4.5'</td>
<td>4.5'</td>
<td>4.5'</td>
</tr>
<tr>
<td>Bandwidth ($\Delta\nu$)</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.37</td>
<td>0.37</td>
<td>0.37</td>
<td>0.37</td>
<td>0.37</td>
<td>0.37</td>
</tr>
<tr>
<td>Transmission</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$\Delta T/T$ sensitivity ($10^{-6}$)</td>
<td>1.6</td>
<td>2.4</td>
<td>3.6</td>
<td>4.3</td>
<td>1.81</td>
<td>2.1</td>
<td>4.6</td>
<td>15.0</td>
<td>144.0</td>
<td>4630</td>
</tr>
</tbody>
</table>

Table 3.1: Summary of the Planck Surveyor satellite frequency channels (G. Efstathiou, private communication). The sensitivity is for a beam pixel after 14 months of observations.

Figure 3.6: An artists impression of the Planck Surveyor satellite. Produced for the Bersanelli et al 1996 phase A study.

used as a cross-check between the two different types of receiver technologies used. The satellite consists of ten frequency channels between 30 and 900 GHz which are summarised in Table 3.1. The four lowest frequency channels consist of HEMT radio receivers while the six highest frequency channels are bolometer arrays. This difference in detector technology was chosen to achieve the best sensitivity to the signal and accounts for the apparent discontinuity in the table between the two 100 GHz channel sensitivities.

Figure 3.6 shows an artist’s impression of the Planck Surveyor satellite. The input data presented in this thesis are simulations of the observations that will be taken by the satellite with the characteristics shown in the above table. These simulations were produced by Francois Bouchet of the Institut d’Astrophysique de Paris in a collaboration with the MRAO.

3.7 The MAP satellite

NASA are also due to launch a new satellite called the Microwave Anisotropy Probe (MAP) in 2000. It is intended to be a follow up of the COBE satellite with full sky coverage but at higher resolution. It has five frequency channels from 20 GHz to 90 GHz which are summarised in Table 3.2 (improvements have also been made to the MAP satellite design but the values quoted here are those available on the NASA MAP web site\(^1\)). The expected results quoted by the MAP team in their

\(^1\)Since the simulations presented here were performed the resolution of the MAP satellite has improved to 12 arc minutes
3.7. THE MAP SATELLITE

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>22</th>
<th>30</th>
<th>40</th>
<th>60</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of detectors</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Angular resolution</td>
<td>54'</td>
<td>39'</td>
<td>32'</td>
<td>23'</td>
<td>17'</td>
</tr>
<tr>
<td>$\Delta T$ sensitivity ($10^{-6}$)</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 3.2: Summary of the MAP satellite frequency channels.

Figure 3.7: Artists impression of the MAP satellite. Produced for the MAP Internet home pages.

Publications usually assume that dust emission will be negligible at these frequencies (this assumption was also made when the COBE data were analysed). It is cheaper to build than the Planck Surveyor satellite and is due to be launched up to six years earlier but the resolution will not be as good. An introduction to the data analysis that is proposed for this satellite can be seen in Wright, Hinshaw and Bennett (1996).

Figure 3.7 shows an artist’s impression of the MAP satellite. Again, the input data presented in this thesis are simulations of the observations that will be taken by MAP with the characteristics shown in the above table. The same input actual sky simulations as used for the Planck Surveyor satellite will be used, to allow a comparison of the results from the two experiments.


Chapter 4

The data from the 5 GHz interferometer and Tenerife experiments

In this chapter I will present the raw and processed data from the four Tenerife experiments and the Jodrell Bank interferometer. Brief analyses done on the raw data itself and comparison with previous surveys are also presented. A further analysis technique that obtains the best information from the data will be discussed in the next chapter and results from this are presented in Chapter 7.

4.1 The Jodrell Bank 5GHz interferometer

4.1.1 Pre-processing

From the complete data set a data subset that is useful must be selected and any data that is obviously not due to real sky fluctuations discarded. The first process is to use an automatic program (developed by Simon Melhuish at Jodrell Bank, see Melhuish et al 1997) that excises the regions in the data contaminated by the Sun and the Moon. The program looks at the time of observations and the position of the Sun and the Moon at that time. It deletes data that is within 47° and 17° of the Sun and Moon respectively. Any abnormal signals that deviate from the mean of the data by more than 3σ are also deleted as these probably arise from noise within the telescope or bad atmospheric conditions.

The two output channels from the interferometer should be in quadrature - however, this is not always the case. Errors in the correlator output lead to an output that can be 80° out of phase rather than 90° and a difference in amplitude between the two channels of 1% was commonly observed. This is easily corrected by multiplying the channels with a suitably chosen matrix and the amplitude and quadrature is restored. Long term baseline drifts caused by instrumental drifts were removed by applying a high-pass filter of one hour (15° Gaussian) in Right Ascension. The final results, binned in 0.5° pixels are stacked together to produce
CHAPTER 4. THE DATA

Figure 4.1: The moon crossing of a typical scan. This is used to calibrate the interferometer.

one low noise data scan per declination. The typical stack contains about 30 days of data.

4.1.2 Calibration

Before any analysis can be performed on the data it is necessary to calibrate the level of the signal and accurately measure the beam shape (the theoretical beam shape will not be achieved unless the experiment is ideal). For both of these purposes the Moon crossings in the data can be used. The position and flux of the Moon is known very accurately and so it is possible to accurately model its contribution to the data. Figure 4.1 shows a typical Moon crossing. In analysing the moon care must be taken as it is slightly resolved by the interferometer and so appears with smaller amplitude than theoretically predicted (but this can be easily modelled). It also moves across the sky during measurements and so appears slightly extended. The Gaussian beam is fitted to the amplitude of the moon crossing and the cosine and sine channels are fitted to the phase of the moon crossing.

To calibrate the interferometer with known sources (of which the moon is an example) it is necessary to convert the flux units normally used for the calibration sources to antenna temperature. The effective area, $A_e$, of a telescope is given by

$$A_e = \epsilon A_p$$  \hspace{1cm} (4.1)

where $A_p$ is the actual physical area of the telescope and $\epsilon$ is the aperture efficiency. The aperture efficiency is given by (Kraus 1982)

$$\epsilon = \frac{\lambda^2 D}{4\pi A_p}$$  \hspace{1cm} (4.2)

where $\lambda$ is the wavelength of the experiment and $D$ is the antenna directivity. In general the antenna directivity depends on the exact beam pattern, $P_n(\alpha, \delta)$ by

$$D = \frac{4\pi}{\int\int_{4\pi} P_n(\alpha, \delta) d\Omega}$$  \hspace{1cm} (4.3)

but we can estimate it by using the model Gaussian beam. From this it is found that $\epsilon = 0.72$. The antenna temperature is then related to the flux by

$$T_A = \frac{\epsilon A_p S}{2k}$$  \hspace{1cm} (4.4)

which gives the flux to temperature conversion of 64 µK/Jy (this is consistent with the result using the beam area in Equation 2.27).

The raw output units of the data from the interferometer are referred to as calibration units. To convert these into antenna temperature we can now compare with predictions of known point sources as well as the moon. The two main point
4.1. JODRELL INTERFEROMETRY

Table 4.1: Some of the sources used in the calibration of the data. Sources marked with a † are highly variable and flux data from a survey carried out by the University of Michigan simultaneously with the 5 GHz survey was used in the calibration.

<table>
<thead>
<tr>
<th>Source</th>
<th>Flux (Jy)</th>
<th>Declination</th>
<th>Right Ascension</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C84†</td>
<td>34</td>
<td>41.3°</td>
<td>49.1°</td>
</tr>
<tr>
<td>3C345†</td>
<td>6</td>
<td>39.9°</td>
<td>250.3°</td>
</tr>
<tr>
<td>4C39.25†</td>
<td>9</td>
<td>39.25°</td>
<td>141.0°</td>
</tr>
<tr>
<td>3C147</td>
<td>10.2</td>
<td>49.8°</td>
<td>84.7°</td>
</tr>
<tr>
<td>3C286</td>
<td>7.3</td>
<td>30.8°</td>
<td>202.2°</td>
</tr>
<tr>
<td>3C48</td>
<td>5.2</td>
<td>32.9°</td>
<td>23.7°</td>
</tr>
</tbody>
</table>

Figure 4.2: The raw data from the cosine channel at 5 GHz for all eleven declinations. The galactic plane crossing is easily seen on this plot.

Sources seen in the data are in the galactic plane. Casiopia A has a flux density of 670 Jy and is at Declination 58.5° and Cygnus A has a flux density of 375 Jy and is at Declination 40.6°. Bright sources in high Galactic latitudes away from the Galactic plane, like those shown in Table 4.1, are easily seen in the data scans and can also be used to check the calibration of the data.

The application of this calibration gives a value of $T_{\text{cal}} = 3.0 \pm 0.2$ K/CAL for the conversion from the interferometer output units (CAL) to degrees Kelvin.

4.1.3 Data processing

The final stacked scans for one of the output channels in the wide spacing interferometer data (1.79 m baseline) are shown in Figure 4.2. The principle Galactic plane crossing at RA $\sim 20.5h$ (308°) and the weak anti–centre crossing at RA $\sim 4h$ (60°) are clearly seen at each declination. Over the full 24 hour (360°) range the errors obtained per pixel and the number of days of data at each declination are summarised in Table 4.2.

At this frequency and resolution, the dominant contributor to the principal crossing in the central sky area is the discrete radio source Cygnus A ($S(5\GHz) \simeq 200$ Jy), with an additional contribution from diffuse emission in the Galactic plane. Other discrete radio sources contribute to the data and for comparison with previous surveys these are assumed to remain at a constant flux level with time. Radio sources at these frequencies are expected to be variable at the $\sim 30\%$ level and consequently radio source variability rather than random noise is the major source of uncertainty in the data. A measure of the uncertainty involved in the assumption that the sources are constant rather than variable can be obtained by testing the method on the discrete sources 3C345 (7.8 Jy) and 4C39 (7.6 Jy), which are clearly detected in all three scans that surround Declination 40° where the sources are located. 3C345 lies at RA 250.3° and Dec. 39°54′11″ whilst 4C39 is located at RA 141.0° and
### Table 4.2: Noise levels and number of days in each stack for the wide spacing interferometer setup.

<table>
<thead>
<tr>
<th>Declination</th>
<th>Mean noise RMS ($\mu$K)</th>
<th>Mean number of days</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.0°</td>
<td>25</td>
<td>42.0</td>
</tr>
<tr>
<td>32.5°</td>
<td>53</td>
<td>15.8</td>
</tr>
<tr>
<td>35.0°</td>
<td>23</td>
<td>80.8</td>
</tr>
<tr>
<td>37.5°</td>
<td>33</td>
<td>30.2</td>
</tr>
<tr>
<td>40.0°</td>
<td>18</td>
<td>162.8</td>
</tr>
<tr>
<td>42.5°</td>
<td>36</td>
<td>53.9</td>
</tr>
<tr>
<td>45.0°</td>
<td>22</td>
<td>84.5</td>
</tr>
<tr>
<td>47.5°</td>
<td>69</td>
<td>8.3</td>
</tr>
<tr>
<td>50.0°</td>
<td>24</td>
<td>72.9</td>
</tr>
<tr>
<td>52.5°</td>
<td>32</td>
<td>33.5</td>
</tr>
<tr>
<td>55.5°</td>
<td>36</td>
<td>39.0</td>
</tr>
</tbody>
</table>

Figure 4.3: Comparison between the raw data (black line) and the predicted point source contribution (red line) from the Green Bank catalogue at Dec. 37.5° for the cosine channel of the interferometer.

Dec. 39°15’24’’. Figure 4.3 shows the data (black line) and predicted discrete source emission (red line) from the Green Bank catalogue for the region RA120° − 270° at Dec 37.5° on an expanded scale; for clarity the $\sim 33 \mu$K error bars have not been shown. The position of the sources 3C345 and 4C39 in the data agree well with the prediction from the Green Bank catalogue and the amplitudes agree to within the expected source variability. Any discrepancies are consistent with the presence of noise and signal due to the Galaxy.

Figure 4.4 shows the amplitude data ($\sqrt{C^2 + S^2}$ where $C$ is the cosine data and $S$ is the sine data) for all eleven declinations compared with the prediction from the Green Bank catalogue. Except for a few variable sources (notably 3C345 at Dec. 37.5° and 35.0°, and the source at the centre of Dec. 50°), this comparison shows a very good agreement between the data and that of the catalogue. Since the Green Bank experiment is only sensitive to sources with an angular size of less than 10.5 arcmin it is fairly safe to say that there is little Galactic emission present in the data. The wide-spacing interferometer data can, therefore, be used as a point source estimation for the other higher frequency experiments at Tenerife. The narrow-spacing data is more sensitive to larger angular scales and, taken together with the wide spacing data, can be used as a good estimate for the Galactic emission. Further analysis of the data is presented in Chapter 7.

Figure 4.4: Comparison between the amplitude of the data and the prediction from the GB catalogue at 4.85 GHz.
4.2 Structure in the Tenerife switched-beam scans

4.2.1 Pre–processing

Data within 50° of the Sun and 30° of the Moon were removed from the raw data of all the Tenerife experiments with an automated process, similar to the interferometer. Data taken in poor atmospheric conditions (about 30% of all data) and any individual pixel that deviated by more than 3σ from the average were also removed. These individual pixels correspond to technical failures in the instrumental system or anomalous sources (like butterflies flying into the horns). The data taken in poor atmospheric conditions was removed by looking at each scan individually. If data appeared to be affected by atmospheric fluctuations then a portion of data around the affected region was removed. Portions of bad data were eliminated by eye because an automated analysis would prove too complicated due to the complexity of the data.

A similar technique to the pre–processing of data from the Jodrell Bank interferometer, by smoothing the data, could have been used to remove long term baseline drifts caused by atmospheric offsets, but it was decided to leave the baselines in and remove them simultaneously with the Maximum Entropy reconstruction described in the next chapter. This method basically finds the best astronomical signal consistent with all the scans, subtracts this from each scan and then performs the smoothing on the residual signal. The final stacked results shown below are therefore after Maximum Entropy processing. However, problems arise when the baseline variations in the raw data are so extreme that they prevent their successful removal in the MEM deconvolution analysis. As noted in Davies et al 1996, this problem is exaggerated at the higher frequencies where the water vapour emission is higher. At these higher frequencies (the 33 GHz experiment in the case of Tenerife observations) it is clear that the variations in baseline are, in certain cases, too extreme for removal and will therefore result in artefacts in the final stacked scan. These artefacts result from poor observing conditions rather than being intrinsic to the astronomy, because such problems occur only for days with severe baselines and appear in a randomly distributed fashion for different days. Removal of such data is essential if the necessary sensitivity to detect CMB fluctuations is to be obtained. This involves the task of examining each raw scan (the best covered declination for the 5° FWHM data sets contains over 200 days of data) and its baseline and deciding if the data are usable. In such cases where the data is un-salvageable, then the data for the full 360° observation are discarded. This ensures that there is no bias introduced by selectively removing features in the scans. After this final stage of editing, the baseline fitting must be repeated for the full remaining data set. The MEM process will now be able to search for a more accurate solution and will produce a new set of more accurate baselines. The coverages of a given declination can now be stacked together and the process repeated until all artefacts of this type are removed. This process is carried out for each of the Tenerife data sets discussed in this thesis but will not be mentioned again.
Table 4.3: Number of independent measurements for the 5° FWHM Tenerife experiments.

### 4.2.2 Stacking the data

The data set consists of 1° bins in right ascension covering a range of declinations and taken over a large number of days (the best scans have nearly 200 days of data). To reduce the amount of data it is necessary to stack all the data for each point that were taken at different days together. This is done after baseline subtraction to avoid addition of atmospheric effects. By taking a weighted mean over the $n_s$ scans (where $n_s$ is the number of days) the final data scan is obtained. The number of days of data at each declination and frequency for the 5° FWHM Tenerife experiment is summarised in Table 4.3. If the data in each 1° bin is given by $y_{ir}$, where $i$ is the bin and $r$ is the day, and the error on $y_{ir}$ is given by $\sigma_{ir}$ then the final stacked data is given by

$$Y_i = \frac{\sum_{r=1}^{n_s} w_{ir} (y_{ir} - b_{ir})}{\sum_{r=1}^{n_s} w_{ir}}$$

(4.5)

where $b_{ir}$ represents any long term baselines that have to be subtracted from the data before stacking and the weighting factor $w_{ir}$ is given by

$$w_{ir} = \frac{1}{\sigma_{ir}^2}.$$ 

(4.6)

The error on the final stack is given by the scatter over the days contributing to each point

$$\sigma_i^2 = \left( \frac{\sum_{r=1}^{n_s} w_{ir} \left[(y_{ir} - b_{ir}) - Y_i\right]^2}{\sum_{r=1}^{n_s} w_{ir} (n_s - 1)} \right).$$

(4.7)

### 4.2.3 Calibration

The data was calibrated using a continuous online noise injection diode so that the data amplitude response remains constant throughout the day’s observing. The output is then in units of CAL (the calibration level). Moon observations and the Galactic plane crossing were then used to calibrate the signal and convert the output
### 4.2. THE TENERIFE SCANS

<table>
<thead>
<tr>
<th>Declination</th>
<th>10 GHz</th>
<th>15 GHz</th>
<th>33 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.0°</td>
<td>-</td>
<td>1.045</td>
<td>-</td>
</tr>
<tr>
<td>32.5°</td>
<td>1.023</td>
<td>1.067</td>
<td>-</td>
</tr>
<tr>
<td>35.0°</td>
<td>1.092</td>
<td>1.039</td>
<td>-</td>
</tr>
<tr>
<td>37.5°</td>
<td>1.057</td>
<td>1.055</td>
<td>-</td>
</tr>
<tr>
<td>40.0°</td>
<td>1.042</td>
<td>1.042</td>
<td>1.166</td>
</tr>
<tr>
<td>42.5°</td>
<td>1.068</td>
<td>1.070</td>
<td>-</td>
</tr>
<tr>
<td>45.0°</td>
<td>1.038</td>
<td>1.040</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.4: Noise enhancement for each declination and frequency of the Tenerife experiment. This extra multiplication factor should be included to account for atmospheric correlations between channels.

The correlated noise is seen in the two channels in all three frequency experiments. The maximum effect is at 33 GHz where an atmospheric signal of $\sim 30\mu$K is seen (Davies et al 1996a). The level of the signal can be calculated by looking at the correlation and cross-correlation between the channels (Gutierrez 1997). Gutierrez (1997) showed that the correlation between the channels only existed on timescales less than 4 minutes (the bin size) and so no significant correlation is found between adjacent positions in RA. The enhancement required for each of the declinations and frequencies is summarised in Table 4.4 (Gutierrez private communication).

#### 4.2.4 Error bar enhancement

Each of the Tenerife experiments contain two independent receivers. These operate simultaneously and look at the same region of sky so as to have a check on the consistency of each receiver (and to reduce the noise by a factor of $\sqrt{2}$). However, the two receivers will also be looking through the same atmospheric signals and so the noise on the two channels will be correlated. This noise is equivalent to a Gaussian noise common to both channels with a coherence time smaller than the binning time, the net effect of which is an enhancement of the error bars (see Gutierrez 1997 or Davies et al 1996a).

#### 4.2.5 The 8.3° FWHM 10 GHz experiment

Between 1984 and 1985 the Tenerife experiment consisted of a double–switching telescope with 8.3° FWHM at 10.45 GHz. Table 4.5 summarises the observations taken during this period. One third of the total sky was covered but the sensitivity...
<table>
<thead>
<tr>
<th>Declination</th>
<th>Number of scans</th>
<th>Mean scan length (hours)</th>
<th>RMS length (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+46.6°</td>
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<td>14.1</td>
<td>3.1</td>
</tr>
<tr>
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<td>16</td>
<td>22.0</td>
<td>9.7</td>
</tr>
<tr>
<td>+39.4°</td>
<td>42</td>
<td>14.1</td>
<td>6.3</td>
</tr>
<tr>
<td>+37.2°</td>
<td>18</td>
<td>14.6</td>
<td>7.6</td>
</tr>
<tr>
<td>+27.2°</td>
<td>17</td>
<td>11.3</td>
<td>3.9</td>
</tr>
<tr>
<td>+17.5°</td>
<td>16</td>
<td>21.1</td>
<td>10.3</td>
</tr>
<tr>
<td>+07.3°</td>
<td>13</td>
<td>9.7</td>
<td>3.3</td>
</tr>
<tr>
<td>+01.1°</td>
<td>52</td>
<td>15.8</td>
<td>11.1</td>
</tr>
<tr>
<td>−02.4°</td>
<td>6</td>
<td>10.7</td>
<td>4.9</td>
</tr>
<tr>
<td>−17.3°</td>
<td>20</td>
<td>8.4</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Table 4.5: Observations with the 8.3° FWHM 10.4 GHz experiment.

Figure 4.5: The 15 scans obtained at Dec = 46.6° displayed as a function of right ascension. Each plot shows the second difference in mK after binning into 1° bins. A running mean has been subtracted from each scan. Long scans are displayed modulo 360°.

reached was not very high as the integration time at each declination was limited. Figure 4.5 shows the full data set for the 46.6° Declination data. The long term baseline drifts are still apparent at this stage. After the baseline has been removed using the maximum entropy technique the data was stacked together and compared with the expected signals (see Chapter 7). At this frequency and angular scale the experiment is more sensitive to Galactic emission than to the CMB radiation and so this data will be used to put constraints on the Galactic contamination to the 5° Tenerife experiments.

The Likelihood results

The statistical properties of the signals present in the data have been analysed using the likelihood function and a Bayesian analysis. This method has been widely used in the past by the Tenerife group (see e.g. Davies et al 1987) and incorporates all the relevant parameters of the observations: experimental configuration, sampling, correlation between measurements, etc. The analysis assumes that both the noise and the signal follow a Gaussian distribution fully determined by their respective auto-correlation function. The source of dominant noise in the data is thermal noise in the receivers which is independent in each data-point (Davies et al 1996) and therefore it only contributes to the terms in the diagonal of the auto-correlation matrix. For this analysis the section of data away from the Galactic plane has been selected otherwise the local contribution would dominate by orders of magnitude over the CMB fluctuations. Also, the analysis has been restricted to data in which
4.2. THE TENERIFE SCANS

Dec. RA σ (µK) Indep. \( \frac{(\ell(\ell+1)C_\ell)^{1/2}}{2\pi} \) (10^{-5})

<table>
<thead>
<tr>
<th>Dec.</th>
<th>RA</th>
<th>σ</th>
<th>Indep.</th>
<th>( \frac{(\ell(\ell+1)C_\ell)^{1/2}}{2\pi} ) (10^{-5})</th>
</tr>
</thead>
<tbody>
<tr>
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<td>96</td>
<td>58</td>
<td>5.0^{+3.3}_{-3.0}</td>
</tr>
</tbody>
</table>

Table 4.6: Statistics of the data used in the analysis. 95% confidence limits are shown.

There is a minimum number of 10 independent measurements for the full RA range (Dec. 7.3° does not have enough data) and to data for which we have a point source prediction (Dec. −17.3° is not covered by the Green Bank survey). This region represents approximately 3000 square degrees on the sky. Table 4.6 presents the sensitivity per beam in the RA range used in this analysis. Also column 4 gives the mean number of independent measurements which contribute to each point. This statistical analysis has been performed directly on the scan data, and not on the MEM deconvolved sky map produced during the baseline subtraction process. Thus, for this section, any effects of using a MEM approach are restricted to the baselines subtracted from the raw data, which will not contain, or affect, any of the astronomical information to which the likelihood analysis is sensitive.

Two different likelihood analyses were made: the first considers the data of each declination independently, and the second considers the full two-dimensional data set. A likelihood analysis was performed assuming a Harrison-Zel’dovich spectrum (so that the covariance matrix is given by Equation 2.20 with n=1), thus the parameter fitted for was \( Q_{RMS-PS} \). Since the \( \ell \) range sampled is small, Equation 3.1 can be used to give an estimate for \( C_\ell \) and this is what is shown in the Table 4.6. The experiment has a peak sensitivity to an \( \ell \) of 14^{+7}_{-6}. The fifth column of Table 4.6 gives, for the one-dimensional analysis, the amplitude of the signal detected with the one-sigma confidence level. The confidence limits on these signals were found by integration over a uniform prior for the likelihood function. These analyses ignore correlations between measurements at adjacent declinations. Therefore a full likelihood analysis, taking this correlation into account, should constrain the signal more efficiently. It should be noted that the two dimensional analysis assumes that the signal has the same origin over the full sky coverage but this may not be the case because of the differing levels of Galactic signal between declinations and across the RA range. In Figure 4.6 the likelihood function resulting from this analysis is shown. It shows a clear, well defined peak at \( \frac{(\ell(\ell+1)C_\ell)^{1/2}}{2\pi} (10^{-5}) = 2.6^{+1.6}_{-1.0} \) (95 % confidence level). This value would correspond to a value of \( Q_{RMS-PS} = 45.2^{+23.8}_{-27.2} \) µK. The results are compatible with the constraints on the signal in each declination considered separately but it is clear that the two dimensional likelihood analysis improves the constraints on the amplitude of the astronomical signal.

A comparison between the results obtained here and the amplitude of the CMB
Figure 4.6: The likelihood function from the analysis of the full data set. There is a clearly defined peak at \( \left( \frac{(\ell+1)}{2\pi} C_\ell \right)^{1/2} (10^{-5}) = 2.6^{+1.3}_{-1.6} \) (95% confidence level).

structure found in Hancock et al (1994) at higher frequencies can be made. They found \( Q_{RMS-PS} \sim 21 \mu K \) in an 5° FWHM switched beam and taking into account the extra dilution a slightly lower level is expected in an 8° FWHM switched beam, assuming a \( n = 1 \) power spectrum. It is seen that the majority of the signal in the 10 GHz, FWHM=8° data is most likely due to Galactic sources. Assuming that the majority of the signal found here is Galactic and using a spatial spectrum of \( C_\ell \propto \ell^{-3} \) (estimated from the Haslam et al 1982 maps) to predict the galactic contamination in a 5° FWHM beam at 10 GHz, then using a full likelihood analysis, it is found that an \( rms \) signal of \( \Delta T_{rms} = 55^{+32}_{-26} \mu K \) is expected. It should be noted that this is an upper limit on the Galactic contribution to the 5° data as the variability of the sources has been ignored when the subtraction was performed (this results in a residual signal from the point sources in the data during the likelihood analysis) and the analysis also includes regions where the Galactic signal is expected to be higher (for example the North Polar Spur). The 5° FWHM Tenerife scans are centred on Dec. 40° and it can be seen from Table 4.6 that this is the region with the lowest Galactic contamination. The results reported in Gutierrez et al (1997), for the 5° FWHM, 10 GHz Tenerife experiment, show that the signal found was \( Q_{RMS-PS} < 33.8 \mu K \) (corresponding to a signal of \( \Delta T < 53 \mu K \)) which is consistent with the prediction here (also taking into account the more significant contribution from the CMB at 5°). This comparison allows a restriction on the \textit{maximum} Galactic contribution to the signal found in Hancock \textit{et al} 1994 to be \( \Delta T_{rms} \sim 18-23 \mu K \) at 15 GHz and \( \Delta T_{rms} \sim 2-4 \mu K \) at 33 GHz depending on whether the contamination is dominated by synchrotron or free-free emission.

### 4.2.6 The 5° FWHM Tenerife experiments

The aim of the 5° FWHM experiments is to produce consistent maps at three operating frequencies (10.45, 14.90 and 33.00 GHz) of the CMB fluctuations and using the frequency coverage to put constraints on the levels of galactic foregrounds (namely synchrotron and free–free which are expected to be the most important at these frequencies). The experiments cover a range between Declinations 27.5° and 45°. The 10.45 GHz, 4.9° FWHM experiment, the 14.9 GHz, 5.2° FWHM experiment and the 33 GHz, 5.0° FWHM experiment have been taking data since 1985, 1990 and 1992 respectively. Table 4.7 summarises the results up to July 1996 (the data presented in this thesis is taken up to August 1997 which includes a new 15 GHz declination scan at 27.5°). It is seen that the full area is not covered by all three experiments but they are continually taking new data to build up a well sampled, high sensitivity data set. Table 4.8 shows the noise levels per 1° bin achieved in this data set.
4.2. THE TENERIFE SCANS

<table>
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</tr>
</tbody>
</table>

Table 4.7: The number of independent measurements in the RA range 161° – 250° in each of the Tenerife data sets.

<table>
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<th>Declination</th>
<th>10 GHz</th>
<th>15 GHz</th>
<th>33 GHz</th>
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</thead>
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<tr>
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<td>33</td>
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<td>40.0°</td>
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<td>42.5°</td>
<td>63</td>
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<td>50</td>
</tr>
<tr>
<td>45.0°</td>
<td>80</td>
<td>26</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.8: The noise per beam in µK for each of the Tenerife final stacked scans.

