NUCLEAR SPIN RELAXATION WITHIN INTERSTELLAR GRAINS

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ABSTRACT

We identify a new mechanism of internal dissipation of rotational kinetic energy in spinning dust grains, arising from the reorientation of nuclear angular momentum, e.g., spins of protons. Grain rotation induces magnetization of the nuclear spin system, with net alignment of nuclear spins parallel to the grain angular velocity. When the grain does not rotate around a principal axis, the nuclear magnetization vector precesses in grain body coordinates, resulting in dissipation of energy. The analogous process involving electron spins was discovered by Purcell and termed “Barnett relaxation”. We revisit the physics of the “Barnett relaxation” process and correct the estimate for the Barnett relaxation rate. We show that nuclear relaxation can be orders of magnitude more important than Barnett relaxation. This finding implies that the processes of “thermal flipping” and “thermal trapping” are important for a broad range of grain sizes.

Subject headings: ISM: Atomic Processes, Dust, Polarization

1. INTRODUCTION

The polarization of starlight, first discovered by Hiltner (1949) and Hall (1949), implies that the axes of grains are partially aligned with the interstellar magnetic field. Alignment of a rotating interstellar grain, whatever its cause may be (see a list of alignment mechanisms in Lazarian, Goodman & Myers 1997), requires the alignment of the grain’s body axes with its angular momentum \( \mathbf{J} \). Purcell (1979) noted that internal relaxation of energy within a rotating grain should cause alignment of the grain’s axis of maximal moment of inertia with \( \mathbf{J} \). In the same pioneering study Purcell calculated the timescales for internal relaxation and found them to be much shorter than the gaseous damping time. He discovered the process that he called “Barnett relaxation” and identified it as apparently the dominant mechanism of internal relaxation.

Barnett relaxation is associated with the Barnett effect, i.e., the spontaneous magnetization of a rotating paramagnetic body (see Landau & Lifshitz 1962). A paramagnetic body has unpaired electron spins and the Barnett effect can be understood in terms of a rotating lattice sharing its angular momentum with the electron spin system. As a result, the rotating body becomes magnetized with magnetization \( \mathbf{M} \) antiparallel to the angular velocity \( \mathbf{\Omega} \). For a freely rotating body \( \mathbf{\Omega} \) and therefore \( \mathbf{M} \) precess about \( \mathbf{J} \) in inertial coordinates; in body coordinates \( \mathbf{\Omega} \) and \( \mathbf{M} \) precess around the body’s axis of maximal moment of inertia (see Purcell 1979). This causes time-varying magnetization of the grain material and therefore entails dissipation.

Spitzer & McGlynn (1979) related the Barnett relaxation time scale with the degree of disorientation of suprathermally (i.e. much faster than thermally) rotating grains during crossovers. The effect of suprathermal rotation was discovered by Purcell (1979), who identified recoils from \( \text{H}_2 \) formation events, occurring at catalytic sites on the grain surface, as the dominant mechanism of suprathermal spin-up.\(^1\)

1. A competing mechanism of spin-up due to radiation torques was recently proposed by Draine & Weingartner (1996, 1997).

\(^2\)For brevity, we will use the term “nuclear spin” to refer to the total nuclear angular momentum \( I \hbar \).

The theory of crossovers was revised recently (Lazarian & Draine 1997, 1999; hereafter LD97, LD99) to include thermal fluctuations within the grain material (Lazarian 1994, Lazarian & Robege 1997). The main conclusion by LD97 is that grains with radii \( a > a_c \approx 1.5 \times 10^{-5} \) cm experience only minor disalignment during crossovers if Barnett relaxation is the dominant dissipational mechanism. As grains are essentially immune from disorientation during their phases of suprathermal rotation, during which paramagnetic relaxation (Davis & Greenstein 1951) aligns them with the magnetic field, the modest disalignment during crossovers entailed nearly perfect alignment of \( a \gtrsim 1.5 \times 10^{-5} \) cm interstellar grains in diffuse clouds with the interstellar magnetic field.

