Two-Step Contribution to Intermediate Energy \((p,p')\) and \((p,n)\) Reactions

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Abstract

We calculate the two-step contribution to \((p,p')\) and \((p,n)\) reactions at intermediate energy. We describe the motion of the incident nucleon with a plane wave and compare the contribution from two-step processes with that from one-step processes. To describe the two-step processes, we extend the response functions to nondiagonal forms with respect to the momentum transfer \(q\).

We performed a numerical calculation for the cross sections of the \(^{12}\text{C}, ^{40}\text{Ca}\)(\(p,p'\)) scattering and the spin longitudinal and spin transverse cross sections of the \(^{12}\text{C}, ^{40}\text{Ca}\)(\(p,n\)) reactions at 346 MeV and 494 MeV. We found that the two-step contribution is appreciable in comparison with the one-step processes in the higher-energy transfer region for the spin longitudinal and the spin transverse \((p,n)\) reactions. We also found that the two-step processes give larger contributions to the spin transverse \((p,n)\) reaction than to the spin longitudinal reaction. This finding is very encouraging to interpret the discrepancy between the DWIA calculation and the experimental results of the spin longitudinal and the spin transverse cross sections.

1 Introduction

In nucleon induced high energy reactions, single-step processes have often been considered as the main contribution to the cross sections. When the excitation energy is low, it is probable that the projectile nucleon collides with the nucleons in the target only once, but as the excitation energy becomes higher, the pre-equilibrium processes, in which the projectile collides with the nucleons several times, are considered to become more effective.

Actually, it has been reported that the single-step process calculation underestimates the scattering cross sections, especially in the large scattering angle region, \([1, 2]\) where the two-step and further multi-step scattering are found to have a large effect.

The multi-step direct reaction (MSDR) was actively studied at the end of the 1970s. Feshbach, Kerman, and Koonin (FKK) \([3, 4]\) developed the framework of multi-step reaction theory. Tamura, Udagawa, and Lenske (TUL),\([1]\) who pointed out a problem in FKK theory, replaced the sum over the excited nuclear eigenstates with that over 1-particle-1-hole (1p-1h) states, introducing a weight function. Another type of formalism was presented by Smith and Wambach, \([5]\) who applied the Glauber approximation to the motion of the projectile and analyzed the forward angle scattering. Kawai et al. \([2, 6, 7, 8]\) presented a multi-step formalism with a local-density (or Thomas-Fermi) approximation for the target and the semiclassical distorted wave (SCDW) approximation for the projectile and the ejectile. They calculated the cross sections of \(^{58}\text{Ni}, ^{90}\text{Zr}\)(\(p,p_x\)) and \((p,n_x)\) reactions at incident energies lower than 200 MeV and determined the scattering angle dependence of the one- and the two-step processes. They extended their calculation to three-step processes \([9]\) and obtained comparable results with one- and two-step processes.
at large angles. De Pace et al. [10] also used the Glauber approximation for the analysis of \((p, n)\) reactions and took account of the spin transfer.

Although multi-step reactions have been studied by various groups, as mentioned above, those with spin dependence have been rarely investigated. In the spin polarized cross sections of the \((p, n)\) reactions, a large discrepancy between the distorted wave impulse approximation (DWIA) calculations and the experimental results has been reported.

In the \((p, n)\) reactions, the spin longitudinal response function \(R_L\) and the spin transverse response function \(R_T\) are extracted from the polarization transfer coefficients. [11, 12, 13, 14] With these observables, the ratio \(R_L/R_T\) is found to be less than 1.

It has been predicted with the random phase approximation (RPA), however, [15] that \(R_L\) is enhanced and the peak of its energy spectrum is shifted downwards, while \(R_T\) is quenched and its peak is shifted upwards. The ratio \(R_L/R_T\) becomes greater than 1 theoretically, and this contradicts the experimental results.

The spin longitudinal cross section \(ID_q\) is roughly reproduced by DWIA with the RPA correlation [14, 16] in the lower excitation energy region, but in the spin transverse cross section \(ID_p\), the estimation amounts to only about half of the experimental result in the whole excitation energy region (see Figs. 1, 2). In the region where the cross sections are underestimated, the multi-step processes are expected to have some contribution.

For this reason, we calculate the two-step spin longitudinal and the spin transverse cross sections of the \((p, n)\) reactions. These reactions were already studied by De Pace et al., but they applied the Glauber approximation, and they assumed that the first step momentum transfer and the second step momentum transfer are nearly parallel to the total momentum transfer. They also assumed that the spin flip occurs only in the first or second step and that in the other step the spin-isospin scalar transition takes place. Further, they neglected the interference between processes with different shares of the momentum transfer in the first and the second steps.

In this paper, we develop a formalism for two-step reactions within the framework of the plane-wave approximation in order to see the relative contributions from one- and two-step processes. We believe that the path lengths of the incident particles in the target nucleus in the one-step and the two-step processes are nearly equal, and therefore the effects of absorption in the one-step processes and in the two-step processes become nearly equal. We treat the spin degree of freedom carefully, taking account of the difference between the direction of the total momentum transfer and that in each step. With this formalism, we calculate the unpolarized cross section of the \((p, p')\) scattering and the spin longitudinal and spin transverse cross sections of the \((p, n)\) reactions at intermediate energy. We investigate the spin dependence of the two-step contribution and partly explain the contradiction in the \(R_L/R_T\) problem. In §2 we present our formalism for one- and two-step processes in detail and study some relations with TUL’s formalism. In §3 we describe the cross sections with spin degrees of freedom. In §4 we explain our method for the numerical calculation. In §5 the results and discussion are given, and in §6 we summarize this paper and give the conclusion.

## 2 Formalism

We consider nucleon induced inelastic scattering and charge exchange reactions in which the target is excited to the continuum region. The total Hamiltonian \(H\) is

\[
H = H_0 + V,
\]

where \(H_0\) is the unperturbed Hamiltonian and \(V\) is the residual interaction between the projectile and the target. They are specified in the projectile-target center of mass (c.m.) system as

\[
H_0 = H_p + U + H_T,
\]

\[
H_p = \sqrt{M_p^2 + \vec{p}^2}, \quad H_T = M_T + H_T^{int} + \frac{\vec{P}^2}{2M_T},
\]

\[
V = \sum_{i=1}^{A} v_{qi} - U \equiv V - U.
\]
\[ H_T^{\text{int}} = H_T^{\text{shell}} + V_T, \]  
\[ H_T^{\text{shell}} = \sum_{i=1}^{A} \left( \frac{p_i^2}{2M} + U_i' \right), \quad V_T = \sum_{i<j} v_{ij} - \sum_{i=1}^{A} U_i', \]

where \( H_p \) and \( H_T \) are the projectile and the target Hamiltonian, respectively, and \( v \) is the effective interaction between particles. The subscript 0 represents the projectile and \( i \) designates the \( i \)-th nucleon in the target. \( U \) is the mean field that the target nucleon creates for the projectile. \( M \) and \( M_T \) are the nucleon mass and the ground state target mass, respectively, \( \hat{p} \) is the projectile momentum operator in the projectile-target c.m. frame, and \( \hat{\boldsymbol{p}}_i \) is the momentum operator of the \( i \)-th nucleon in the intrinsic frame of the target. The \( U_i' \) are the mean fields for the nucleons in the target.

The \( T \)-matrix is given in the ‘solved’ Lippman-Schwinger form
\[ T = V + V \frac{1}{E^+ - H} V, \]  
where \( E^+ = E + i\epsilon \). Using (2.1) and expanding \( 1/(E^+ - H) \) in \( V \), one obtains
\[ T = \sum_{j=1}^{\infty} V(G_0 V)^{j-1}, \]
with
\[ G_0 = \frac{1}{E^+ - H_0}. \]

Then, the \( T \)-matrix elements become
\[ T_{n_0}^{j} = \sum_{j=1}^{\infty} T_{n_0}^{(j)}, \]
with the \( j \)-th order \( T \)-matrices
\[ T_{n_0}^{(j)} = \langle \chi_i^{(+)}/\Phi_n | V(G_0 V)^{j-1} | \Phi_0 \rangle |\chi_i^{(+)}/\rangle, \]
where \( \chi_i^{(+)} \) and \( \chi_i^{(-)} \) are the distorted waves of the initial and final channel, and the eigenstates of the target nucleus \( |\Phi_n \rangle \) obey the equation
\[ H_T^{\text{int}} |\Phi_n \rangle = E_n^{\text{int}} |\Phi_n \rangle. \]

Here \( n = 0 \) denotes the ground state, and we set \( E_0^{\text{int}} = 0 \). The \( j \)-th term of (2.10) describes the \( j \)-step processes.