Dec. 35° at 10 and 15 GHz

The data set consisting of the Declination 35° scan at 10 GHz and 15 GHz has been analysed separately to the remainder of the data as an example of the analysis techniques possible on the individual scans (see Gutierrez et al 1997). The declination 40° was analysed previously (see Hancock et al 1994) but new data has been added since then. This region was chosen as it contains one of the best coverage of high Galactic latitudes (see Table 4.7). The remaining data (including the 35° data) will be analysed with Maximum entropy in a full two-dimensional analysis in Chapter 7. The analysis is concentrated in the region RA=161° – 250° which is at Galactic latitude $b > 40°$ where the CMB signal is not buried beneath foreground contributions. Figure 4.7 shows the stacked data in the region of interest. First the possible sources of non-CMB foregrounds in the data have been considered; the contribution due to known point-sources has been calculated using the Kuhr et al (1981) catalogue and the Green Bank sky survey (Condon & Broderick 1986); this was complemented with the Michigan and Metsahovi monitoring programme. In this band of the sky the most intense radio-source at high Galactic latitude is 1611+34 (RA=16°11′48″, Dec.=34°20′20″) with a flux density $\sim 2$ Jy at 10 and 15 GHz. After convolution with the triple beam of the experiment this represents peak amplitudes of $\sim 60$ and $\sim 30$ µK at 10 and 15 GHz respectively, which are in the limit of the detection of each data-set. The predicted $rms$ of the point-source contribution from the remaining unresolved sources, using the above surveys, along the scan is 26 and 11 µK at 10 and 15 GHz. Therefore, even considering uncertainties in the prediction of
this order, this can only be responsible for a small fraction of the detected signals as will be discussed below. In the following analysis this point source contribution has been subtracted from the data scans. The contamination by a foreground of unresolved radio-sources has been predicted to have an $\text{rms} \lesssim 30$ and $\lesssim 15 \, \mu K$ at 10 and 15 GHz respectively (Franceschini et al 1989) consistent with the findings here. In previous papers (Davies, Watson and Gutiérrez 1996) the unreliability of the predictions of the diffuse Galactic foreground using the low frequency surveys at 408 MHz (Haslam et al 1982) and 1420 MHz (Reich & Reich 1988) has been demonstrated. It is possible to estimate the magnitude of such a contribution from the 8° FWHM Tenerife experiment. At 15 GHz it was shown that the maximum contribution from the Galactic foregrounds was $\sim 20 \, \mu K$.

The Likelihood results

The likelihood analysis has been applied to the data at 10 GHz and 15 GHz in the range RA=161° − 250°. Assuming a Harrison-Zel’dovich spectrum for the primordial fluctuations the likelihood curve for the 15 GHz data shows a clear peak with a likelihood $L/L_0 = 5.5 \times 10^4$ in a signal of $\Delta T_{\text{RMS}} \sim 32 \, \mu K$ which corresponds to an expected power spectrum normalisation $Q_{\text{RMS}−\text{PS}} = 20.1 \, \mu K$ at an $\ell$ of 18$^{+7}_{−7}$. The maximum Galactic contribution to this data set was shown to be $\Delta T_{\text{rms}} = 23 \mu K$ (from the analysis of the 8° data) which cannot account for the signal seen here. Analysing the curve in a Bayesian sense with a uniform prior, a value of $Q_{\text{RMS}−\text{PS}} = 20.1^{+7.1}_{−5.1} \, \mu K$ (68 % confidence limit) was obtained. These results do not depend greatly on the precise region analysed; for instance analysing the sections RA=161° − 230° or RA=181° − 250°, the results show $Q_{\text{RMS}−\text{PS}} = 20.3^{+7.3}_{−7.3} \, \mu K$ and $Q_{\text{RMS}−\text{PS}} = 20.0^{+8.0}_{−6.0} \, \mu K$ respectively. As a consequence of the noisy character of the 10 GHz data, there is no evidence of signal in the likelihood analysis of the data at this frequency; instead a limit on $Q_{\text{RMS}−\text{PS}} \leq 33.8 \, \mu K$ (95 % confidence limit) is obtained which is compatible with the amplitude of the signal detected at 15 GHz. The signal at 15 GHz is only slightly larger than the level of the signal present in the COBE DMR data ($Q_{\text{RMS}−\text{PS}} = 18 \pm 1.5 \, \mu K$, Bennett et al 1996) and this indicates that, if the CMB fluctuations do indeed correspond to the standard COBE DMR normalised Harrison-Zel’dovich model, the possible foreground contamination would contribute with $\Delta T_{\text{RMS}} \lesssim 16 \, \mu K$ at 15 GHz.

A joint likelihood analysis (Gutiérrez et al 1995) was run between the data presented here at 15 GHz and those at Dec.= 40° presented in Hancock et al 1994. The angular separation between both strips is similar to the beam size of the individual antennas so a partial correlation between the Dec.=+35° and Dec.=+40° scans is expected. The region between RA=161° − 230° at Dec.=+40° was chosen so as to exclude the variable radio-source 3C 345 (RA$\sim 250°$) that was clearly detected in the data. Analyses of models with a power spectrum $P \propto k^n$ for the primordial fluctuations (tilted models) were performed independently. Figure 4.8 shows the am-

Figure 4.7: The Dec 35° data at 10 GHz and 15 GHz in the region RA=161° − 250°.
4.2. THE TENERIFE SCANS

Figure 4.8: The constraints on $Q_{RMS-PS}$ given by the Dec. 35° data.

Figure 4.9: The declination 27.5° data at 15 GHz. The top figure shows the stacked data. The principal Galactic plane crossing is clearly visible at RA $\sim$ 300°. The middle figure shows the typical errors across the scan. It is seen that in the best region a sensitivity of $\sim$ 60$\mu$K has been achieved. The bottom scan shows the number of independent data points that have gone into the scan. It is seen that no data was taken of the Galactic anti-centre.

The full data set

The remaining data taken with the 5° FWHM experiments will be analysed simultaneously to give a better constraint on underlying signals. As the sampling rate of the Tenerife declinations scans is less than a beam width, the adjacent scans will have strong correlations which can be used to put a stronger constraint on the underlying sky signal. As an example of the quality of the new data taken recently, the declination 27.5° data is shown in Figure 4.9. The Galactic plane is clearly seen at RA $\sim$ 300° whereas the Galactic anti-centre has not been observed. At the positions where more data was take (bottom figure) it is easily seen that the errors on the final stacked scan are lowest (middle figure) as expected. Due to the level of noise in this scan at present no obvious point sources are visible but more data is due to be taken over the next year to increase the sensitivity.

Figures 4.10 to 4.12 show the stacked scans (with errors) for each of the declinations at each of the frequencies. The stacking was done after baseline removal with MEM. The accuracy of the data is easily seen through the observation of point sources. For example, the two main point sources shown in this region of the scans are 3C345 at RA 250.3° and Dec. 39°54′11″ and 4C39 at RA 141.0° and Dec. 39°15′24″. These are both clearly seen (4C39 is on the very left of the scans) in the 10 GHz and 15 GHz scans. Unfortunately due to the increased level of atmospheric contribution to the scans at 33 GHz it has been impossible to complete the analysis of the declinations at this frequency in time for this thesis. Therefore, only the old declination 40° data (which is the highest sensitivity data at this frequency and was presented in Hancock 1994) is included here. It is expected that the final set of 33 GHz data will be available by Easter of 1998.

Figure 4.10: The stacked scans at 10 GHz for the Tenerife 5° FWHM experiments.
CHAPTER 4. THE DATA

Figure 4.11: The stacked scans at 15 GHz for the Tenerife 5° FWHM experiments.

Figure 4.12: The stacked scan at 33 GHz for the Tenerife 5° FWHM experiments.

The Likelihood results

The likelihood analysis that was applied to the Dec. 35° above was then applied to the full data set. It has already been seen that the 15 GHz and 33 GHz data sets should be relatively free from Galactic contamination and so the results from the likelihood analysis to these two data sets should give very good constraints on the level of the CMB. The 10 GHz data set is more sensitive to Galactic free-free and synchrotron and so a higher level of fluctuations is expected in this data set (although if the CMB and Galactic fluctuations are exactly in anti-phase then a lower level would be achieved but this is highly unlikely to be the case).

The results from the likelihood analyses are shown in Table 4.9 and Figures 4.13 and 4.14. As is seen the level of all of the 15 GHz and 33 GHz data sets are ∼20μK which is consistent with a CMB origin for the fluctuations. The higher level in the 10 GHz channel is also seen. Unfortunately due to the noisy level of the 10 GHz channel only 95% upper limits were possible on most of the declination data sets.

It is possible to analyse the data sets simultaneously at each frequency as each declination is correlated with its neighbours (the beam is wider than the declination strip separation). A two dimensional likelihood for the 10 GHz data set (consisting of the 6 declinations) and the 15 GHz data set (consisting of the 8 declinations) was performed. As in the Dec. 35° case the likelihood curve for the full two dimensional 15 GHz data set is a ridge in the $n/Q_{\text{RMS-PS}}$ plot and so no constraints on $n$ are possible with the Tenerife data alone. However, for $n = 1$ the full two dimensional analysis gives $Q_{\text{RMS-PS}} = 22^{+5}_{-3} \mu$K. The COBE results showed a level of $18 \pm 2 \mu$K for the CMB anisotropy (Bennett et al 1996) and this is consistent with the data presented here for a Harrison-Zel’dovich spectrum (scale invariant). A likelihood analysis of the two data sets together can also be performed to put constraints on the slope of the primordial spectrum and this is done below.

A separate analysis of the same data set was made in parallel by Carlos Gutierrez (Gutierrez, private communication) and the results from this analysis are shown in Table 4.10. The main difference between the two analysis was a different point source catalogue was used for the predictions but the results are consistent with those presented here. The two point source catalogues used were taken at different times and so a difference is expected between the two predictions due to the variability of some of the sources. To eliminate even this small discrepancy a new point source data set has become available. This includes observations taken of each of the point sources within the Tenerife field over the last ten years and so a simultaneous fit for

Figure 4.13: The likelihood results for the individual declination scans at 10 GHz for the Tenerife 5° FWHM experiment.
4.2. THE TENERIFE SCANS

Table 4.9: Results from the one dimensional likelihood analysis of the Tenerife 5° FWHM experiments in the RA range 160° to 230°. All are for $Q_{RMS-PS}$ in $\mu$K and show the 68% confidence limits.

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Figure 4.14: The likelihood results for the individual declination scans at 15 GHz for the Tenerife 5° FWHM experiment.

In terms of the $rms$ temperature fluctuations the results from the full two dimensional likelihood analysis of the Tenerife 10 GHz and 15 GHz 5° FWHM data set is shown in Figure 4.15. The peak of the likelihood for the 10 GHz data gives $\Delta T_{rms} = 53^{+13}_{-13} \mu$K and that for the 15 GHz gives $\Delta T_{rms} = 39^{+5}_{-5} \mu$K. When comparing this with the prediction from the 8° FWHM Tenerife experiment for the upper limit to the Galactic contamination ($\Delta T_{rms} = 55^{+32}_{-26} \mu$K at 10 GHz and $< 23 \mu$K at 15 GHz) it is seen that the majority of the signal at 10 GHz is likely to be Galactic in origin whereas the majority of the 15 GHz signal must come from other sources (namely the CMB). A full analysis of the Tenerife data would include a simultaneous analysis of all data from the full frequency range of the Tenerife experiments as well as data from other experiments more sensitive to the foregrounds (e.g. the Galactic surveys at lower frequencies or the DIRBE dust map at a higher frequency). This is work that is in progress utilising the new multifrequency Maximum Entropy Method (see Chapter 5) but for present purposes it is useful to make the assumption that the 15 GHz signal is CMB alone (which has been shown to be a good assumption).

Assuming that the majority of the signal in the 15 GHz data set is CMB and not...
Table 4.10: Results from the one dimensional likelihood analysis of the Tenerife 5° FWHM experiments in the RA range 160° to 230°, done in parallel to those presented in this thesis (Gutierrez, private communication). All are for $Q_{RMS-PS}$ in µK and show the 68% confidence limits.

<table>
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<td>$\leq 33$</td>
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<td>$59^{+22}_{-17}$</td>
<td>$21^{+14}_{-10}$</td>
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Galactic in origin it is possible to use this data set as a constraint on models. The Dec +40° data presented in Hancock et al (1997) was used to put constraints on the value of $n$, the spectral index of the power spectrum of the initial fluctuations. This data set is now larger and so the errors have been reduced. The data was compared with the COBE data so as to put a greater constraint on the spectral index. In the likelihood analysis performed it was found that $n = 1.10^{+0.25}_{-0.30}$ (at 68% confidence) which is consistent with the result from COBE alone (Tegmark & Bunn 1995).

The same method can be applied to the full data set. To simplify the analysis it is assumed that there are no correlations between the COBE and Tenerife data sets. There is a large overlap between the filter functions of the two experiments and the region of sky observed by Tenerife is also observed by COBE. Therefore, there should be significant correlations between the two data sets for these pixels. However, for the majority of the COBE pixels there is no correlation with the Tenerife data and so, as a first attempt, this approximation is valid. The assumption of no cross-correlation between the two data sets allows separate inversions of the covariance matrices and so all that is required in the joint likelihood analysis is a multiplication of the COBE likelihood curve (Tegmark, private communication) by the Tenerife likelihood curve produced above. The result from this joint likelihood analysis is shown in Figure 4.16. The peak of the likelihood curve is at $n = 1.19$ and $Q_{RMS-PS} = 19.3\mu K$.

It is also possible to integrate over one of the parameters to put a constraint on the other. Integrating over $Q_{RMS-PS}$ it was found that $n = 1.11^{+0.2}_{-0.2}$ (68% confidence limits) and integrating over $n$ it was found that $Q_{RMS-PS} = 19.9^{+3.3}_{-3.2}\mu K$ (68% confidence limits). It is noted that the slope between the Tenerife and COBE scales is expected to be slightly greater than one (up to 10%) even for the scale invariant spectrum because of the contribution to the Tenerife data set from the tail of the Doppler peak. However, even without this enhanced slope the joint analysis is consistent with a Harrison-Zel’dovich spectrum. For $n = 1$ it was found
Figure 4.16: The likelihood results for the combination of COBE and Tenerife data sets. This is very preliminary analysis as no cross-correlations between the two data sets have been taken into account.

Figure 4.17: The levels of fluctuations found from various CMB experiments. The solid line is a $\chi^2$ minimisation for the points. The two dashed lines correspond to altering the Saskatoon results by ±14%. Evidence for a Doppler peak is clearly visible and it is seen that the Tenerife data is in agreement with the results from other experiments. The vertical bars on each point correspond to the error on the levels calculated and the horizontal bars correspond to the window function of the experiments.

that $Q_{RMS-PS} = 22.2^{+4.4}_{-4.2}\mu K$. This can also be compared with the joint likelihood analysis for the 15 GHz and 33 GHz Tenerife Dec. 40° data which gives $Q_{RMS-PS} = 22.7^{+8.3}_{-5.7}\mu K$ for $n = 1$ which is consistent with the joint COBE and 15 GHz Tenerife likelihood analysis. Currently work is being done into calculating the full likelihood results for the joint analysis incorporating all of the cross-correlations between the COBE and Tenerife data sets.

By placing the results from Tenerife for the level of the CMB onto the power spectrum graph, along with the results from other experiments, it can be seen that the standard CDM model (with inflation) is consistent with the findings. Figure 4.17 shows the levels of fluctuations found by various CMB experiments. The solid line is a polynomial fit to the points that minimises $\chi^2$ and the two dashed lines correspond to altering the Saskatoon experiment calibration by 14% (the quoted uncertainty in the results). It is clearly seen that there is evidence for the Doppler peak in the spectrum (see Hancock et al 1996).
Chapter 5

Producing sky maps

The purpose of any Microwave Background experiment is to produce a map of the fluctuations present on the last scattering surface. However, the methods used to obtain the sensitivity required involve scanning, beam switching and interferometry and so a way of working back from the data to the underlying real sky is needed. Similar problems occur when more than one component is present in the data and a method of separating out the different signals is required.

In this Chapter analysis techniques that can be used to find the best astronomical signals from a given data set are presented. The four methods to be described are CLEAN (Section 5.1), Singular Value Decomposition (Section 5.2), a new Maximum Entropy Method (Section 5.4) and the Wiener filter (Section 5.6). In the following Chapters the methods are used to analyse the data from each of the experiments described in Chapter 3.

The Maximum Entropy Method has been introduced previously in Hancock (1994). A fuller account of the method and the choice of parameters is given in this chapter, as well as new error calculations and new applications to interferometry and multiple frequency observations. Therefore, the equations that appear in Chapter 4 of Hancock (1994) are repeated here (Equations 5.9 to 5.22) so that the full method can be followed.

In general, the data from a CMB experiment will take the form of the true sky convolved with the instrumental response matrix with any baseline variations or noise terms added on. It is assumed that the observations obtained from a particular experiment have been integrated into discrete bins. For the $i$-th row and the $j$-th column, the data, $y_i^{(j)}$, recorded by the instrument can be expressed as the instrumental response matrix $R_i^{(j)}(i', j')$ acting on the true sky $x(i', j')$: 

$$ y_i^{(j)} = \sum_{(i', j')} R_i^{(j)}(i', j') x(i', j') + \epsilon_i^{(j)}, \quad (5.1) $$

where $i'$ and $j'$ label the true sky row and column position respectively. The $\epsilon_i^{(j)}$ term represents a noise term, assumed to be random, uncorrelated Gaussian noise.

It is immediately clear from Equation 5.1 that the inversion of the data $y_i^{(j)}$ to obtain the two-dimensional sky distribution $x(i', j')$, is singular. The inverse, $R^{-1}$, of the instrumental response function does not exist, unless the telescope samples
all of the modes on the sky. Consequently there is a set of signals, comprising the annihilator of \( R \), which when convolved with \( R \) gives zero. Furthermore, the presence of the noise term \( \epsilon \) will effectively enlarge the annihilator of \( R \) allowing small changes in the data to produce large changes in the estimated sky signal. It is, therefore, necessary to use a technique that will approximate this inversion.

### 5.1 CLEAN

The CLEAN technique is an operation which reduces the data into a set of point source responses. The point source responses are then convolved with the beam to obtain the ‘CLEAN’ map. For the two dimensional data a two dimensional CLEAN was implemented. Firstly the algorithm looks for the peak value in the data amplitude (in the case of the interferometer this is a complex amplitude) and notes its position, \((i, j)\). It then subtracts the normalised beam centred on \((i, j)\) multiplied by \(\gamma\) times the peak value, where \(\gamma\) is the loop gain \((\gamma < 1)\) set by the user, from the data. This process is then repeated iteratively. If this is allowed to continue ad–infinitum all the data would be fitted by the point source beam responses but this has no advantage over the original data so a criterion must be set as to when to halt the iterations. Two methods can be used to halt the iterations. Either the number of iterations can be set \((N_{\text{iter}})\) or the peak amplitude of the residual map can be set so that no points below this will be fitted. The final responses data set is then convolved with the beam to produce a cleaned map. The obvious problem with this method is that it eliminates all fluctuations in the data below the threshold level (or below the level set by \(N_{\text{iter}}\)). However, it does reconstruct all large sources very quickly.

### 5.2 Singular Value Decomposition

In singular value decomposition (SVD) the solution to Equation 5.1 can be approximated by solving

\[
y_i^{(j)} = \sum_{(i', j')} R_i^{(j)}(i', j') \hat{x}(i', j')
\]  

(5.2)

where \(\hat{x}\) is the best estimate (given the noise and beam convolution) of the underlying true sky \(x\). The solution is found by minimising the residual

\[
r = \sum_{(i, j)} \left| y_i^{(j)} - \sum_{(i', j')} R_i^{(j)}(i', j') \hat{x}(i', j') \right|^2 .
\]  

(5.3)

To find an approximate inverse of the \(R\) matrix, so that the best \(\hat{x}\) can be found, it is written as the product of three matrices.

\[
R = UVW^T
\]  

(5.4)
where $\mathbf{U}$ and $\mathbf{V}$ are both orthogonal matrices and $\mathbf{W}$ is purely diagonal with elements $w_k$. The formal inverse of $\mathbf{R}$ is therefore easily found and is equal to

$$\mathbf{R}^{-1} = \mathbf{V} \left[ \operatorname{diag} \left( \frac{1}{w_k} \right) \right] \mathbf{U}^T. \tag{5.5}$$

The singular values of $\mathbf{R}$ (comprising the annihilator) are easily seen as the values of $w_k$ that are zero. To overcome this problem the SVD analysis sets a condition number which is defined as the ratio of the largest $w_k$ to the smallest $w_k$. Any $w_k$ below the minimum value set by the condition number (so that it is close to zero and therefore represents a possible singular value) has the corresponding $1/w_k$ set to zero. The condition number is usually set to machine precision and so rounding errors and singularities are removed making the approximate inverse possible to calculate.

The SVD analysis does not take into account the level of noise in the data vector and so may have the tendency to amplify the noise. A method of regularising this is required.

### 5.3 Bayes’ theorem and the entropic prior

There are many degenerate sky solutions that are consistent with the data but it is desired to assume as little as possible about the true sky so as to avoid bias towards any particular model. By considering the problem from a purely Bayesian viewpoint, the most probable sky distribution given our data set and some prior information is desired. Given a hypothesis $H$, data $D$ and background information $I$, then Bayes’ theorem states that the posterior probability distribution $\Pr(H|DI)$ is the product of the prior probability $\Pr(H|I)$ and the likelihood $\Pr(D|HI)$, with some overall normalising factor called the evidence $\Pr(D|I)$:

$$\Pr(H|DI) = \frac{\Pr(H|I) \Pr(D|HI)}{\Pr(D|I)}. \tag{5.6}$$

The likelihood $\Pr(D|HI)$ is fully defined by the data, but there is freedom to choose the prior $\Pr(H|I)$. The most conservative prior assumption is to simply take the sky fluctuations to be small at some level. It is therefore desired to find the sky that contains the least amount of information, the one closest to being smooth, that is still consistent with the data set. This forms the basis of the so called “maximum entropy methods” (MEM) (Gull 1989, Gull & Skilling 1984) used in the reconstruction of images. If the class of sky models is restricted to those with this property then this leads to a prior of form (Skilling 1989),

$$\Pr(H|I) \propto \exp(\alpha S(f, m)) \tag{5.7}$$

for an image $f(\xi)$ and prior information $m(\xi)$, where $\xi$ is the position on the sky. The regularising parameter, $\alpha$, is a dimensional constant that depends on the scaling of the problem and $S(f, m)$ is the cross entropy. For a positive, additive distribution (PAD), the cross entropy is given by (Skilling 1988):
where \( m(\xi) \) can also be considered a constraint on \( f(\xi) \) such that when the entropic prior dominates, the aposteriori sky map \( f(\xi) \) does not deviate significantly from \( m(\xi) \). This form is chosen for the entropy to ensure that the global maximum of the entropy at \( f(\xi) = m(\xi) \) is zero and it is the only form of entropy consistent with coordinate and scaling invariance (Skilling 1988) which does not introduce any biases away from a small fluctuation model.

5.3.1 Positive and negative data reconstruction

Due to the log entropy term, this standard form is inapplicable in the more general case of an image \( x(\xi) \) that can take both positive and negative values. Various methods in the past have been tried to reconstruct data containing negative peaks. Laue, Skilling and Staunton (1985) proposed a two channel MEM, which involved splitting the data into positive and negative features and then reconstructing each separately but not taking into account any continuity constraint between the two. This method is inappropriate for differencing experiments as the positive and negative features originate from the beam shape and not from separate sources. White and Bunn (1995) have proposed adding a constant onto the data to make it wholly positive. They use the Millimetre-wave Anisotropy Experiment (MAX) data to reconstruct a \( 5^\circ \times 2^\circ \) region of sky. As simulations performed show, this method introduces a bias towards positive (or negative if the data are inverted) reconstruction. The reason for this is that the added constant has to be small enough so that numerical errors are not introduced into the calculations but a lower constant will give less range for the reconstruction of negative features and so the most probable sky will be a more positive one. A method to overcome both of these problems is proposed here (see Jones et al 1998 and Maisinger et al 1997).

Consider the image to be the difference between two PADS:

\[
x(\xi) = u(\xi) - v(\xi),
\]

so that the expression for the cross entropy becomes:

\[
S(u, v, m) = \int d\xi \left[ u(\xi) - m_u(\xi) - u(\xi) \ln \left( \frac{u(\xi)}{m_u(\xi)} \right) \right] +
\int d\xi \left[ v(\xi) - m_v(\xi) - v(\xi) \ln \left( \frac{v(\xi)}{m_v(\xi)} \right) \right].
\]

With the prior thus defined, it is possible to calculate the probability of obtaining the reconstructed sky \( x \) (the hypothesis \( H \)) given \( y \) (the data \( D \)):

\[
\Pr(x|y) \propto \Pr(x) \Pr(y|x),
\]

and then maximise \( \Pr(x|y) \) to obtain the most likely 2-D image of the microwave sky.
5.4 MEM IN REAL SPACE

Figure 5.1: The data from scan 5 of Figure 4.5, Chapter 4, displayed on an expanded temperature scale against RA bin number. Long time scale variations in the mean level are evident in the RAW scan (bottom panel). The middle panel shows the baseline fit found by the method of Section 5.4. The top panel shows the baseline corrected scan. The bin numbers exceed 360 since the scan begins near the end of an LST day, and the data are not folded modulo 360°.

5.4 Maximum Entropy deconvolution in real space

The application of Maximum Entropy to experiments with large sky coverage will now be discussed.

5.4.1 Long period baseline drifts.

In Figure 5.1, one of the Dec = +46.6° scans from the 8° FWHM Tenerife experiment, a slow variation in baseline (with a peak to peak amplitude of \( \sim 2 \) mK) is distinctly evident. Most of the scans obtained from this and the other Tenerife experiments, show these variations, to a greater or lesser degree, and therefore their removal is an important part of the analysis. These long period baselines vary both along a given scan and from day to day, clearly indicating that they are due, in the main, to atmospheric effects, with a possible contribution from diurnal variations in the ambient conditions. The remainder of this section concentrates on the 8° FWHM Tenerife experiment but the other Tenerife experiments are easily analysed in the same way with minor modification to the equations derived (note that the baseline variations are removed from the Jodrell Bank interferometer in the pre-processing stage of analysis). The time scale for these baseline variations appears to be several hours. With variations of this kind included, Equation 5.1 can be written as

\[
y_i^{(j)} = \sum_{(i',j')} R_{i'}^{(j)}(i',j') x(i',j') + \epsilon_i^{(j)} + b_i^{(j)}
\] (5.11)

where \( b_i^{(j)} \) represents the long term baseline variation. It is possible to examine this quantitatively by calculating the transfer function of the experiment, which defines the scales of real structures on the sky to which the telescope is sensitive. Variations produced on scales other than these will be entirely the result of non-astronomical (principally atmospheric) processes and should be removed.

As the Earth rotates the beams are swept in RA over a band of sky centred at a constant declination. For present illustrative purposes, it is sufficient to approximate the beams as one-dimensional in RA, with the beam centre and the East and West throw positions lying at the same declination. The beam shape for each individual horn is well represented by a Gaussian with dispersion \( \sigma = \text{FWHM}/2\sqrt{2\ln 2} = 3.57°: \)

\[
B(\theta) = \exp \left( -\frac{\theta^2}{2\sigma^2} \right),
\] (5.12)
and the beam switching in right ascension may be expressed as a combination of delta functions:

\[ S(\theta) = \delta(0) - \frac{1}{2}(\delta(\theta_b) + \delta(-\theta_b)), \]

(5.13)

for a switch angle \( \theta_b = 8.3^\circ \) in RA. So, the beam pattern is

\[ R(\theta) = B(\theta) * S(\theta). \]

Thus, the transfer function, (i.e. the Fourier transform of the beam pattern) is just:

\[ g(k) = 2\sqrt{2\pi}\sigma \exp \left( -\frac{k^2\sigma^2}{2} \right) \sin^2 \left( \frac{k\theta_b}{2} \right). \]

(5.14)

In Figure 5.2, the transfer function for waves of period \( \theta = 2\pi/k \) is plotted. As a function of declination the \( \theta \) co-ordinate must be multiplied by a factor of \( \sec \delta \) because a true angle \( \theta \) on the sky covers \( \sim \theta/\cos \delta \) in right ascension. The peak response of the instrument is to plane waves of period \( \sim 22^\circ \sec \delta \), i.e. individual peaks/troughs with FWHM \( \sim 7^\circ \). The response drops by a factor 10 for structures with periods greater than \( \sim 7 \) hours and less than \( \sim 40 \) minutes. The long period cutoff is due to cancellation of the large-scale structures in the beam differencing pattern, while the short period cutoff is simply due to dilution of structures within a single beam. The cutoff on large scales in particular is significant for the analysis, since it tells us that variations in the data on time scales \( > \sim 7 \)h \( \sec \delta \) are almost certainly due to long time scale atmospheric effects, or terrestrial and environmental effects, rather than being intrinsic to the astronomical sky. Thus identification and removal of such ‘baseline’ effects is important. By using the whole data set to calculate the most probable astronomical sky signal with maximum entropy deconvolution, it is possible to simultaneously fit a long-time scale Fourier component baseline to each scan. Then a stack of \( n \) days of data at a given declination to obtain a final scan with a \( \sim \sqrt{n} \) improvement in sensitivity to true astronomical features is possible.

5.4.2 The beam

Note that in the case of the Tenerife and Jodrell experiments, it is not necessary to re-define the beam matrix for each position in RA (which requires a matrix \( R_{ij}^{(j)} \) for the \( i \)-th bin in RA and \( j \)-th bin in declination) since the beam is translationally invariant in the RA direction. However, it is clear that the beam shape projected into RA and Dec co-ordinates will be a function of declination. The beam matrix in Equation 5.1 can be written as \( R_{ij}^{(j)}(i',j') \), for declination \( j \). This simplifies the problem slightly but does not invalidate the use of MEM for a more general experiment.