LD99 showed that for grains smaller than \( a_c \) the crossovers are mostly performed via the process of “thermal flipping” and this would entail a high degree of grain alignment if it were not for the associated process of “thermal trapping”; the latter occurs when thermal flipping is so rapid as to effectively average out the systematic torques due to \( \text{H}_2 \) formation, etc. We note that the value of \( a_c \) was calculated in LD97 and LD99 assuming Barnett relaxation to be the dominant mechanism of internal energy dissipation. In the presence of more effective relaxation mechanisms the coupling between rotational and vibrational degrees of freedom would be larger thermal flipping should be facilitated, and the value of \( a_c \) would be increased.

In this paper we show that the dynamics of the nuclear spins\(^2\) leads to an important new process of internal dissipation in a rotating grain. Although the role of nuclear spins for paramagnetic relaxation was mentioned in the classic study by Jones & Spitzer (1968), and their role in developing a Barnett moment in a non-paramagnetic substance was pointed out by Purcell (1979), the internal relaxation associated with these spins was overlooked.

Below we describe this new magneto-mechanical effect, which we term “nuclear relaxation”. We show that under a broad range of conditions this process dominates the dissipa-
tion of rotational kinetic energy in a rotating grain. As a result, “nuclear relaxation” plays an essential role in the dynamics and alignment of interstellar grains.

2. DYNAMICS OF NUCLEAR SPINS

Purcell (1978) noted that an analog of the Barnett effect exists for nuclear spins. If a rotating body has initially an equal number of nuclear spins directed parallel and anti-parallel to the angular velocity \( \vec{\Omega} \), it can decrease its kinetic energy, at constant total angular momentum \( \vec{J} \), if some of the angular momentum is transferred to the nuclear spin system. Increasing the projection of the nuclear angular momentum along \( \vec{J} \) by \( +h \) (at constant \( J \)) reduces the rotational kinetic energy by \( h\vec{\Omega} \). If the rotating body is allowed to come into thermal equilibrium (without exchanging angular momentum) with a heat reservoir of temperature \( T_{\text{dust}} \), then particles of spin \( S \) develop a net alignment per particle

\[
\frac{\sum_{m=-S}^{S} m \exp(mh\vec{\Omega}/kT_{\text{dust}})}{\sum_{m=-S}^{S} \exp(mh\vec{\Omega}/kT_{\text{dust}})} .
\]

Note that this does not depend on the magnetic moment \( \mu \).

In Table 1 we list a number of non-zero spin nuclei which could be important for interstellar grains. Carbonaceous grains will contain \( ^{13}C \) at the \( \sim 1\% \) level (\( n \approx 10^{21} \text{ cm}^{-3} \)), and \( ^{2}H \) at perhaps the \( 10\% \) level (\( n \approx 10^{22} \text{ cm}^{-3} \)). Silicate grains will contain \( ^{29}\text{Si} \) at the \( \sim 1\% \) level (\( n \approx 10^{21} \text{ cm}^{-3} \)), and could also contain comparable abundances of \( ^{57}\text{Fe} \), \( ^{27}\text{Al} \), and \( ^{55}\text{Mn} \), as well as \( ^{1}H \). For purposes of discussion we will assume the nuclear spin system to consist of protons with \( n_{n} = 10^{22} \text{ cm}^{-3} \).

As the number of parallel and antiparallel spins becomes different the body develops magnetization. The relation between \( \vec{\Omega} \) and the strength of the “Barnett-equivalent” magnetic field \( H_{\text{BE}}^{(n)} \) (Purcell 1979) that would cause the same nuclear magnetization (in a nonrotating body) is given by

\[
H_{\text{BE}}^{(n)} = \frac{h}{g_{n} \mu_{N}} \vec{\Omega} ,
\]

where \( g_{n} \) is the so-called nuclear \( g \)-factor (see Morrish 1980), and \( \mu_{N} \equiv e\hbar/2m_{e}c \) is the nuclear magneton, smaller than the Bohr magneton of the electron to proton mass ratio, \( m_{e} / m_{p} \).