Here we neglect the interference terms with \( j \neq j' \). The different order \( T \)-matrices mainly excite different number particle-hole states. Then, the final state of \( T^{(j)} \) is different from that of \( T^{(j') \prime} \). Hence the contribution from the interference terms can be neglected. Then the double-differential cross section becomes
\[ \frac{\partial^2 \sigma}{\partial \Omega_2 \partial \epsilon_f} \approx \sum_{j=1}^{\infty} \frac{\partial^2 \sigma^{(j)}}{\partial \Omega_2 \partial \epsilon_f} = K \sum_{j=1}^{\infty} X^{(j)}, \]
\[ K \equiv \sum_{j=1}^{\infty} |T_{n_0}^{(j)}|^2 \delta(\omega - (E_i - E_f)), \]
where \( \omega = \epsilon_i - \epsilon_f \) is the energy transfer, \( k_i \) and \( k_f \) are the projectile and ejectile asymptotic wave number, \( \mu_i (\mu_f) \) is the reduced energy of the projectile (ejectile),
\[ \mu_i = \frac{\epsilon_i E_i}{\epsilon_i + E_i}, \quad \mu_f = \frac{\epsilon_f E_f}{\epsilon_f + E_f}, \]
and \( \epsilon_i (\epsilon_f) \) and \( E_i (E_f) \) are the projectile (ejectile) and the target (residual) nucleus energies,
\[ \epsilon_i = \sqrt{M^2 + k_i^2}, \quad \epsilon_f = \sqrt{M^2 + k_f^2}, \]
\[ E_i = M_T + \frac{k_i^2}{2M_T}, \quad E_f = M_T + E_n^{\text{int}} + \frac{k_f^2}{2M_T}. \]
2.1 One-step processes

We are mainly interested in the relative importance of the one- and the two-step processes. Therefore we adopt a plane-wave approximation for the distorted waves $\chi^{(1)}$ and $\chi^{(-)}$. Then the one-step $T$-matrix becomes

$$T^{(1)}_{n_0}(k_f, k_i) = \langle k_i|\langle \Phi_n|V|\Phi_0\rangle|k_f\rangle = \langle k_i|\langle \Phi_n|V|\Phi_0\rangle|k_i\rangle$$

(2.18)

for $n \neq 0$. The effective interaction $V$ between the projectile and the target can be written as

$$V(r_0) = \sum_{i=1}^{A} \rho_0(r_0 - r_i) = \int \rho(r) \rho(r) dr,$$

(2.19)

where $\rho(r) = \sum_{i=1}^{A} \delta(r - r_i)$. For the moment we suppress the spin-isospin dependence of the interaction $v$ to make the argument simple. From §3.1 we explicitly introduce the spin dependence.

In the impulse approximation the effective interaction $v(r_0 - r)$ between the projectile and the nucleon in the target is approximated by

$$v(r_0 - r) = \int t_{NN}(p)e^{ip\cdot(r_0 - r)} \frac{dp}{(2\pi)^3},$$

(2.20)

where $t_{NN}(p)$ is the nucleon-nucleon transition matrix (NN $t$-matrix) for the momentum transfer $p$. Then

$$\langle k_i|V(r_0)|k_i\rangle = t_{NN}(k_f - k_i)\rho(k_f - k_i).$$

(2.21)

Defining the momentum transfer as $q \equiv k_f - k_i$, one finds

$$T_{n_0}^{(1)}(k_f, k_i) = t_{NN}(q)|\Phi_n|\rho(q)|\Phi_0\rangle,$$

(2.22)

where $\rho(q)$ is the density operator in the momentum space. This is the $T$-matrix in the $t\rho$ approximation, which is a simple version in various impulse approximations. The NN $t$-matrix is, in fact, expressed as a function of $q$, $Q = k_i + k_f$, and the incident energy in the laboratory frame $K_{lab}$, but we suppressed the $Q$ dependence, and we do not write $K_{lab}$ explicitly.

Substituting (2.22) into (2.14), one obtains $X^{(1)}$ as

$$X^{(1)} = |t_{NN}(q)|^2 \sum_{n \neq 0} |\langle \Phi_n|\rho(q)|\Phi_0\rangle|^2 \delta(\omega - (E_f - E_i)),$$

(2.23)

$$= \frac{\sqrt{s}}{M_R} |t_{NN}(q)|^2 R(q, \omega^{int}),$$

(2.24)

where $R(q, \omega^{int})$ is the response function of the density fluctuation

$$R(q, \omega^{int}) \equiv \sum_{n \neq 0} \langle \Phi_0|\rho^{\dagger}(q)|\Phi_n\rangle\langle \Phi_n|\rho(q)|\Phi_0\rangle \delta(\omega^{int} - E_n^{int}).$$

(2.25)

The quantity $M_R$ ($= M_T + E_n^{int}$) is the mass of the residual nucleus, and $s = (E_i + E_f)^2$. The response function defined in (2.25) is the strength per unit energy with respect to the energy transfer in the intrinsic frame $\omega^{int}$, while the double-differential cross section is the cross section per unit energy with respect to the energy transfer in the projectile-target c.m. system $\omega$. The factor $|d\omega^{int}/d\omega| = \sqrt{s}/M_R$ is the variable transformation coefficient derived from the equation [18]

$$\epsilon_f = \frac{s + M_f^2 - M_R^2}{2\sqrt{s}}.$$

(2.26)

Thus the one-step double-differential cross section can be written as a product of the square of the NN $t$-matrix and the response function.

Since the exact nuclear states $|\Phi_n\rangle$ are complicated, we cannot calculate the response functions by using the expression (2.25). To obtain a calculable expression for the response functions, we adopt two
approximations of TUL.[1] They assumed that |Φ₀⟩ is the 0-particle-0-hole (0p-0h) state and divided the excited state |Φₙ⟩ into 1p-1h states |Φₖ⟩ and other more complicated states and wrote it as

\[ |Φₙ⟩ = \sum_B a_B^{(n)} |Φₖ⟩ + δ|Φ₀⟩, \quad n \neq 0. \] (2.27)

With the equation

\[ \langle Φₙ|ρ(q)|Φ₀⟩ = \sum_B a_B^{(n)*} \langle Φₖ|ρ(q)|Φ₀⟩, \] (2.28)

which implies that the density operator ρ(q) can excite the target only to 1p-1h states, (2.25) becomes

\[ R(q, ω_{int}) = \sum_{n \neq 0} \sum_{B′B} \langle Φ₀|ρ(q)|Φₖ⟩ a_B^{(n)*} a_B^{(n)} \langle Φₖ|ρ(q)|Φ₀⟩ δ(ω_{int} - E_{n}). \] (2.29)

Then we use the TUL statistical approximation

\[ \sum_{n \neq 0} a_B^{(n)*} a_B^{(n)} δ(ω_{int} - E_{n}) = δ_{BB'} c_B(ω_{int}), \] (2.30)

by assuming that the \( a_B^{(n)} \) are random with respect to \( n \) for a fixed \( B \). For \( B = B' \), the left-hand side of (2.30) becomes \( c_B(ω_{int}) \), the probability per unit energy that the state |Φₖ⟩ is mixed in at the excitation energy \( ω_{int} \). Then we obtain the relation

\[ R(q, ω_{int}) \simeq \sum_B c_B(ω_{int}) \langle Φ₀|ρ(q)|Φₖ⟩ \langle Φₖ|ρ(q)|Φ₀⟩. \] (2.31)

This is calculable with a reasonable assumption for \( c_B(ω_{int}) \).

### 2.2 Two-step processes

The two-step \( T \)-matrix in the plane wave approximation can be written as

\[ T_{n0}^{(2)} (k, k_i) = \sum_{n′ \neq 0} \langle k_i| |Φₙ⟩|V|Φₙ⟩Gₙ′⟨Φₙ′⟩|V|Φ₀⟩|k⟩, \] (2.32)

where the propagator \( Gₙ′ \) is

\[ Gₙ′ = \left< Φₙ′ \left| \frac{1}{E⁺ - H₀} \right| Φₙ′ \right>. \] (2.33)

We rewrite (2.32) as

\[ T_{n0}^{(2)} (k, k_i) = \sum_{n′ \neq 0} \frac{dk_m}{(2π)^3} \frac{dk_m'}{(2π)^3} \langle k_i| |Φₙ⟩|V|Φₙ⟩|k_m⟩⟨k_m'Gₙ′|k_m||k_m⟩⟨k_f| |Φₙ'⟩|V|Φ₀⟩|k_i⟩. \] (2.34)

Since we are interested in high incident energy reactions, we can assume that the mean field \( U \) has a small effect on the particle. We assume that the momentum of the propagating particle does not vary during the sequential collisions, and thus

\[ U = U₀(k_m) δ(k_m - k_m'). \] (2.35)

The Green’s function becomes diagonal with respect to \( k_m \), so we can write the \( T \)-matrix as

\[ T_{n0}^{(2)} (k, k_i) = \sum_{n′ \neq 0} \frac{dk_m}{(2π)^3} \langle k_i| |Φₙ⟩|V|Φₙ⟩|k_m⟩Gₙ′(k_m)|k_m⟩⟨k_m| |Φₙ'⟩|V|Φ₀⟩|k_i⟩, \] (2.36)

where

\[ Gₙ′(k_m) \equiv \frac{1}{E⁺ - U₀(k_m) - \sqrt{M^2 + k_m^2} - \left( M_T + E_{n}^{int} + \frac{k_m^2}{2M_T} \right)}. \] (2.37)