For example, in the case of the Tenerife experiment
\[ R^{(j)}(i', j') = C \left[ \exp \left( -\frac{\theta_C^2}{2\sigma^2} \right) - \frac{1}{2} \left( \exp \left( -\frac{\theta_E^2}{2\sigma^2} \right) + \exp \left( -\frac{\theta_W^2}{2\sigma^2} \right) \right) \right] \] (5.15)

where \( \theta_C \), \( \theta_E \), and \( \theta_W \) represent the true angular separation of the point \((i', j')\) from the beam centre and the East and West throw positions respectively. The normalisation of the beam matrix is determined by \( C \) and is implemented with respect to a single beam. The angles \( \theta_C \), \( \theta_E \), and \( \theta_W \) can be calculated using spherical geometry. If the beam is centred at Dec. \( \delta^{(j)} \) and \( \alpha_0 \) is the (arbitrary) RA origin for the definition of all the beams, for a source at a general \((\alpha, \delta)\) corresponding to the grid point \((i', j')\) the distance from the main beam centre is

\[ \theta_C = \arccos \left( \sin \delta^{(j)} \sin \delta + \cos(\alpha_0 - \alpha) \cos \delta^{(j)} \cos \delta \right). \] (5.16)

There are also two other beams (with half the amplitude of the central beam), due to the beamswitching and mirror wagging, a distance \( \theta_b \) (the beamthrow) either side of the central peak. These have RA centres given by \( \theta_E \) or \( \theta_W \) as in Equation 5.15 defined as

\[ \cos(\theta_{E or W} - \alpha_0) = \frac{\cos \theta_b - \sin^2 \delta^{(j)}}{\cos^2 \delta^{(j)}}, \]

and (fairly accurately for the beamthrow used in practice) Dec’s of \( \delta^{(j)} \) still. Thus their angular distances from the source at \((i', j')\) can be worked out from Equation 5.16, yielding \( \theta_{E or W} \), and the final \( R^{(j)} \) entry computed from Equation 5.15.

### 5.4.3 Implementation of MEM

If the assumption of random Gaussian noise if made, the likelihood is given by:

\[ \Pr(y|x) \propto \exp \left( -\frac{\chi^2}{2} \right), \] (5.17)

where \( \chi^2 \) is the misfit statistic (Equation 5.22). Thus in order to maximise \( \Pr(x|y) \) it is simply necessary to minimise the function

\[ F = \chi^2 - \alpha S \] (5.18)

where a factor of two has been absorbed into the constant \( \alpha \). Thus the process is to iterate to a minimum in \( F \) by consistently updating the 2-D reconstructed sky \( x(i', j') \). For the Tenerife data set long term baselines for each scan are also built up simultaneously. The baselines are represented by a Fourier series

\[ b_{ir}^{(j)} = C_{ir}^{(j)} + \sum_{n=1}^{n_{\text{max}}} \left[ C_{nr}^{(j)} \cos \left( \frac{2n\pi i}{l} \right) + D_{nr}^{(j)} \sin \left( \frac{2n\pi i}{l} \right) \right] \] (5.19)

for the \( r \)-th scan at the \( j \)-th declination with \( n_{\text{max}} \) baseline coefficients to be fitted. The basis data vector index \( i \) runs from 1 to \( l \), the length of the data sets (3 × 24\(^h\) for the Tenerife 8° FWHM data set). Thus to obtain a minimum period solution \( z \)
7th sec $\delta$ (Section 5.4.1) we must limit the number of baseline coefficients, $n_{\text{max}}$ in Equation 5.19 to less than 9 (for the case $\delta = 40^\circ$) for the Tenerife $8^\circ$ FWHM data set and less than 16 for the Tenerife $5^\circ$ FWHM data set.

For the $r$th Tenerife scan, Equation 5.11 may be written as

\[ y_{ir}(j) = y_{ir}^{\text{pred}(j)} + b_{ir}(j) + \epsilon_{ir}(j), \quad (5.20) \]

with the baseline variation $b$ included and

\[ y_{ir}^{\text{pred}(j)} = \sum_{i',j'} R^{(j)}(i', j') x(i' + i, j'), \quad (5.21) \]

is the predicted signal produced by the telescope in the absence of noise and baseline offsets. The beam matrix $R$ is now defined with respect to an origin (which was chosen to be zero) in the $i'$ direction as it is translationally invariant in RA. This gives us the term $x(i' + i, j')$ instead of $x(i', j')$. So, the $\chi^2$ for the problem is

\[ \chi^2 = \sum_{j=1}^{n_{\text{decs}}} \sum_{i=1}^{n_{\text{ra}}} w_{ic}(j) (y_{ic}^{\text{pred}(j)} - y_{ic}^{\text{obs}(j)})^2, \quad (5.22) \]

for a total number of declinations $n_{\text{decs}}$, total number of RA bins $n_{\text{ra}}$ and observed stacked data values $y_{ic}^{\text{obs}(j)}$ with weighting factor $w_{ic}(j)$, which are a weighted average over the $ns$ scans with the baseline $b_{ir}(j)$ subtracted from each of the scans $y_{ir}^{(j)}$ ($r$ is an index running over the $ns$ scans). In the absence of data $w_{ir}(j)$, for each individual scan, is set to zero and when data is present it is given by the inverse of the variance for the data point. It is a fairly simple task to compute $y_{ir}^{\text{pred}(j)}$, since it is possible to use prior knowledge of the geometry of the instrument to calculate the expected response function $R^{(j)}(i', j')$ for each $i'$, $j'$, at RA $i$ and declination $j$, thus $\chi^2$ is fully defined.

For the Jodrell Bank interferometer there are two orthogonal channels that should be included. As the entropy term depends on an unconvolved map this does not change with the introduction of these two orthogonal terms. However, for the $i$–th bin in right ascension and the $j$–th bin in declination the data, the $\chi^2$ term becomes

\[ \chi^2 = \sum_{j=1}^{n_{\text{decs}}} \sum_{i=1}^{n_{\text{ra}}} w_{ic}(j) (y_{ic}^{\text{pred}(j)} - y_{ic}^{\text{obs}(j)})^2 + \sum_{j=1}^{n_{\text{decs}}} \sum_{i=1}^{n_{\text{ra}}} w_{is}(j) (y_{is}^{\text{pred}(j)} - y_{is}^{\text{obs}(j)})^2, \quad (5.23) \]

for a total number of declinations $n_{\text{decs}}$, total number of RA bins $n_{\text{ra}}$ and observed data value $y_{ic/s}^{\text{obs}(j)}$ with weighting factor $w_{ic/s}(j)$ taken to be the inverse of the variance of the pixel. The cosine and sine channels are given by

\[ y_{ic}^{(j)} = \sum_{(i',j')} C^{(j)}(i', j') x(i' + i, j') + \epsilon_{ic}^{(j)}, \quad (5.24) \]

and
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\[ y_{is}^{(j)} = \sum_{(i', j')} S_{i', j'}^{(j)} x_{i' + i, j' + j'}^{(j)} + \epsilon_{is}^{(j)}, \]  
(5.25)

where \( i' \) and \( j' \) label the true sky right ascension bin and declination bin respectively, and \( C \) and \( S \) are the beam matrices for the cosine and sine channel respectively. The beams are once more defined with respect to an arbitrarily chosen origin of \( i = 0 \) as they are translationally invariant in RA. The \( \epsilon_{is}^{(j)} \) term represents a noise term, assumed to be random Gaussian noise.

Applying the requirements of continuity in entropy with respect to \( u \) (so that the partial gradients with respect to \( u \) and \( v \) are equal and opposite, which leads to \( uv = m^2 \)) to Equation 5.9 implies an entropy term, \( S(x, m) \), of form:

\[ \sum_{i', j'} \left[ \psi_{i', j'} - 2m_{i', j'} - x_{i', j'} \ln \left( \frac{\psi_{i', j'} + x_{i', j'}}{2m_{i', j'}} \right) \right], \]
(5.26)

where \( \psi_{i', j'} = u_{i', j'} + v_{i', j'} = (x_{i', j'}^2 + 4m_{i', j'}^2)^{1/2} \). Using our prior information that the sky fluctuations have zero mean, \( m_{i', j'} = m_u = m_v \) (from Equation 5.9) was chosen, and the minima of the large entropy case corresponds to \( x = (1 - \frac{m_u}{m_v}) u = 0; m_{i', j'} \) can therefore be considered as a level of ‘damping’ imposed on \( x_{i', j'} \) rather than a default model as in the positive-only MEM. A large value of \( m \) allows large noisy features to be reconstructed whereas a small value of \( m \) will not allow the final sky to deviate strongly from the zero mean.

Thus, if the value of the regularising parameter \( \alpha \) and the ‘damping’ term \( m \) is known then \( F \) is determined and the best sky reconstruction is that for which \( \partial F/\partial x_{ij} = 0, \forall x_{ij} \). This is most easily implemented by applying one-dimensional Newton-Raphson iteration simultaneously to each of the \( x_{ij} \) to find the zero of the function \( G(x) = \partial F/\partial x \). This means that \( x \) is updated from the \( n \)-th to the \((n + 1)\)-th iteration by

\[ x_{lm}^{n+1} = x_{lm}^n - \gamma \left( \frac{G(x_{lm}^n)}{\frac{\partial G}{\partial x_{lm}}|_{x_{lm}^n}} \right). \]
(5.27)

Convergence towards a global minimum is ensured by setting a suitable value for the loop gain \( \gamma \) and updating \( x_{lm} \) only if \( \frac{\partial G}{\partial x_{lm}}|_{x_{lm}^n} \) is positive. The differential with respect to the sky model is easily found analytically. For \( \chi^2 \) it is found that

\[ \frac{\partial^2 \chi^2}{\partial x_{lm}^2} = 2 \sum_k \sum_i w_i^{(k)} \left( R^{(k)}(l - i, m)(y_i^{pred(k)} - y_i^{obs(k)}) \right) \]
(5.28)

and, considering only the diagonal terms that are of concern here,

\[ \frac{\partial^2 \chi^2}{\partial x_{lm}^2} = 2 \sum_k \sum_i w_i^{(k)} \left( R^{(k)}(l - i, m) \right)^2 \]
(5.29)

and for the entropy term, \( S \), it is found that
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\[ \frac{\partial S}{\partial x_{lm}} = - \ln \left( \frac{u_{lm}}{m_{lm}} \right) = - \ln \left( \frac{\psi_{lm} + x_{lm}}{2m_{lm}} \right) \] (5.30)

and

\[ \frac{\partial^2 S}{\partial x_{lm}^2} = - \frac{1}{u_{lm} + v_{lm}} = - \frac{1}{\psi_{lm}}. \] (5.31)

In the case of the Tenerife data where the long-term baseline fluctuations are left in the data to be analysed by MEM, it is possible to simultaneously fit for the parameters of the baselines. The best reconstruction of the microwave sky is calculated and subtracted from each individual scan and then an atmospheric baseline is fitted to each scan. To fit for the baseline parameters \( C^{(j)}_{0r}, C^{(j)}_{nr} \) and \( D^{(j)}_{nr} \) as expressed in Equation 5.19 it is sufficient to implement a simultaneous but independent \( \chi^2 \) minimisation on each of these to obtain the baseline for the \( r \)-th scan.

From the Bayesian viewpoint minimising \( \chi^2 \) is just finding the maximum posterior probability by using a uniform prior. This is also done with a Newton-Raphson iterative technique with a new loop gain, \( \gamma_b \).

5.4.4 Errors on the MEM reconstruction

The posterior probability of the MEM reconstruction can be written as

\[ \Pr(H|DI) \propto \exp \left[ - \left( \chi^2 - \alpha S \right) \right] \] (5.32)

and expanding the exponent around its minimum gives

\[ \chi^2 - \alpha S = (\chi^2 - \alpha S)_{\text{min}} + x^\dagger \nabla \left( \chi^2 - \alpha S \right)_{\text{min}} + \frac{1}{2} x^\dagger \nabla^2 \left( \chi^2 - \alpha S \right)_{\text{min}} x. \] (5.33)

At the minima (the maximum probability) \( \nabla (\chi^2 - \alpha S) = 0 \) and so, to second order, Equation 5.32 can be written as

\[ \Pr(H|DI) = \Pr(H_{MP}|DI) \exp \left[ - \frac{1}{2} x^\dagger \nabla^2 \left( \chi^2 - \alpha S \right)_{\text{min}} x \right] \] (5.34)

where \( H_{MP} \) is the most probable reconstruction of \( H \) given the data, \( D \), and information, \( I \) (i.e. it is the MEM solution). This is a Gaussian in \( x \) with covariance matrix given by the inverse Hessian, \( M^{-1} = (\nabla^2 (\chi^2 - \alpha S)_{\text{min}})^{-1} \). For the reconstruction of data from CMB experiments the Hessian is given by

\[ M_{lk} = R_{il} R_{lk} + \frac{\alpha}{x_{lk}} \delta_{lk} \] (5.35)

where \( R \) is the beam and \( x \) is the sky reconstruction. This follows directly from considering all of the terms, on and off the diagonal, in Equations 5.29 and 5.31. The second differential of the entropy is a diagonal matrix, however, the second differential of the \( \chi^2 \) is not diagonal. The errors on the reconstruction, therefore, involve the inversion of a large matrix that is computationally intense. For the case...
of the Planck surveyor this would involve the inversion of a $(400 \times 400 \times 5)^2$ matrix. For this reason the simpler method of performing Monte Carlo simulations was used to obtain an estimate on the errors. However, in the Fourier plane the problem is dramatically simplified (see Section 5.5).

### 5.4.5 Choosing $\alpha$ and $m$

In this MEM approach the entropic regularising parameter, $\alpha$, controls the competition between the requirement for a smoothly varying sky and the noisy sky imposed by the data. The larger the value of $\alpha$ the more the data are ignored. The smaller the value of $\alpha$ the more noise is reconstructed. A choice of $\alpha$ that will take maximum notice of the data vectors containing information on the true sky distribution, while using the beam response shape to reject the noisy data vectors is required. In some sense, the entropy term may be thought of as using the prior information that the sky is continuous at some level to fill in for the information not sampled by the response function, thereby allowing the inversion process to be implemented.

Here, $m$ is chosen to be of similar size to the $\text{rms}$ of the fluctuations so that the algorithm has enough freedom to reconstruct the expected features. Increasing/decreasing $m$ by an order of magnitude from this value does not alter the final result significantly so that the absolute value of $m$ is not important. This is different to a positive–only MEM because in that case $m$ is chosen to be the default model (the value of the sky reconstruction in the absence of data) and is therefore more constrained by the problem itself. In the case of positive/negative MEM, as $m$ is the default model on the two channels and not on the final sky, there is a greater freedom in its choice.

For a data set with independent data points, $\alpha$ is completely defined in a Bayesian sense (see Section 5.5.3). For data sets containing a large number of non-independent points (as in the case of real sky coverages that contain beam convolutions so that the data set is too large to allow inversion of the Hessian; see Section 5.5.3), the optimum choice of $\alpha$ is somewhat controversial in the Bayesian community and while several methods exist (Gull 1989, Skilling 1989) it is difficult to select one above the others that is superior. The criterion that $\chi^2 - \alpha S = N$, where $N$ is the number of data points that are fitted in the convolved sky, is used here. If any of the data points are weighted to zero, as the galactic plane crossing is in the cases considered here, these points should not be included in $N$. Increasing/decreasing $\alpha$ by a factor of ten decreases/increases the amplitude of the fluctuations derived in the final analysis by $\leq 5\%$. The value of $\alpha$ is decreased in stages until $\chi^2 - \alpha S = N$; experience has shown that a convergent solution, for the 10 GHz, 5° FWHM Tenerife experiment, is best obtained with the typical parameter values given in Table 5.1. Below this value for $\alpha$ the noisy features in the data have a large effect and the scans are poorly fitted. This can be seen in Figure 5.3 where $\chi^2 - \alpha S$ begins to flatten out as $\alpha$ is lowered further than the chosen value (shown as a cross). From this figure it is easy to chose the value of $\alpha$. Note that any significance cannot be attached to the absolute value of $\alpha$, since it is a parameter that depends on the scaling of the problem. Also shown in Figure 5.3 is the effect of varying $m$, the ‘damping’ term.
### Table 5.1: The parameters used in the MEM inversion.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$4 \times 10^{-2}$</td>
</tr>
<tr>
<td>$m$</td>
<td>50 $\mu$K</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\gamma_b$</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Figure 5.3: The effect of varying $m$ and $\alpha$ on the final $\chi^2 - \alpha S$ of the MEM algorithm. The cross shows the values chosen for the final reconstruction presented in Chapter 7 of the 10 GHz $5^\circ$ Tenerife experiment.

It is seen that a larger value of $m$ requires a larger value of $\alpha$ to produce the same result. This is because the increased freedom in the reconstruction due to $m$ must be compensated for by a larger restriction due to $\alpha$.

#### 5.4.6 Galactic extraction

By including different frequency information (for example data from the 10, 15 and 33 GHz beam switching Tenerife experiments, the 5 GHz interferometer at Jodrell Bank and the 33 GHz interferometer at Tenerife, or the multifrequency channels of the Planck surveyor satellite) it should also be possible to separate Galactic foreground emission from the intrinsic CMB signals utilising the different power law dependencies. It is possible to rewrite Equation 5.1 for the full multi–frequency data set. For the $k$-th frequency channel of an experiment with $m$ frequencies

$$y_k(r) = \sum_{l=1}^{n} \sum_r Q_{kl}(r, r') x_l(r') + \epsilon_k(r) \quad (5.36)$$

where $x_l(r')$ is the $l$-th component (e.g. CMB, dust emission, point source emission etc.). $Q_{kl}(r, r')$ is therefore a combination of the beam response matrix and the frequency spectrum of each component. The drawback in using this method is that the frequency spectrum of the individual components needs to be known before the method can be implemented. However, in most cases, the spectrum is known to a good approximation and, if desired, MEM can be used to find the most probable spectrum for any unknown components although this requires much longer computational time. The MEM treatment follows directly from this equation similarly to the single frequency MEM, except that there are $n$ entropy terms (one for each component) and $m \chi^2$ terms (one for each frequency channel).
5.5 Maximum entropy in Fourier space

If the sky coverage of a particular experiment is not too large, so that the sphericity is negligible and a Fourier transform can be used, then the problem becomes simplified. This may also be possible with larger sky coverages but will require a more sophisticated decomposition of the sky harmonics (e.g. full sky spherical harmonic decomposition, for which the main limitation is computational time). Therefore, in Fourier space it is useful to look at the full Maximum Entropy problem again.

If the Fourier transform of Equation 5.36 is taken then we find

\[ \hat{y}_k(k) = \sum_l \hat{Q}_{kl}(k, k') \hat{x}_l(k') + \hat{\epsilon}_l(k) \tag{5.37} \]

where \( k \) is the Fourier mode. The convolution between \( Q \) and \( x \) has been replaced by a multiplication. This in itself represents a substantial simplification of all of the calculations involved in implementing the MEM approach. It is seen that each Fourier mode is independent of each other Fourier mode and so the Maximum Entropy algorithm can be implemented on a pixel by pixel basis (where each pixel is now a separate Fourier mode). However, it is also possible to include further information to MEM using this method.

If the correlation of \( x \) is known then it may be useful to use this extra information to further constrain the image reconstruction. We define the Fourier transform of the signal covariance matrix as

\[ \hat{C}_{ll'}(r) = \langle \hat{x}_l(r) \hat{x}_{l'}(r) \rangle \tag{5.38} \]

where \( \hat{C}_{ll'} \) is real with dimensions \( n \times n \) (the number of components). The diagonal elements of \( \hat{C} \) contain the ensemble average power spectra of the different components at a reference frequency \( \nu_0 \) and the off-diagonal terms contain the cross power spectra between components. The known correlations can be incorporated into MEM through the use of an intrinsic correlation function (ICF) and a set of independent hidden variables (Gull & Skilling 1990 and Hobson et al. 1998a). This is most easily achieved by the inclusion of lower triangular matrix \( \hat{L} \) such that

\[ \hat{x} = \hat{L} \hat{h}. \tag{5.39} \]

\( \hat{L} \) is obtained by performing a Cholesky decomposition (Press et al. 1992) of the matrix \( \hat{C} \) such that \( \hat{L} \hat{L}^T = \hat{C} \). Thus, the components of \( \hat{h} \) are uncorrelated and of unit variance and so

\[ \langle \hat{x} \hat{x}^\dagger \rangle = \langle \hat{L} \hat{h} \hat{h}^\dagger \hat{L}^T \rangle = \hat{L} \langle \hat{h} \hat{h}^\dagger \rangle \hat{L}^T = \hat{L} \hat{C} \hat{L}^T = \hat{C} \tag{5.40} \]

as required. Now the ‘default’ model in the MEM is a measure on \( \hat{h} \) and not \( \hat{x} \) and so the \textit{rms} expected level is simply unity.

Equation 5.37 can now be written as

\[ \hat{y}_k(k) = \sum_{l,m} \hat{Q}_{kl}(k, k') \hat{L}_{lm}(k') \hat{h}_m(k') + \hat{\epsilon}_l(k) \tag{5.41} \]
and $\hat{L}$ can be absorbed into $\hat{Q}$ so that it now contains frequency and spatial information for each of the foregrounds and the CMB. Now the MEM problem becomes the reconstruction of $\hat{h}$. Equation 5.9 follows directly except that now the difference between $u$ and $v$ is used to reconstruct $\hat{h}$.

### 5.5.1 Implementation of MEM in Fourier space

The problem is now exactly the same as before except all the terms are complex. The log term in the entropy does not allow for complex reconstructions but it is noted that the real and imaginary parts of the sky are independent of each other and so it is possible to split them up into two different PADS. Therefore, one complex reconstruction is split into a total of four channels; $u_{\text{real}}$, $v_{\text{real}}$, $u_{\text{imag}}$ and $v_{\text{imag}}$. The $\chi^2$ term is easily calculable from the reconstruction $\hat{h}$ as it depends linearly upon it (through the multiplication). Therefore, the reconstruction for each $k$-mode can be found separately.

If there is an initial guess for the distribution of $\hat{h}$ then it is possible to incorporate this into the MEM algorithm in two different ways. The first way is to incorporate the information into the covariance matrix and allow MEM to reconstruct the best hidden variables given the covariance matrix. In the absence of any initial guess at the shape of the covariance matrix, it is set to a flat model with the total power chosen to be the same as that in the input maps. If it is noted that the difference between $m_u$ and $m_v$ is the ‘default’ level for $\hat{h}$ (the real and imaginary parts of $\hat{h}$ need to be considered separately here) then another method for taking into account an initial guess is possible. Taking a uniform background common to both default values (this is taken as the $\text{rms}$ of $\hat{h}$) the initial guess at $\hat{h}$ is split into a positive part (added onto $m_u$) and a negative part (added onto $m_v$) so that $m_u - m_v$ is equal to the guess. Without the initial guess then it can be seen that $\hat{h}$ represents phase information and so both $m_u$ and $m_v$ are assigned with unit amplitude. In the absence of data $\hat{h}$ will default to the original guess.

The function $F$ is now fully defined and the maximum of the posterior probability is required. To find the minimum of $F$, which is now dependent on a very small number of variables, a quick minimisation routine is required. The variable metric minimisation (Press et al 1992), which uses the first derivative of the function and estimates the second derivative, approaches the minimum quadratically. This proved to be the most efficient method of minimising in Fourier space (other methods were tried and comparison between the various minimisations showed that each method was consistent).

### 5.5.2 Updating the model

After the posterior probability has been maximised an estimate of the underlying sky signal $\hat{x}$ is obtained. This could be used as the best sky reconstruction but it is also possible to use this estimate as the initial guess to MEM to find a better reconstruction. The guess can either be used to update the covariance matrix or the default sky models as described above. It is then possible to iterate until no
significant change is observed between iterations (a 3% change in any pixel flux was used as a measure of whether the image had converged or not in the simulations in the following Chapter).

### 5.5.3 Bayesian $\alpha$ calculation

A common criticism of MEM has been the arbitrary choice of $\alpha$, the Lagrange multiplier. However, it is possible to completely define $\alpha$ using a Bayesian approach. If the problem is thought of as trying to find the maximum probability with respect to $\hat{h}$ and $\alpha$, given some background information $I$, it is possible to first integrate over all possible $\hat{h}$ values to find that

$$\Pr(\alpha|\hat{y},I) = \frac{1}{\Pr(\alpha|I)} \int \Pr(\hat{h}, \hat{y}, \alpha|I) \frac{d^N \hat{h}}{\det[\psi]^{1/2}}$$

and maximise this in a similar way to maximising over $\hat{h}$ before. The metric on the space of positive/negative distributions is given by $-\nabla_h \nabla_h S(h,m)$ which leads to the invariant volume, $\det[\psi]^{-1/2}$, in the integral (this can be derived by considering the difference between two Poisson distributions; see Hobson & Lasenby 1998).

The full posterior probability can be written as

$$\Pr(\hat{h}, \hat{y}, \alpha|I) = \Pr(\hat{h}|\alpha,I) \Pr(\hat{y}|\hat{h}, \alpha,I) \Pr(\alpha|I)$$

and expanding this one finds

$$\Pr(\hat{h}, \hat{y}, \alpha|I) = \frac{\Pr(\alpha|I)}{Z_s Z_L} \exp \left( \alpha (S_1 + S_2 + ... S_N) - \frac{1}{2} \chi^2 \right)$$

where $Z_s$ is the normalisation constant for the entropy term and $Z_L$ is that for the likelihood term. The total entropy is the sum of the entropy for each of the separate maps that go into $\hat{h}$.

If one Taylor expands each term about its minimum (e.g. $S = S_{\text{min}} + \frac{1}{2} \hat{h} \nabla^2 S_{\text{min}} \hat{h} + ...$) then it is found that, to second order,

$$Z_s = \left(2\pi\right)^{\frac{N}{2}} \alpha^{\frac{N}{2}}$$

where $N$ is the number of independent variables to be found. For example, $N = 6$ in the case of a data set made from three input maps (i.e. $\hat{h}$ consists of three channels) if the calculation is performed on each Fourier mode separately (remembering that each of the three channels is made up of a real and an imaginary channel) and $N = 6n_p$, where $n_p$ is the number of pixels in each map, if the calculation is performed on all Fourier modes simultaneously. The function to be minimised is given by

$$F(h) = F(h)_{\text{min}} - \frac{1}{2} \hat{h} [\psi^{-\frac{1}{2}}] A[\psi^{-\frac{1}{2}}] \hat{h}$$

where $A = [\psi^{\frac{1}{2}}] M [\psi^{\frac{1}{2}}]$ and $M$ is the Hessian and $\psi = -\nabla^2 S$. The probability distribution for $\alpha$ is given by Equation 5.42 and so substituting in the expansions it is found that
70  

CHAPTER 5. PRODUCING SKY MAPS

Figure 5.4: The convergence of $\alpha$ for a typical experiment (in this case the analysis of Planck surveyor data). The vertical axis is $2\alpha S_{\text{tot}} + N - \alpha \text{Tr}A^{-1}(\alpha)$ and the Bayesian $\alpha$ is found on the point of intersection.

\[
\Pr(\alpha|\hat{y}, I) = \frac{1}{Z_L} \frac{(2\pi)^{\frac{N}{2}}(\det A)^{-\frac{1}{2}}}{(2\pi)^{\frac{N}{2}}\alpha^{-\frac{N}{2}}} \exp \left( \alpha(S_1 + S_2 + ... S_N) - \frac{1}{2} \chi^2 \right) \quad (5.47)
\]

The most probable $\alpha$ can now be calculated by taking the derivative of the log of Equation 5.47 and equating to zero.

\[
(S_1 + S_2 + ... S_N) + \frac{N}{2\alpha} \frac{d}{d\alpha} \log \det A = 0 \quad (5.48)
\]

and noting that the differential of $A$ with respect to $\alpha$ is just the identity matrix it is found that

\[
S_{\text{tot}} + \frac{N}{2\alpha} - \frac{1}{2} \text{Tr}A^{-1} = 0 \quad (5.49)
\]

Rearranging Equation 5.49, the most probable value for $\alpha$ at each Fourier mode is the solution to

\[
2\alpha S_{\text{tot}} + N = \alpha \text{Tr}A^{-1}(\alpha) \quad (5.50)
\]

It should be noted that $A$ is also a function of $\alpha$, so the solution of Equation 5.50 is non-trivial. An iterative approach to the problem is necessary. First solve

\[
\alpha_{\text{new}} = \frac{n}{\text{Tr}A^{-1}(\alpha_{\text{old}}) + 2S_{\text{tot}}} \quad (5.51)
\]

and then use this new $\alpha$ to perform a new minimisation until convergence is reached for $\alpha$. Figure 5.4 shows the typical convergence around the minima for $\alpha$. This method for calculating $\alpha$ can only be used when one minimisation of $F(\hat{h})$ has been performed. Thus an initial guess for $\alpha$ is required. This will be described in the next section. It is noted that this can be incorporated with the update procedure described in Section 5.5.2 until global convergence is reached. In real space this method is infeasible, as it involves the inversion of the Hessian, which, with convolutions, is a very large matrix.

5.5.4 Errors on the reconstruction

In the Fourier domain the inverse of the Hessian (the inverse of the Fourier transform of Equation 5.35) is much easier to find than in real space. Instead of the large matrix inversions that were necessary with the convolution it is now possible to do each $k$-mode separately. This is used to put errors on the power spectrum (or the full two dimensional Fourier transform of the underlying sky).
5.6. THE WIENER FILTER

Figure 5.5: The short dashed line shows the value of $\alpha S$ for the case in which $\alpha = 2$. The horizontal scale is the standard deviation of the reconstruction away from the model value (in this case taken to be unity). It is seen that the maximum value of the entropy is zero and the algorithm will default to this value in the absence of data. Given the data, the algorithm will try to maximise the entropy and so will reconstruct data with smaller amplitudes (the range in $x$). The equivalent plot for the Wiener filter is shown as the solid line and it can be seen that MEM approaches Wiener for small values of $x$, whereas at large amplitudes, a greater range of $x$ can be reconstructed by MEM. The long dashed line shows the results for a Bayesian $\alpha$ calculation on the Planck surveyor analysis and it can be seen that the Wiener filter falls far short of the required dynamic range for reconstruction.

The full multi-channel MEM has been derived from information theoretic considerations within the context of Bayes’ theorem. A different approach to the choice of prior probability will now be discussed and its connection to MEM will be highlighted.