Consider a classical picture that would correspond to the concept of the Barnett-equivalent magnetic field. Imagine the (proton) spins as small positively charged tops within the rotating body. In body coordinates their motion is given by

\[
\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \vec{\Omega} ,
\]

where \( \vec{\Omega} \) is the angular velocity of the body in an inertial frame, and the prime signifies differentiation with respect to the rotating frame. It is evident that this equation is similar to the equation of motion in an inertial frame if a “Barnett-equivalent” magnetic field defined by eq. (2) were present. In other words, the motion of spins in the rotating body-coordinate frame is as though a fictitious “Barnett-equivalent” magnetic field \( H_{\text{BE}}^{(n)} \) were present in a nonrotating body.

3. NUCLEAR SUSCEPTIBILITY

The zero-frequency paramagnetic susceptibility of the nuclear spins with magnetic moments \( \mu_{n} = g_{n} \mu_{N} \) is given by Curie’s Law (Morrish 1980)

\[
\chi_{n}(0) = \frac{n_{n} \mu_{n}^{2}}{3kT} = 4.1 \times 10^{-11} \left( \frac{\mu_{N}}{\mu_{n}} \right)^{2} \left( \frac{n_{n}}{10^{22} \text{ cm}^{-3}} \right) \left( \frac{15 \text{ K}}{T} \right) .
\]

We wish to estimate the time scale on which the net alignment of the nuclear spin system will respond to changes in the direction of \( \vec{\Omega} \). When an oblate grain rotates freely about an axis that does not coincide with its principal axis of largest moment of inertia \( \vec{a} \), \( \vec{\Omega} \) precesses about \( \vec{J} \). Due to the Barnett effect the magnetization of nuclear spins should follow \( \vec{J} \). Interactions within the nuclear spin system (i.e., coupling of one nuclear spin to the magnetic field arising from other nuclear spins) conserve spin angular momentum, and therefore cannot change the direction of magnetization. However, the component of \( \vec{J} \) along \( \vec{J} \) is constant, and so even very slow exchange of angular momentum between the lattice and the nuclear spin system will result in magnetization of the nuclear spin system along \( \vec{J} \). Since \( \vec{J} \) precesses in body coordinates, even magnetization along \( \vec{J} \) will have dissipation associated with it.

Consider first the coupling of the nuclear spins to the electron spin system.\(^{1}\) The latter is coupled to the lattice on the “spin-lattice” timescale \( \tau_{sl} \). If we assume that the electron spin system reacts relatively quickly to the change in rotational velocity \( \vec{\Omega} \), then the nuclear spin system will become aligned by exchange of angular momentum with the electron spin system.

The “internal” magnetic field due to the electron spin system is \( H_{i} \approx 3.8n_{e} \mu_{e} \) (van Vleck 1937), where \( n_{e} \) is the density of “unpaired” electrons, and \( \mu_{e} \approx \mu_{B} \) is the magnetic moment per unpaired electron, where \( \mu_{B} = e\hbar/2m_{e}c \) is the Bohr magneton. Because the electrons themselves precess with characteristic frequency \( \omega_{e} = \mu_{B}H_{i}/\hbar \), where \( \mu_{B} \approx 2 \) is the electron “\( g \)-factor”, the correlation time for \( H_{i} \) is of order \( \omega_{e}^{-1} \). The fluctuating magnetic field causes the nuclear spins to undergo a random walk; each step lasts \( \sim \omega_{e}^{-1} \), and the change in direction of the nuclear moment per step is \( \delta \omega \approx (g_{n} \mu_{N}H_{i}/\hbar)\omega_{e}^{-1} \ll 1 \). The initial phase relationship is lost after a time \( \tau_{ne} \approx (1/\delta \omega)^{2} \omega_{e}^{-1} \). Therefore the time for the nuclear spins to decorrelate with their initial direction – which is also the time scale on which they “relax” to the new alignment of eq.(1) – is

\[
\tau_{ne} \approx \frac{h\mu_{e}}{3.8n_{e}g_{e}^{2}\mu_{N}^{-1}} \approx 3.0 \times 10^{-4} \left( \frac{2.7}{g_{e}} \right)^{2} \left( \frac{10^{22} \text{ cm}^{-3}}{n_{e}} \right) s .
\]

The nuclear spin system can exchange angular momentum with the electron spin system on the timescale \( \tau_{ne} \).