5
Now we use the two TUL assumptions. The first one is

\[
\langle \Phi_n' | \mathcal{V} | \Phi_0 \rangle = \sum_C a_C^{(n')*} \langle \Phi_C | \mathcal{V} | \Phi_0 \rangle.
\]  

(2.38)

The other is the ‘spectator assumption’,

\[
\langle \Phi_n | \mathcal{V} | \Phi_n' \rangle = \sum_B a_B^{(n)*} \langle \Phi_B | \mathcal{V} | \Phi_0 \rangle,
\]

(2.39)

which implies that a collision always occurs by creating a new particle-hole pair and that the particle-hole pairs which are present in |\Phi_n'\rangle play only the role of spectators. Substituting (2.38), (2.39) and (2.21) into (2.36) we obtain

\[
T_{n0}^{(2)}(k_i, k_i) = \sum_{n' \neq 0} \sum_{B, C} \int \frac{dq_1}{(2\pi)^3} a_{Bn'}^{(1)n} t_{NN}(q_1 - q_1) \langle \Phi_B | \rho(q - q_1) | \Phi_0 \rangle \\
\times G_{n'}(k_i + q_1) a_C^{(n)*} t_{NN}(q_1) \langle \Phi_C | \rho(q_1) | \Phi_0 \rangle,
\]  

(2.40)

where \( q_1 = k_m - k_i \).

Next we proceed to calculate \( X^{(2)} \) of (2.14). It becomes

\[
X^{(2)}(n) = \frac{\sqrt{s}}{M_R} \sum_{n' \neq 0} \delta(\omega_{n'} - E_{n'}^*) \sum_{BC} \sum_{n''} \sum_{BC'} \int \frac{dq_1}{(2\pi)^3} \int \frac{dq_1'}{(2\pi)^3} a_B^{(n'*)} a_{B'n''} a_C^{(n'*)} a_{C'n''}
\times t_{NN}(q_1 - q_1') t_{NN}(q - q_1') \langle \Phi_0 | \rho(q - q_1') | \Phi_B \rangle \langle \Phi_B | \rho(q - q_1') | \Phi_0 \rangle \\
\times G_{n'}(k_i + q_1) G_{n''}(k_i + q_1) t_{NN}(q_1) \langle \Phi_0 | \rho(q_1') | \Phi_C \rangle \langle \Phi_C | \rho(q_1) | \Phi_0 \rangle.
\]

(2.41)

Substituting the TUL assumption

\[
\sum_{n \neq 0} \delta(\omega_{n} - E_{n}^*) a_{B'n''}^{(n)*} a_{C'n''}^{(n)*} = \delta_{BB'} \delta_{n'n''} c_{B'n''}(\omega_{n}^*)
\]

(2.42)

which is analogous to (2.30), we obtain

\[
X^{(2)}(n) = \frac{\sqrt{s}}{M_R} \sum_{n' \neq 0} \sum_{BC} \sum_{n''} \int \frac{dq_1}{(2\pi)^3} \int \frac{dq_1'}{(2\pi)^3} \times c_{B'n''}(\omega_{n}^*) a_C^{(n'*)} t_{NN}(q_1 - q_1') t_{NN}(q - q_1')
\]

\[
\times \langle \Phi_0 | \rho(q - q_1') | \Phi_B \rangle \langle \Phi_B | \rho(q - q_1') | \Phi_0 \rangle G_{n'}(k_i + q_1) G_{n''}(k_i + q_1')
\times t_{NN}(q_1) \langle \Phi_0 | \rho(q_1') | \Phi_C \rangle \langle \Phi_C | \rho(q_1) | \Phi_0 \rangle.
\]

(2.43)

With the identity \( \int d\omega \delta(\omega - (E_m - E_1)) = 1 \), \( X^{(2)} \) becomes

\[
X^{(2)}(n) = \frac{\sqrt{s}}{M_R} \sum_{n' \neq 0} \sum_{BC} \int \frac{dq_1}{(2\pi)^3} \int \frac{dq_1'}{(2\pi)^3} \times \delta(\omega_1 - (E_m - E_1)) c_{B'n''}(\omega_{n}^*) a_C^{(n'*)} a_{C'n''}
\times t_{NN}(q_1 - q_1') t_{NN}(q - q_1') \langle \Phi_0 | \rho(q - q_1') | \Phi_B \rangle \langle \Phi_B | \rho(q - q_1') | \Phi_0 \rangle \\
\times G(k_i + q_1, \omega_1) G^*(k_i + q_1') t_{NN}(q_1) t_{NN}(q_1') \langle \Phi_0 | \rho(q_1') | \Phi_C \rangle \langle \Phi_C | \rho(q_1) | \Phi_0 \rangle,
\]

(2.44)

where \( E_m \) is the target energy in the intermediate state,

\[
E_m = M_T + E_{n'}^{int} + \frac{k_m^2}{2M_T}.
\]

(2.45)
Here we have eliminated the $n'$ dependence of $G_{n'}(k_i + q_1)$ as
\[
G(k_i + q_1, \omega_1) = \frac{1}{E^+ - U_0(k_m) - \sqrt{M^2 + k_m^2} - \left(M_T + \frac{k_i^2}{2M_T} + \omega_1\right)}.
\] (2.46)

Rewriting the delta function with $\omega_{1\text{int}}^\prime$, (2.44) becomes
\[
X^{(2)} = \frac{s}{M_R^2} \sum_{n' \neq 0} \sum_{BCC'} \int d\omega_1 \int \frac{d\mathbf{q}_1}{(2\pi)^3} \int \frac{d\mathbf{q}_1'}{(2\pi)^3} \delta(\omega_{1\text{int}}^\prime - E_{n'}^\prime) c_{Bn'}(\omega_{1\text{int}}^\prime) a_C^{(n')} a_{C'}^{(n')} 
\times t_{NN}(\mathbf{q} - \mathbf{q}_1) t_{NN}^\dagger(\mathbf{q} - \mathbf{q}_1') \langle \Phi_0 | \rho(\mathbf{q} - \mathbf{q}_1) \Phi_0 \rangle 
\times G(k_i + q_1, \omega_1) G^*(k_i + q_1', \omega_1) t_{NN}(\mathbf{q}_1) t_{NN}^\dagger(\mathbf{q}_1') \langle \Phi_0 | \rho(\mathbf{q}_1') \Phi_C \rangle \langle \Phi_0 | \rho(\mathbf{q}_1) \Phi_0 \rangle.
\] (2.47)

Using the approximation
\[
c_{Bn'}(\omega_{1\text{int}}^\prime) \simeq c_B(\omega_{1\text{int}}^\prime - \omega_{1\text{int}}^\prime),
\] (2.48)

which results from the assumption (2.39) that the particle-hole pairs which are present in the intermediate state have nothing to do with creating a new particle-hole pair, we apply the TUL approximation (2.30) as
\[
X^{(2)} = \frac{s}{M_R^2} \int d\omega_1 \int \frac{d\mathbf{q}_1}{(2\pi)^3} \int \frac{d\mathbf{q}_1'}{(2\pi)^3} 
\times t_{NN}(\mathbf{q} - \mathbf{q}_1) t_{NN}^\dagger(\mathbf{q} - \mathbf{q}_1') 
\times \sum_{B} c_B(\omega_{1\text{int}}^\prime - \omega_{1\text{int}}^\prime) \langle \Phi_0 | \rho(\mathbf{q} - \mathbf{q}_1') \Phi_B \rangle \langle \Phi_B | \rho(\mathbf{q} - \mathbf{q}_1) \Phi_0 \rangle 
\times G(k_i + q_1, \omega_1) G^*(k_i + q_1', \omega_1) t_{NN}(\mathbf{q}_1) t_{NN}^\dagger(\mathbf{q}_1') \langle \Phi_0 | \rho(\mathbf{q}_1') \Phi_C \rangle \langle \Phi_0 | \rho(\mathbf{q}_1) \Phi_0 \rangle.
\] (2.49)

Consequently, we obtain
\[
X^{(2)} = \frac{s}{M_R^2} \int d\omega_1 \int \frac{d\mathbf{q}_1}{(2\pi)^3} \int \frac{d\mathbf{q}_1'}{(2\pi)^3} 
\times t_{NN}(\mathbf{q} - \mathbf{q}_1) t_{NN}^\dagger(\mathbf{q} - \mathbf{q}_1') R(\mathbf{q} - \mathbf{q}_1, \mathbf{q} - \mathbf{q}_1'; \omega_{1\text{int}}^\prime - \omega_{1\text{int}}^\prime) 
\times G(k_i + q_1, \omega_1) G^*(k_i + q_1', \omega_1) t_{NN}(\mathbf{q}_1) t_{NN}^\dagger(\mathbf{q}_1') R(\mathbf{q}_1, \mathbf{q}_1'; \omega_{1\text{int}}^\prime),
\] (2.50)

where $R(\mathbf{q}, \mathbf{q}'; \omega_{1\text{int}}^\prime)$ is expressed as
\[
R(\mathbf{q}, \mathbf{q}'; \omega_{1\text{int}}^\prime) = \sum_{A} c_A(\omega_{1\text{int}}^\prime) \langle \Phi_0 | \rho(\mathbf{q}') \Phi_A \rangle \langle \Phi_A | \rho(\mathbf{q}) \Phi_0 \rangle.
\] (2.51)

Here we have extended the response function to a nondiagonal form with respect to the momentum transfer $\mathbf{q}$. The last three factors in (2.50), which is the product of two NN $t$-matrices and the response function, represent the first collision, and with the next product of the Green’s functions, the particle in the intermediate state propagates in the target nucleus. Then the second collision occurs with the remaining three factors.