5.6 The Wiener filter

The most conservative prior, $Pr(H|I)$, in Equation 5.6 may not the best choice in the presence of added information (see Zaroubi et al 1995). If the form of the hypothesis $H$ is known then this information can also be used to further constrain its reconstruction. Taking $m_u = m_v = m = 1$ and assuming that the levels of the fluctuations are small, so that $\hat{h}$ is small, the entropy can be rewritten, to second order, as

$$\alpha S(\hat{h}) = - \sum_k \frac{\alpha \hat{h}^2(k)}{4}$$

(5.52)

where the sum is over the Fourier modes. Using $\hat{x} = \hat{L}\hat{h}$, the prior probability is therefore found to be

$$Pr(\hat{x}|I) = \exp \left( -\frac{\alpha}{4} \hat{x}^\dagger (\hat{L}^T \hat{L})^{-1} \hat{x} \right).$$

(5.53)

Noting that $\hat{L}^T \hat{L} = \hat{C}$ this is equivalent to a Gaussian prior if $\alpha = 2$. Therefore, in the limit of small fluctuations it is seen that Maximum Entropy reduces to a quadratic form fully defined by a Gaussian covariance probability. This is the Wiener filter. Figure 5.5 shows the difference in the range of amplitude in the reconstruction that Wiener and MEM allow. It is seen that there is very little difference between the $\alpha = 2$ MEM and the Wiener filter out to about three standard deviations. If the fluctuations are not necessarily Gaussian then it is possible to take $\alpha = 2$ as our starting guess in the full Maximum Entropy approach and then use Bayesian alpha calculations to properly define the Lagrangian multiplier in subsequent iterations.

For the Gaussian case the prior probability has become
\[ \Pr(\hat{x}|I) \propto \exp \left( -\frac{1}{2} \hat{x}^\dagger \hat{C}^{-1} \hat{x} \right) \]  

(5.54)

and the matrix \( \hat{C} \) is the covariance matrix of the model given by

\[ \hat{C} = \langle \hat{x}^\dagger \hat{x} \rangle. \]  

(5.55)

The problem then becomes the minimisation of

\[ F = \chi^2 + \hat{x}^\dagger \hat{C}^{-1} \hat{x} \]  

(5.56)

This minimisation can be done in the same way as MEM through an iterative algorithm or it can be solved analytically. In full,

\[ F = (\hat{y} - \hat{Q} \hat{x})^\dagger \hat{N}^{-1} (\hat{y} - \hat{Q} \hat{x}) + \hat{x}^\dagger \hat{C}^{-1} \hat{x} \]  

(5.57)

where \( \hat{Q} \) is the instrumental response matrix, from Equation 5.36 and \( \hat{N} \) is the variance matrix of the data. Minimising with respect to \( \hat{x} \), the solution is

\[ \hat{x}' = \hat{C} \hat{Q}^\dagger (\hat{Q} \hat{C} \hat{Q}^\dagger + \hat{N})^{-1} \hat{y}. \]  

(5.58)

The Wiener filter, \( W \), that finds the best linear approximation to \( \hat{x}_i(\mathbf{r}) \) has resulted:

\[ \hat{x}'_i(\mathbf{r}) = W_{ij}(\mathbf{r}, \mathbf{r}') \hat{y}_j(\mathbf{r}') \]  

(5.59)

where \( i \) is now running over all the pixels in the component maps, \( j \) is over all the pixels in the data maps and the Wiener filter is given by

\[ W = \hat{C} \hat{Q}^\dagger (\hat{Q} \hat{C} \hat{Q}^\dagger + \hat{N})^{-1}. \]  

(5.60)

This can be easily written in the hidden variable space, as for MEM, but for illustrative purposes it is left in Fourier space. The Bayesian probability can now be written in terms of this filter (completing the square of Equation 5.57 can also be used to find the Wiener filter):

\[ \Pr(\hat{x}|\hat{y}I) \propto \exp \left( -\frac{1}{2} [\hat{x} - W \hat{y}]^\dagger (\hat{C}^{-1} + \hat{Q}^\dagger \hat{N}^{-1} \hat{Q}) [\hat{x} - W \hat{y}] \right) \]  

(5.61)

which is seen to have the quadratic form of the simplified MEM approach. From this it is easily seen that the covariance matrix on this reconstruction is given by

\[ H_w^{-1} = (\hat{C}^{-1} + \hat{Q}^\dagger \hat{N}^{-1} \hat{Q})^{-1}. \]  

(5.62)

The Wiener filter can now be considered as minimising the variance of the reconstruction errors of the Fourier components given by

\[ \langle \hat{\Delta}_i(\mathbf{k})^2 \rangle = \langle |\hat{x}'_i(\mathbf{k}) - \hat{x}_i(\mathbf{k})|^2 \rangle. \]  

(5.63)

It is well known that the Wiener filter smooths out the fluctuations at low flux levels (corresponding to levels below the noise). This has the effect of reducing the
power in a map. Therefore, the Wiener filter cannot be used in an iterative fashion as it will tend to zero.

Both MEM and the Wiener filter (the quadratic approximation to MEM) in the Fourier plane have a disadvantage over the real space MEM (Wiener in real space involves the inversion of very large matrices and so is infeasible to run). As the calculations are done on a pixel by pixel basis in Fourier space, the number of pixels at each frequency has to be the same. Due to the different pixelisation usually seen at different frequency experiments this requires an additional first step of re-pixelisation. In real space the full MEM analysis does not require this extra pixelisation as all the separate pixelisations can be taken into account in the matrix $Q$ which does not have to be square. Also, the Wiener filter requires the same number of pixels in the output maps as in the input maps and so cannot be used to reconstruct a map in irregularly sampled data (as in the case of the Tenerife or Jodrell Bank data).

### 5.6.1 Errors on the Wiener reconstruction

It has already been shown that the covariance matrix (the inverse Hessian) for the Wiener reconstruction is given by Equation 5.62. Therefore, the assignment of errors on the Wiener filter reconstruction is straightforward. It should be noted however, that the simple propagation of errors in the Wiener filter are a direct result of the Gaussian assumption of the initial covariance structure. This differs from the MEM error calculation as the MEM approximates the peak of the probability to be Gaussian to calculate the errors and not the whole probability distribution as in the Wiener filter case.

### 5.6.2 Improvements to Wiener

An ‘improvement’ to the Wiener filter has been proposed by a number of authors (see, for example, Rybicki & Press 1992, Bouchet et al 1997, Tegmark 1997). They propose a rescaling of the power spectrum (either by dividing the power spectrum by a quality factor or by the introduction of a Lagrange multiplier into the prior probability) so that the total reconstructed power is equal to the total input power of the maps. It has been shown that the power spectrum reconstructed by the standard Wiener filter is a biased estimation of the true power spectrum (for example Bouchet et al 1997) in that it weights the reconstructed power spectrum by signal/(signal+noise). This bias can be quantified by introducing a quality factor for each component $l$, $S_l(k)$, given by

$$S_l(k) = \sum_m W_{lm}(k)Q_{ml}(k)$$  \hspace{1cm} (5.64)

where $W_{lm}(k)$ is the Wiener matrix at Fourier mode $k$, $Q_{lm}(k)$ is the instrumental response matrix and the sum is over the frequencies of the experiment. The quality factor varies between unity (in the absence of noise) and zero. If $\hat{x}_l(k)$ is Wiener reconstruction of the $l$th component and $k$th Fourier mode and $\hat{x}_l(k)$ is the exact value it can be shown (Bouchet et al 1997) that
\[ \langle |\hat{x}\prime_l(k)|^2 \rangle = S_l(k) \langle |\hat{x}_l(k)|^2 \rangle . \] (5.65)

It is seen that the power spectrum of the reconstruction is an underestimate of the actual power spectrum by the quality factor. By rescaling the Wiener matrix it can easily be seen that the resulting power spectrum will be unbiased. However, it is found that the variance of the reconstructed maps increases as they are noisier.

The other proposed variant on Wiener filter is the introduction of a Lagrange multiplier into the prior probability that scales the input power spectrum. It has been suggested (Tegmark 1997) that this parameter should be chosen to obtain a desired signal to noise ratio in the final reconstruction. However, if it is noted that this Lagrange multiplier has exactly the same role as the MEM \( \alpha \) it is seen that this parameter is completely defined in a Bayesian sense similarly to \( \alpha \). The inclusion of this Lagrange multiplier simply results in the Wiener filter becoming the quadratic approximation to MEM without the automatic setting of \( \alpha \) due to the absolute value of the Gaussian probability covariance matrix.

In Wiener filtering the introduction of the quality factor results in noisier maps. The addition of the Lagrange multiplier just results in the quadratic approximation to MEM and as MEM and Wiener are indistinguishable in the Gaussian case there is no need to perform both methods. Therefore, as the final product of the analysis presented in this thesis is intended to be the real sky maps, only classic Wiener and the full MEM will be tested in the next chapter.

The methods discussed in this chapter will be applied to simulated data in the following chapter to test their relative strengths at analysing data from CMB experiments. The best method(s) will then be used to analyse the data and produce actual CMB maps from the experiments discussed in Chapter 4 in Chapter 7.
Chapter 6

Testing the algorithms

In this chapter the analysis techniques presented in the previous chapter are tested by applying them to various CMB data sets. The best overall method is then used in the following chapter to analyse data from the experiments presented in Chapter 3 and to produce maps of the sky at various frequencies.

6.1 Comparison between CLEAN and MEM

Simulations were performed of the CLEAN algorithm and the MEM algorithm using the 5 GHz Jodrell Bank interferometer as an example. The simulations using CLEAN were performed by Giovanna Giardino (see Giardino 1995). Data sets were produced from the Green Bank catalogue by convolving them with the interferometer beam. Gaussian noise was then added at varying levels to the scans to simulate the atmosphere and instrumental noise. Eleven declinations were simulated and ten ‘observations’ of each declination were made. The six noise levels chosen were 15, 25, 35, 45, 55 and 65 µK as these correspond to the range of noise levels expected in the real data set. Only the central RA range (161° – 240°) was analysed to reduce computing time. Figure 6.1 shows the input simulated data (dotted line) with the MEM reconstruction (solid line) and the CLEAN reconstruction (dashed line). This particular simulation is for Declination 30° and a noise level of 25 µK. It was chosen at random from the full set of simulations. As can be seen from the figure the MEM result appears to follow the data more closely than the CLEAN result.

Figure 6.2 shows the mean of the difference between the noise free simulated map and the reconstructed maps from MEM and various CLEAN reconstructions. For the ideal case the mean would be zero for the reconstruction (i.e. it reconstructed the exact simulated data). As can be seen the CLEAN results deviate from zero by a few micro Kelvin whereas the MEM result is centred around zero. CLEAN also gets worse as the noise is increased because it cannot tell the difference between a

Figure 6.1: Comparison at one declination of the MEM reconstruction (solid line), the CLEAN reconstruction (dashed line) and the simulated data (dotted line).
CHAPTER 6. TESTING THE ALGORITHMS

Figure 6.2: Mean of difference between the noise free map and the reconstructed maps. The MEM reconstructed differences are shown by the filled circles. CLEAN has been run for different values of the parameter $N_{\text{iter}}$ and $\gamma$. $N_{\text{iter}} = 10000$ for all curves except for the hollow squares, for which $N_{\text{iter}} = 20000$. Stars refer to $\gamma = 0.02$, hollow squares to $\gamma = 0.06$, filled triangles to $\gamma = 0.1$ and filled squares to $\gamma = 0.25$ ($\gamma$ is the loop gain in each iteration of the CLEAN algorithm). The large error bars are due to the number of realisations per noise level being limited to 10.

Figure 6.3: Standard deviation of the difference between the amplitude of the noise free map and the maps reconstructed with CLEAN and MEM. The symbols are as in the previous figure.

noise peak and a data peak whereas the MEM process does not get any better or worse.

The errors in the final map have also been calculated (by use of a Monte Carlo technique) and these are shown in Figure 6.3. As can be seen the different CLEAN processes all have a similar error in their reconstructions but MEM has a much lower error. This means that the MEM reconstruction will also be more consistent between data sets with different noise realisations, which is essential when analysing data that has many scans to be simultaneously analysed (as in the case of Tenerife). The MEM error line is also flatter than the CLEAN results (and also flatter than a line with unit gradient) which means that not only does MEM perform better than CLEAN on all noise scales but MEM actually does relatively better at extracting signals when the noise level is higher.

The final test of the algorithm is to check whether the fluctuations reconstructed by the two methods are present in the original map. It may be the case that the MEM method has reconstructed the correct amplitude of the signal but has put all of the features in the incorrect locations. To test this a correlation coefficient method which correlates the levels of the fluctuations between the input and output maps was used. Two maps will have a correlation coefficient of one if they are identical. As can be seen in Figure 6.4 the MEM reconstructions are very close to unity and so it is seen with confidence that MEM is reconstructing the simulations very accurately. The CLEAN results, however, shows that at low noise level it closely follows the simulations but at high noise levels the correlation drops dramatically. From this analysis it is concluded that MEM out-performs a simple CLEAN routine on all reconstruction properties of the maps. The CLEAN technique is, therefore, not used in subsequent analysis.

Figure 6.4: The correlation coefficient between the noise free map and the reconstructions from MEM and CLEAN. The symbols are the same as in the previous figures.
6.2 The Planck Surveyor simulations

The MEM Galactic extraction algorithm was tested on simulated observations from the Planck Surveyor satellite (see Hobson et al 1998a). To constrain any of the foregrounds in CMB data it is necessary to have a large frequency coverage. The Planck Surveyor satellite covers a range from 31 GHz to 847 GHz and has a high angular resolution (see Chapter 3) so that it will sample all of the foregrounds that were mentioned in Chapter 2.

6.2.1 The simulations

The simulations described here include six different components as input for the observations (see Bouchet, Gispert, Boulanger & Puget 1997 and Gispert & Bouchet 1997). The main component ignored in these simulations are extra–galactic point sources. Very little information is known about the distribution of point sources at the observing frequencies of the Planck Surveyor. The usual method for predicting point source contamination is to use observations at IRAS frequencies (> 1000 GHz) or low frequency surveys (< 10 GHz) and extrapolate to intermediate frequencies. This has obvious problems and so predictions are unreliable. For small sky coverage point source subtraction is performed by making high–resolution, high–flux–sensitivity observations of the point sources at frequencies close to the CMB experiment (see for example, O'Sullivan et al 1995). For all sky observations, point source removal is more complicated as it is difficult to make the required observations of the point sources. For the Planck satellite it is anticipated that the point sources can be subtracted to a level of 1 Jy at each observing frequency, and it may be possible to subtract all sources brighter than 100 mJy at intermediate frequencies where the CMB emission peaks (De Zotti et al 1997). De Zotti et al (1997) find that the number of pixels affected by point sources to be low and that the level of fluctuations due to unsubtracted sources is also very low. Therefore, no modelling of extra–galactic point sources will be made here. Recently, surveys at 350 GHz and 660 GHz (Smail, Ivison & Blain, 1997) have confirmed previous estimates of the contribution made by point sources (De Zotti et al. 1997). However, these surveys were in specially selected regions (gravitationally lensed objects) and so may be an over-estimate of the actual point source contribution. The MEM algorithm described here has been applied to simulations with point sources (Hobson et al. 1998b) and it was seen that their inclusion alters the conclusions reached here very little.

The six components used are the CMB, kinetic and thermal SZ, dust, bremsstrahlung and synchrotron. A detailed discussion of the simulations used is given by Bouchet et al (1997) and Gispert & Bouchet (1997). These have reasonably well defined spectral characteristics and this information can be used, together with the multifrequency observations, to distinguish between them. Both MEM and Wiener filtering are used to attempt a reconstruction of the components from simulated data taken with 14 months of satellite observation. The simulations are constructed on $10^\circ \times 10^\circ$ fields with 1.5′ pixels. Two models of the CMB are used. The first used is a stan-
standard CDM model with $H_0 = 50\text{km s}^{-1}\text{Mpc}^{-1}$ and $\Omega_b = 0.05$. The second is a string model produced by Francois Bouchet. The SZ component (thermal and kinetic) was produced by Aghanim et al (1997) using a Press-Schechter formalism (Press & Schechter 1974) which gives the number density of clusters per unit redshift, solid angle and flux density interval. The gas profiles of individual clusters were modelled as King $\beta$–model (King 1966), and their peculiar velocities were drawn at random from an assumed Gaussian velocity distribution with a standard deviation at $z = 0$ of 400 km s$^{-1}$. Galactic dust emission and bremsstrahlung have been shown to have a component which is correlated with 21cm emission from HI (Kogut et al 1996, Boulanger et al 1996). To model this correlation two IRAS 100 $\mu$m maps were used; one for HI correlated emission and one for HI uncorrelated. The components are then related by

$$HI_{\text{corr}}(\nu) = A [0.95 f_D(\nu) + 0.5 f_B(\nu)]$$

(6.1)

and

$$HI_{\text{uncorr}}(\nu) = B [0.05 f_D(\nu) + 0.5 f_B(\nu)]$$

(6.2)

where $f_D$ is the frequency dependence of the dust emission and $f_B$ is the frequency dependence of the bremsstrahlung. Therefore, the dust component was modelled by addition of 95% of the correlated HI IRAS map and 5% of the uncorrelated map and extrapolating to lower frequencies assuming a black body temperature of 18 K and an emissivity of 2 for the dust. The bremsstrahlung (or free–free) component was modelled by using 50% of the HI correlated IRAS map and 50% of the uncorrelated IRAS map. The combined map was scaled so that the $\text{rms}$ amplitude of the bremsstrahlung at 53 GHz was 6.2 $\mu$K and a temperature spectral index of $\beta = 2.16$ was assumed. No spatial template is available at sufficiently high resolution for the synchrotron maps. The simulations were modelled by using the low frequency Haslam et al maps at 408 MHz, which have a resolution of 0.85$'$ and adding small scale structure that follows a $C_l \propto l^{-3}$ power spectrum. A temperature spectral index of $\beta = 2.9$ was assumed.

The components used were all converted to equivalent thermodynamic temperature, for comparison purposes, from flux using the equation

$$\Delta I(\mu) = \Delta T x^4 e^x (e^x - 1)^2$$

(6.3)

where $x = h\nu/kT$ and $T = 2.726K$. The flux is originally in units of Wm$^{-2}$Hz$^{-1}$sr$^{-1}$ and all programs use flux units rather than temperature. The six input components (CMB, kinetic SZ, thermal SZ, dust, bremsstrahlung and synchrotron) are shown in Figures 6.5 (for the CDM simulation) and 6.6 (for the string simulation). Each component is plotted at 300 GHz and has been convolved with a Gaussian of 4.5$'$ FWHM, the highest resolution of the Planck Surveyor. It is noted that the IRAS 100 $\mu$m maps used as templates for the Galactic dust and free–free emission appear quite non–Gaussian and the imposed correlation between these two foregrounds is clearly seen. The azimuthally averaged power spectra of the input maps (CDM
realisation) are shown in Figure 6.7. It is easily seen that the power in the maps is suppressed for $\ell > 2000$. This is due to the finite resolution.

The final design of the Planck Surveyor satellite is still to be decided and significant improvements have recently been made to the proposed sensitivities. Therefore, these recent improvements will be used here to simulate observations (see Table 3.1 for full details on the frequency channels used). Simulated observations were produced by integrating the emission due to each physical component across each waveband, assuming the transmission is uniform across the band. At each frequency the beam was assumed to be Gaussian with the appropriate FWHM. Care is needed to include the effect of the waveband integration in the MEM and Wiener algorithms but this can easily be done in the conversion matrix described in Section 5.4.6. The frequency channels now consist of a noise-free flux in Wm$^{-2}$.

Isotropic Gaussian noise was added to each frequency channel with the typical $rms$ expected from 14 months of observations. Figure 6.8 shows the $rms$ fluctuations at each observing frequency due to each physical component after convolution with the appropriate beam. The $rms$ noise per pixel at each frequency is also plotted. It is seen that only the dust and CMB emissions are above the noise level (although the thermal SZ is extremely non–Gaussian and has many peaks above the noise level). The kinetic SZ has the same spectral characteristics as the CMB, but the effect of the beam convolution at the different frequencies on a point source distribution is more pronounced. The data was created on maps with pixels that assumed a spatial sampling of FWHM/2.4 at each frequency. That meant that for the $10^\circ \times 10^\circ$ sky area there were $320 \times 320$ pixels at the highest frequency and $44 \times 44$ pixels at the lowest frequency. As the calculations were performed in the Fourier plane it was necessary to repixelise the maps onto a common resolution although it is noted that this is not necessary for the MEM algorithm in real space. The final ten frequency channels (now with pixel noise added) are shown in Figure 6.9.

### 6.2.2 Singular Value Decomposition results

The simple linear inversion of Singular Value Decomposition (S.V.D.) was used on the two different data sets (one for the CDM realisation and one for the string realisation of the CMB) from the simulated Planck Surveyor data. The final reconstructed map for the CDM and strings simulations are shown in Figures 6.10 and 6.11 and are summarised in Table 6.1. The grey scales of the reconstructions and the input maps are the same. It was not possible to attempt a separate reconstruction of the CMB and kinetic SZ effect as these have the same frequency dependence. However, six plots are shown for easy comparison with the input maps. As can be seen from the figures and tables, the S.V.D. performs quite well on both the CMB (CDM and strings) and the dust channels but fails to reconstruct the other channels. As the input maps used are known it is possible to calculate the residuals for each reconstruction. This is defined as

$$e_{rms} = \left[ \frac{1}{N} \sum_{i=1}^{N} \left( T_{rec}^i - T_{true}^i \right)^2 \right]^{1/2},$$

(6.4)
Figure 6.5: The six input maps used in the simulations for the data taken by the Planck Surveyor. This is for the CDM model of the CMB. They are shown at 300 GHz in $\mu$K. a) CDM simulation, b) Kinetic SZ, c) Thermal SZ, d) Dust emission e) Free-free emission and f) Synchrotron emission.

Figure 6.6: The six input maps used in the simulations for the data taken by the Planck Surveyor. This is for the string model of the CMB. They are shown at 300 GHz in $\mu$K. a) String simulation, b) Kinetic SZ, c) Thermal SZ, d) Dust emission e) Free-free emission and f) Synchrotron emission.

Figure 6.7: The azimuthally-averaged power spectra of the input maps shown in Figure 6.5 at 300 GHz.

Figure 6.8: The rms thermodynamic temperature fluctuations at each Planck Surveyor observing frequency due to each physical component, after convolution with the appropriate beam and using a sampling rate of FWHM/2.4. The rms noise per pixel at each frequency channel is also plotted.

Figure 6.9: The ten channels in the simulated data from the Planck Surveyor. The noise and lower resolution is clearly seen in the low frequency channels. The noise levels represent an improvement for the LFI since Bersanelli et al 1996 was published. The units are in $\mu$K equivalent integrated over the band.
6.2. THE PLANCK SURVEYOR SIMULATIONS

Table 6.1: Results from the Singular Valued Decomposition analysis of simulated data taken by the Planck Surveyor for a string CMB signal and CDM CMB signal.

<table>
<thead>
<tr>
<th>Component</th>
<th>Input $\Delta T$ in $\mu$K</th>
<th>SVD reconstruction $\Delta T$ in $\mu$K</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMB strings</td>
<td>Max. 305.53</td>
<td>261.50</td>
</tr>
<tr>
<td></td>
<td>Min. -338.20</td>
<td>-386.63</td>
</tr>
<tr>
<td></td>
<td>Rms. 69.43</td>
<td>77.81</td>
</tr>
<tr>
<td>CMB CDM</td>
<td>Max. 272.45</td>
<td>315.30</td>
</tr>
<tr>
<td></td>
<td>Min. -277.82</td>
<td>-319.15</td>
</tr>
<tr>
<td></td>
<td>Rms. 73.94</td>
<td>81.03</td>
</tr>
<tr>
<td>Dust</td>
<td>Max. 647.88</td>
<td>626.26</td>
</tr>
<tr>
<td></td>
<td>Min. -316.68</td>
<td>-354.97</td>
</tr>
<tr>
<td></td>
<td>Rms. 174.15</td>
<td>173.72</td>
</tr>
<tr>
<td>T-SZ</td>
<td>Max. 75.08</td>
<td>88.97</td>
</tr>
<tr>
<td></td>
<td>Min. 0.00</td>
<td>-59.48</td>
</tr>
<tr>
<td></td>
<td>Rms. 4.82</td>
<td>15.40</td>
</tr>
<tr>
<td>Free-free</td>
<td>Max. 2.04</td>
<td>4.96</td>
</tr>
<tr>
<td></td>
<td>Min. -1.61</td>
<td>-9.42</td>
</tr>
<tr>
<td></td>
<td>Rms. 0.67</td>
<td>2.74</td>
</tr>
<tr>
<td>Synchrotron</td>
<td>Max. 0.23</td>
<td>1.70</td>
</tr>
<tr>
<td></td>
<td>Min. -0.24</td>
<td>-0.94</td>
</tr>
<tr>
<td></td>
<td>Rms. 0.08</td>
<td>0.51</td>
</tr>
</tbody>
</table>

where $T_{\text{rec}}^i$ is the reconstructed temperature of pixel $i$, $T_{\text{true}}^i$ is the input temperature and $N$ is the total number of pixels. The values for the residuals are shown in Table 6.2. The desired accuracy of the Planck Surveyor is 5$\mu$K (Bersanelli et al. 1996) and it is seen that the SVD analysis of the data falls far short of this sensitivity.

The residuals are a rather crude method of quantifying the accuracy of the reconstructions. A more useful technique is to look at the amplitude of the reconstructed maps as compared to the input maps. Usually this plot consists of a collection of points but to make things clearer three contour levels are plotted. The 68%, 95% and 99% distribution of the reconstructed amplitudes are shown as a function of the input amplitude. These plots can be used to give an estimate on the accuracy of the reconstructed maps. A perfect reconstruction would be a diagonal line with unit gradient. Figures 6.12 and 6.13 show the plots for the SVD analysis. The spread of points in this plot do not respond to the respective residual values for each map as the residual calculation also takes into account how far away from the diagonal the points are whereas this plot only shows the deviation away from the best fit line. The gradient of the best fit line is shown in Table 6.2. From this table it is easily seen that the dust and CDM realisation are reproduced quite well (the gradient is close to unity). However, it is seen that the reconstruction of the CDM realisation of the CMB is more accurate than that of the strings realisation. This is due to the
Table 6.2: The rms of the residuals and the gradients of the best-fit straight line through the origin for the comparison plots in the SVD reconstructions shown in Figs 6.10 and 6.11.

extra Gaussian features from the noise that are introduced during the SVD analysis having a larger effect on the non-Gaussian string reconstruction than on the Gaussian CDM reconstruction. It is clear that other methods of analysis should be investigated to obtain better results.

6.2.3 MEM and Wiener reconstructions

In the previous chapter it was seen that, in the Fourier domain, it is possible to give the MEM and Wiener algorithms either the full correlation matrix of the components, an estimate of the correlation matrix, or no information on the spatial distribution (except for a starting guess on the total power in the map). Two different levels of information were tested. The first gives the methods the full correlation matrix (including cross-correlations) by using the input maps to construct the average correlations. The second assumes that nothing about the spatial distribution is known and the correlation matrix is assumed to be flat. These two cases represent the two extremes that the real analysis of Planck Surveyor satellite data will take. They are presented as useful constraints and the actual analysis performed will be somewhere between the results presented here. For MEM, the Bayesian $\alpha$ was found for each of the cases and the model was updated between iterations until convergence was reached (less than 5% change in any of the pixel flux levels). With no correlation information the ICF was also updated between iterations. For Wiener no update was attempted as classic Wiener has a tendency to suppress power and so updating would cause the reconstruction to tend to zero.

The results of the MEM and Wiener analyses can be seen in Figures 6.14-6.29. Each analysis is grouped into four figures. The first and second show the reconstructions of the MEM and Wiener algorithms. The third and fourth show the accuracy of the reconstruction (similarly to the SVD analysis). Each of the reconstructions have been convolved to the experimental resolution (4.5'). A comparison of these figures with the true input components in Figures 6.5 and 6.6 clearly shows that the dominant input components (i.e. the CMB and the dust emission) are faithfully reconstructed in all cases. Table 6.2.3 shows the range and $rms$ of the reconstruc-
## 6.2. THE PLANCK SURVEYOR SIMULATIONS

<table>
<thead>
<tr>
<th>Component</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\Delta T) in (\mu K)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CMB strings</td>
<td>Max.</td>
<td>305.53</td>
<td>298.34</td>
<td>285.16</td>
<td>280.56</td>
</tr>
<tr>
<td></td>
<td>Min.</td>
<td>-338.20</td>
<td>-328.64</td>
<td>-327.33</td>
<td>-330.10</td>
</tr>
<tr>
<td></td>
<td>Rms.</td>
<td>69.43</td>
<td>69.25</td>
<td>68.97</td>
<td>69.04</td>
</tr>
<tr>
<td>CMB CDM</td>
<td>Max.</td>
<td>272.45</td>
<td>275.18</td>
<td>276.44</td>
<td>277.54</td>
</tr>
<tr>
<td></td>
<td>Min.</td>
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<td>-290.56</td>
<td>-289.59</td>
<td>-291.99</td>
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<tr>
<td></td>
<td>Rms.</td>
<td>73.94</td>
<td>73.88</td>
<td>73.70</td>
<td>73.78</td>
</tr>
<tr>
<td>Dust</td>
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<td>645.63</td>
<td>645.37</td>
<td>643.94</td>
</tr>
<tr>
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<td>Min.</td>
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<td>-320.87</td>
<td>-321.22</td>
<td>-324.32</td>
</tr>
<tr>
<td></td>
<td>Rms.</td>
<td>174.15</td>
<td>174.15</td>
<td>174.14</td>
<td>174.15</td>
</tr>
<tr>
<td>K-SZ</td>
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<td>5.16</td>
<td>0.87</td>
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</tr>
<tr>
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<td>Min.</td>
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<td>-1.64</td>
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<td>-</td>
</tr>
<tr>
<td></td>
<td>Rms.</td>
<td>0.86</td>
<td>0.25</td>
<td>0.20</td>
<td>-</td>
</tr>
<tr>
<td>T-SZ</td>
<td>Max.</td>
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<td>53.16</td>
<td>39.52</td>
<td>48.22</td>
</tr>
<tr>
<td></td>
<td>Min.</td>
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<td>-5.01</td>
<td>1.05</td>
<td>-3.56</td>
</tr>
<tr>
<td></td>
<td>Rms.</td>
<td>4.82</td>
<td>4.29</td>
<td>2.84</td>
<td>4.22</td>
</tr>
<tr>
<td>Free-free</td>
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<td>2.16</td>
<td>1.44</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>Min.</td>
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<td>-1.72</td>
<td>-0.73</td>
<td>-0.85</td>
</tr>
<tr>
<td></td>
<td>Rms.</td>
<td>0.67</td>
<td>0.57</td>
<td>0.50</td>
<td>0.39</td>
</tr>
<tr>
<td>Synchrotron</td>
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<td>0.24</td>
<td>0.14</td>
<td>0.02</td>
</tr>
<tr>
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<td>Min.</td>
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<td>-0.25</td>
<td>-0.10</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>Rms.</td>
<td>0.08</td>
<td>0.09</td>
<td>0.04</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 6.3: Results from the Planck simulations. (a) are the input values, (b) are the reconstructed values from the full MEM with ICF information, (c) are the reconstructed values from the full Wiener filter with ICF information, (d) are the reconstructed values from the full MEM with no ICF information and (e) are the reconstructed values from the full Wiener filter with no ICF information.