Curiously enough a quicker nuclear spin-spin relaxation arises from the direct interaction of nuclear moments. The magnetic field of the neighboring nuclei is \( \sim 3.8n_{n} \mu_{n} \), and the nuclear spin-spin relaxation time is

\[
\tau_{nn} \approx \frac{\hbar}{3.8g_{n}n_{n}\mu_{N}^{2}} = \tau_{ne} \left( \frac{g_{n}}{g_{e}} \right) \left( \frac{\mu_{N}}{\mu_{n}} \right) \left( \frac{n_{e}}{n_{n}} \right) \approx 0.58\tau_{ne} \left( \frac{n_{e}}{n_{n}} \right) .
\]

For grains with high percentage of \( ^{1}H \), \( \tau_{nn} \) may be a factor of several smaller than \( \tau_{ne} \). The net nuclear relaxation rate \( \tau_{-1}^{-1} = \tau_{nn}^{-1} + \tau_{ne}^{-1} \).

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\(^{1}\)We use the term “electron spin system” to refer to the system of paramagnetic ions and electrons.

\(^{2}\)This effect could be called the “nuclear Barnett effect” as opposed to the ordinary or “electron Barnett effect”.

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\(H_{\text{BE}}^{(n)}\) denotes the magnetic field due to the Barnett-equivalent effect.
For an applied magnetic field of constant magnitude, rotating at frequency $\omega$, the magnetic susceptibility contributed by the nuclear spin system can be estimated to be (see Draine & Lazarian 1999)

$$\chi_n(\omega) \approx \chi_n(0) \frac{1}{(1 - i \omega \tau_n/2)^2}$$

(7)

$$\chi''_n(\omega) \approx \Im [\chi_n(\omega)] = \chi_n(0) \frac{\omega \tau_n}{1 + (\omega \tau_n/2)^2}$$

(8)

4. BARNETT RELAXATION REVISITED

Purcell (1979) assumed that the dissipation arising from the Barnett effect in a rotating grain is the same as would occur if the Barnett equivalent magnetic field were rotating in a stationary grain. The analogy, however, is not complete. Consider a freely rotating oblate grain with eigenvalues of the moment of inertia tensor $I_{\parallel}, I_{\perp}, I_1$, with $I_{\parallel} > I_{\perp}$. Let $\hat{a}$ be the principal axis of largest moment of inertia, $I_1$. If the grain is not rotating about a principal axis, then in inertial coordinates, $\hat{a}$ and the angular velocity $\Omega$ each precess around the In grain body coordinates, the angular velocity $\Omega$ precesses around $\hat{a}$; the frequency of this precession is

$$\omega_1 = (h - 1)\Omega \cos \theta_\Omega = \frac{(h - 1)J \cos \theta_J}{I_{\parallel}}$$

(9)

(see Purcell 1979), where $h \equiv I_{\parallel}/I_{\perp}$, $\theta_\Omega$ is the angle between $\Omega$ and $\hat{a}$, and $\theta_J$ is the angle between $J$ and $\hat{a}$.\(^5\)

Let $\tau_{\Omega1} \approx 10^{-8}$ s be the electron spin-lattice coupling time. In the limit $\omega_1 \tau_{\Omega1} \ll 1$, the electron spin system will respond to the instantaneous angular velocity, with a magnetization $\chi^e_{BE}$, where $\chi^e_{BE} \equiv h\Omega/g_{\mu_B} H$ is the Barnett-equivalent field for the electrons. In body coordinates, this magnetization has a component $\chi^e_{BE} \sin \theta_J$ which is rotating around $\hat{a}$, and as a result there will be energy dissipation at a rate $dE/dt = \chi''_n(\omega_1) V(H^e_{BE})^2 \sin^2 \theta_{\Omega} f(\omega_1, \theta_J)$.