### 3 Formalism with spin

We are concerned with the unpolarized cross section of the $(p, p')$ scatterings and the spin longitudinal and spin transverse cross sections of the $(p, n)$ reactions. Hence we must treat the spin of the nucleon
where $\mu$, with the target angular momentum $J_T$. The symbol $\text{Tr}$ represents the trace of the spin substates of the scattering nucleon and $T'$ denotes the summation of all allowed initial and final states of the target,

$$\text{Tr}^{'TT} = \sum_{0,n} \langle \Phi_0 | T' | \Phi_n \rangle \langle \Phi_n | T | \Phi_0 \rangle \delta (\omega - (E_f - E_i)).$$

The observables $D_i$ introduced by Bleszynski et al. [19] are

$$ID_0 = \frac{1}{2J_T + 1} K T y T' [T_0 T'_0],$$

$$ID_n = \frac{1}{2J_T + 1} K T y T' [T_n T'_n],$$

$$ID_q = \frac{1}{2J_T + 1} K T y T' [T_q T'_q],$$

$$ID_p = \frac{1}{2J_T + 1} K T y T' [T_p T'_p].$$

We refer to $ID_q$ and $ID_p$ as the ‘spin longitudinal’ and ‘spin transverse’ cross sections respectively.

3.1 One-step processes

The NN $t$-matrices can be decomposed generally as

$$t_{NN}(q) = \sum_{\mu \bar{\mu}} \sigma_{\mu 0} \sigma_{\bar{\mu} 0} t_{\mu \bar{\mu}}(q),$$

with $\mu, \bar{\mu} = u, q, n, p$, where $\sigma_{j \mu} = I_j, \sigma_{\bar{\mu} 0} = \sigma_j \cdot \bar{\mu} (\mu \neq u), (j = 0, i)$. The operator $\sigma_i$ denotes the spin operator of the nucleon in the target. With this equation, (2.24) can be rewritten as

$$X^{(1)} = \sum_{\mu \bar{\mu}} X^{(1)}_{\mu \bar{\mu}} \sigma_{0 \mu} \sigma_{0 \bar{\mu}},$$

$$X^{(1)}_{\mu \bar{\mu}} = \frac{\sqrt{2}}{M_R} \sum_{\mu' \bar{\mu}'} t^{* \mu' \bar{\mu}'}(q) R_{\mu' \bar{\mu}'}(q, \omega_{\text{int}}) t_{\mu \bar{\mu}}(q),$$

where

$$R_{\mu' \bar{\mu}'}(q, \omega_{\text{int}}) \equiv \sum_B c_B(\omega_{\text{int}}) \langle \Phi_0 | \rho_{\mu'}(q) | \Phi_B \rangle \langle \Phi_B | \rho_{\bar{\mu}'}(q) | \Phi_0 \rangle,$$

$$\rho_{\bar{\mu}'}(q) \equiv \sum_{i=1}^{A} e^{-iq \cdot r_i} \sigma_{\bar{\mu}'}.$$
The unpolarized, the spin longitudinal and the spin transverse cross sections, \( I, \, ID_q \) and \( ID_p \), can be rewritten as
\[
I = K \sum_\mu X^{(1)}_{\mu\mu}, \quad ID_q = K X^{(1)}_{qq}, \quad ID_p = K X^{(1)}_{pp},
\]
respectively.

### 3.2 Two-step processes

In the spin dependent two-step formalism, \( X^{(2)} \) in (2.50) becomes
\[
X^{(2)} = \int d\omega_1 \int \frac{dq_1}{(2\pi)^3} \int \frac{dq_1'}{(2\pi)^3} \]
\[
\times \sum_{\mu_2\mu_2',\mu_1\mu_1'} X^{(2)}_{\mu_2\mu_2';\mu_1\mu_1'} \left( \frac{1}{2} \text{Tr}(\sigma_0 \sigma_0' \sigma_{0\mu_2} \sigma_{0\mu_1}) \right),
\]
\[
X^{(2)}_{\mu_2\mu_2';\mu_1\mu_1'} = \frac{s}{M^2} \sum_{\rho_2\rho_2'} \int f_{\rho_2\rho_2'}(q_1)f_{\rho_2\rho_2'}(q_1') \frac{1}{2} \text{Tr}(\sigma_0 \sigma_0' \sigma_{0\rho_2} \sigma_{0\rho_1}),
\]
\[
\times G(k_1 + q_1; \omega_1) G^*(k_1 + q_1'; \omega_1)
\]
\[
\times R_{\rho_2\rho_2'}(q - q_1, q - q_1'; \omega - \omega_1) R_{\rho_2\rho_2'}(q_1, q_1'; \omega_1),
\]
where \( \mu_1, \mu_1' = u, q_1, n_1, p_1 \) and \( \mu_2, \mu_2' = u, q_2, n_2, p_2 \), and so on. Here we defined the unit vectors
\[
\hat{q}_2 = \frac{q - q_1}{|q - q_1|}, \quad \hat{n}_2 = \frac{k_1 \times \hat{q}_2}{|k_1 \times \hat{q}_2|}, \quad \hat{p}_2 = \hat{q}_2 \times \hat{n}_2.
\]

We also extended the response function to nondiagonal form with respect to the spin direction as
\[
R_{\rho_2\rho_2'}(q, q'; \omega) = \sum_A \delta_{\rho_2} \langle \phi_A(q) | \rho_2 \rho_2'(q') | \phi_A(q) \rangle.
\]

The unpolarized, the spin longitudinal, and the spin transverse cross sections become
\[
I = K \frac{1}{2} \text{Tr} X^{(2)} = K \int d\omega_1 \int \frac{dq_1}{(2\pi)^3} \int \frac{dq_1'}{(2\pi)^3} \]
\[
\times \sum_{\mu_2\mu_2',\mu_1\mu_1'} X^{(2)}_{\mu_2\mu_2';\mu_1\mu_1'} \left( \frac{1}{2} \text{Tr}(\sigma_0 \sigma_0' \sigma_{0\mu_2} \sigma_{0\mu_1}) \right),
\]
\[
ID_q = K \int d\omega_1 \int \frac{dq_1}{(2\pi)^3} \int \frac{dq_1'}{(2\pi)^3} \]
\[
\times \sum_{\mu_2\mu_2',\mu_1\mu_1'} X^{(2)}_{\mu_2\mu_2';\mu_1\mu_1'} \left( \frac{1}{2} \text{Tr}(\sigma_0 \sigma_0' \sigma_{0\mu_2} \sigma_{0\mu_1}) \right) \left( \frac{1}{2} \text{Tr}(\sigma_0 \sigma_0' \sigma_{0\rho} \sigma_{0\rho}) \right),
\]
\[
ID_p = K \int d\omega_1 \int \frac{dq_1}{(2\pi)^3} \int \frac{dq_1'}{(2\pi)^3} \]
\[
\times \sum_{\mu_2\mu_2',\mu_1\mu_1'} X^{(2)}_{\mu_2\mu_2';\mu_1\mu_1'} \left( \frac{1}{2} \text{Tr}(\sigma_0 \sigma_0' \sigma_{0\mu_2} \sigma_{0\mu_1}) \right) \left( \frac{1}{2} \text{Tr}(\sigma_0 \sigma_0' \sigma_{0\rho} \sigma_{0\rho}) \right),
\]
respectively.

### 4 Practical method of calculation

#### 4.1 Various processes of two-step reactions

We calculate \((p, p')\) inelastic scattering and \((p, n)\) charge exchange reactions at 346 and 494 MeV. In the two-step \((p, n)\) reaction, the \((p, n)\) charge exchange collision occurs in the first step or in the second step.
(see Fig. 3). If the \((p,n)\) charge exchange collision occurs in the first step, the second step has to be a \((n,n')\) collision, and if the first step is a \((p,p')\) collision, then the second step has to be a \((p,n)\) collision.

Moreover, there are two cases in non-charge-exchange \((p,p')\) and \((n,n')\) collisions, because the nucleon in the target nucleus that participates in the collisions can be a proton or a neutron. Consequently, we must consider four cases in the two-step \((p,n)\) reaction.

For two-step \((p,p')\) scattering, we must consider five cases (see Fig. 4). If the \((p,n)\) charge exchange collision occurs in the first step, then the second step has to be a \((n,p)\) collision. In this case, the nucleon in the target nucleus that takes part in the first step collision has to be a neutron and that in the second step collision has to be a proton. This is the only case in which charge exchange collisions occur in the two-step \((p,p')\) scattering.