Table 6.3 shows the residuals for each of the maps given by Equation 6.4.

Perhaps most importantly, the CMB has been reproduced extremely accurately (to within 6\(\mu K\)). The residual errors on the MEM reconstruction of the CMB, as compared to the Wiener reconstruction, are slightly smaller. The residual errors for the other components are similar for the MEM and Wiener reconstructions, but are always slightly lower for the MEM algorithm. The difference between the two methods is seen to be greatest for the components that are known to be non-Gaussian in nature (dust, free-free, synchrotron and the thermal SZ effect). There is little difference between the reconstructions of the kinetic SZ (see below for discussion on the SZ reconstructions). For the full ICF information case the free-free emission, which is highly correlated with the dust, has been reconstructed reasonably well,
Figure 6.10: The results from the S.V.D. analysis of simulated data taken by the Planck Surveyor for a CDM CMB signal. The maps have been convolved with a 4.5′ Gaussian beam as this is the lowest resolution that the experiment is sensitive to.

Figure 6.11: The results from the S.V.D. analysis of simulated data taken by the Planck Surveyor for a string CMB signal.

Figure 6.12: A comparison between the input flux in each pixel and the output flux in that pixel for the SVD reconstructed maps with full correlation information for the CDM model of the CMB. A perfect reconstruction would be a diagonal line. The three contours enclose 68%, 95% and 99% of the pixels. If no contours are seen then no reconstruction was possible.

Figure 6.13: A comparison between the input flux in each pixel and the output flux in that pixel for the SVD reconstructed maps for the string model of the CMB.

<table>
<thead>
<tr>
<th>Component</th>
<th>(a) (µK)</th>
<th>(b) (µK)</th>
<th>(c) (µK)</th>
<th>(d) (µK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMB (CDM)</td>
<td>5.9</td>
<td>6.0</td>
<td>6.1</td>
<td>7.5</td>
</tr>
<tr>
<td>CMB (strings)</td>
<td>6.2</td>
<td>6.4</td>
<td>7.2</td>
<td>10.2</td>
</tr>
<tr>
<td>Kinetic SZ</td>
<td>0.9</td>
<td>0.9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Thermal SZ</td>
<td>3.9</td>
<td>4.1</td>
<td>4.4</td>
<td>4.6</td>
</tr>
<tr>
<td>Dust</td>
<td>1.6</td>
<td>1.9</td>
<td>1.9</td>
<td>2.1</td>
</tr>
<tr>
<td>Free-Free</td>
<td>0.3</td>
<td>0.4</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>Synchrotron</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 6.4: The rms of the residuals in the a) MEM with full ICF information, b) Wiener with full ICF information, c) MEM with no ICF information and d) Wiener with no ICF information reconstructions.
containing most of the main features present in the true input map. It is noted that the Wiener reconstruction of the free-free emission follows the dust emission more closely than the MEM reconstruction which has also reconstructed some free-free emission that is uncorrelated with the dust. For the case of no prior ICF information it is seen that no significant reconstruction of the free-free emission is made with the Wiener analysis whereas a smooth version has been reconstructed by MEM. The recovery of the synchrotron emission is less impressive. With full information MEM and Wiener can reconstruct the synchrotron to some degree but the significance of the features is not very high (this is seen in the very broad correlations between input amplitude and reconstructed amplitude in Figure 6.16).

Again, the residual errors on the MEM and Wiener reconstructions do not show all of the information available about the reconstructions. The extent of the deviation of the correlation plots away from the diagonal also contains information. Table 6.5 shows the gradients of the best fit lines for the MEM and Wiener reconstructions with full and no prior ICF information. From these best fit lines it is seen that the CMB and dust emission are both reconstructed very well. However, it is seen that MEM always outperforms the Wiener on the CMB reconstruction when no prior ICF information was given. This is due to Wiener underestimating the temperature of each pixel in the CMB channel. It is clearly seen that the thermal SZ is consistently reconstructed with a lower amplitude than the true amplitude. Even though the Wiener filter has been given the full prior ICF information it is still outdone by MEM with no prior information in the case of this highly non-Gaussian effect. The range of values reconstructed by Wiener is lower than that for MEM (the spread is smaller around the best fit line), however, the residual errors for the Wiener are always larger than those for MEM because of the smaller amplitudes reconstructed. The fit for MEM and Wiener can be improved by assuming the reconstruction is at a lower resolution (which is likely to be the case as there is little information on the SZ at the highest frequencies where the resolution is highest). For example, the reconstruction of the thermal SZ effect has a much stronger correlation with the input maps if it is assumed that the resolution of the data is 10.6′ (the resolution of the frequency channel where the thermal SZ effect is most dominant in the Planck Surveyor data). At this resolution the error is then 3\mu K.

Visually, the string realisation of the CMB appears to be reconstructed very accurately with all the non-Gaussian features still apparent in all cases. Comparing the residual errors from the string realisation with that of the CDM realisation, it is seen that even when MEM is used the non-Gaussian process is reconstructed slightly less accurately (a 10% increase in the residual errors in both cases considered). However, when the Wiener filter is applied the difference is much more enhanced (a 10% and 50% increase in the residual errors for the case of full and no prior correlation information respectively). In all cases the gradient of the MEM reconstruction is closer to unity than that of Wiener, but it is seen that the Gaussian realisation is more accurately reconstructed.

As a test of the power of MEM the frequency dependence of various components was also checked. There was an obvious minimum in the value of $\chi^2$ for the actual values of the dust temperature and emissivity as there is a lot of information on the
Table 6.5: The gradients of the best-fit straight line through the origin for the correlation plots of the reconstructions.

Figure 6.14: The six reconstructed channels from the MEM analysis of the Planck simulated data for a CDM model of the CMB using full correlation information.

Figure 6.15: The six reconstructed channels from the Wiener analysis of the Planck simulated data for a CDM model of the CMB using full correlation information.

Figure 6.16: A comparison between the input flux in each pixel and the output flux in that pixel for the MEM reconstructed maps with full correlation information for the CDM model of the CMB.

Figure 6.17: A comparison between the input flux in each pixel and the output flux in that pixel for the Wiener reconstructed maps with full correlation information for the CDM model of the CMB.

Figure 6.18: The six reconstructed channels from the MEM analysis of the Planck simulated data for a strings model of the CMB using full correlation information.

Figure 6.19: The six reconstructed channels from the Wiener analysis of the Planck simulated data for a strings model of the CMB using full correlation information.

Figure 6.20: A comparison between the input flux in each pixel and the output flux in that pixel for the MEM reconstructed maps with full correlation information for the strings model of the CMB.

Figure 6.21: A comparison between the input flux in each pixel and the output flux in that pixel for the Wiener reconstructed maps with full correlation information for the strings model of the CMB.
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Figure 6.22: The six reconstructed channels from the MEM analysis of the Planck simulated data for a CDM model of the CMB using no correlation information. The plot shows the kinetic SZ (figure (b)) for ease of comparison even though no attempt was made to reconstruct this effect.

Figure 6.23: The six reconstructed channels from the Wiener analysis of the Planck simulated data for a CDM model of the CMB using no correlation information.

Figure 6.24: A comparison between the input flux in each pixel and the output flux in that pixel for the MEM reconstructed maps with no correlation information for the CDM model of the CMB.

Figure 6.25: A comparison between the input flux in each pixel and the output flux in that pixel for the Wiener reconstructed maps with no correlation information for the CDM model of the CMB.

Figure 6.26: The six reconstructed channels from the MEM analysis of the Planck simulated data for a strings model of the CMB using no correlation information.

Figure 6.27: The six reconstructed channels from the Wiener analysis of the Planck simulated data for a strings model of the CMB using no correlation information.

Figure 6.28: A comparison between the input flux in each pixel and the output flux in that pixel for the MEM reconstructed maps with no correlation information for the strings model of the CMB.

Figure 6.29: A comparison between the input flux in each pixel and the output flux in that pixel for the Wiener reconstructed maps with no correlation information for the strings model of the CMB.
dust emission from the higher frequency channels. The free–free and synchrotron spectral indices were less tightly constrained as the $\chi^2$ depends mainly on the CMB and dust emissions. It should therefore be possible to constrain the spatial and frequency dependence of the dust emission but more assumptions, or higher sensitivity, is required to constrain the other Galactic components.

### 6.2.4 SZ reconstruction

The thermal SZ effect can be used to calculate the value of $H_0$ (Grainge et al 1993, Saunders 1997 etc.). However, for this to be possible the shape of the cluster must be known. Any algorithm used to reconstruct the information about the thermal SZ effect must closely follow the true shape of the underlying cluster and not just reconstruct the $\text{rms}$ or the amplitude. It was seen in the reconstructed maps that the MEM algorithm reconstructs the thermal SZ effect in more clusters than the Wiener filter. With no information on the correlation function the Wiener filter performs very badly in reconstructing the non-Gaussian emission. Figure 6.30 shows some of the typical thermal SZ profiles reconstructed by the MEM and Wiener algorithms with full and no correlation information. They are compared with the input profiles convolved with a Gaussian of size $10.6'$, the resolution of the 100 GHz channel (the frequency at which the fractional contribution of the thermal SZ effect is largest). It is easily seen that the full MEM analysis does reconstruct the cluster profiles closer to their truer shape than the Wiener filter. Thus, as expected, the Gaussian assumption of the Wiener filter leads to poor reconstructions of highly non-Gaussian fields as compared with MEM. It is also seen that even with no prior information on the correlations, the MEM algorithm still reconstructs a reasonable approximation to the true profiles.

At first sight it appears that the MEM reconstructions contain some spurious features as compared to the input profiles. This is seen most dramatically in the top panel of Figure 6.30 for the full prior ICF case. The central cluster appears to have an extra feature on the right hand side. In fact, this phenomenon illustrates the care that should be taken when interpreting SZ profiles found this way, since the feature is actually present in the input map, but has almost been smoothed out by the $10.6'$ convolution. The reason it is still present in the reconstruction is that the effective resolution of the MEM and Wiener reconstructions can vary across the map, depending on the level of the pixel noise and the other components. Therefore, in some regions a degree of super-resolution is possible whereas in regions where the pixel noise, or emission from the other components, is large the super-resolution does not occur. This was tested with different pixel noise realisations and it was seen that the areas of super-resolution did indeed move across the map.

The kinetic SZ can be used to put constraints on the interactions of galaxy clusters through their velocity distributions. No reconstruction was attempted when no information about the correlations was given, as the frequency spectrum of the kinetic SZ is the same as that for the CMB. Even in the case when full correlation information was given the kinetic SZ emission is not recovered particularly well. The reconstruction as compared to the input kinetic SZ emission is shown in Figure 6.31.
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Figure 6.30: Typical reconstructions of SZ profiles. The solid line is the input cluster convolved to 10.6′ (the resolution at which the sensitivity to the SZ effect is a maximum). The dashed line is the reconstruction from the MEM analysis using (a) full correlation information and (b) no correlation information. The dotted line is the reconstruction from the Wiener analysis using (a) full correlation information and (b) no correlation information. As can be seen the Wiener result is a much smoother reconstruction than the MEM result, especially in the case with no correlation information when the Wiener does not reconstruct any significant features but the MEM still follows the true profiles very well.

Figure 6.31: Comparison between the input simulation of the kinetic SZ effect convolved to 23′ (the resolution at which the fractional contribution of the kinetic SZ is highest in the data) and the MEM reconstruction of the effect.

Comparing the spatial location of the recovered kinetic SZ profiles to that of the thermal SZ it is seen that only the large kinetic SZ effects that occur on pixels also associated with a large thermal SZ are reconstructed. Therefore, it is seen that the recovery of the kinetic SZ is highly dependent on the information given to the MEM algorithm prior to the analysis and the data is not sufficiently sensitive to allow the kinetic SZ to be extracted by itself. This is easily seen as the largest kinetic SZ effect (in the lower right quadrant of the figures) is not reconstructed as it is not associated with a large thermal SZ effect. For a better reconstruction extra information is required, as well as the frequency and average spatial spectra. This may come in the form of the positions of the clusters (through the thermal SZ effect) and then using the information that the SZ effect occurs mainly on angular scales below any contribution from the CMB. However, this is very difficult to incorporate in an automatic algorithm and so no further analysis was attempted.

6.2.5 Power spectrum reconstruction

Figures 6.32 and 6.33 show the azimuthally averaged power spectra for the MEM and Wiener reconstructions with full prior correlation information respectively. The 68% confidence intervals obtained from the inversion of the Hessian are also shown. It is seen that the 68% confidence intervals always encompass the input power spectra. It should be noted that the confidence intervals shown are for the reconstructed maps given that the data is representative of the full sky and do not take into account sample or cosmic variance. For the components with the most amount of information (namely the CMB and dust) the confidence intervals are correspondingly smaller whereas for components with little information (as in the case of synchrotron) the confidence intervals are much larger. It is seen that the MEM and Wiener confidence intervals are fairly similar but this is to be expected as the Hessian calculation for both comes from a Gaussian assumption.

The reconstruction of the CMB is very good out to an ℓ of about 1500 for both MEM and Wiener. Beyond ℓ ~ 1500 Wiener begins to visibly underestimate the true
power but MEM is still accurate to $\ell \sim 2000$. The dust emission is reconstructed very well out to an $\ell$ of about 3000 for the MEM reconstruction and 2000 for the Wiener reconstruction, the extra information over the CMB coming from the increased resolution at the higher frequencies. The white noise spectra of the thermal SZ and kinetic SZ is much more closely followed by the MEM reconstruction although both algorithms have lost some of the power from the free-free and synchrotron reconstructions.

For the case with no prior ICF information the difference between the MEM and Wiener reconstruction is more easily seen. The power spectrum reconstructions for this case are seen in Figure 6.34 and 6.35. Even in the CMB reconstruction it is seen that the Wiener-produced power spectrum is consistently below the true spectrum even at very low $\ell$. The MEM reconstruction is accurate out to $\ell \sim 1500$ at which point it drops rapidly to zero. This rapid drop, seen in most of the reconstructions, is a result of continually updating the ICF in an area for which there is little information in the data. For the thermal SZ it is seen that the MEM reconstruction is reasonably accurate out to $\ell \sim 1000$ but the Wiener filter is only reasonably accurate out to $\ell \sim 200$. No reconstruction of the kinetic SZ was attempted. The dust power spectrum reconstruction is accurate out to $\ell \sim 1000$ but the Wiener filter is only reasonably accurate out to $\ell \sim 200$. No reconstruction of the kinetic SZ was attempted. The dust power spectrum reconstruction is accurate out to $\ell \sim 2000$ for MEM and $\ell \sim 3000$ for Wiener. However, it is seen that the Wiener reconstruction has a spurious bump in the power spectrum at $\ell \sim 2000 - 5000$ which overestimates the power for the CMB and this is again seen for the dust. Thus it is unclear whether the reconstruction from the Wiener filter beyond $\ell \sim 2000$ is accurate. Out to $\ell \sim 70$ the MEM reconstruction approximates the free-free true power spectrum but for Wiener, and the two reconstructions of the synchrotron emission, the power spectrum is always underestimated.

6.3 The MAP simulations

Four years prior to the launch of the Planck Surveyor NASA is due to launch its MAP satellite. This is a cheaper alternative to Planck, with less frequency coverage and lower angular resolution. As a test of the relative strength of the two experiments the same analysis that was performed on the Planck Surveyor was performed on the MAP satellite. The same simulations as the Planck analysis were used so that a direct comparison between the two satellites would be possible. The input maps for MAP were the same as those for the Planck, although the resolution of MAP is about 4 times worse than that of Planck and so the results are expected to be smoother (see Figure 6.36). The resolution of the MAP satellite has improved considerably since these simulations were performed and the latest design has a best resolution of about 2 times that of Planck (see Jones, Hobson, Lasenby & Bouchet 1998 for simulations with the current design). The true test of the sensitivity of MAP is again in the flux reconstruction of features that it is sensitive to. A fit to the six channels was again attempted. Only the CDM model of the CMB is shown here as the string simulations show very similar results.
Figure 6.32: The reconstructed power spectra (dark line) with errors compared to the input power spectra (faint line) of the Planck Surveyor CDM CMB simulation. The errors are calculated using the Gaussian approximation for the peak of the probability distribution. This reconstruction was produced by MEM with full ICF information.

Figure 6.33: The reconstructed power spectra (dark line) with errors compared to the input power spectra (faint line) of the Planck Surveyor CDM CMB simulation. The errors are calculated using the Gaussian approximation for the full probability distribution. This reconstruction was produced by Wiener filtering with full ICF information.

Figure 6.34: The reconstructed power spectra (dark line) with errors (dotted line) compared to the input power spectra (faint line) of the Planck Surveyor CDM CMB simulation. The errors are calculated using the Gaussian approximation for the peak of the probability distribution. This reconstruction was produced by MEM with no ICF information.

Figure 6.35: The reconstructed power spectra (dark line) with errors (dotted line) compared to the input power spectra (faint line) of the Planck Surveyor CDM CMB simulation. The errors are calculated using the Gaussian approximation for the full probability distribution. This reconstruction was produced by Wiener filtering with no ICF information.

Figure 6.36: The six input maps convolved to the highest resolution of the MAP satellite for comparison with the reconstructions. No reconstruction of the kinetic SZ was attempted as the signal at this resolution is negligible but it is still plotted to allow easy comparison with the Planck results.
6.3.1 MEM and Wiener results

The MEM and Wiener analyses of the MAP simulations were carried out with the same levels of information as the Planck analysis. The results of the analyses can be seen in Figures 6.37-6.44. Again, each analysis is grouped into four figures. The first and second show the reconstructions of the MEM algorithm and the Wiener filter respectively. The third and fourth show the accuracy of the reconstruction.

It is easily seen that there is no information in the MAP data on any of the foregrounds that the Planck Surveyor is sensitive to. Table 6.6 shows the $\text{rms}$ of the reconstructions from the simulations. The lower frequency coverage of MAP results in no information available on the dust emission (this is seen when no correlation information is given and the reconstruction is just set to zero by both MEM and Wiener). Even though the low frequency channels should contain information on the free-free and synchrotron emissions none is recovered. This is due to the lower resolution and sensitivity that MAP has. MAP was never designed to extract information on the SZ effect and does not contain channels near the critical 217 GHz frequency. Hence, no information on either of the SZ effects was reconstructed. However, the CMB is reconstructed fairly well. There is a very strong correlation for both the MEM and Wiener filter. Little difference is seen between these two reconstructions as the resolution is not large enough to pick them out and everything appears Gaussian (this was true in both the CDM and string simulations; the level of Gaussianity in the string simulation at this resolution is tested in Chapter 8). The 68% confidence intervals on the CMB reconstruction are $19\mu K$ and $29\mu K$ for the reconstructions with full correlation information and no correlation information respectively. The gradient of the best-fit lines for the correlation plots are 0.97 and 0.91 for the full prior ICF information and no prior ICF information respectively. It is seen that the reconstructions with full prior correlation information are more accurate than those with no assumed correlations by $10\mu K$. Therefore, a much better reconstruction of the CMB can be achieved with added information on its spatial distribution in contrast to the Planck Surveyor simulations where there was enough information in the data to reconstruct the CMB at a high degree of accuracy (the full and no prior correlation information reconstructions having a 68% confidence interval of $6\mu K$ and $7\mu K$ respectively). The Planck Surveyor appears to be three times more sensitive to the CMB than MAP (although the sensitivity and resolution of both satellites is not finalised).

6.4 MEM and Wiener: the conclusions

The simulations of the Planck Surveyor and MAP satellite data were analysed by both MEM and Wiener filtering. It was found that there were no differences between the two in the case of a fully Gaussian data set (see the MAP analysis where all non-Gaussian effects were negligible). However, when there was a non-Gaussian effect present, whether in a foreground or in the map to be reconstructed, there was a significant improvement when MEM was used as opposed to Wiener. The difference between the reconstructions using full and no prior information was less marked.
6.4. MEM AND WIENER: THE CONCLUSIONS

Figure 6.37: The six reconstructed maps from the MEM analysis of the MAP simulated data for a CDM model of the CMB using full correlation information.

Figure 6.38: The six reconstructed maps from the Wiener analysis of the MAP simulated data for a CDM model of the CMB using full correlation information.

Figure 6.39: A comparison between the input flux in each pixel and the output flux in that pixel for the MEM reconstructed maps with full correlation information for the CDM model of the CMB. The contours are as before.

Figure 6.40: A comparison between the input flux in each pixel and the output flux in that pixel for the Wiener reconstructed maps with full correlation information for the CDM model of the CMB.

Figure 6.41: The six reconstructed maps from the MEM analysis of the MAP simulated data for a CDM model of the CMB using no correlation information.

Figure 6.42: The six reconstructed maps from the Wiener analysis of the MAP simulated data for a CDM model of the CMB using no correlation information.

Figure 6.43: A comparison between the input flux in each pixel and the output flux in that pixel for the MEM reconstructed maps with no correlation information for the CDM model of the CMB.

Figure 6.44: A comparison between the input flux in each pixel and the output flux in that pixel for the Wiener reconstructed maps with no correlation information for the CDM model of the CMB.
Table 6.6: Results from the MAP simulations. (a) are the input values, (b) are the reconstructed values from the full MEM with ICF information, (c) are the reconstructed values from the full Wiener filter with ICF information, (d) are the reconstructed values from the full MEM with no ICF information and (e) are the reconstructed values from the full Wiener filter with no ICF information.
6.5. TENERIFE SIMULATIONS

for the MEM analysis than for the Wiener analysis. This is a very useful property because a smaller range of possible reconstructions means that the analysis is less sensitive to the prior information (which may be incorrect in the real analysis). It has been shown that the Wiener approach can be considered as the quadratic approximation to MEM and so even in the Gaussian case Wiener will be no better than MEM. Therefore, MEM will be used to analyse the Tenerife and Jodrell data in the next Chapter. The following section will test the power of MEM to analyse the Tenerife data set.

6.5 Testing the MEM algorithm on the Tenerife data set

Before applying the MEM algorithm to the real data, simulations were carried out to test its performance. Two-dimensional sky maps were simulated using a standard cold, dark matter model \( \left( H_o = 50 \text{ km s}^{-1}, \Omega_b = 0.1 \right) \) with an \( \text{rms} \) signal of 22\( \mu \text{K} \) per pixel (normalised to COBE second year data, \( Q_{\text{rms-PS}} = 20.3 \mu \text{K} \), see Tegmark & Bunn 1995). Observations from the sky maps were then simulated by convolving them with the Tenerife 8\( ^\circ \) FWHM beam. Before noise was added the positive/negative algorithm was tested by analysing the data and then changing the sign of the data and reanalysing again. In both cases the same, but inverted, reconstruction was found for the MEM output and so it is concluded that this method of two positive channels introduces no biases towards being positive or negative. Various noise levels were then added to the scans before reconstruction with MEM. The two noise levels considered here are 100\( \mu \text{K} \) and 25\( \mu \text{K} \) on the data scans, which represent the two extrema of the data that are expected from the various Tenerife configurations (100\( \mu \text{K} \) for the 10 GHz, FWHM=8\( ^\circ \) data and 25\( \mu \text{K} \) for the 15 and 33 GHz, FWHM=5\( ^\circ \) data).

Figure 6.45 shows the convolution of one of the simulations with the Tenerife beam and the result obtained from MEM with the two noise levels. The plots are averaged over 30 simulations and the bounds are the 68\% confidence limits (simulation to simulation variation). As seen, MEM recovers the underlying sky simulation to good accuracy for both noise levels, with the 25\( \mu \text{K} \) result the better of the two as expected. Figure 6.46 shows the reconstructed intrinsic sky from two of the simulations after 60 Newton–Raphson iterations of the MEM algorithm as compared with the real sky simulations convolved in an 8\( ^\circ \) Gaussian beam. Various common features are seen in the three scans like the maxima at RA 150\( ^\circ \), minima at RA 170\( ^\circ \) and the partial ring feature between RA 200\( ^\circ \) and 260\( ^\circ \) with central minima at RA 230\( ^\circ \), Dec. +35\( ^\circ \). All features larger than the \( \text{rms} \) are reconstructed in both the 25\( \mu \text{K} \) and 100\( \mu \text{K} \) noise simulations. However, there is a some freedom in the algorithm to move these features within a beam width. This can cause spurious features to appear at the edge of the map when the guard region (about 5\( ^\circ \)) around the map contains a peak (this can be seen in the map as a decaying tail away from the edge). For example, the feature at RA 230\( ^\circ \), Dec 50\( ^\circ \) has been moved down by a few degrees out of the guard region in the 100\( \mu \text{K} \) noise simulation so it appears
Figure 6.45: The solid line shows the sky simulation convolved with the Tenerife 8° experiment. The bold dotted line in the top figure shows the MEM reconstructed sky after deconvolution with the Tenerife beam, averaged over simulations of the MEM output from a simulated experiment with 25μK Gaussian noise added to each scan. Also shown are the 68% confidence limits (simulation to simulation variation; dotted lines) on this deconvolution. The bold dotted line in the bottom figure shows the deconvolution averaged over simulations with 100μK Gaussian noise added to each scan. The 68% confidence bounds (dotted lines) are also shown for this scan.

Figure 6.46: The top figure is the simulated sky convolved with an 8° Gaussian beam. The middle and bottom figures are the reconstructed skies from the 25μK and 100μK noise simulations (see text) respectively. They are all convolved with a final Gaussian of the same size.

There is a tendency for the MEM algorithm to produce super-resolution (Narayan & Nityananda 1986) of the features in the sky so that even though the experiment may not be sensitive to small angular scales the final reconstruction appears to have these features in it. Even though this effect is only minor, care must be taken not to interpret these features as actual sky features but instead the maps should be convolved back down with a Gaussian to the size of the features that are detectable by the experiment in consideration. This has been done with the two lower plots in Figure 6.46, so that a direct comparison between all three is possible. By comparison of these plots it is seen that the reconstructed sky obtained from the MEM algorithm does give a good description of the actual sky.

As an indicator of the error on the final sky reconstruction from the MEM, a histogram of the fractional difference between the input and output map temperatures is plotted in Figure 6.47. If the initial temperature at pixel $(i, j)$ is given by $T_{input}$ and the temperature at the same pixel in the output reconstructed map (after convolution with a Gaussian beam to avoid superresolution) is given by $T_{recon}$ then

$$\frac{T_{recon} - T_{input}}{T_{input}}$$

is put into discrete bins and summed over all $(i, j)$. The final histogram is the number of pixels within each bin. The output map has been averaged over pixels within the beam FWHM as features have freedom to move by this amount. A graph centred on -1 would mean that the output signal is near zero and the amplitude is too small while a graph centred on 0 would mean the reconstruction is very accurate. As can be seen both graphs (Figure 6.47 (a) and (c) for the 25μK and 100μK noise simulations respectively) can be well approximated by a Gaussian centred on a value just below zero. This means that the MEM has a tendency to ‘damp’ the data which is expected and this ‘damping’ increases with the level of noise (∼10% ‘damping’ for the 25μK simulation and ∼20% for the 100μK simulation). From the integrated
Figure 6.47: Histograms of the errors in the reconstruction of the simulated sky maps. (a) shows the $\frac{T_{\text{recon}} - T_{\text{input}}}{T_{\text{input}}}$ for the 25µK noise simulation and (c) shows the integrated $|\frac{T_{\text{recon}} - T_{\text{input}}}{T_{\text{input}}}|$ for (a). (b) and (d) are the corresponding plots for the 100µK noise simulation.

plots (Figure 6.47 (b) and (d)) MEM can be expected to reconstruct all features with better than 50% accuracy a half of the time for the 25µK noise simulation and a third of the time for the 100µK noise simulation.

6.6 Discussion

As seen from simulations performed in Section 6.2, the positive/negative MEM algorithm performs very well recovering the amplitude, position and morphology of structures in both the reconvolved scans and the two-dimensional deconvolved sky map. No bias, other than the ‘damping’ enforcement, is introduced into the results from the methods described here and so this is the best of the methods tried to use when making maps using microwave background data, as the bare minimum of prior knowledge of the sky is required. Even with the lowest signal to noise ratio (the 100µK noise simulation which corresponds to our worse case in the Tenerife experiments) all of the main features on the sky were reconstructed. Using this method it was possible to put constraints on the galactic contamination for other experiments at higher frequencies, which is essential when trying to determine the level of CMB fluctuations present.