In the opposite limit $\omega_1 \tau_{\Omega1} \gg 1$, the electron spin system is almost decoupled from the lattice. The electron spins interact magnetically, but the total spin of the electrons is conserved, and cannot follow $\Omega$ as it precesses around $J$. However, since the spin lattice coupling will not be zero, eventually the electron spin system will become magnetized in response to the (stationary) component of $\Omega$ along $J$, with a magnetization $\chi^e_{BE} \cos \theta_J$. This magnetization has a component $\chi^e_{BE} \cos (\theta_J - \theta_J) \sin \theta_J$ perpendicular to $\hat{a}$, and in body coordinates this component of the magnetization will rotate at frequency $\omega_1$. We can write the energy dissipation rate as

$$dE/dt = \chi''_{n1} V(H_{BE}^e)^2 \sin^2 \theta_{\Omega} f(\omega_1, \theta_J)$$

(10)

where

$$f \approx \frac{1 + (\omega_1 \tau_{\Omega1})^2 \sin^2 \theta_J \cos^2 (\theta_J - \theta_J) / \sin^2 \theta_J}{1 + (\omega_1 \tau_{\Omega1})^2}$$

(11)

gives the asymptotic behavior for $\omega_1 \tau_{\Omega1} \ll 1$ and $\omega_1 \tau_{\Omega1} \gg 1$.

The kinetic energy of the grain is $(J^2/2I_1)[(1 + (h - 1) \sin^2 \theta_J]$. Thus, $dE/d\theta_J = J_2 \sin \theta_J$, and Barnett relaxation entails

$$\frac{d\theta_J}{dt} = -h(1 - J \cos \theta_J) \left( \frac{h}{g_{\mu_B}} \right) \frac{(\omega_1 \tau_{\Omega1})^2 \sin \theta_J \cos \theta_J f(\omega_1, \theta_J)}{2}$$

(12)

\(^5\)The two are related via $\tan \theta_\Omega = h \tan \theta_J$.

The Barnett relaxation is slower than estimated by Purcell (1979) by the factor $f < 1$. However, this factor is of order unity, and we will take $f = 1$ in the remaining discussion.

For an $a \times a \sqrt{3} \times a \sqrt{3}$ brick, Barnett relaxation gives

$$\tau_{BR} \approx 2.1 \times 10^9 \rho^2 a^7 5 \left( \frac{J_d}{J} \right)^2 \left[ 1 + \left( \frac{\omega_1 T_d}{2} \right)^2 \right] s$$

(13)

where $\rho \equiv \rho/(2 \ g \ cm^{-3})$, $a \equiv a/10^{-5} \ cm$ and $J_d \equiv [I_1 k T_d/(h - 1)]^{1/2}$ is a characteristic angular momentum appearing in the theory of crossovers (LD97, LD99).

5. NUCLEAR RELAXATION IN A ROTATING GRAIN

Nuclear relaxation is more effective than Barnett relaxation. The time for nuclear relaxation to align $\hat{a}$ with $J$ is

$$\tau_{NR} = \frac{I_b I_2}{(h - 1) V J^2} \frac{g_{\mu_B}^{2} \mu_b}{h^2} \frac{1}{\chi_n(0) \tau_{n}} \left[ \frac{\omega_1 \tau_{n}}{2} \right]^2$$

(14)

For an $a \times b \times b$ brick with $b/a = \sqrt{3}$, we have $h - 1 = 1/2$, and

$$\tau_{NR} \approx 610 \rho^2 a^7 5 \left( \frac{J_d}{J} \right)^2 \left( \frac{n_2}{n_1} (\frac{g_{\mu_B}^{2}}{3.1}) \right)^2 \frac{[1 + (\omega_1 T_d)^2]}{2} s$$