However, if no charge exchange collision occurs, the nucleons struck in the first step and the second step can be a proton or a neutron. Hence we can consider four cases in the two-step \((p,p')\) scattering that include no charge exchange collisions.

### 4.2 Green’s function

Our aim is to compare the two-step contribution with the one-step contribution. We adopted the plane wave approximation for the motion of the projectile and the ejectile. Thus in the one-step processes no absorption was included. We believe that the path lengths of the incident particles in the target nucleus in the one-step and in the two-step processes are nearly equal, and the effects of absorption become nearly equal in these processes. Therefore we should also remove the effect of absorption from the Green’s function. Then the effect of absorption in the two-step processes become zero, as in the one-step processes. We use the relativistic expression for the Green’s function and apply the on-energy shell approximation as

$$G(k_m) \simeq -i\delta \left( E - U_0 - \sqrt{M^2 + k_m^2} - \sqrt{(M_T + \omega_l^{\text{int}})^2 + k_m^2} \right).$$  \hspace{1cm} (4.1)

We take a representative value for \(k_m\) in \(U_0\) and set \(U_0 = -3.5\) MeV[20] and 5.5 MeV[21] for \(K_{\text{lab}} = 346\) MeV and 494 MeV, respectively.

Substituting (4.1) into (3.16), we obtain, for instance, from (3.19)

$$I = K \int_0^\omega d\omega_1 \int_0^\infty \frac{q_1^2}{(2\pi)^3} dq_1 \int_{-1}^1 d(\cos\theta_1) \int_0^{2\pi} d\phi_1 \int_0^\infty \frac{q_1'^2}{(2\pi)^3} dq_1' \int_{-1}^1 d(\cos\theta_1') \int_0^{2\pi} d\phi_1'$$

$$\times \sum_{\mu,\mu'} \sum_{\mu_1,\mu_1'} X^{(2)}_{\mu\mu_1,\mu'\mu_1'} \frac{1}{2} \text{Tr}(\sigma_0\mu_1')\sigma_0\mu_2'\sigma_0\mu_2'\sigma_0\mu_1),$$  \hspace{1cm} (4.2)

where

$$X^{(2)}_{\mu\mu_1,\mu'\mu_1'} = \frac{\pi^2 s}{M^2} \sum_{\mu_2,\mu_2',\mu_1,\mu_1'} \sum$$

$$\times T_{\mu,\mu_1}'(K_{\text{lab}},q_1')T_{\mu,\mu_2}'(K_{\text{lab}},q - q_1')T_{\mu_1,\mu_2'}(K_{\text{lab}},q - q_1)T_{\mu_1,\mu_1'}(K_{\text{lab}},q_1)$$

$$\times \delta \left( E - U_0 - \sqrt{M^2 + (k_i + q_1)^2} - \sqrt{(M_T + \omega_l^{\text{int}})^2 + (k_i + q_1)^2} \right)$$

$$\times \delta \left( E - U_0 - \sqrt{M^2 + (k_i + q_1')^2} - \sqrt{(M_T + \omega_l^{\text{int}})^2 + (k_i + q_1')^2} \right)$$

$$\times R_{\mu_2\mu_2'}(q - q_1, q - q_1'; \omega_l^{\text{int}})R_{\mu_1\mu_1'}(q_1, q_1'; \omega_l^{\text{int}}).$$  \hspace{1cm} (4.3)

We set \(z\)-axis parallel to the direction of \(k_i\). With the delta functions, we can carry out the integration of \(\cos\theta_1\) and \(\cos\theta_1'\) analytically.

Other integrations are carried out numerically. The mesh sizes were chosen as \(\Delta \omega_l^{\text{int}} = 10.0\) MeV, \(\Delta q_1^{\text{int}} = 0.2\) (1/fm) and \(\Delta \phi_1 = 15.0^\circ\), where \(q_1^{\text{int}} = \{(A - 1)/A\}q_1\), with which we can obtain sufficiently accurate results. The integration range of \(q_1^{\text{int}}\) is restricted up to 4.4 (1/fm), beyond which the response functions are negligibly small.
Here we have written the energy dependence of the NN $t$-matrices explicitly and assumed that the projectile has kinetic energy $K_{\text{lab}}$ in the first collision and $K'_{\text{lab}}$ in the second collision, where

\[ K'_{\text{lab}} = K_{\text{lab}} - \omega_{\text{1lab}}, \quad \omega_{\text{1lab}} = \frac{-t - M_T^2 + M_B^2}{2M_T}, \quad t = \omega^2 - q_1^2. \tag{4.4} \]

### 4.3 NN $t$-matrices

We use the method of Love and Franey [22] to calculate the NN $t$-matrix. The derivations of the formulae are given in detail in the Appendix. We use the parameter values of Ref. [23] and treat the exchange terms by replacing $Q_{\text{cm}}$ with $k_1$, as is suggested in Ref. [22].

The scattering amplitude $f_{\text{NN}}(K_{\text{lab}}, q_{\text{cm}})$ is written as [24]

\[ f_{\text{NN}}(K_{\text{lab}}, q_{\text{cm}}) = (A_0 + A_1(\tau_0 \cdot \tau_i)) + (B_0 + B_1(\tau_0 \cdot \tau_i))\sigma_0 \cdot \hat{n}_{\text{cm}} \sigma_i \cdot \hat{n}_{\text{cm}} \]
\[ + (C_0 + C_1(\tau_0 \cdot \tau_i))\sigma_0 \cdot \hat{n}_{\text{cm}} + (C_0 + C_1(\tau_0 \cdot \tau_i))\sigma_i \cdot \hat{n}_{\text{cm}} \]
\[ + (E_0 + E_1(\tau_0 \cdot \tau_i))\sigma_0 \cdot \hat{q}_{\text{cm}} \sigma_i \cdot \hat{q}_{\text{cm}} \]
\[ + (F_0 + F_1(\tau_0 \cdot \tau_i))\sigma_0 \cdot \hat{Q}_{\text{cm}} \sigma_i \cdot \hat{Q}_{\text{cm}}, \tag{4.5} \]

where

\[ \hat{q}_{\text{cm}} = \frac{k'_{\text{cm}} - k_{\text{cm}}}{|k'_{\text{cm}} - k_{\text{cm}}|}, \quad \hat{n}_{\text{cm}} = \frac{k_{\text{cm}} \times k'_{\text{cm}}}{|k_{\text{cm}} \times k'_{\text{cm}}|}, \quad \hat{Q}_{\text{cm}} = \hat{n}_{\text{cm}} \times \hat{n}_{\text{cm}}, \tag{4.6} \]

and $k_{\text{cm}} (k'_{\text{cm}})$ is the projectile (ejectile) momentum in the NN c.m. system.

The relation between the NN $t$-matrices and the NN scattering amplitudes in the NN c.m. frame is

\[ t_{\text{NN}}(K_{\text{lab}}, q_{\text{cm}}) = \eta f_{\text{NN}}(K_{\text{lab}}, q_{\text{cm}}), \tag{4.7} \]
\[ \eta = \frac{-4\pi}{\sqrt{M^2 + k_{\text{cm}}^2}}. \tag{4.8} \]

Therefore the $t_{\mu \nu}$ in (3.9) are given by

\[ t_{uu} = \eta(A_0 + A_1(\tau_0 \cdot \tau_i)), \tag{4.9} \]
\[ t_{nn} = \eta(B_0 + B_1(\tau_0 \cdot \tau_i)), \tag{4.10} \]
\[ t_{nu} = \eta(C_0 + C_1(\tau_0 \cdot \tau_i)), \tag{4.11} \]
\[ t_{un} = \eta(C_0 + C_1(\tau_0 \cdot \tau_i)), \tag{4.12} \]
\[ t_{q\bar{q}} = \eta(E_0 + E_1(\tau_0 \cdot \tau_i)), \tag{4.13} \]
\[ t_{pp} = \eta(F_0 + F_1(\tau_0 \cdot \tau_i)), \tag{4.14} \]

and all others are 0.

Calculating the expectation values of the isospin operators, we get the amplitudes for proton-proton scattering as

\[ t_{uu} = \eta(A_0 + A_1), \ldots. \tag{4.15} \]

For neutron-neutron scattering these are the same as for proton-proton scattering.

Other types of NN collisions are proton-neutron charge exchange collisions and proton-neutron non-charge exchange collisions. Their amplitudes are

\[ t_{uu} = 2\eta A_1, \ldots, \tag{4.16} \]
\[ t_{uu} = \eta(A_0 - A_1), \ldots, \tag{4.17} \]

respectively.