It is clear that this approach works well and provides a useful technique for extracting the optimum CMB maps from both current and future multi-frequency experiments. This will become of ever increasing importance as the quality of CMB experiments improves.
Chapter 7

Foregrounds and CMB features in the sky maps

In this chapter I will use the maximum entropy method to produce maps of the sky at various frequencies from the data taken by the Jodrell Bank and Tenerife experiments. The possible origin of some of the features on the maps will be discussed.

7.1 The Jodrell Bank 5GHz interferometer

7.1.1 Wide–spacing data

The wide–spacing interferometer at Jodrell Bank has a baseline of 1.79m. The transfer function of the beam (i.e. the Fourier transform of the beam) is shown in Figure 7.1. It can be seen that in the RA direction it is sensitive to $\sim 1.5'$ scales whereas in the declination direction it is sensitive to $\sim 8'$ scales. At this frequency (5 GHz) the largest signals present will be those from extra–galactic point sources on the smaller angular scales and galactic synchrotron emission on the larger angular scales. This data will therefore be used as a prediction for point source contribution in the results from the other experiments and as a possible constraint on galactic emission. Simulations performed for the level of the CMB fluctuations that are expected in the data from a CDM dominated Universe with $H_0 = 50 \text{km s}^{-1} \text{Mpc}^{-1}$ and $\Omega_b = 0.03$ give $3.0 \pm 1.1 \mu K$ (the error is the standard deviation over 300 simulations). This is far below the noise and so it is ignored in the analysis presented here.

The MEM algorithm was used to extract the best information from the data using both sine and cosine channels as constraints as described in the previous chapter. The full deconvolved map was not used as there is no simple consistent method to analyse the different scale dependencies in the two directions (the map appears to be stripped in the RA direction due to the larger resolution) and thus compare the result with other data from experiments such as Tenerife. The map

Figure 7.1: The cosine beam power sensitivity as a function of inverse degrees. The sine beam has the same pattern but contains different phase information.
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Figure 7.2: Comparison between the MEM reconstruction (solid line) and the Clean reconstruction (dashed line) of the data (dotted line). The noise RMS in the scans are a) 25µK, b) 18µK and c) 36µK.

Figure 7.3: MEM reconvolution (green line) of the cosine channel of the Jodrell Bank 5 GHz wide spacing interferometer compared to the raw data (black line with error bars showing the one sigma deviation across scans).

presented here is the result of combining the cosine and sine channel output from MEM to obtain the amplitude and is therefore still convolved in a beam with 8° FWHM. The analysis was restricted to the high Galactic latitude data as there is too much confusion in the Galactic plane to allow any constraints on the CMB. The Galactic plane crossing is also an order of magnitude larger than the fluctuations seen in the rest of the scan and so the MEM algorithm has a tendency to fit this region better at the expense of the ‘interesting’ regions.

To check that MEM was reconstructing the data correctly the results were visually compared to the CLEAN results. This comparison is shown in Figure 7.2 and it is seen that the MEM result follows the noisy data more closely than the CLEAN result, as was found in the simulations.

Figures 7.3 and 7.4 show the MEM reconstructions of the data compared to the raw stacked data for the cosine and sine channels respectively. It is easily seen that the MEM reconvolution does follow the raw data very well in each declination. Figure 7.5 shows the MEM reconstructed 2–D sky map for the high Galactic latitude region (RA 130° to 260°). The error on this map is calculated by use of Monte-Carlo simulations and can be read directly from Figure 6.3. As the average error on the input data is 37 µK the error on the 2–D sky map is 10 µK. The point sources used as a check for the calibration correspond to the three largest peaks in this plot; 3C345 at RA 250°, Dec. 39° with a flux of 6 Jy (≈ 400µK); 4C39 at RA 141°, Dec. 39° with a flux of 9 Jy (≈ 550µK); 3C286 at RA 200°, Dec. 30° with a flux of 7 Jy (≈ 450µK). The amplitude of the data does not correspond to the exact predictions as the sources are variable to ≈ 30% but all agree to within this factor. Other sources can be used to check the calibration and are clearly seen in the data (e.g. 3C295 at RA 210°, Dec. 52°).

Figure 7.6 is a comparison between the MEM output of the data and the predicted point source contribution by using the low frequency GB catalogues. The 1.4 GHz and 4.9 GHz catalogues were extrapolated by performing a pixel by pixel fit for the spectral index. There is very good agreement between the MEM output and the prediction as expected since the interferometer is very sensitive to point sources. The difference between the GB prediction and the data is shown in Fig-
7.1. THE JODRELL BANK 5GHZ INTERFEROMETER

Figure 7.5: MEM reconstructed 2-D sky map of the wide-spacing Jodrell Bank interferometer data at high latitude.

Figure 7.6: Comparison between the GB catalogue (contour) and the MEM reconstruction (grey-scale). A different grey scale is used on all comparison plots (compared to the above plot) to enhance features in the MEM reconstruction for easier comparison. The contour levels are set at 10 equal intervals between the 0.0mK and 0.2mK. The green contours are above 0.1mK and the red contours are below 0.1mK. A larger region is also plotted to allow easier comparison of features at the edges of the maps.

Figure 7.7. Notice that the maximum amplitude of the difference is 280 µK and the two largest peaks occur at the positions of the two variable sources, 3C345 and 4C39. This difference may therefore be due to the variability of the sources.

The other dominant source of radiation that could contribute at this frequency and angular scale is that arising from synchrotron sources. For example, the discrepancy between the GB prediction and the data, for the region close to 3C345, could also be due to Galactic emission as well as the variability of the source. This is a region where Galactic emission has been detected previously. To see the extent to which synchrotron contaminates the maps, the low frequency surveys of Haslam et al, 1982, and Reich & Reich, 1988, were extrapolated up to 5 GHz. This is done to obtain a crude estimate on the Galactic spectral index as it has already been noted that the artefacts in these surveys make extrapolation difficult. Figure 7.9 shows the extrapolated 408 MHz survey map compared to the MEM output and Figure 7.8 shows the extrapolated 1420 MHz survey map. The two extrapolations were done by assuming a synchrotron power law with spectral index of -2.75. As the two surveys are already convolved in a beam with 0.85° FWHM (the 1420 MHz survey was convolved to the same resolution as the 408 MHz survey), care must be taken when carrying out the extrapolation. The interferometer beam will reduce the amplitude of the prediction by a factor that depends on the fringe visibility and this can be calculated from the Fourier transforms. The Fourier transform of the cosine channel can be shown to be

\[ R(\hat{u}, v) = \frac{1}{2} \sigma^2 \left[ \exp \left( -\frac{\sigma^2}{2} \left((u - u_o)^2 + v^2\right) \right) + \exp \left( -\frac{\sigma^2}{2} \left((u + u_o)^2 + v^2\right) \right) \right] \] (7.1)

where \( u_o = 2\pi b/\lambda \) is the fringe spacing for the interferometer with baseline \( b \), operating at wavelength \( \lambda \), and \( \sigma \) is the dispersion of the interferometer. The Fourier transform of the sine channel is exactly the same except that the difference of the

Figure 7.7: Difference between the GB catalogue prediction and the MEM reconstruction. Notice that the main differences occur at the positions of the two main, variable point sources, 3C345 and 4C39.
two exponentials is required. Multiplying Equation 7.1 with the Fourier transform of a Gaussian (so that a convolution is applied in real space) corresponding to the FWHM of the surveys (0.85°) and calculating the maximum visibility of the source (i.e. when the telescope is pointing directly at the source) it is found that

\[ V = \sigma^2 \frac{S_\circ}{a^2 + \sigma^2} \exp \left[ -\left( \frac{u^2}{2} \right) \left( \frac{\sigma^2 a^2}{a^2 + \sigma^2} \right) \right] \]  

(7.2)

where \( a \) is the dispersion of the Galactic survey used (0.85° FWHM corresponds to 0.33° dispersion) and \( S_\circ \) is the actual flux of the source. With \( \lambda = 6\text{cm} \), \( b = 1.79\text{m} \), \( \sigma = 3.4° \) and \( a = 0.33° \), as in the case of the wide spacing data, it is found that \( V = 0.91S_\circ \). Therefore, after the convolution of the 1420 MHz survey with the interferometer beam has been performed it is necessary to multiply by a factor of 1/0.91 = 1.1 and all results quoted here have taken this into account.

The lack of obvious correlations between the results and those of the low frequency surveys (except where the point sources are seen in both surveys) may be due to errors in the surveys. As previously discussed, the baselevels of the low frequency surveys are uncertain to about 10% and as the area considered is in the region of the survey where the intensity is at a minimum this is where the baselevel is expected to have maximum effect on the extrapolations. However, if it is assumed that the extrapolation is correct then the absence of correlation must result from a steepening of the spectral index of the synchrotron from the low frequency surveys and so the galactic emission is not as predicted. The frequency dependence and the spatial variance in the steepening of the spectrum is unknown without any intermediate frequency surveys. Column 2 of Table 7.1 summarises the \( \text{rms} \) of the data from this experiment in the high Galactic latitude region (RA: 130° to 260°). The error on the data is calculated by using the combination of errors for the cosine, \( \sigma_c \), and sine, \( \sigma_s \), channels.

\[
\sigma_{\text{amp}}^2 = \left( \frac{\delta A}{\delta y_c} \sigma_c \right)^2 + \left( \frac{\delta A}{\delta y_s} \sigma_s \right)^2
\]

(7.3)

where \( y_s \) and \( y_c \) are the sine and cosine channel responses respectively and \( A = \sqrt{y_s^2 + y_c^2} \). This gives
7.1. THE JODRELL BANK 5GHZ INTERFEROMETER

<table>
<thead>
<tr>
<th>Dec.</th>
<th>Data</th>
<th>Error</th>
<th>MEM</th>
<th>GB</th>
<th>Galactic (1420 MHz)</th>
<th>Galactic (408 MHz)</th>
</tr>
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<tr>
<td>30°</td>
<td>0.141</td>
<td>0.028</td>
<td>0.133</td>
<td>0.097</td>
<td>2.48</td>
<td>138.0</td>
</tr>
<tr>
<td>32°</td>
<td>0.169</td>
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<td>0.123</td>
<td>0.078</td>
<td>2.05</td>
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<tr>
<td>35°</td>
<td>0.152</td>
<td>0.023</td>
<td>0.154</td>
<td>0.093</td>
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<td>101.4</td>
</tr>
<tr>
<td>37°</td>
<td>0.223</td>
<td>0.033</td>
<td>0.199</td>
<td>0.156</td>
<td>1.44</td>
<td>100.3</td>
</tr>
<tr>
<td>40°</td>
<td>0.188</td>
<td>0.020</td>
<td>0.189</td>
<td>0.181</td>
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<td>105.3</td>
</tr>
<tr>
<td>42°</td>
<td>0.168</td>
<td>0.039</td>
<td>0.127</td>
<td>0.127</td>
<td>1.39</td>
<td>113.3</td>
</tr>
<tr>
<td>45°</td>
<td>0.096</td>
<td>0.021</td>
<td>0.085</td>
<td>0.067</td>
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<td>115.3</td>
</tr>
<tr>
<td>47°</td>
<td>0.145</td>
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<td>1.76</td>
<td>124.0</td>
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<td>0.094</td>
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<tr>
<td>52°</td>
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<td>0.108</td>
<td>0.076</td>
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<td>194.1</td>
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<tr>
<td>55°</td>
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<td>0.116</td>
<td>0.055</td>
<td>2.03</td>
<td>112.3</td>
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<td>Total</td>
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<td>0.037</td>
<td>0.140</td>
<td>0.110</td>
<td>1.95</td>
<td>145.6</td>
</tr>
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</table>

Table 7.1: Summary of the results found from the 5 GHz, wide spacing interferometer at the high Galactic latitude region (RA: 130° to 260°). All values are in mK. The predictions were found by convolving the survey maps with the interferometer beam (see text).

\[ \sigma_{\text{amp}} = \sqrt{\frac{y_c^2 \sigma_c^2 + y_s^2 \sigma_s^2}{y_c^2 + y_s^2}}. \]  

These errors are shown in column 3 of Table 7.1. Column 4 shows the \textit{rms} of the MEM reconstruction of the data. The remaining 3 columns are the predictions obtained by convolving each survey with the interferometer beam. The Galactic survey results are quoted at their observing frequencies.

7.1.2 Narrow–spacing data

After 1994 the baseline of the interferometer was changed to 0.702 m. This meant that the experiment was now more sensitive to the large scale Galactic fluctuations. Together, the two data sets can be used to put a very tight constraint on the Galactic emission as well as a point source prediction for other experiments. Simulations performed showed that the level of CMB fluctuations that are expected in the data from a CDM dominated Universe with \( H_0 = 50 \text{km s}^{-1} \text{Mpc}^{-1} \) and \( \Omega_b = 0.03 \) are \( 5.2 \pm 1.6 \mu K \) (the error is the standard deviation over 300 simulations). This is still below the noise and is ignored here.

The same procedure was used as in the wide–spacing data analysis. Again, the comparison between the MEM reconvolved data and the raw data show very good agreement and this can be seen in Figures 7.10 and 7.11. Figure 7.12 shows the MEM reconstructed 2–D sky map for the high Galactic latitude region (RA 130° to 260°). Using Figure 6.3 the error on the 2–D sky map is 8 \( \mu K \). The point sources
Figure 7.10: MEM reconvolution (green line) of the cosine channel of the Jodrell Bank 5 GHz narrow spacing interferometer compared to the raw data (black line with error bars showing the one sigma deviation across scans).

Figure 7.11: MEM reconvolution (green line) of the sine channel of the Jodrell Bank 5 GHz narrow spacing interferometer compared to the raw data (black line with error bars showing the one sigma deviation across scans).

seen with the wide–spacing interferometer are again clearly visible in the narrow–spacing data. Figure 7.13 shows the comparison between the point source prediction and the MEM reconstruction. However, there are now larger sources that are not due to point source contributions (e.g. the region about RA 170°, Dec. 35°). These are most likely produced by the extra sensitivity to large scale structure that the narrow–spacing data has and are therefore most likely to be Galactic in origin. The dilution of the beam given by Equation 7.2 becomes $V = 0.56S$. Therefore, after the convolution of the two low frequency surveys for comparison, it is necessary to multiply by a factor of $1/0.56 = 1.8$ and all results quoted here have taken this into account.

Figure 7.14 and Figure 7.15 show the two low frequency surveys extrapolated with a synchrotron power law compared to the narrow–spacing MEM reconstruction. It is seen that there is little correlation between the 1420 MHz survey and the MEM reconstruction (ignoring the point source contributions that are seen in both maps) but there are some common features in the 408 MHz and the MEM reconstruction (although these are saturated in the contours). For example, there is a possible Galactic feature at RA 170°, Dec. 35° which is common to both the 408 MHz and MEM reconstruction but it appears at a higher level in the 408 MHz survey (the contour levels are saturated) which may be an indication that it is a steepened synchrotron source. The 1420 MHz survey is considered to be of poorer quality in the low level Galactic emission (high Galactic latitude) than the 408 MHz survey as it suffers from more striping effects. This could account for the discrepancy between the two predictions. Table 7.2 summarises the results for this experiment in the high Galactic latitude region (RA 130° to 260°).

7.1.3 Joint analysis

It is possible to combine the two data sets to extract the most likely common underlying sky for both the narrow and wide spacings. This is a very good test of the consistency of the experiment. The joint MEM analysis described in the Chapter 5 can be used to combine the narrow and wide–spacing and extract the most likely underlying sky common to the two experiments. Table 7.3 summarises the results
7.1. THE JODRELL BANK 5GHz INTERFEROMETER

Figure 7.13: Comparison between the GB catalogue (contour) and the MEM reconstruction (grey-scale).

Figure 7.14: Comparison between the 1420 MHz survey map extrapolated with a uniform spectral index and the MEM reconstruction. The grey-scale shows the MEM and the contours show the 1420 MHz survey.

Figure 7.15: Comparison between the 408 MHz survey map extrapolated with a uniform spectral index and the MEM reconstruction. The grey-scale shows the MEM and the contours show the 408 MHz survey.

<table>
<thead>
<tr>
<th>Dec.</th>
<th>Data</th>
<th>Error</th>
<th>MEM recons.</th>
<th>GB prediction</th>
<th>Galactic (1420 MHz)</th>
<th>Galactic (408 MHz)</th>
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</thead>
<tbody>
<tr>
<td>30°</td>
<td>0.164</td>
<td>0.031</td>
<td>0.149</td>
<td>0.116</td>
<td>3.14</td>
<td>130.1</td>
</tr>
<tr>
<td>32°</td>
<td>0.229</td>
<td>0.032</td>
<td>0.196</td>
<td>0.116</td>
<td>3.01</td>
<td>189.9</td>
</tr>
<tr>
<td>35°</td>
<td>0.211</td>
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<td>0.206</td>
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<td>2.36</td>
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</tr>
<tr>
<td>37°</td>
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<td>1.91</td>
<td>179.9</td>
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<tr>
<td>40°</td>
<td>0.150</td>
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<td>42°</td>
<td>0.173</td>
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</tr>
<tr>
<td>47°</td>
<td>0.147</td>
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<td>2.24</td>
<td>189.6</td>
</tr>
<tr>
<td>50°</td>
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<td>0.161</td>
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<td>52°</td>
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<td>3.04</td>
<td>192.2</td>
</tr>
<tr>
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<td>0.090</td>
<td>2.57</td>
<td>166.2</td>
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<tr>
<td>Total</td>
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<td>0.030</td>
<td>0.169</td>
<td>0.119</td>
<td>2.59</td>
<td>190.8</td>
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Table 7.2: Summary of the results found from the 5 GHz, narrow spacing interferometer at the high Galactic latitude region (RA: 130° to 260°). All values are in mK. The predictions were found by convolving the survey maps with the interferometer beam.
Table 7.3: Summary of the results found from the joint MEM analysis of the two 5 GHz interferometer data sets at the high Galactic latitude region (RA: 130° to 260°). All values are in mK.

<table>
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<th>Dec.</th>
<th>WS MEM result</th>
<th>NS MEM result</th>
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<tr>
<td>30°</td>
<td>0.134</td>
<td>0.150</td>
</tr>
<tr>
<td>32°</td>
<td>0.122</td>
<td>0.193</td>
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<tr>
<td>35°</td>
<td>0.155</td>
<td>0.204</td>
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<tr>
<td>37°</td>
<td>0.198</td>
<td>0.185</td>
</tr>
<tr>
<td>40°</td>
<td>0.189</td>
<td>0.162</td>
</tr>
<tr>
<td>42°</td>
<td>0.127</td>
<td>0.122</td>
</tr>
<tr>
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<td>0.087</td>
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<tr>
<td>50°</td>
<td>0.094</td>
<td>0.161</td>
</tr>
<tr>
<td>52°</td>
<td>0.108</td>
<td>0.168</td>
</tr>
<tr>
<td>55°</td>
<td>0.116</td>
<td>0.147</td>
</tr>
<tr>
<td>Total</td>
<td>0.140</td>
<td>0.168</td>
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from the combination of the two data sets. By comparing this to Table 7.1 and 7.2 it can be seen that the results from the joint analysis are almost identical to those from the individual analyses. This means that the narrow and wide–spacing data sets are consistent and can be used to put constraints on Galactic emission. Any discrepancies between the analysis are a result of the variability in the point sources.

To predict the level of Galactic fluctuations at higher frequencies the spectral index dependence of the foregrounds is required. To calculate this it is possible to compare the results from the joint analysis to the predictions from the lower frequency surveys. Firstly the point sources must be subtracted from the map reconstruction to leave the Galactic contribution. The prediction from the Green Bank catalogue was subtracted from the narrow and wide–spacing data to leave the residual signal. Due to the variability of the sources this residual will be an upper limit on the Galactic contribution and only by having continuous source monitoring would the results be better. We will use the residual signal for the narrow–spacing data (where the Galactic signal is expected to be higher and so a smaller error will be obtained). The \( \text{rms} \) of this signal is \( 73 \pm 23 \mu K \) (where the error on the MEM reconstruction comes from comparison with simulations and a 30% variability in the major sources is assumed). When comparing this to the signal of \( 2.59 \pm 0.26 \text{ mK} \) (error from a 10% error in the survey) at 1420 MHz and \( 190.8 \pm 19 \text{ mK} \) at 408 MHz it is found that the average spectral index from 1420 MHz to 5 GHz is \( 2.8 \pm 0.4 \) and from 408 MHz to 5 GHz is \( 3.1 \pm 0.4 \). These results are in agreement with previous predictions of the Galactic spectral index at a range of angular scales. Bersanelli et al (1996) found that the spectral index between 1420 MHz and 5 GHz was \( 2.9 \pm 0.3 \) on a 2° angular scale and Platania et al (1997) found that the spectral index between 1420 MHz and a range of frequency data between 1 GHz and 10 GHz...
7.1. THE JODRELL BANK 5GHZ INTERFEROMETER

Figure 7.16: The variation in the derived spectral index using the narrow-spacing data and the 408 MHz survey. The contour level is at the \emph{rms} of the map.

Figure 7.17: The variation in the derived spectral index using the narrow-spacing data and the 1420 MHz survey. The contour level is at the \emph{rms} of the map.

gave a spectral index of $2.8 \pm 0.2$ on an $18^\circ$ angular scale. This is an indication of a steepening synchrotron spectral index. To take the analysis one step further it is possible to compare the surveys pixel by pixel to obtain a map of the spectral index dependencies.

Figure 7.16 shows the spectral index variation as predicted by comparison between the 408 MHz survey and the narrow-spacing data over the high Galactic region. The \emph{rms} spectral index is $3.3 \pm 0.5$ where the error is now the \emph{rms} over the map. This is consistent with the above result. Figure 7.17 shows the result for the 1420 MHz survey prediction. The \emph{rms} spectral index is $3.1 \pm 0.9$. The regions where there is a large deviation away from the \emph{rms} spectral index (which results in the large variance) are usually associated with variable point sources (e.g. 3C345) that have not been fully removed from the 5 GHz interferometer or Galactic survey data. All the results so far are consistent with a steepened synchrotron source being the dominant contributor to the data. It is also possible that the shallower index between 1420 MHz and 5 GHz than between 408 MHz and 5 GHz could be due to the increasing importance of free–free emission, although further frequency measurements are necessary to confirm this. Therefore, the low frequency surveys should not be used (especially the 1420 MHz survey) as a prediction for Galactic contribution without taking into account the synchrotron steepening. The new 5 GHz data presented here represents an intermediate step in the frequency coverage between the low frequency Galactic surveys and the higher frequency CMB experiments. It is therefore a very useful check on Galactic models and can be used to make estimates of Galactic contamination in CMB experiments.

It should be noted that the level of Galactic emission predicted here may be too large as the artefacts in the surveys may enhance the fluctuations. Also, the 1420 MHz and 408 MHz surveys contain contributions from the point sources (for example 3C295) and so the level of Galactic fluctuations predicted will be too large. If it is assumed that 50% of the 408 MHz survey \emph{rms} is due to these effects (i.e. $135 \pm 20$ mK at 408 MHz for the narrow spacing data) then the spectral index constraint is $\beta = 3.0 \pm 0.4$ which still corresponds to a steeping synchrotron spectrum. This does not take into account the residual effect from the point sources in the 5 GHz prediction and so should be taken as a lower limit on the spectral index. This does not alter the conclusion that the low frequency surveys should not be used by themselves as a prediction for galactic emission as these artefacts, or the presence of point sources, only make the prediction worse.

If it is assumed that the signal remaining after subtracting the Green Bank point source prediction from the data is due to Galactic emission alone then it is possible to make a prediction for the level of contamination in the Tenerife data. As the
interferometer is only sensitive to a certain range of angular scales it is necessary to assume something about the spatial variation of the synchrotron source. A spectrum of $l^{-3}$ is used spatially and the steepened synchrotron spectrum is assumed to hold until the higher Tenerife frequencies. Therefore, considering the 5° FWHM Tenerife experiments, it is expected that the 10 GHz data will be contaminated by 30 $\mu$K of synchrotron emission, the 15 GHz data will be contaminated by 10 $\mu$K of synchrotron emission and the 33 GHz data will be contaminated by 1 $\mu$K of synchrotron emission. However, these figures are very approximate as the interferometer is sensitive to two different scales in the RA and Dec. direction and this has not been taken into account and will reduce the contamination level. Also the increasing importance of the free–free emission has been ignored which will increase the Galactic contamination level.

7.2 The Tenerife experiments

The MEM programs were applied to all Tenerife data described in Chapter 4. This is an ongoing process and the results shown here are not final but represent the current iteration of the analysis.

7.2.1 Reconstructing the sky at 10.4 GHz with 8° FWHM

To apply the MEM deconvolution process described to the data from the 10.4 GHz, 8° FWHM, Tenerife experiment it is necessary to select parameters that not only achieve convergence of the iterative scheme, but also make the fullest use of the data. The amplitude of the fluctuations that are of interest is at least two orders of magnitude smaller than the magnitude of the signal produced during the major passage through the Galactic plane region ($\sim 45$ mK at $\sim$ Dec. $+40^\circ$). Clearly, any baseline fitting and reconstruction will be dominated by this feature at the expense of introducing spurious features into the regions which are of interest. For this reason the data (Table 7.4) corresponding to the principal Galactic plane crossing are not used in the reconstruction.

In contrast, the anti-centre crossing ($\sim$ RA 60° at $\sim$ Dec. $+40^\circ$) corresponding to scanning through the Galactic plane, but looking out of the Galaxy, is at an acceptable level ($\lesssim 5$ mK) and is a useful check on the performance and consistency of the observations. With the parameters set as in Chapter 5, Table 5.1, $\chi^2$ demonstrates a rapid convergence. For example, the change in $\chi^2$ after 120 iterations of MEM is $\Delta\chi^2/\chi^2 \approx -9 \times 10^{-4}$ while the change in $\chi^2_{\text{base}}$ is $\Delta\chi^2_{\text{base}}/\chi^2_{\text{base}} \approx -2 \times 10^{-4}$.

The fitted baselines are subtracted from the raw data set to provide data free from baseline effects, allowing the scans for a given declination to be stacked together to provide a single high sensitivity scan. Figure 7.18 shows the stacked results for the 8° experiment at each declination compared with the reconvolution of the MEM result with the beam. The weak Galactic crossing is clearly visible at RA=$50^\circ - 100^\circ$. At lower declinations this crossing shows a complex structure with peak amplitudes $\sim$ a few mK. Only positions on the sky with more than ten independent measurements have been plotted. The data with better sensitivities are those at Dec.$=+39.4^\circ$ and
7.2. THE TENERIFE EXPERIMENTS

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</tr>
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</tr>
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<td>+17.5°</td>
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<td>+07.3°</td>
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Table 7.4: The Galactic plane regions excised at each declination.

Figure 7.18: The stacked scans at each declination displayed as a function of right ascension. Again the plots are the second difference binned into 4° bins and the 68% confidence limits. The main Galactic plane crossing has been excluded, and only positions on the sky in which we have more than ~10 independent measurements have been plotted. Also shown (solid line) is the reconvolved result from MEM overlayed onto each declination scan.

+1.1°.

The sky is not fully sampled with this data set (as seen in Figure 7.18) but the MEM uses the continuity constraints on the data to reconstruct a two-dimensional sky model. In Figure 7.19, the sky reconstruction is shown. Although a rectangular projection has been used for display, the underlying computations use the full spherical geometry for the beams (as described in Chapter 3). The anti-centre crossings of the Galactic plane are clearly visible on the right hand side of the image, while one should recall that the principal Galactic crossing has been excised from the data. It is clearly seen that there is apparent continuity of structure between adjacent independent data scans which are separated by less than the 8° beam width (see the higher declination strips in the plot where the data are more fully sampled). Where the data are not fully sampled (the lower declinations) the MEM has reverted to zero as expected and this is seen as ‘striping’ along declinations in the reconstructed map.

Figure 7.19: MEM reconstruction of the sky at 10.4 GHz, as seen by the Tenerife 8.4° FWHM experiment.
Figure 7.20: Comparison between the MEM reconstructed sky convolved in the Tenerife beam (solid line), the predicted point source contribution at Dec.=1.1° (dashed line) and the Tenerife data (dotted line with one sigma error bars shown). The source observed is 3C273 (RA= 12h26m33s, Dec.= +02°19′43″).

7.2.2 Non-cosmological foreground contributions

Point sources

The contribution of discrete sources to this data set has been estimated using the Kühr et al (1981) catalogue, the VLA calibrator list and the Green Bank sky surveys (Condon & Broderick 1986); sources \( \lesssim 1 \) Jy at 10.4 GHz were not included in the analysis. The response of the instrument to these point sources has been modelled by converting their fluxes into antenna temperature (1 Jy is equivalent to 12 \( \mu \)K for the experiment), convolving these with the triple beam of the instrument and sampling as for the real data (see the details in Gutiérrez et al 1995). The two main radio sources at high Galactic latitude, expected in the Tenerife scans are 3C273 (RA=186.6°, Dec.=+02°19′43″) with a flux density at 10 GHz of \( \sim 45 \) Jy; this object should contribute with a peak amplitude \( \Delta T \sim 500 \) \( \mu \)K in the triple beam to the data at Dec.=+1.1°, and 3C84 (RA=49.1°, Dec.=+41°19′52″) with a flux density at 10 GHz of \( \sim 51 \) Jy. Figure 7.20 presents a comparison between the MEM result reconvolved in the Tenerife triple beam, the data and the predicted contribution of the radio source 3C273. A diffuse Galactic contribution near the position of this point source accounts for the differences in amplitude and shape of the radio source prediction and the data (see below). The radio sources 3C273 and 3C84 have also been detected in the deconvolved map of the sky shown in Figure 7.19. For example, 3C273 is clearly seen in the reconstructed map. Also clearly detected are 3C345 (RA=250.3°, Dec.=+39°54′11″) and 4C39 (RA=141.0°, Dec.=+39°15′23″) in both the reconvolved scans and the deconvolved map. Many other features are seen in the deconvolved map but these may be swamped by the Galactic emission so it cannot be said with confidence that any originate from point sources. For example, features at Dec.\( \sim +40° \), RA\( \sim 180° \), Dec.\( \sim +17.5° \), RA\( \sim 240° \) and Dec.\( \sim +1.1° \), RA\( \sim 220° \) do not correspond to any known radio sources (see Figure 7.18). The additional contribution by unresolved radio sources has been estimated to be \( \Delta T/T \sim 10^{-5} \) at 10.4 GHz (Franceschini et al 1989) in a single beam. This will be less in the Tenerife switched beam and is not considered in the analysis presented here.

