Comment on “Determination of pion-baryon coupling constants from QCD sum rules”

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Abstract

In this comment, we propose possible errors in constructing the continuum contribution in the sum rule studied by M. C. Birse and B. Krippa, Phys. Rev. C. 54 (1996) 3240.

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In Ref. [1], Birse and Krippa [BK] have calculated $\pi NN$, $\pi \Sigma \Sigma$, and $\pi \Sigma \Lambda$ coupling constants using QCD sum rules. In this comment, we want to point out that they might not treat the continuum contribution properly. If the continuum contribution is treated properly within their analysis, the results provided in Ref. [1] need to be significantly modified.

BK considered the two-point correlation function

$$\Pi(p) = i \int d^4x e^{ip \cdot x} \langle 0 | T[\eta_p(x)\bar{\eta}_n(0)] | \pi^+(k) \rangle.$$ (1)

Here $\eta_p$ is the proton interpolating field of Ioffe [2] and $\eta_n$ is the neutron interpolating field. For $\pi \Sigma \Sigma$ and $\pi \Sigma \Lambda$ couplings, they use the interpolating fields for $\Sigma^+$, 0 and $\Lambda$ obtained from the nucleon interpolating field by using SU(3) rotation. They constructed the sum rules in the leading order of the pion momentum $k$ and considered the structure $k\gamma_5$.

In the conventional QCD sum rules, the continuum contribution is modeled such that its spectral density is given by a step function which starts from a threshold, say $S_\pi$. According to the duality argument, the coefficient of the step function is obtained from the terms containing $\ln(-p^2)$ in the OPE. More specifically, the spectral density for the continuum is

$$\rho^{\text{phen}}(s) \equiv \rho^{\text{ope}}(p^2) \theta(p^2 - S_\pi),$$

where $\rho^{\text{ope}}$ is the spectral density of the OPE.

The sum rule equation after the Borel transformation is given by

$$\int_0^\infty ds e^{-s/M^2} [\rho^{\text{ope}}(s) - \rho^{\text{phen}}(s)] = 0,$$ (2)

where $M$ is the Borel mass. Within this approach, it is straightforward to construct the sum rule equations. Then it is easy to prove that $E_2$, $E_1$, and $E_0$ factors appearing in Eqs.(17),(28),(29) and in the numerator of Eq.(36) of Ref. [1] should be replaced as follows,

$$E_2 \rightarrow E_1; \quad E_1 \rightarrow E_0; \quad E_0 \rightarrow 1.$$ (3)

To be more specific, let’s consider the second term in Eq.(17) of Ref. [1] and prove the replacement of $E_1 \rightarrow E_0$. This term comes from Eq.(13) whose form can be written as $C \ln(-p^2)$ where $C$ is a constant. The corresponding spectral density is $C \theta(p^2)$ which provides the continuum spectral density as $C \theta(p^2 - S_\pi)$. Then the integration over $p^2$ with the Borel weight,

$$\int_0^\infty dp^2 e^{-p^2/M^2} C\theta(p^2 - S_{\pi N}) = C \ M^2 e^{-S_{\pi N}/M^2}.$$ (4)

And the corresponding integration for the OPE part yields $CM^2$. By moving the continuum part to the OPE side, one obtains the term

$$CM^2(1 - e^{-S_{\pi N}/M^2}).$$ (5)

Note that the factor $(1 - e^{-S_{\pi N}/M^2})$ is the definition of $E_0$, not $E_1$! This $E_0$ factor should appear in the second term of Eq.(17) instead of $E_1$. The third replacement, $E_0 \rightarrow 1$, is easy to see because this $E_0$ factor in Ref. [1] comes from the OPE term of $1/p^2$. Obviously, the spectral density of $1/p^2$ is just a delta function, $\delta(p^2)$, and it can not contribute to the continuum. Similarly, one can derive the replacement, $E_2 \rightarrow E_1$.

Then how these replacements affect their results? In figure 1 of Ref. [1], after these replacements, the dashed line now varies from 13 to 30 the solid line varies from 7.5 to
5. Therefore, the curves one gets after the replacements are clearly different from what BK provided in Ref. [1], which of course changes their results substantially. Additional discussion on this sum rule using the correct continuum can be found in Ref. [3].

According to Ref. [4], it is claimed that the use of a double dispersion relation [5] is crucial in deriving those continuum factors in question. Within a double dispersion relation, the perturbative spectral density is claimed to be of the form,

\[ \rho(s_1, s_2) = b(s_1)\delta(s_1 - s_2). \]  

(6)

The function \( b(s) \) in Eq.(6) is determined via the relation,

\[ \int_0^\infty ds_1 \int_0^\infty ds_2 \frac{\rho(s_1, s_2)}{(s_1 - p^2)(s_2 - p^2)} = \int_0^\infty \frac{b(s)}{(s - p^2)^2} = Cln(-p^2) \]  

(7)

The second relation is satisfied for \( b(s) = -Cs \), up to some subtraction terms.

According to Ref. [4], the continuum contribution from \( Cln(-p^2) \) is obtained by putting Eq.(6) (with \( b(s) = -Cs \)) into the double dispersion integral,

\[ \Pi(p^2) \equiv \int_{S_\pi}^\infty ds_1 \int_{S_\pi}^\infty ds_2 \frac{\rho(s_1, s_2)}{(s_1 - p^2)(s_2 - p^2)} = -\int_{S_\pi}^\infty \frac{Cs}{(s - p^2)^2} \]

\[ = C\left[ ln(S_\pi - p^2) - \frac{S_\pi}{S_\pi - p^2} + \cdots \right] \]  

(8)

Then BK performed a single Borel transformation, transferred this continuum to the OPE side, and obtained the factor, \( E_1 \). Similarly, one can get the factors, \( E_0 \) and \( E_2 \), by following the similar steps.

However, in this derivation, it is important to note that the term, \( S_\pi/(S_\pi - p^2) \), in Eq. (8) is crucial in obtaining the \( E_1 \) factor. Without this, one would obtain \( E_0 \), the same factor that we have derived using the conventional method mentioned above. The situation is similar in producing the factors, \( E_0 \) and \( E_2 \), in BK sum rule. What is the physical meaning of this new term? If one takes the discontinuity of the correlator, \( \Delta\Pi \equiv \Pi(p^2 + i\epsilon) - \Pi(p^2 - i\epsilon) \), then the new term yields a pole at \( p^2 = S_\pi \) in addition to a step-like continuum coming from the term \( ln(S_\pi - p^2) \). The step-like continuum is well understood from the QCD duality but the pole at the continuum threshold does not have any physical meaning. This pole is produced by the mathematical way of constructing the continuum and therefore needs to be subtracted out. Otherwise, the duality of QCD in constructing the continuum in QCD sum rule can not be satisfied by the presence of this pole at the continuum threshold. Once the pole subtracted out, then the factors in question can be replaced as we suggested.

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REFERENCES