(15)
Internal Relaxation

Figure 1 shows $\tau_{\mathrm{NR}}$, $\tau_{\mathrm{BR}}$ and the overall relaxation time $\tau = (\tau_{\mathrm{NR}}^{-1} + \tau_{\mathrm{BR}}^{-1})^{-1}$ as a function of grain size, for grains rotating with $J/J_d = 1$ and $J = 10^{1/2} J_d$. For slow rotation rates (large $a$) the timescale $\tau_{\mathrm{NR}}$ is $\sim 10^6$ times shorter than $\tau_{\mathrm{BR}}$. As a result the critical grain size for which thermal fluctuations dominate the dynamics of crossovers is increased by an order of magnitude. We conclude that “thermal flipping” (LD99) must therefore be an essential part of crossover dynamics for all grains with $a \lesssim 10^{-4}$ cm.

6. DISCUSSION

The first striking question is why an effect so feeble as nuclear magnetism can be so important in terms of internal relaxation. The answer is that spins align in a rotating body because of their angular momentum, not because of their magnetic moments. The magnetic moment enters only as a means for the spins to exchange angular momentum with the lattice. The coupling within the electron spin system is very effective, for the spins to exchange angular momentum with the lattice. The answer is that spins align in a rotating body coordinates. In the case of the nuclear spin system, however, the value of the nuclear magnetic moment is large enough to provide significant spin-spin coupling, but weak enough so that there is a significant lag in the nuclear spin alignment when $\Omega$ precesses around the grain axis $\hat{a}$: the coupling is “just right” for nuclear relaxation to be extremely effective for $10^{-5} \lesssim a \lesssim 10^{-4}$ cm grains. The efficiency of nuclear relaxation drops for sufficiently high frequencies and therefore Barnett relaxation dominates for $a \gtrsim 5 \times 10^{-6}$ cm, as seen in Figure 1.

LD99 found that “thermal flipping” was an important element of grain dynamics for sufficiently small grains. Assuming the Barnett effect to dominate internal dissipation, they estimated that thermal flipping would be important for $a \lesssim 10^{-5}$ cm grains, and they speculated that for $a \gtrsim 5 \times 10^{-6}$ cm grains, thermal flipping would be so rapid as to interfere with the ability of systematic torques to spin these grains up to suprathermal rotation rates.

In the present paper we have shown that internal relaxation associated with nuclear spin alignment can be many orders of magnitude more rapid than due to the Barnett effect for $a \gtrsim 10^{-5}$ cm grains. As a result, the phenomena of thermal flipping and thermal trapping become important for grains as large as $\sim 10^{-4}$ cm.

7. CONCLUSIONS

The results of this study can be summarized as follows:

1. Freely rotating interstellar grains are subject to “nuclear relaxation” when their angular momentum $\mathbf{J}$ is not along a principal axis.

2. For slowly rotating interstellar grains the dissipation due to nuclear relaxation is many orders of magnitude higher than that due to Barnett relaxation.

3. The critical size $a_c$ below which thermal fluctuations are effective during a crossover increases to $\sim 10^{-4}$ cm. Thermal flipping and thermal trapping must therefore be essential elements of the dynamics of all $a \lesssim 10^{-4}$ cm interstellar grains.

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REFERENCES

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Table 1
Nuclear Properties$^a$

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$I^b$</th>
<th>Abund.(%)</th>
<th>$\mu_N/\mu_N^c$</th>
<th>$g_\mu$</th>
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<tr>
<td>$^1$H</td>
<td>1/2</td>
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<td>2.675</td>
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<td>$^{13}$C</td>
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<tr>
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$^a$From Robinson (1991)

$^b$Total angular momentum quantum number

$^c$\(\mu_N \equiv e\hbar/2m_pc = 5.05 \times 10^{-24}\ \text{erg/gauss.}\)