Diffuse Galactic contamination

The contribution of the diffuse Galactic emission in the data can be estimated in principle using the available maps at frequencies below 1.5 GHz. The 408 MHz (Haslam et al 1982) and 1420 MHz (Reich & Reich 1988) surveys were used; unfortunately the usefulness of these maps is limited because a significant part of the high Galactic latitude structure evident in them is due to systematic effects as already discussed (also see Davies, Watson & Gutiérrez 1996). Only in regions (such as cross-
nings of the Galactic plane) where the signal dominates clearly over the systematic uncertainties, is it possible to estimate the expected signals at higher frequencies. With this in mind, these two maps were converted to a common resolution ($1^\circ \times 1^\circ$ in right ascension and declination respectively) and convolved in the triple beam response.

This contribution at 408 and 1420 MHz can be compared with the data at 10.4 GHz to determine the spectral index of the Galactic emission in the region where these signals are high enough to dominate over the systematic effects in the low frequency surveys. A power law spectra ($T \propto \nu^{-\beta}$) for the signal with an index independent of the frequency, but varying spatially was assumed. The signals in the Galactic anti-centre are weaker than those for the Galactic plane crossing and are mixed up with several extended structures, but even in this case it is possible to draw some conclusions about the spectral index in this region. It was found that $\beta = 3.0 \pm 0.2$ between 408/1420 MHz and $\beta = 2.1 \pm 0.4$ between 1420/10400 MHz which indicates that free-free emission dominates over synchrotron at frequencies $\gtrsim 1420$ MHz in the Galactic plane. Taking this together with the results from the 5 GHz interferometer it is seen that synchrotron dominates for frequencies up to 5 GHz and then free-free will dominate. One of the stronger structures in the region away from the galactic plane is at RA $\sim 180^\circ - 200^\circ$, Dec $\sim 0^\circ$ and therefore the main contribution should be to the data at Dec=$1^\circ.1$. This structure at 408 MHz, assuming a slightly steepened synchrotron spectral index of $\beta = 2.8$, gives a predicted peak amplitude at 10.4 GHz of $\sim 500 \mu$K; it is believed that this is responsible for the distortion between the measurements at Dec=$1^\circ.1$ and the predictions for the radio source 3C 273.

7.2.3 The Dec $35^\circ$ 10 and 15 GHz Tenerife data.

A first direct comparison of the Tenerife and COBE DMR data at Dec.$= +40^\circ$, which also included the 33 GHz data, was made by Lineweaver et al (1995) who demonstrated a clear correlation between the data-sets and showed the presence of common individual features. Bunn, Hoffman & Silk (1995) applied a Wiener filter to the two-year COBE DMR data assuming a CDM model. They obtained a weighted addition of the results at the two more sensitive frequencies (53 and 90 GHz) in the COBE DMR data, and used the results of this filtering to compute the prediction for the Tenerife experiment over the region $35^\circ \leq \text{Dec.} \leq 45^\circ$. At high Galactic latitude the most significant features predicted for the Tenerife data are two hot spots with peak amplitudes $\sim 50 - 100 \mu$K around Dec.$=+35^\circ$ at RA $\sim 220^\circ$ and $\sim 250^\circ$. A comparison between the reconvolved results of the data from the Tenerife 15 GHz, $5^\circ$ FWHM experiment, using Maximum Entropy and the COBE data has been made, and this prediction is plotted in Figure 7.21. The solid line shows the reconvolved results at 15 GHz after subtraction of the known point source contribution. The two most intense structures in these data agree in amplitude and position with the predictions from 53 and 90 GHz (dashed line), with only a slight shift in position for the feature at RA=$250^\circ$. A possible uncertainty by a factor as large as 2 in the contribution of the point-source 1611+34 would
Figure 7.21: Comparison between the 15 GHz data (solid line) and the COBE prediction by Bunn et al 1995 (dashed line).

Figure 7.22: A multifrequency MEM reconstruction of the two fluctuations (white spots on the bottom colour contour plot), which Bunn et al (1995) predicted should be seen by the Tenerife experiments using COBE results, from the 10 GHz and 15 GHz channels. 0 and 50 on the y–axis correspond to 65° and 15° in declination respectively. 0 and 50 on the x–axis correspond to 210° and 260° in right ascension respectively. The vertical axis is in arbitrary units. Only the Dec +35° data was used to constrain this reconstruction so the fluctuations fall to zero away from this declination.

change only slightly the shape and amplitude of this second feature. As a test the multifrequency MEM was applied to the 10 GHz and 15 GHz Tenerife data at this declination. The program is currently in development and so it was not possible to apply it to the full two dimensional data set in the time allowed. However, the application of multi-MEM to this declination can be used as a test of the power that it will have in analysing the full two dimensional data set. Figure 7.22 clearly shows the two features predicted by Bunn, Hoffman & Silk (1995) at Dec +35° as reconstructed by the multifrequency MEM algorithm using the 10 GHz and 15 GHz data simultaneously.

7.2.4 The full 5° FWHM data set

The MEM deconvolution was applied to the full data set at each frequency. This represents a large portion of the sky at the two lower frequencies and so it is possible to produce sky maps covering a large area. Figures 7.23 and 7.24 show the MEM reconvolved data compared to the raw stacked data at each declination for the 10 GHz and 15 GHz Tenerife experiments respectively. At both frequencies all of the declinations were analysed simultaneously utilising the continuity across the sky. As can be seen the MEM result falls within the one sigma confidence limits at each declination. There are some discrepancies between declinations where, because the data were taken at different times, the variability of the sources leads to a different flux contribution. This can be seen clearly at RA 250° where the variable source (the predicted source contribution is shown as the red line) has become smaller in amplitude between the data acquisition of Dec. 37.5° and that of Dec. 40°. The only way to allow for this is to make simultaneous observations of all sources and subtract their flux from the raw data. This is work in progress (see Figure 2.7).

Figure 7.23: Comparison of the MEM reconvolved data (green line) and the raw stacked data (black line) for each of the declinations at 10 GHz. Also shown (red line) is the expected point source contribution from an extrapolation of the 1.4 GHz and 5 GHz Green Bank point source catalogue.
Figure 7.24: Comparison of the MEM reconvolved data (green line) and the raw stacked data (black line) for each of the declinations at 15 GHz. Also shown (red line) is the expected point source contribution from an extrapolation of the 1.4 GHz and 5 GHz Green Bank point source catalogue.

Figure 7.25: The reconstruction of the sky at 10 GHz using MEM on the Tenerife 5° Tenerife experiment data. The pixels are 1° × 1°.

As only one declination is currently available at 33 GHz no map reconstruction was possible and so the MEM algorithm was only used to subtract the long term baseline variations. The result from this subtraction was shown in Figure 4.12. However, the two dimensional map reconstructions at 10 GHz and 15 GHz, which are fully sampled in both declination and right ascension in this region, are shown in Figures 7.25 and 7.26. The switched beam pattern has been removed from the data to produce these map reconstructions but they are still convolved in a 5° beam. Only the central region away from the Galactic plane crossings is shown as this is the area where it may be possible to identify CMB features.

**Point source contribution**

The main radio sources in this region of the sky are 3C345 (RA 250°, Dec. 39°, ∼ 8 Jy at 10 GHz and 15 GHz), 4C39 (RA 141°, Dec. 39°, ∼ 9 Jy at 10 GHz and 15 GHz) and 3C286 (RA 200°, Dec. 30°, 4.5 Jy at 10 GHz and 3.5 Jy at 15 GHz). The two larger sources are clearly visible with peak amplitudes of ∼ 400µK at 10 GHz. 4C39 is seen with a peak amplitude of 300µK and 3C345 with an amplitude of 250µK at 15 GHz. The expected amplitude (using Equation 2.25) is 300µK for 3C345 and 350µK for 4C39 at both frequencies. The discrepancy between the observed flux and the expected flux (taken from the Kuhr catalogue) is easily accounted for when it is noted that both sources are ∼ 50% variable. 3C286 is less well defined at 10 GHz as it occurs at the very edge of the observed region but there is still evidence for a source at the expected position with a peak amplitude of ∼ 100µK (the expected peak is 170µK). In the 15 GHz reconstructed map 3C286 is more clearly defined and can be seen with a peak amplitude of 140µK (the expected peak is 130µK). All other sources in the region have peak amplitudes at least five times smaller than the three discussed here and are therefore well below the noise.

**Galactic source contribution**

As the comparison between the low frequency Galactic maps and the 5 GHz interferometer maps was so poor it was decided that an extrapolation up to 10 GHz
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would be impossible. Therefore, a comparison of the 5 GHz narrow spacing map and the 10 GHz Tenerife map was made by eye to check for any possible common features. One feature which is clearly detected in both the 5 GHz (at 0.3mK in the interferometer beam) and 10 GHz (at 0.2mK in the 5° Gaussian beam) maps lies just above Dec. 35° at RA 170° (this feature is also detected in the 1420 MHz and 408 MHz maps as well). This feature was assumed to be Galactic in origin in the 5 GHz map and also seems to be Galactic in origin in the 10 GHz map (as it has vanished in the 15 GHz map). Taking into account the different beam sizes (FWHM of the Jodrell interferometer is 8° and that of the 10 GHz Tenerife experiment is 5°) it is possible to calculate an approximate spectral index for the feature. The peak amplitude at 5 GHz was ∼ 280 µK and at 10 GHz was ∼ 140 µK and so the spectral index is $\beta = 2.3 \pm 0.5$. This would indicate a Galactic origin and it is more likely that the feature is free-free emission. This agrees with the findings of the 8° FWHM experiment that the majority of Galactic emission between 5 GHz and 10 GHz was free-free emission in origin.

The 10 GHz map is generally expected to contain more Galactic features (indeed the rms values of the 10 GHz data are larger than the 15 GHz data which would indicate additional emission processes are contributing to the data) but it is very difficult to assign each feature to being Galactic or cosmological in origin without a simultaneous analysis of the 10 GHz and 5 GHz (and possibly lower frequency data). This is now work in progress with the new multi-MEM procedure but due to the size and complexity of the problem it was not possible to produce any results in time for the publication of this thesis.

CMB features in the map

It is very difficult to decide whether a particular feature is cosmological in origin or whether it originates from one of the foregrounds considered here. Without the completion of the full analysis now in progress it is only possible to speculate on the origin of the features detected. By comparing the various frequencies from the Tenerife experiments or by comparison with other experiments it is possible to make a good ‘guess’ at whether a particular feature is CMB or not. It was seen that the 15 GHz Tenerife data set is expected to have a maximum Galactic contribution of 10 µK which is well below the noise and so the 15 GHz reconstruction is predominantly CMB. The 15 GHz data can be used by itself, as a first approximation, to put constraints on CMB features. One example of a possible CMB feature is visible at RA 180° and Dec. 40°. This feature is detected at the same amplitude in the 10 GHz, 15 GHz and 33 GHz data sets and so is a clear candidate for being CMB in origin (this was first reported in Hancock et al 1994). Another example are the features at Dec. 35° between RA 210° and 250° at 15 GHz which appear as two positive features separated by a large negative feature. These also appear in the COBE 53 GHz and 90 GHz maps and were used for the prediction by Bunn et al (1995) which was shown in Figure 7.21. With such a large frequency coverage indicating that the features do have the correct spectral dependence to be CMB in origin these are probably the best candidates in literature today for CMB features.
The following chapter introduces some of the techniques used to analyse the maps produced here in an attempt to automatically characterise the features without the need for a comparison by eye.
Chapter 8

Analysing the sky maps

In the preceding chapter the data were processed by MEM to produce a two dimensional partial sky map of the CMB fluctuations. In this chapter I will explore some of the main procedures that are in use to analyse CMB maps to get the maximum amount of information from them.

8.1 The power spectrum

A simple way of comparing maps is to look at their power spectra. By Fourier transforming the temperature fluctuation distribution underlying theories can be tested by comparing the predicted spectrum with the observed. However, there are problems in implementing this. The main problem is the spherical nature of the sky. A simple Fourier transform is not possible unless the sky area is small enough so that the spherical nature of the sky can be ignored. To overcome this, a high resolution experiment can be used to survey a small area of sky. The power spectra from $10^\circ \times 10^\circ$ patches of simulated Planck Surveyor data were plotted in Chapter 6. As the sky area covered decreases sample variance will quickly dominate the errors. Sample variance was not plotted in the power spectra of Chapter 6 as the purpose of these plots was not to compare the result with theory but to compare the reconstructed power spectra with the true, input power spectra. Otherwise, it is necessary to use spherical harmonics to transform the map into $\ell$ space (see Chapter 2). This has problems caused by the nature of data acquisition. Without full sky coverage (which is never possible because of the Galactic plane contamination of the data, which needs to be excluded when testing the CMB parameters) there will always be artefacts present due to the absence of data. Window functions (for example, the cosine bell) can be used to reduce their effects but they cannot be completely eliminated. The window functions also have the effect of reducing the number of data points that are used in the analysis (the ones at the edge of the map are weighted down) and so have the effect of increasing the errors on the final parameter estimation. So, instead of trying to predict what the theory looks like using the data, it is better to use the theory to try and predict what the data should look like, as all artefacts can then be easily incorporated, and then compare this prediction with the real data. This is done using the likelihood function (see
Chapter 4 for likelihood results).

The power spectrum is a useful test for the cosmological parameters in a given theory. However, it is fairly straight-forward to construct a map with Gaussian fluctuations and one with non-Gaussian fluctuations (based on the string model for example) that have the same power spectrum. Therefore, it is necessary to use further tests for non-Gaussianity and the remainder of this chapter attempts to summarise some of these tests.

### 8.2 Genus and Topology

When presented with a map of any description the eye automatically searches for shapes within that image. It would therefore be logical to construct an algorithm that will do this but in a statistical manner. By using the topology of an object it is possible to group together shapes with similar mathematical properties. There are many ways of defining the topological parameters of an object. The shapes in a two dimensional map can be characterised by their area, circumference or curvature. The mean curvature of a map is also known as the Genus.

#### 8.2.1 What is Genus?

In three dimensions (see Gott et al. 1986) an object will have a genus of +1 if it is similar to a torus and a genus of -1 if it is similar to a sphere. In two dimensions (see Gott et al. 1990) an object will have a genus of +1 if it is similar to a coin and a genus of -1 if it is similar to a ring. The genus of a map is simply the sum of the genus of each of the shapes in that map. If a two dimensional map is a perfect sponge shape, so that there are an equal number of coin and ring shapes, then the total average genus is zero and we have a perfect Gaussian field. In one dimension the genus is simply taken as the number of up crossings above a certain threshold (see Coles & Barrow 1987).

Genus was first applied in cosmology to large scale structure surveys. Many such surveys (e.g. de Lapparent, Geller & Huchra 1986, Schectman et al. 1992, Jones et al. 1994) are presently being analysed in this manner and the results from these will be of great interest in their own right. However, topology offers a unique way for comparing the fluctuations present in the large scale structure with those in the microwave background. This comparison between today’s anisotropies and their precursors will lead to information on the evolution of the universe through gravitational interactions. I have developed algorithms to apply the genus statistic to pixellised CMB maps and the results from the application of these algorithms will be presented here. Firstly, it is useful to derive the expected form of the genus for the case of a purely Gaussian process.

In two dimensions the genus, $G$, of the surface is given by

$$ G = \text{No. isolated high density regions} - \text{No. isolated low density regions} \quad (8.1) $$

In the case of a CMB map this corresponds to setting a threshold temperature and calculating the number of fluctuations above that threshold minus those below the
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The genus can also be defined in terms of the curvature of the contours that enclose the shapes. Consider a contour, C, enclosing an excursion region (defined as the region in a map above, or below, the threshold) counterclockwise. The curvature of the contour, along its length s, is defined as

\[ \kappa(s) = \frac{1}{R} \]  

where \( R \) is the radius of a circle which, when placed so that its perimeter lies on \( s \), has the same curvature as the contour. The radius of curvature, \( R \), is defined as being positive if the circle is on the same side of the contour as the enclosed region and negative if the circle is on the other side. The total curvature is the integral along \( s \),

\[ K = \int_C \kappa ds \]  

The genus is then defined as

\[ G = \frac{1}{2\pi} \int_C \kappa ds. \]  

For example, consider a contour enclosing a simple, circular, high temperature region (like a coin). The radius of curvature around the contour will always be equal to the radius of the coin and, as the circle is on the same side of the contour as the enclosed region, it will be positive. Therefore, \( \kappa \) is a constant \((1/R)\) and the integral around \( s \) is equal to the perimeter of the coin \((2\pi R)\). The genus is therefore equal to 1. For a circular contour surrounded by a high temperature region (like the inside of a ring), the radius of curvature is still equal to the radius of the ring but now it is defined as being negative (the enclosed region is on the outside of the circle). Therefore, the genus is now equal to -1. There will also be contours that cross the edge of the map being analysed and in this case the genus will be fractional.

The genus of an object is usually quoted as a function of the threshold level set in computing the excursion region. There are many different methods to derive the expected functional form for the genus of a two dimensional map. Adler (1981) derived the form of the genus for general geometrical problems and Bardeen et al (1986) and Doroshkevich (1970) use the Euler–Poincare statistic to derive the frequency of high density peaks in Gaussian fields. All CMB maps are produced on a pixellised grid and so it is more advantageous to follow the derivation set out in Hamilton et al (1986; hereafter HGW86) which applies the genus approach to three dimensional smoothed large scale structure surveys.

HGW86 use tessellated polyhedra to analyse large scale structure data. By smoothing the density function of the large scale structure they are able to calculate the mean density in octahedra. This gives the objects in their maps regular, repeating shapes that make it easier to calculate the genus. In two dimensions, as in CMB maps, the simplest form of tessellation to use are square pixels (although this does lead to an ambiguity in assigning genus; see below). Each octahedron (or pixel in two dimensions) is then either above or below the threshold level set. The
Figure 8.1: The 16 different configurations possible around a vertex in a pixellised map. The shaded region corresponds to pixels that are above the threshold level and are, therefore, within a high density excursion region.

Figure 8.2: Case (d) in Figure 8.1 has two possible configurations around the vertex leading to two different genus.

surface of the excursion region is then the surface of the octahedra. As stated in HGW86, the curvature of any polyhedra is only non–zero at its vertices and so it is relatively easy to find the total curvature by summing up the vertex contributions. As the number of polyhedra approaches infinity (so that they are infinitely small) then the genus calculated in this way approaches the true genus of the objects being analysed.

Consider a map, in two dimensions, made up of square pixels of size \( d \times d \). Each pixel has four vertices which touch four other pixels. The number of vertices per unit volume, \( N_{\text{vol}} \), is therefore

\[
N_{\text{vol}} = \frac{N_{\text{vert}} \times \text{Area}}{N_{\text{pixels}}} = \frac{1}{d^2},
\]

where \( N_{\text{pixels}} \) is the number of pixels at each vertex. Now consider the possible configurations about the vertices. Figure 8.1 shows the sixteen possibilities around each vertex. The genus of the vertex in each case is easily calculated. For (a) the genus is zero in both 4 low density and 4 high density cases. For (b) the genus is +1/4 for the 3 low density and 1 high density case, and -1/4 for the 3 high density and 1 low density complimentary case. For (c) the genus is zero. For (d) the genus is slightly ambiguous. If we consider the two high density pixels to be connected and separating the two low density pixels then the genus is -2/4 but if the two low density regions are connected and they separate the two high density regions then the genus is +2/4. These two possibilities are shown in Figure 8.2. To account for this ambiguity the genus is assigned randomly as \( \pm 2/4 \) for this case. For a CMB map the density of a pixel is just the average temperature within that pixel and from now on will be referred to as such.

The genus defined above can be shown to be correct in the simplest cases: consider a single high temperature pixel in a sea of low temperature pixels. Each vertex will then correspond to case (b) in Figure 8.1 and contribute +1/4 to the total genus. Summing over each of the vertices gives \( 4 \times +1/4 = +1 \) which is the expected result from Equation 8.1. The total expected theoretical genus for any pixellised map can be calculated by summing the genus contribution for each vertex multiplied by the probability that the vertex has that configuration.

Assume that the temperature distribution in the CMB map is Gaussian and define fluctuations from the mean as

\[
\delta = \frac{T - \bar{T}}{T}
\]
where $\bar{T}$ is the average temperature in the full sky map. In the case of CMB maps $\bar{T} = 2.73$K, the temperature of the blackbody spectra, but this is already subtracted from the maps in most cases. The probability of each vertex configuration can now be calculated. Label the four pixels around a vertex in a clockwise direction as 1, 2, 3 and 4 and define the correlation function as

$$\xi_{ij} = <\delta_i \delta_j>$$

where $i$ and $j$ are over the four pixels. Now define that probability function

$$f(\delta_1, \delta_2, \delta_3, \delta_4) = \frac{1}{4\pi^2 (\det[\xi])^{1/2}} \exp \left( -\frac{1}{2} \sum_{ij} \xi_{ij}^{-1} \delta_i \delta_j \right)$$

(8.8)

where $\det[\xi]$ is the determinant of the covariance matrix $\xi_{ij}$. It is easily seen that, from symmetry the matrix $\xi$ can be written as

$$\begin{pmatrix}
\xi(0) & \xi_{12} & \xi_{13} & \xi_{14} \\
\xi_{12} & \xi(0) & \xi_{12} & \xi_{13} \\
\xi_{13} & \xi_{12} & \xi(0) & \xi_{12} \\
\xi_{14} & \xi_{12} & \xi_{13} & \xi(0)
\end{pmatrix}.$$  

(8.9)

Note that $\xi_{ii} = \xi(0)$ which in turn is the square of the pixel temperature $rms$ from $\xi_{ii} = <\delta_i^2>$. In terms of the continuous correlation function over the map $\xi_{12} = \xi(d)$ and $\xi_{13} = \xi(d\sqrt{2})$. As the pixels become smaller and smaller a Taylor expansion can be made for $\xi(r)$

$$\xi(r) = \xi(0) + \frac{r^2}{2!}\xi^{(2)} + \frac{r^4}{4!}\xi^{(4)} + \ldots$$

(8.10)

where

$$\xi^{(n)} = \left| \frac{d^n \xi(r)}{dr^n} \right|_{r=0}.$$  

(8.11)

It is now possible to derive the functional form of the genus.

If the probability that pixel 1 is above the threshold temperature, $\delta_c$, and pixels 2, 3 and 4 are below $\delta_c$ is equal to $p_1$ then

$$p_1 = \int_{-\infty}^{\delta_c} \int_{-\infty}^{\delta_c} \int_{-\infty}^{\delta_c} \int_{-\infty}^{\delta_c} f(\delta_1, \delta_2, \delta_3, \delta_4) d\delta_1 d\delta_2 d\delta_3 d\delta_4.$$  

(8.12)

Similarly, the probability that pixels 1 and 2 are above $\delta_c$ and pixel 3 and 4 are below $\delta_c$ is given by

$$p_{12} = \int_{-\infty}^{\delta_c} \int_{-\infty}^{\delta_c} \int_{-\infty}^{\delta_c} \int_{-\infty}^{\delta_c} f(\delta_1, \delta_2, \delta_3, \delta_4) d\delta_1 d\delta_2 d\delta_3 d\delta_4.$$  

(8.13)

The remaining probabilities follow in an analogous way. By symmetry $p_1 = p_2 = p_3 = p_4$, $p_{12} = p_{14} = p_{23} = p_{34} = p_{13} = p_{24}$ and $p_{123} = p_{234} = p_{341} = p_{124}$. Combining this with Equation 8.5 the genus per unit area is given by
\[ G = \frac{1}{d^2} \sum_i g_i p_i \]  

where \( i \) runs over the vertices and \( g_i \) is the genus of the pixel with configuration \( i \) and probability \( p_i \). Using the symmetry of the probabilities the genus is

\[ G = \frac{1}{d^2} (0 \times p_{\text{none}} + \left(\frac{1}{4}\right) \times 4p_1 + 0 \times p_{12} + \frac{1}{2} \times \left(\frac{2}{4}\right) \times 2p_{12} + \] 

\[ \frac{1}{2} \times \left(\frac{2}{4}\right) \times 2p_{12} + \left(-\frac{1}{4}\right) \times 4p_{123} + 0 \times p_{1234} = \frac{(p_1 - p_{123})}{d^2} \]  

(8.15)

and so only \( p_1 \) and \( p_{123} \) need to be calculated. The algebra required in Equation 8.12 is very long and in the past has been relegated to computer programs like MACSYMA (see Melott et al 1986, hereafter MCHGW). MCHGW perform the analysis for hexagon shaped pixels which eliminates the ambiguity in case (d), Figure 8.1. However, hexagonal pixels are not used in general CMB experiments (although see Tegmark 1996 for a recent hexagonal pixel projection of the COBE data) and so the analysis here is restricted to square pixels. MCHGW quote the result for square pixels to fourth order in pixel size after the Taylor expansion of all \( \xi \) terms have been performed. They find, to second order in pixel size,

\[ G = \frac{1}{(2\pi)^{3/2}} \left( -\frac{\xi^{(2)}}{\xi(0)} \right) \nu e^{-\nu^2/\nu} \left( 1 + \frac{d^2}{24} \left( -\frac{\xi^{(2)}}{\xi(0)} \right) [3 - \nu^2 - \frac{\xi^{(4)}}{(\xi^{(2)})^2}] + \ldots \right) \]  

(8.16)

where \( \nu \) is given by

\[ \nu = \frac{\delta_c}{\xi(0)^{1/2}}. \]  

(8.17)

and is the number of standard deviations of the temperature rms that \( \delta_c \) is from the mean of the map. As the pixel size approaches zero the genus approaches the result for a non–pixellised map which was shown by Adler (1981, p115) to be

\[ G \propto \nu \exp \left( -\frac{\nu^2}{2} \right). \]  

(8.18)

The term in \( d^2 \) in Equation 8.16 can, therefore, be thought of as an error on the genus calculated from a pixellised map due to the pixellisation.

For random Gaussian fluctuations the area fraction covered by fluctuations above a certain threshold, \( \delta_c \), is given by

\[ f = \int_0^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\nu^2/2} d\nu = \frac{1}{2} \text{erfc} \left( \frac{\nu}{\sqrt{2}} \right), \]  

(8.19)

where \( \text{erfc}(x) \) is the complementary error function. This is an easier definition to use when implementing the algorithm in the case of a pixellised map, as it is trivial to set the \( f \) highest temperature pixels as the excursion region. It is this definition which is used in the analysis to construct the contours, but all graphs will be plotted as a function of \( \nu \).
Figure 8.3: Genus of CDM simulations (error bars are the \textit{rms} over the simulations) compared with the theoretical expected genus for a Gaussian distributed function (dashed line).

Figure 8.4: Genus of string simulations (error bars are \textit{rms} over the simulations) compared with the theoretical expected genus for a Gaussian distributed function (dashed line).

The expected form for a random Gaussian temperature map has been calculated. The genus from the data, calculated on pixellised maps using the definitions shown in Figure 8.1, can be compared to this and if it differs significantly from the expected curve then the underlying field is non–Gaussian.

8.2.2 Simulations

To test the genus algorithm and its power at distinguishing between the origin of fluctuations, simulated data was used. This also tests the relative merit of each of the experiments for distinguishing between Gaussian and non–Gaussian origins for the temperature fluctuations. Simulations for the Planck Surveyor experiment were used. The genus of the full sky will closely follow that of the theoretical curve (to within cosmic variance) if it is Gaussian distributed. However, most experiments do not have sufficient sky coverage to allow for this. Therefore, the simulations performed here are for a smaller patch of the sky. The simulations were made at 300 GHz.

Figure 8.3 shows 9 regions analysed using genus for a Cold Dark Matter simulation compared to the theoretical curve for a Gaussian process. The regions used for these plots are 50 \times 50 pixels (a total of 1.25° \times 1.25°). Figure 8.4 shows 9 regions analysed for a string simulation. There is no significant difference to the eye between these plots. Figure 8.5 shows the results for the SZ analysis. The SZ genus appears to be shifted to the left of the theoretical genus. This shift implies that there are more excursion regions above the \textit{rms} of the map than below it (the area under the curve in the positive \textit{v} region is larger than that in the negative \textit{v} region). The SZ effect is made up of point source features and so there are indeed more excursion regions above the \textit{rms}.

To test the significance of the genus analysis a $\chi^2$ fit to the recovered genus was performed. The $\chi^2$ level was minimised with respect to the amplitude of the theoretical genus. Table 8.1 shows the results for each of the maps and whether that map is assigned to be Gaussian or not. As can be seen from the table each channel is assigned correctly to Gaussian or non-Gaussian for the full range of \textit{v}.

Figure 8.5: Genus of SZ simulations (error bars are the \textit{rms} over the simulations) compared with the theoretical expected genus for a Gaussian distributed function (dashed line).
Table 8.1: $\chi^2$ obtained from the calculated genus for each of the simulations compared to the predicted genus for a Gaussian distributed function.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$-1 &lt; \nu &lt; 1$</th>
<th>$-2 &lt; \nu &lt; 2$</th>
<th>$-3 &lt; \nu &lt; 3$</th>
<th>Gaussian?</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDM</td>
<td>$1.0 \pm 0.5$</td>
<td>$1.1 \pm 0.6$</td>
<td>$3.3 \pm 2.6$</td>
<td>Yes</td>
</tr>
<tr>
<td>Strings</td>
<td>$1.3 \pm 0.8$</td>
<td>$1.2 \pm 0.5$</td>
<td>$30 \pm 16$</td>
<td>No</td>
</tr>
<tr>
<td>SZ</td>
<td>$18 \pm 4$</td>
<td>$60 \pm 7$</td>
<td>$673 \pm 89$</td>
<td>No</td>
</tr>
</tbody>
</table>

Figure 8.6: One of the string simulations produced by Pedro Ferreira at a) 1.0′, b) 4.5′, c) 10′ and d) 17′ resolution.