We note that the isospin operators of the struck nucleons are included in the response functions. Hence we use the $t$-matrix amplitudes for the charge exchange collision as

\[ t_{uu} = \sqrt{2}\eta A_1, \ldots, \tag{4.18} \]

instead of (4.16).
4.4 Response functions

We obtained a simple expression for the response function in (2.31). We take account of the weight function $c_B(\omega)$ through a complex potential for the single particles, which is of a Woods-Saxon-type central potential with the derivative form spin-orbit potential as

$$U'(r) = -(V + iW) \frac{1}{1 + \exp\left(\frac{r - R_0}{a_0}\right)}$$

$$- 2 \left(\frac{1}{m_x}\right)^2 V_{ls} \frac{\exp\left(\frac{r - R_{SO}}{a_{SO}}\right)}{r} \left[1 + \exp\left(\frac{r - R_{SO}}{a_{SO}}\right)\right] r (1 \cdot s) + V_{Coul},$$

(4.19)

where the radius parameters are $r_0 = r_{SO} = r_{Coul} = 1.27$ fm, the diffusenesses are $a_0 = a_{SO} = 0.67$ fm, the potential depths are determined by the binding energy of the last occupied single particle level, and the imaginary part of the potential is $W = 5.0$ MeV. [25] The depth of the $1 \cdot s$ part is chosen to be 6.5 MeV (10.0 MeV) for $^{12}$C ($^{40}$Ca). [20] The response functions used in the DWIA calculation depicted in Figs. 1 and 2 include the effects of the RPA correlation with the $\Delta$ degree of freedom as well as the effective mass and the spreading width of hole states. However, here we calculate the response functions with no correlations in order to extract the effect of the nuclear reaction mechanism exclusively.

We extended the response functions to nondiagonal form with respect to the momentum transfer $q$ in the two-step formalism. The spin scalar isoscalar response function in the nondiagonal form is defined as [26]

$$R(q, q'; \omega^{\text{int}}) \equiv \sum_{B \neq 0} \left\langle \Phi_0 \sum_i e^{i q' r_i} \Phi_B \right| \left| \sum_j e^{-i q r_j} \Phi_B \right\rangle \delta(\omega^{\text{int}} - E_B^{\text{int}}).$$

(4.20)

The response functions which include the spin operators are generally written in the form

$$R_{\alpha' \alpha}(q, q'; \omega^{\text{int}}) \equiv \sum_{B \neq 0} \left\langle \Phi_0 \left| \sum_i \sigma_i \cdot \hat{u}' e^{i q' r_i} \Phi_B \right| \sum_j \sigma_j \cdot \hat{u} e^{-i q r_j} \Phi_B \right\rangle \delta(\omega^{\text{int}} - E_B^{\text{int}}).$$

(4.21)

Here the response function has also been extended to the nondiagonal form with respect to the spin direction.

If the spin-orbit force is negligible and the target nucleus is spin-saturated, the single particle states can be written as $|n, l, m, s = 1/2, \mu\rangle$ with the principal quantum number $n$, the angular momentum $l$, its third component $m$, the spin $s$, and its third component $\mu$. Then the response function can be separated into the spin part and the orbital part as

$$R_{\alpha' \alpha}(q, q'; \omega^{\text{int}}) \equiv \sum_n \sum_{l_n m_n} \sum_{\mu_p \mu_h} \left[ \left\langle \mu_h | \sigma \cdot \hat{u}' | \mu_p \right\rangle \left\langle \mu_p | \sigma \cdot \hat{u} | \mu_h \right\rangle \right]$$

$$\times \left[ \left\langle n_l h_k m_h | e^{i q' r} | n_p l_p m_p \right\rangle \left\langle n_p l_p m_p | e^{-i q r} | n_h l_h m_h \right\rangle \right] \delta(\omega^{\text{int}} - E_B^{\text{int}}),$$

(4.22)

where $p$ and $h$ denote a particle and a hole, respectively. The spin part becomes $\text{Tr}[\sigma \cdot \hat{u}' \sigma \cdot \hat{u}] = 2 \hat{u}' \cdot \hat{u}$, while that of the response function without the spin operators becomes the trace of the unit matrix. Thus we get the relation

$$R_{\alpha' \alpha}(q, q'; \omega^{\text{int}}) = (\hat{u}' \cdot \hat{u}) R(q, q'; \omega^{\text{int}}).$$

(4.23)

This relation is plotted in Fig. 5, and we judge it reasonable.
If only one spin operator is included in the response function, then the spin part becomes \( \text{Tr}[\sigma \cdot \hat{u}] = 0 \), and the response functions become

\[
R_u(q, q'; \omega^{\text{int}}) \equiv \sum_{B \neq 0} \left< \Phi_0 \sum_i e^{i q \cdot r_i} \right| \Phi_B \left| \sum_j \sigma_j \cdot \hat{u} e^{-i q \cdot r_j} \right| \Phi_0 \right> \delta(\omega^{\text{int}} - E_B^{\text{int}})
= 0,
\]

\[
R_{\bar{u}}(q, q'; \omega^{\text{int}}) \equiv \sum_{B \neq 0} \left< \Phi_0 \sum_i \sigma_i \cdot \hat{\bar{u}} e^{i q' \cdot r_i} \right| \Phi_B \left| \sum_j e^{-i q \cdot r_j} \right| \Phi_0 \right> \delta(\omega^{\text{int}} - E_B^{\text{int}})
= 0.
\]

(4.24)

We cannot calculate the response functions for charge exchange collisions with these relations. Therefore we must prepare two more response functions. One is for the \((p, n)\) collision and the other is for the \((n, p)\) collision. The difference among these three response functions is seen in Fig. 6. These response functions also satisfy the relations

\[
R^+_{u, a}(q, q'; \omega^{\text{int}}) = (\hat{u}' \cdot \hat{u}) R^+(q, q'; \omega^{\text{int}}),
\]

\[
R^-_{u, a}(q, q'; \omega^{\text{int}}) = (\hat{u}' \cdot \hat{u}) R^-(q, q'; \omega^{\text{int}}),
\]

(4.25)

(4.26)

where

\[
R^+(q, q'; \omega^{\text{int}}) \equiv \sum_{B \neq 0} \left< \Phi_0 \sum_i e^{i q \cdot r_i \tau_-} \right| \Phi_B \left| \sum_j e^{-i q \cdot r_j \tau_+} \right| \Phi_0 \right> \delta(\omega^{\text{int}} - E_B^{\text{int}}),
\]

\[
R^-(q, q'; \omega^{\text{int}}) \equiv \sum_{B \neq 0} \left< \Phi_0 \sum_i e^{i q \cdot r_i \tau_+} \right| \Phi_B \left| \sum_j e^{-i q \cdot r_j \tau_-} \right| \Phi_0 \right> \delta(\omega^{\text{int}} - E_B^{\text{int}}),
\]

(4.27)

with \( \tau_\pm = (\tau_x \pm i \tau_y)/\sqrt{2} \). The response functions \( R^+ \) and \( R^- \) are for the \((p, n)\) and \((n, p)\) collisions, respectively. With these three response functions, it is sufficient to calculate the various response functions that appear in (4.3). Using the 'spectator assumption', we assume that the target is in the ground state of \(^{12}\text{C} (^{40}\text{Ca})\), even when \((n, n')\) or \((n, p)\) collision occurs in the second step.

We must consider the struck nucleons when we use the response function for \((p, p')\) collisions. If we write the response function for \((p, p')\) collisions as (4.20), the struck nucleon can be a proton or a neutron. However, when we calculate the cross sections, we distinguish a proton from a neutron, as shown in Figs. 3 and 4. Therefore we must divide the spin scalar isoscalar response function by 2 when we apply this response function to (4.3).

### 4.5 Cross sections

Using the spin-dependent form of the cross sections in (3.14), the one-step unpolarized, the spin longitudinal and transverse cross sections are written as

\[
I = K \frac{\sqrt{\pi}}{M_R} \sum_{\mu \bar{\mu}} |t_{\mu \bar{\mu}}(q)|^2 R(q, \omega^{\text{int}}),
\]

(4.28)

\[
ID_q = K \frac{\sqrt{\pi}}{M_R} |t_{qq}(q)|^2 R_{qq}(q, \omega^{\text{int}}),
\]

(4.29)

\[
ID_p = K \frac{\sqrt{\pi}}{M_R} |t_{pp}(q)|^2 R_{pp}(q, \omega^{\text{int}}),
\]

(4.30)

respectively. Here we have used the approximation \( q \simeq q_{\text{cm}} \) which was justified in Ref. [17]. The summations of \( \mu \) and \( \bar{\mu} \) run over only \((\mu, \bar{\mu}) = (u, u), (n, n), (u, n), (u, u), (q, q)\) and \((p, p)\), and we have used the relation \( R_{\mu \bar{\mu}}(q, \omega^{\text{int}}) = R(q, \omega^{\text{int}}) \), which is derived from (4.23).
We now explain the method of calculating the two-step cross sections given in (3.19)–(3.21). The quantity \( \chi^{(2)}_{\mu_1\mu_2\mu_1\mu_2} \) has four NN \( t \)-matrices, each of which generates 6 terms. Therefore we encounter 6\(^4\) (\( = 1296 \)) terms for each isospin-dependent process explained in §4.1. However, with the approximation (4.24) we can eliminate many terms, and finally we have to take only 400 terms (see Fig. 7). We see that the response functions are common except the scalar products that appear from the spin parts (see (4.23) and so on).