However, when the central range of $\nu$ is considered, the SZ effect is the only one to be assigned as non-Gaussian. This is to be expected as the non-Gaussian features of the string simulations are on small angular scales (line discontinuities) and these will only show up in the genus at the extremities of the map where there are not enough fluctuations for the average effect to appear Gaussian (by the central limit theorem). Therefore, it is possible to say that for a highly non-Gaussian process (like the SZ effect) the genus algorithm can easily distinguish the non-Gaussian effects, whereas for a process in which the central limit theorem dominates (a large area of CMB anisotropies produced by strings) genus can only distinguish the non-Gaussian effects in the peaks of the distribution.

As a further use of the genus algorithm, simulations of the CMB maps produced by strings were used to compare the proposed satellite experiments. Table 8.2 shows the minimised $\chi^2$ for the genus from 50 string simulations produced by Pedro Ferreira at different resolutions. One of the simulations is shown in Figure 8.6 at the four different resolutions considered here. The Planck Surveyor is expected to have a maximum resolution of 4.5 arc minutes and the MAP satellite will have a maximum resolution of 17 arc minutes\(^1\). From the table it can be seen that with a beam of 4.5′ the non-Gaussian nature of the strings is clearly seen at 15 times the expected $\chi^2$ for a Gaussian process, whereas at 17′ the non-Gaussian nature is still seen but at a much reduced level. Again, it is seen that without a very high resolution experiment ($\sim 1′$) the non-Gaussian nature of the string simulation is only seen in the extrema of the temperature distribution. It should be noted that no noise was added to these simulations so these represent the best scenario for any experiment with these resolutions.

It is also possible to use the genus algorithm to check for differences between non-Gaussian theories. Three possible sources of non-Gaussian effects are monopoles, strings and textures. Simulations provided by Neil Turok were used to test the power of the genus algorithm for distinguishing between these three theories of non-

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\(^1\)Since the simulations presented here were performed the resolution of the MAP satellite has improved to 12 arc minutes
8.2. GENUS AND TOPOLOGY

<table>
<thead>
<tr>
<th>Beam FWHM</th>
<th>$-1 &lt; \nu &lt; 1$</th>
<th>$-2 &lt; \nu &lt; 2$</th>
<th>$-3 &lt; \nu &lt; 3$</th>
<th>Gaussian?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>$4.5 \pm 1.6$</td>
<td>$3.3 \pm 1.1$</td>
<td>$69 \pm 13$</td>
<td>No</td>
</tr>
<tr>
<td>4.5</td>
<td>$0.9 \pm 0.5$</td>
<td>$0.9 \pm 0.4$</td>
<td>$15 \pm 5$</td>
<td>No</td>
</tr>
<tr>
<td>10.0</td>
<td>$0.9 \pm 0.5$</td>
<td>$0.9 \pm 0.4$</td>
<td>$4.8 \pm 1.7$</td>
<td>No</td>
</tr>
<tr>
<td>17.0</td>
<td>$1.0 \pm 0.5$</td>
<td>$0.9 \pm 0.4$</td>
<td>$3.1 \pm 0.8$</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 8.2: $\chi^2$ obtained from the calculated genus over 50 string simulations provided by Pedro Ferreira. They are convolved with different beams and are over a $2^\circ \times 2^\circ$ patch of the sky.

Figure 8.7: Genus from 9 of the 30 simulations of the CMB from a monopole theory. The errors are the 68% confidence limits over those 30 simulations.

Gaussian anisotropies. Figures 8.7 to 8.9 show the genus for nine of the simulations of the monopoles, strings and textures. By eye there does not seem to be a great deal of difference between the three defect models. However, with the theoretical Gaussian model fitted (by minimising the $\chi^2$ as before), the difference is more obvious. Table 8.3 shows the results from the $\chi^2$ calculation for each model. As can be seen each model requires the extremities of the temperature distribution to be distinguished from a Gaussian process. It is also possible to see that the defect process that deviates most from Gaussian is string theory. All three are easily discernible from Gaussian at a very large significance. Table 8.4 shows the likelihood values for each of the non-Gaussian models used here. One test simulation of each process was compared to each input model to see if the genus statistic could correctly identify the model. For these particular cases, it is seen that the underlying input model for each test simulation is correctly identified although there is a possibility that the texture and monopole maps may be mistaken for each other. The addition of noise will reduce the differences slightly but it has been shown that MEM can reconstruct the CMB to a very high degree of accuracy so it is not expected to effect the results significantly.

8.2.3 The Tenerife data

So far the genus algorithm has been applied to simulated data from future experiments. Now the genus algorithm will be used on simulated data from an existing experiment, the Tenerife switched-beam experiment. Figure 8.10 shows the normalised genus averaged over 30 maps taken from a Gaussian realisation of the CMB for the Tenerife experiment. This is seen to have the expected form of Equation

Figure 8.8: Genus from 9 of the 30 simulations of the CMB from a string theory. The errors are the 68% confidence limits over those 30 simulations.
Figure 8.9: Genus from 9 of the 30 simulations of the CMB from a texture theory. The errors are the 68% confidence limits over those 30 simulations.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\chi^2$ $-1 &lt; \nu &lt; 1$</th>
<th>$-2 &lt; \nu &lt; 2$</th>
<th>$-3 &lt; \nu &lt; 3$</th>
<th>Gaussian?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopoles</td>
<td>1.0 ± 0.5</td>
<td>1.5 ± 0.6</td>
<td>274 ± 31</td>
<td>No</td>
</tr>
<tr>
<td>Strings</td>
<td>1.0 ± 0.5</td>
<td>0.9 ± 0.3</td>
<td>1558 ± 106</td>
<td>No</td>
</tr>
<tr>
<td>Textures</td>
<td>0.8 ± 0.4</td>
<td>1.1 ± 0.5</td>
<td>645 ± 85</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 8.3: $\chi^2$ obtained from the calculated genus over 30 simulations provided by Neil Turok of the CMB expected from different defect models. They are convolved with a 4.5’ beam to simulate the best results possible from the Planck Surveyor.

<table>
<thead>
<tr>
<th>Input model</th>
<th>Monopole</th>
<th>Strings</th>
<th>Textures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopoles</td>
<td>0.16</td>
<td>$2.3 \times 10^{-11}$</td>
<td>0.11</td>
</tr>
<tr>
<td>Strings</td>
<td>$2.0 \times 10^{-15}$</td>
<td>0.44</td>
<td>$8.1 \times 10^{-10}$</td>
</tr>
<tr>
<td>Textures</td>
<td>0.42</td>
<td>$6.5 \times 10^{-8}$</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Table 8.4: The likelihood results for the three different non-Gaussian simulations. A peak value of 1 is obtained if the test model has the exact genus of the input model. The genus of the input model was found by averaging over 32 test models of each defect process. It is seen that the strings model is easily distinguished from the other two defect processes (see text).
Figure 8.10: Average genus from 30 2 dimensional simulations of the CMB with the experimental configuration of the Tenerife experiment over a $10^\circ \times 100^\circ$ area of the sky. A standard deviation (st dev) of 0 corresponds to 50% of the area being high density and 50% low and a standard deviation of 3 corresponds to 98% of the area being low density and 2% high. The genus has been normalised to one at its maximum.

Figure 8.11: Genus of one of the 2 dimensional simulations used as the input for the next figure.

8.18.

Simulated Tenerife observations of one of the Gaussian realisations were performed for a $10^\circ \times 100^\circ$ area of the sky. The genus of the input map used is shown in Figure 8.11. The data was then analysed with the genus algorithm and the average over 30 noise realisations is shown in Figure 8.12. As can be seen from this plot, even though the error bars are large, the point of intersection with the y-axis is well recovered. This point corresponds to half the pixels being classed as high temperature and half as low temperature. It is intrinsically dependent on the smaller fluctuations in the data as well as the large ones and MEM is seen to be performing very well on all amplitudes in this reconstruction. In this realisation it is seen that the two regions (high and low temperature) are not completely equivalent as expected in a Gaussian case but as this is just one realisation from an ensemble this is to be expected. Therefore, when using the genus to analyse maps care must be taken to include both the errors from the noise realisations (Figure 8.12) and the errors from the sample variance (Figure 8.10) before any conclusion about the non-Gaussian nature of the underlying process is reached.

The genus algorithm can also be used to provide extra proof that observations have detected real astronomical fluctuations and are not noise dominated. The amplitude of a genus curve is proportional to the amount of structure present within the map so even though the noise map has the same form as the CMB map (for Gaussian CMB) the amplitude will be different. By simulating noise maps Colley, Gott & Park (1996) and Smoot et al (1994) show that the COBE maps have more structure in them than expected from pure noise at a level of over four standard deviations. They also show that there is no significant deviation away from Gaussian fluctuations, although this only rules out highly non-Gaussian processes, as most expected non–Gaussian fluctuations will approach Gaussian on this angular scale due to the central limit theorem. The genus algorithm will now be applied to the Tenerife data set in a similar way.

Figure 8.12: Genus of the output map after 30 simulations with the Tenerife configuration with different noise realisations. v=0 corresponds to 50% of the area being high density and 50% low, v=3 corresponds to 98% of the area being low density and 2% high.
CHAPTER 8. ANALYSING THE SKY MAPS

Figure 8.13: Genus of the 15 GHz Tenerife MEM reconstructed map. The 68% confidence limits shown are calculated from the Monte-Carlo simulations (results in Figure 8.12).

The real data

As the 10 GHz data set is expected to be contaminated by Galactic emission only the 15 GHz data set will be used here. The map shown in Figure 7.26 was used to test the power of the genus algorithm. The region between RA 131° and 260° was analysed to test for Gaussianity. Figure 8.13 shows the result for this analysis. A preliminary attempt at subtracting the effect of the two point sources in this region was made prior to the genus analysis and it can be seen that the non-Gaussian behaviour expected due to these sources does not show in this figure. The average $\chi^2$ for the difference between the theoretical curve and the genus from the 15 GHz data is 0.38. It is seen that the MEM reconstruction of the Tenerife 15 GHz data is completely consistent with a Gaussian origin (less than one sigma deviation away from the theoretical Gaussian curve). However, this does not mean that it is inconsistent with a non-Gaussian origin. As in the COBE analysis only highly non-Gaussian processes can be ruled out as most expected non-Gaussian fluctuations will approach Gaussian at the scales that Tenerife is sensitive to.

8.2.4 Extending genus: the Minkowski functionals

Recent advances (Schmalzing & Buchert 1997, Kerscher et al 1997, Mecke, Buchert & Wagner 1994, and references therein) in the analysis of Large Scale Structure data sets using integral geometry have led to an interest in this area of statistics in the CMB community (see Winitski & Kosowsky 1998 and Schmalzing & Gorski 1998). Large Scale Structure and CMB data sets both need the following requirements for a functional that can describe them: the functional must be independent of the orientation or position in space (motion invariance), must be additive (so that the functional of the combination of two data sets is the addition of the two separate functionals minus their intersection) and must have conditional continuity (the functional of a pixellised data set must approach the true functional of the underlying process as the pixels are made smaller). These three requirements taken together were shown by Hadwiger (1957) to lead to only $d + 1$ functionals in a $d$-dimensional space that would completely describe the data set. These are the Minkowski functionals. For a CMB data set (in 2-dimensional space) there are three Minkowski functionals; surface area, boundary length and the Euler characteristic (or genus).

As has already been shown, the genus of the CMB maps holds a great deal of information and so it is expected that the inclusion of the two other Minkowski functionals will allow further discrimination between theories. Tests on the three-dimensional Minkowski functions applied to Large Scale Structure data sets (Jones, Hawthorn & Kaiser 1998) have shown that the addition of the other Minkowski functionals does indeed increase the power to discern between underlying theories. Other groups have already began to test the Minkowski functionals on CMB data
8.3. Correlation functions

Instead of looking at the morphology of the temperature distribution using Minkowski functionals (which include genus) it is possible to use statistical techniques to measure the distribution in space of pixel fluxes. This is done using correlation functions. It has been shown that any correlation function can be expressed in terms of the Minkowski functionals (for example, see Mecke, Buchert & Wagner 1994). However, the correlation functions do contain useful properties and so they will be discussed here. I will summarise the two, three and four point correlation functions and apply the two and four point functions to various data sets.

8.3.1 Two point correlation function

The two point correlation function is a measure of the average product of temperature fluctuations in two directions. For two pixels in a CMB map, \( i \) and \( j \), separated by an angle \( \beta \), the two point correlation function is given by

\[
V(\beta) = \left\langle \left( \frac{\Delta T_i}{T_\circ} \right) \left( \frac{\Delta T_j}{T_\circ} \right) \right\rangle
\]

(8.20)

and it is easily seen that when \( \beta = 0 \), so that \( i = j \), \( V(0) \) is the variance of the data. The variance is very easily calculated and is used as a check at each of the data reduction stages. It is also fitted for in the likelihood function.

On the raw data the two point correlation function can be used as a test of the origin of the emission detected in a CMB experiment. By applying the weighted two point correlation function to the Tenerife data it is possible to test whether the data is consistent with noise or whether there is some underlying signal present. The weighted two point correlation function is given by

\[
C(\theta) = \frac{\sum_{i,j} \Delta T_i \Delta T_j w_i w_j}{\sum_{i,j} w_i w_j}
\]

(8.21)

where \( w_i \) and \( \Delta T_i \) are the weight \( (1/\sigma_i^2) \) and double-differenced temperature of the Tenerife data set. In this form the two point correlation function is also known as the auto-correlation function.

Figure 8.14 presents the auto-correlation of the 15 GHz Tenerife data in the region at RA=161°–250°. The error-bars were determined by Monte-Carlo techniques. The errors on each data point were estimated by assuming a random Gaussian process with the appropriate \( \text{rms} \) given by the Tenerife data set (i.e. the noise). The data point was then displaced by this amount and a new data set was constructed for which the auto-correlation function was found. This was done over 1000 noise realisations and the errors show the 68% confidence limits over these realisations. These
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CHAPTER 8. ANALYSING THE SKY MAPS

Figure 8.14: The auto–correlation of the 15 GHz data in the region at RA=161° – 250°. The solid line is the best fit to the data, the short dashed line shows the expected region of correlation for a Harrison-Zel’dovich spectrum CMB model and the long dashed line shows the expected region of correlation for the case of pure noise. The errors are calculated using a Monte-Carlo technique. This figure was produced by Carlos Gutierrez.

Techniques were also used to obtain the confidence bands in the case of pure uncorrelated noise (long-dashed lines) and the expected correlation (short-dashed line) in the case of a Harrison-Zel’dovich spectrum for the primordial fluctuations with an amplitude corresponding to the signal of maximum likelihood (see Chapter 4). Clearly this model gives an adequate description of the observed correlation whilst the results are incompatible with pure uncorrelated noise. The cross-correlation between the data at 10 GHz and 15 GHz is inconclusive as it is dominated by the noisy character of the 10 GHz data.

A variation to the two point correlation function that is commonly used (see, for example, Kogut et al 1995 and Kogut et al 1996) is the extrema correlation function. Instead of applying the two point correlation function to the full data set, the peaks (and troughs) of the data set are found. A peak is defined as any pixel ‘hotter’ than the neighbouring pixels and a trough is any pixel ‘colder’ than the neighbouring pixels. The correlation function between these peaks and troughs is then found. It can be separated into three different analyses; a) peak-peak auto-correlation (and trough-trough auto-correlation), b) peak-trough cross-correlation and c) extrema cross-correlation (the correlation between all extrema regardless of whether they are a peak or trough). Kogut et al (1995) use the extrema two point correlation function to analyse the COBE 53 GHz map and use the likelihood function to predict whether the peaks are from a Gaussian or non-Gaussian source. They find that the COBE result is most likely to have originated from a Gaussian distribution of fluctuations although the significance of their analysis is very difficult to compute.

The theoretical predictions for the two point and three point correlation functions can be found in Bond & Efstathiou (1987) and Falk, Rangarajan & Srednicki (1993) show the predictions for the full two point correlation function and the collapsed three point correlation function (see below) for inflationary cosmologies. The main problem with any of the correlation function analysis techniques is that they do not take into account any noise or foregrounds present. Therefore, it is necessary to perform the analysis on the MEM processed map or simulations of the noise and foregrounds must be performed to evaluate their effect. Kogut et al (1995) use Monte Carlo simulations of the noise added onto different models for the CMB (they do not include foregrounds) to evaluate the significance of their result.

8.3.2 Three point correlation function

The three point correlation function is similar to the two point correlation function except that it takes the product between three pixels. The angular separation be-
8.3. CORRELATION FUNCTIONS

tween pixels is now $\alpha$ (between $i$ and $j$), $\beta$ (between $i$ and $k$) and $\gamma$ (between $j$ and $k$) which gives

$$S(\alpha, \beta, \gamma) = \langle \left( \frac{\Delta T_i}{T_o} \right) \left( \frac{\Delta T_j}{T_o} \right) \left( \frac{\Delta T_k}{T_o} \right) \rangle$$

and when $\alpha = \beta = \gamma = 0$, so that $i = j = k$, $S$ is defined as the skewness of the data. The skewness of the data is slightly more sensitive to non-Gaussian features than the two point correlation function.

The collapsed three point correlation function ($\beta = \alpha$ and $\gamma = 0$) was used by Gangui & Mollerach (1996) to analyse the COBE results. They found that defects could not be ruled out using the COBE data but a higher resolution experiment could distinguish between Gaussian fluctuations and those arising from textures. Falk et al (1993) also show that the collapsed three point correlation function is not sensitive enough to be detected by COBE for generic models (those without specially chosen parameters to make the three point correlation function artificially large).

8.3.3 Four point correlation function

The four point correlation function is very rarely used in full. Instead it is used in the collapsed form when the separation between pixels is zero. This is defined as the kurtosis (see, for example, Gaztanaga, Fosalba & Elizalde 1997) and is equal to

$$K = \frac{\left\langle \left( \frac{\Delta T}{T_o} \right)^4 \right\rangle - 3 \left\langle \left( \frac{\Delta T}{T_o} \right)^2 \right\rangle^2}{\left\langle \left( \frac{\Delta T}{T_o} \right)^2 \right\rangle^2}$$

The kurtosis is a good discriminatory test between Gaussian and non-Gaussian features but it can only be applied effectively in data with little or no noise and minimal foregrounds. For the Gaussian case the kurtosis should tend to zero.

Gaztanaga et al (1997) use the kurtosis to analyse data from the MAX, MSAM, Saskatoon, ARGO and Python CMB experiments (all are sensitive to angular scales around the first Doppler peak). They find that there is a very large kurtosis for each of the experiments and a Gaussian origin of the fluctuations is ruled out at the one sigma level. However, the analysis does not allow for any systematic errors or foreground effects and these could alter the results greatly.

Application to simulated data

The kurtosis was applied to the Planck simulation maps. Table 8.5 shows the kurtosis values for the analysis. The kurtosis of the reconstructions is compared to that of the input maps convolved with the highest resolution of the experiment. As can be seen the kurtosis of the SZ effect is very high as expected for a strongly non-Gaussian feature. After the simulated observations and analysis was performed it can be seen that the MEM result reconstructs the kurtosis very well. This implies
Table 8.5: The kurtosis for the input simulations and reconstructions from the analysis of the Planck and MAP simulations shown in Chapter 6. The MEM and Wiener reconstructions are for the case of full ICF information. The input maps for the two experiments are convolved to their highest resolution.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>CDM CMB</th>
<th>Strings CMB</th>
<th>SZ effect</th>
<th>Dust map</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planck input</td>
<td>0.03</td>
<td>1.53</td>
<td>-1.56</td>
<td>0.15</td>
</tr>
<tr>
<td>MEM reconstruction</td>
<td>0.04</td>
<td>1.49</td>
<td>-1.36</td>
<td>0.15</td>
</tr>
<tr>
<td>Wiener reconstruction</td>
<td>0.05</td>
<td>1.49</td>
<td>3.31</td>
<td>0.16</td>
</tr>
<tr>
<td>MAP input</td>
<td>-0.17</td>
<td>1.61</td>
<td>-0.62</td>
<td>0.17</td>
</tr>
<tr>
<td>MEM reconstruction</td>
<td>-0.02</td>
<td>2.11</td>
<td>-1.31</td>
<td>-0.92</td>
</tr>
<tr>
<td>Wiener reconstruction</td>
<td>0.10</td>
<td>1.59</td>
<td>-1.67</td>
<td>-0.95</td>
</tr>
</tbody>
</table>

that MEM is reconstructing the non-Gaussianity closely. The Wiener results are less impressive. In each non-Gaussian process the Wiener reconstruction is worse than the MEM reconstruction and most markedly for the SZ effect where the input map had a kurtosis of -1.56 and Wiener filtering recovered a kurtosis of 3.31. The results for the MAP simulation show that the kurtosis can also distinguish between the Gaussian and non-Gaussian origin of the CMB fluctuations at this lower resolution but cannot reconstruct the dust or SZ very well. This is due to the lack of frequency coverage for the latter processes and not due to the resolution of the experiment. Therefore, MEM is better at reconstructing the non-Gaussian features than Wiener filtering (as was expected from the results in Chapter 6) and both MAP and Planck should be able to distinguish between a Gaussian and non-Gaussian process for the origin of the CMB. It should be noted that the string simulations used here did not contain a Gaussian background which is expected to be present due to the effects prior to recombination and any possible reionisation that may have occurred.

The following Chapter will attempt to summarise the results presented in this thesis and bring them together in a coherent fashion.
In the beginning there was only darkness, dust and water. The darkness was thicker in some places than in others. In one place it was so thick that it made man. The man walked through the darkness. After a while, he began to think.

Creation myth from the Pima tribe in Arizona
Chapter 9

Conclusions

In this final chapter I will attempt to bring together the various aspects of the work discussed in this thesis. A brief review of the main results found and their implications for cosmology will be undertaken in the first section, while the future of CMB experiments is discussed in the final section.

9.1 Discussion

Observations from the 5 GHz interferometer at Jodrell Bank and the 10, 15 and 33 GHz switched beam experiments at Tenerife have been presented and analysed. Simulations of observations from the proposed Planck surveyor and MAP satellites have also been performed.

The Jodrell Bank interferometer covers an area of the sky between declinations +30° and +55° while the Tenerife experiments cover an area between declinations +30° and +45°. There are over 100 independent measurements for the average pixel in right ascension at each of the declinations sampled in both experiments. The noise per beam for each of the experiments are \( \sim 20 \mu K \) for the 5 GHz data, \( \sim 50 \mu K \) for the 10 GHz data, \( \sim 20 \mu K \) for the 15 GHz data and \( \sim 30 \mu K \) for the 33 GHz data. Taken together these form very good constraints on the CMB fluctuations as well as the Galactic foregrounds and point source contribution. Figure 9.1 shows the level of Galactic foreground emission expected in a 5° FWHM CMB experiment at frequencies between 408 MHz and 33 GHz. The points used in the generation of this plot are the 408 MHz and 1420 MHz surveys and predictions from the 5 GHz Jodrell Bank interferometer and the 10 GHz, 8° FWHM Tenerife experiments. It is seen that at frequencies below 5 GHz synchrotron emission dominates the foregrounds, whereas at frequencies above 5 GHz free-free emission dominates.

The level of CMB fluctuation found using the 15 GHz Tenerife data set is \( Q_{RMS-PS} = 22^{+5}_{-3} \mu K \) (68 % confidence) at an \( \ell \) of \( 18^{+9}_{-7} \) which is consistent with findings from the COBE data. Combining the likelihood results from the COBE and

Figure 9.1: Level of Galactic foreground in a 5° FWHM experiment predicted by the 408 MHz, 1420 MHz, 5 GHz and 10 GHz (8° FWHM) surveys.
Figure 9.2: Recent results from various CMB experiments. Shown is the predicted CMB level from the 15 GHz Tenerife experiment derived here. The solid line is the prediction (normalised to COBE) for standard CDM with $\Omega_\odot = 1.0$, $\Omega_b = 0.1$ and $H_\odot = 45 \text{ km s}^{-1}\text{Mpc}^{-1}$. The Saskatoon points have a 14% calibration error.

Tenerife data sets more stringent constraints on the level of CMB can be found. This analysis gave $Q_{RMS-PS} = 19.9^{+3.5}_{-3.2}\mu K$ for the level of the Sachs-Wolfe plateau. It was also possible to put constraints on the spectral index which gave $n = 1.1^{+0.2}_{-0.2}$ (at 68% confidence). For an $n = 1$ spectrum it was found that $Q_{RMS-PS} = 22.2^{+4.4}_{-4.2}\mu K$ which can be compared to the result from the Dec. 40° 33 GHz Tenerife data which gives $Q_{RMS-PS} = 22.7^{+8.3}_{-5.7}\mu K$ for $n = 1$. These likelihood results are all consistent with a CMB origin for the structure within the data and span a frequency range of between 15 GHz and 90 GHz. Common features between the COBE and Tenerife data were found leading to the conclusion that actual CMB features are observed in the two experiments. Figure 9.2 shows the most recent results from various CMB experiments around the world (figure provided by Graca Rocha). The Tenerife result calculated here is plotted. The solid line is the predicted curve for a Cold Dark Matter Universe with $\Omega_\odot = 1$, $H_\odot = 45 \text{ Mpc km}^{-1}\text{s}^{-1}$ and $\Omega_b = 0.1$. Taking the Tenerife data together with the results from other experiments, sensitive to smaller angular scales, shows evidence for a Doppler peak as expected for inflation.

A new technique for analysing data from CMB experiments was presented. Positive/negative Maximum Entropy was used to extract the most information out of each data set. With the Tenerife experiment the long term atmospheric baseline variations were removed from the data scans as well as the triple beam pattern produced by the switching of the beam. Sky maps at 5° resolution at 10 GHz and 15 GHz, and 8° at 10 GHz, were produced. With the Jodrell Bank experiment it was possible to analyse the two data sets (from the two different baselines) independently to produce two sky maps at 5 GHz and 8° resolution. This analysis was compared to the CLEAN technique and it was shown that the MEM outperforms CLEAN in all areas of map reconstruction. Using the new MEM technique it was possible to analyse the two baseline data sets simultaneously to show that they were consistent.

The MEM technique was also applied to simulated data from both the Planck Surveyor and MAP satellites. Using multi-frequency information it was shown that it should be possible to extract information on the CMB to high accuracy ($6 \mu K$ for map reconstruction and out to $l \sim 2000$ for power spectrum reconstruction) with, or without, any knowledge on the spatial distribution of the foregrounds. The multi-MEM technique was also compared to Single-Valued Decomposition and the Wiener filter and it was found that multi-MEM always outperforms SVD and if any of the foregrounds (or the CMB itself) is non-Gaussian in structure then multi-MEM also outperforms the Wiener filter.

The final chapter introduced some of the techniques that are used to analyse the CMB sky maps once they have been produced. It was seen that all the techniques reviewed have advantages. The most promising test for non-Gaussianity appears to
be Minkowski functionals as these incorporate all of the other techniques together in just three functionals (for a two dimensional map). Using the auto-correlation function it was shown that there is an excess signal present in the 15 GHz Tenerife sky map that is not due to noise alone. The Genus analysis of this map showed that it was consistent with a Gaussian process.

9.2 The future of CMB experiments

Within the next ten years a new generation of CMB experiments will be in operation. These include both space based (like the Planck Surveyor and MAP), balloon based (like TopHat and Boomerang) as well as ground based (like the Very Small Array which is based on a similar design to the CAT interferometer). It has been shown here that the Planck surveyor will produce very accurate maps of the CMB fluctuations. It will also allow very tight constraints on the level of Galactic foreground emissions which can be subtracted from other experiments. The recent ESA report on the Planck Surveyor show that the sensitivity of the satellite has improved since the simulations presented here were performed and so the accuracy will be even higher.

Ground–based measurements have already proven to provide tight constraints on the level of the CMB anisotropies and so the VSA (a 15 element interferometer that will measure the CMB anisotropy at high angular resolution) should also perform very well. In conjunction with existing ground based telescopes constraints on the CMB anisotropies will increase considerably prior to the launch of either satellite. The Tenerife experiment will continue to take measurements at all three frequencies with the aim of having a final two dimensional map with very low noise and the Jodrell Bank interferometer is currently taking data for a new baseline. With such a wealth of data at many frequencies (5 - 900 GHz) conventional analysis techniques will have to be refined. The maximum entropy algorithm described here can cope with multiple frequencies, varying pixel size, multiple component fitting and a very high level of noise. This ‘multi-MEM’ is now in the process of being applied to the Tenerife and Jodrell Bank data described in this thesis. The data from the two experiments are also being combined with data from COBE and lower frequency surveys (the 408 MHz, 1420 MHz and 2300 MHz surveys) using the multi-MEM in an analogous way to the Planck Surveyor analysis presented here, to put better constraints on the CMB at the large angular scales covered by Tenerife and COBE ($\ell < 30$).

Another area of CMB research which is becoming increasingly of interest is the comparison with large scale structure. All theories that explain the shape of the power spectrum of CMB anisotropies also make predictions for the evolution of these anisotropies into the present large scale structure. By attempting to match the two power spectra on the different scales more constraints can be put on the exact form of the underlying matter. This is now being done by various groups although research into this area is still in its very early stages.

With new high quality data and the ability to extract the CMB signal from the
foreground contamination, very tight constraints on the cosmological parameters should be achievable.

\[ Eureka? \]

C. Lineweaver on the COBE discovery
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