Next we carry out the summation of 400 products of four NN \( t \)-matrices with the scalar products. The traces of the spin operators of the incident nucleon are easily calculated as, for instance,

\[
\frac{1}{2} \text{Tr}(\sigma_\mu \sigma_{0\mu_2} \sigma_{0\mu_1}) = s \cdot q,
\]

\[
\frac{1}{2} \text{Tr}(\sigma_\mu \sigma_{0\mu_2}) = s \cdot p,
\]

with the relation

\[
\sigma_{0\mu_2} = I_0 s_0 + \sigma_0 \cdot s,
\]

where \( s_0 \) is a real number and \( s \) is a complex vector. After the summation we calculate the five-fold integration of the sum multiplied by the spin scalar response functions.

In fact, the spin scalar response functions are also common if charge exchange collision occurs in the same step in the \((p, n)\) reactions. As for the \((p, p')\) scattering, they are common to the four processes that have no charge exchange collision (see Figs. 3 and 4). Hence, in a practical calculation, we also carry out the summation of the products of four NN \( t \)-matrices with the scalar products in these processes, in which the spin scalar response functions are common, and after the summation we calculate the five-fold integration.

5 Results

In Fig. 8 we display the double differential cross sections of the \((p, p')\) scattering and the spin longitudinal and the spin transverse \( ^{12}\text{C}(p, n) \) reactions at 494 MeV as functions of the energy transfer \( \omega_{\text{lab}} \) in the laboratory frame. In this calculation the momentum transfer \( q_{\text{int}} \) in the intrinsic frame of the target is 1.55 (1/fm) (\( q_{\text{lab}} \) in the laboratory frame is 1.69–1.72 (1/fm)). The scattering angle corresponding to this momentum transfer is about 18\(^\circ\). [11, 12, 13]

In the \( ^{12}\text{C}(p, p')\) scattering, the peak of the one-step cross section is around \( \omega_{\text{lab}} = 60 \) MeV in this no-correlation calculation, and the contribution of the two-step processes there is only around 0.5% of the one-step process. The two-step cross section is still increasing around \( \omega_{\text{lab}} = 125 \) MeV, and it becomes about 6.0% of the one-step process there. These results can be seen in panel (d), which indicates the ratios of the two-step cross sections to the one-step cross sections. The contribution of the two-step processes to the \( ^{12}\text{C}(p, p')\) scattering is negligibly small. However, in the \( ^{12}\text{C}(p, n) \) reactions, there are considerable contributions from the two-step processes. In the spin longitudinal \( ^{12}\text{C}(p, n) \) reaction, the contribution of the two-step processes is 3.5% of the one-step process at \( \omega_{\text{lab}} = 60 \) MeV and 28% at \( \omega_{\text{lab}} = 125 \) MeV. For the spin transverse \( ^{12}\text{C}(p, n) \) reaction, the two-step contribution becomes larger than that for the spin longitudinal reaction, and at \( \omega_{\text{lab}} = 60 \) and 125 MeV the ratios become 9.0% and 68%, respectively. From these results we can say that the two-step processes become more effective as the energy transfer increases in the quasi-elastic region.

Figure 9 displays the double differential cross section at 346 MeV of the same reactions. The momentum transfer \( q_{\text{int}} \) is 1.52 (1/fm) (\( q_{\text{lab}} \) is 1.66–1.68 (1/fm)). It corresponds to a scattering angle of 22\(^\circ\).[14] The contributions from the two-step processes become larger than those at 494 MeV. In the \((p, p')\) scattering, the ratio at \( \omega_{\text{lab}} = 60 \) MeV is less than 1% and at \( \omega_{\text{lab}} = 125 \) MeV it is about 11%. In the spin longitudinal \( ^{12}\text{C}(p, n) \) reaction, the ratio is close to 40% at \( \omega_{\text{lab}} = 125 \) MeV, while that of the spin transverse \( ^{12}\text{C}(p, n) \) reaction exceeds 75% there. The contribution to the spin transverse reaction is much larger than that to the spin longitudinal reaction, as in the case of 494 MeV.

We assume that the \( E \)-term and the \( F \)-term in the NN \( t \)-matrices mainly determine the spin longitudinal and the spin transverse cross sections, respectively, in the two-step processes. The difference between the two-step contributions for the spin longitudinal and the spin transverse cross sections may
be due to the momentum transfer dependence of the $E$-term and the $F$-term which are shown in Fig. 10. The dominant real part of the $E$-term changes sign at $0.7$ (1/fm), while the $F$-term never becomes 0. The square of NN $t$-matrices for proton-proton and non-charge exchange neutron-proton scattering decreases monotonically as a function of the momentum transfer. Thus the contribution from the $E$-term becomes smaller than that from the $F$-term through the integration of the momentum transfers.

The results for $^{40}\text{Ca}$ at 494 and 346 MeV are given in Figs. 11 and 12, respectively. Since $^{40}\text{Ca}$ is a spin-saturated nucleus, the relation (4.23) becomes a better approximation. We guess that the results are more reliable than those for $^{12}\text{C}$. The two-step contributions in comparison with the one-step contributions are larger than those for the $^{12}\text{C}$ case. The momentum transfer $q_{\text{int}}$ for the reactions at 494 MeV is $1.655$ (1/fm) ($q_{\text{lab}}$, is $1.69–1.71$ (1/fm)). For the $(p, p')$ scattering and the spin longitudinal and spin transverse $^{40}\text{Ca}(p, n)$ reactions, the ratios are $7.3\%$, $45\%$ and $87\%$ at $\omega_{\text{lab}} = 120$ MeV, respectively.

At 346 MeV the momentum transfer $q_{\text{int}}$ is $1.625$ (1/fm) ($q_{\text{lab}}$, is $1.66–1.68$ (1/fm)). The ratios reach $13\%$ for the $(p, p')$ scattering, $49\%$ for the spin longitudinal $(p, n)$ reaction and $88\%$ for the spin transverse reaction at $\omega_{\text{lab}} = 120$ MeV. We can say that the contribution from the two-step processes is larger at 346 MeV than at 494 MeV only for the $(p, p')$ scattering in the $^{40}\text{Ca}$ case. Here we have confirmed again that the two-step contribution to the spin transverse reaction is much larger than that to the spin longitudinal reaction.

6 Summary and conclusion

We constructed a formalism for the two-step direct reaction within the framework of the plane wave approximation in order to compare its contribution with the one-step process. With this approximation we factored the nucleon-nucleus $T$-matrix into the NN $t$-matrix and the transition density in the one-step formalism. In the two-step formalism we expressed the two-step cross section with a nondiagonal response function with respect to the momentum transfer $q$.

In the Green’s function we removed the effect of absorption, because the motion of the projectile and the ejectile was described with plane waves. We applied the on-energy shell approximation to the Green’s function.

With this formalism, we calculated the cross sections of the $^{12}\text{C}$, $^{40}\text{Ca}(p, p')$ scattering and the spin longitudinal and spin transverse cross sections of the $^{12}\text{C},^{40}\text{Ca}(p, n)$ reactions at 346 and 494 MeV. The scattering angles were set to $22^\circ$ and $18^\circ$, respectively, for comparison with the experiments at LAMPF[13] and RCNP.[14]

In $(p, p')$ scattering, the contributions from the two-step processes were found to be small. However, there are appreciable contributions from two-step processes in the $(p, n)$ reactions. In the spin longitudinal $(p, n)$ reaction, the two-step contribution is about $30\%–50\%$ of the one-step cross section, while the ratio for the spin transverse $(p, n)$ cross sections become about $70\%–90\%$ around $\omega_{\text{lab}} = 120$ MeV. We believe that the difference between the two-step contributions for the spin longitudinal cross section and the spin transverse cross section is due to the difference between the momentum transfer dependences of the $E$-term and the $F$-term in the NN scattering amplitude.

We found that the contributions of the two-step processes become larger as the energy transfer increases in the quasi-elastic region. We also found that the two-step contributions in comparison with the one-step contributions are larger in the $^{40}\text{Ca}$ case than in the $^{12}\text{C}$ case.

It has been reported that in DWIA calculations the cross sections of the spin longitudinal $(p, n)$ reactions are underestimated beyond the quasi-elastic peak region, and for the spin transverse $(p, n)$ reactions, the calculations amount to only half of the experimental results. We give the DWIA results multiplied by the ratios of the sum of the one- and the two-step cross sections to the one-step cross section for $^{12}\text{C}$ in Fig. 13 and for $^{40}\text{Ca}$ in Fig. 14. From these figures we can clearly see that the theoretical results including the two-step contribution are closer to the experimental results than the DWIA results, particularly for $ID_q$ at 494 MeV.

However, our results are still insufficient for explaining all of the discrepancy. In those regions where the cross sections are still underestimated, the response functions that include 2p-2h correlations may have some effect. Further multi-step processes must be investigated in those regions. A two-step calculation with full distortion is of course needed to obtain a more quantitatively reliable conclusion.
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Appendix

We used the method of Love and Franey [22] to calculate the NN \( t \)-matrix. It is written in the form

\[
t_{NN}(K_{lab}, q) = [\tilde{V}_{SO}^C(q) - \tilde{V}_{SO}^C(Q)]P_{SO} + [\tilde{V}_{SE}^C(q) + \tilde{V}_{SE}^C(Q)]P_{SE}
+ [\tilde{V}_{TO}^C(q) - \tilde{V}_{TO}^C(Q)]P_{TO} + [\tilde{V}_{TE}^C(q) + \tilde{V}_{TE}^C(Q)]P_{TE}
+ i\frac{q}{4}[Q\tilde{V}_{LSO}^L(q) + q\tilde{V}_{LSO}^L(Q)][(\sigma_0 + \sigma_i) \cdot \hat{n}P_{TO}
+ i\frac{q}{4}[Q\tilde{V}_{LSE}^L(q) - q\tilde{V}_{LSE}^L(Q)][(\sigma_0 + \sigma_i) \cdot \hat{n}P_{TE}
= [\tilde{V}_{TNO}^T(q)S_{12}^ODD(q) - \tilde{V}_{TNO}^T(Q)S_{12}^ODD(\hat{Q})]
- [\tilde{V}_{TNE}^T(q)S_{12}^EVEN(q) + \tilde{V}_{TNE}^T(Q)S_{12}^EVEN(Q)],
\]

(A.1)

where

\[
q = 2k_{cm}\sin(\theta/2),
\]
\[
Q = 2k_{cm}\cos(\theta/2),
\]

and

\[
P_{SO} = P_{s=0}P_{t=0} = \frac{1 - \sigma_0 \cdot \sigma_i}{4} 1 - \tau_0 \cdot \tau_i,
\]
\[
P_{SE} = P_{s=0}P_{t=1} = \frac{1 - \sigma_0 \cdot \sigma_i}{4} 3 + \tau_0 \cdot \tau_i,
\]
\[
P_{TO} = P_{s=1}P_{t=1} = \frac{3 + \sigma_0 \cdot \sigma_i}{4} 3 + \tau_0 \cdot \tau_i,
\]
\[
P_{TE} = P_{s=1}P_{t=0} = \frac{3 + \sigma_0 \cdot \sigma_i}{4} 1 - \tau_0 \cdot \tau_i,
\]
\[
S_{12}^ODD(q)P_{t=1} = (3\sigma_0 \cdot \hat{q}\sigma_i \cdot \hat{q} - \sigma_0 \cdot \sigma_i)\frac{3 + \tau_0 \cdot \tau_i}{4},
\]
\[
S_{12}^EVEN(q)P_{t=0} = (3\sigma_0 \cdot \hat{q}\sigma_i \cdot \hat{q} - \sigma_0 \cdot \sigma_i)\frac{1 - \tau_0 \cdot \tau_i}{4},
\]

(A.2)

(A.3)

(A.4)

(A.5)

(A.6)

(A.7)

(A.8)

(A.9)

with the projectile wave number \( k_{cm} \) in the NN c.m. system. The potential depths are given as

\[
\tilde{V}_{s^C}^C(k) = 4\pi \sum_i \frac{V_{s^C,i}^C(R_k^i)^3}{1 + (kR_k^i)^2},
\]
\[
\tilde{V}_{LS^\pi}^L(k) = 8\pi \sum_i \frac{V_{LS^\pi}^Lk(R_k^i)^5}{[1 + (kR_k^i)^2]^2},
\]
\[
\tilde{V}_{TNS^\pi}^T(k) = 32\pi \sum_i \frac{V_{TNS^\pi}Tk^2(R_k^i)^7}{[1 + (kR_k^i)^2]^3},
\]

(A.10)

(A.11)

(A.12)

where \( k = q \) or \( Q \), \( s \) is the spin singlet (S) or spin triplet (T), and \( \pi \) is the parity in the NN system. The summations over \( i \) are taken over several ranges. It is these values, \( V_{s^C,i}^C, V_{LS^\pi}^L \) and \( V_{TNS^\pi}^T \) that are given in the table in Ref. [23].

Defining

\[
t_{SO}^C = \tilde{V}_{SO}^C(q) - \tilde{V}_{SO}^C(Q),
\]
\[
t_{SE}^C = \tilde{V}_{SE}^C(q) + \tilde{V}_{SE}^C(Q),
\]
\[
t_{TO}^C = \tilde{V}_{TO}^C(q) - \tilde{V}_{TO}^C(Q),
\]

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\[ t_{\text{TE}}^C = \tilde{V}_{\text{TE}}^C(q) + \tilde{V}_{\text{TE}}^C(Q), \]
\[ t_{\text{LSO}}^C = \frac{1}{4}[Q\tilde{V}_{\text{LSO}}^C(q) + q\tilde{V}_{\text{LSO}}^C(Q)], \]
\[ t_{\text{LSE}}^C = \frac{1}{4}[Q\tilde{V}_{\text{LSE}}^C(q) - q\tilde{V}_{\text{LSE}}^C(Q)], \]
\[ t_{\text{TNO}}^C = \tilde{V}_{\text{TNO}}^C(q), \tilde{V}_{\text{TNO}}^C(Q), \]
\[ t_{\text{TNE}}^C = \tilde{V}_{\text{TNE}}^C(q), \tilde{V}_{\text{TNE}}^C(Q), \]  
(A.13)

and noting Eqs. (4.5) and (4.7), one obtains the relations

\[ \eta_{A0} = \frac{1}{16}(t_{\text{SO}}^C + 3t_{\text{SE}}^C + 9t_{\text{TO}}^C + 3t_{\text{TE}}^C), \]  
(A.14)

\[ \eta_{A1} = \frac{1}{16}(-t_{\text{SO}}^C + t_{\text{SE}}^C + 3t_{\text{TO}}^C - 3t_{\text{TE}}^C), \]  
(A.15)

\[ \eta_{B0} = \frac{1}{16}(-t_{\text{SO}}^C - 3t_{\text{SE}}^C + 3t_{\text{TO}}^C + t_{\text{TE}}^C), \]
\[ + \frac{1}{4}(3t_{\text{TNO}}^C(q) + t_{\text{TNE}}^C(q) - 3t_{\text{TNO}}^C(Q) + t_{\text{TNE}}^C(Q)), \]  
(A.16)

\[ \eta_{B1} = \frac{1}{16}(t_{\text{SO}}^C - t_{\text{SE}}^C + t_{\text{TO}}^C - t_{\text{TE}}^C) \]
\[ + \frac{1}{4}(t_{\text{TNO}}^C(q) - t_{\text{TNE}}^C(q) - 3t_{\text{TNO}}^C(Q) - t_{\text{TNE}}^C(Q)), \]  
(A.17)

\[ \eta_{C0} = \frac{1}{4}(3t_{\text{LSO}}^C + t_{\text{LSE}}^C), \]  
(A.18)

\[ \eta_{C1} = \frac{1}{4}(t_{\text{LSO}}^C - t_{\text{LSE}}^C), \]  
(A.19)

\[ \eta_{E0} = \eta_{B0} + \frac{1}{4}(-9t_{\text{TNO}}^C(q) - 3t_{\text{TNE}}^C(q)), \]  
(A.20)

\[ \eta_{E1} = \eta_{B1} + \frac{1}{4}(-3t_{\text{TNO}}^C(q) + 3t_{\text{TNE}}^C(q)), \]  
(A.21)

\[ \eta_{F0} = \eta_{B0} + \frac{1}{4}(9t_{\text{TNO}}^C(Q) - 3t_{\text{TNE}}^C(Q)), \]  
(A.22)

\[ \eta_{F1} = \eta_{B1} + \frac{1}{4}(3t_{\text{TNO}}^C(Q) + 3t_{\text{TNE}}^C(Q)). \]  
(A.23)

The corresponding \( t \)-matrix in the nucleon-nucleus system can, to a good approximation, be written by replacing \( Q \) with the wave number of the projectile in the nucleon-nucleus system \( k_i \), which is expressed as

\[ k_i^2 \equiv M^2 A \beta \left[ \frac{1 + \alpha}{1 + \beta} \right], \quad \alpha \equiv \frac{K_{\text{lab}}}{2M}, \quad \beta \equiv \frac{4\alpha A}{(A + 1)^2}, \]  
(A.24)

where \( A \) is the mass number of the target nucleus.

Moreover, for calculating nucleon-nucleus scattering, an \( A \) dependent kinematic modification is required. This modification is provided by the transformation of the \( t \)-matrices given by

\[ t_{\text{NA}} = \frac{\epsilon_{\text{cm}}^2}{\epsilon_i} t_{\text{NN}}, \]  
(A.25)

\[ \epsilon_{\text{cm}}^2 = M^2(1 + \alpha), \quad \epsilon_i^2 = M^2 + k_i^2, \quad \epsilon_{i}^2 = M^2 + \left( \frac{k_i}{A} \right)^2, \]  
(A.26)

where \( \epsilon_i \) (\( \epsilon_t \)) is the total energy of the incident (target) nucleon in the nucleon-nucleus system, and \( \epsilon_{\text{cm}} \) is the total energy of the incident nucleon in the NN system.
References