Sneutrino Vacuum Expectation Values and Neutrino Anomalies
Through Trilinear R-parity Violation

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Abstract
Neutrino mass spectrum is reanalyzed in supersymmetric models with explicit trilinear $R$ violation. Models in this category are argued to provide simultaneous solution to the solar and atmospheric neutrino anomalies. It is shown specifically that large mixing and hierarchical masses needed for the vacuum solution of neutrino anomalies arise naturally in these models without requiring any additional symmetries or hierarchies among the trilinear couplings.

1. Introduction:
The hypothesis of neutrino oscillations has gained acceptance after careful observation of the atmospheric muon neutrino deficit at the Superkamioka [1]. If neutrinos do oscillate then both the solar and the atmospheric neutrino deficits can be simultaneously understood in terms of oscillations among the three known neutrinos. This however requires the presence of two large mixing [2] angles among three neutrino states. Many different theoretical models [3] have been proposed in this context. Supersymmetry (SUSY) provides a framework where both the largeness of mixing and hierarchy in masses can be naturally understood.

Supersymmetric extension of the standard model contains the following lepton number violating terms:

$$W_L = \lambda'_{ijk} L_i Q_j D_k^c + \lambda_{ijk} L_i L_j E_k^c + \epsilon_i L_i H_2^c . \quad (1)$$

These naturally lead to neutrino masses [4]. The neutrino spectrum in this model has been extensively studied [5–10] in the literature in recent times. It has been shown [5,6]
that bilinear SUSY violating interactions provide very economical framework which can simultaneously accommodate hierarchical masses and large mixing. In contrast, the neutrino spectrum implied by the trilinear interactions $\lambda, \lambda'$ may appear arbitrary a priori due to very large number of such couplings. It was emphasized by Drees et al. [7] that this is not the case and neutrino spectrum could be quite predictive even in models with trilinear couplings. This can be understood from eq.(1) which in the absence of bilinear terms is invariant under a global U(1) symmetry with U(1) charges 1, -1, -2 for the fields $L, D^c, E^c$ respectively. This symmetry would thus prevent generation of neutrino masses if it was not broken by the down quark and charged lepton Yukawa couplings $h^D_k$ and $h^E_k$ respectively. This means that the neutrino masses generated by the trilinear interactions in eq.(1) are always accompanied by the above Yukawa couplings and hierarchy in the latter gets decoded into the neutrino masses if all the trilinear couplings are assumed to be similar in magnitude. In the limit of keeping only the b quark Yukawa coupling, one combination of neutrino fields namely $\lambda'_{33} \nu_i$, obtains a mass and this combination would contain large mixing of all the states if $\lambda'_{33}$ are comparable for different $i$. The other mass would arise when the strange quark Yukawa coupling is turned on and one thus naturally gets [7],

$$\frac{m_{\nu_b}}{m_{\nu_s}} \sim \frac{m_s}{m_b},$$

reproducing the hierarchy needed to understand the solar and atmospheric neutrino anomalies simultaneously. The natural expectation of this scenario is large mixing among all three neutrinos and this is not favored by the more likely small angle MSW [11] solution for the solar neutrino deficit. This led to imposition of ad-hoc discrete symmetries in [7] to prevent unwanted trilinear couplings reducing the attractiveness of the scenario.

It is clear from the forgoing discussion that more natural possibility with the trilinear couplings of similar magnitudes is to have large mixing among all the neutrinos. This however then favors the vacuum solution to the solar neutrino problem in which case the hierarchy among neutrino masses is required to be stronger than displayed in eq.(2). A careful analysis of the neutrino spectrum reveals that under the standard assumptions, the neutrino mass hierarchy resulting in models with only trilinear $R$ violating couplings at
a high scale is indeed stronger than the one in eq.(2). It can be strong enough to get the vacuum solution for the solar neutrino problem. This coupled with large mixing among neutrinos alleviates any need to postulate discrete symmetry as in [7] and makes the trilinear lepton number violation an attractive means to understand neutrino anomalies.

The key feature leading to a different conclusion compared to [7] is the observation that the presence of trilinear interactions in the original superpotential at a high scale, induces [12] terms linear in the sneutrino fields in the effective potential at the weak scale. These sneutrino fields then obtain vacuum expectation value (vev) and cause neutrino-neutralino mixing. Neutrino mass generated through this mixing dominates over the loop mass considered in [7] and in other works [9,13]. This alters the neutrino mass hierarchy compared to eq.(2). We discuss these issues quantitatively in the following.

2. Sneutrino Vevs and Neutrino Masses

For definiteness, we shall concentrate on the trilinear interactions containing $\lambda'$ couplings and comment on the inclusion of the $\lambda$ couplings latter on. The presence of non-zero $\lambda'_{ijk}$ is known to induce two separate contributions to the neutrino masses and we discuss them in turn.

A. Tree level mass

We adopt the conventional supergravity framework [14] according to which the structure of the superpotential dictates the structure of the soft SUSY breaking terms. Thus with only trilinear L-violating interactions, the soft terms do not contain bilinear terms at a high scale. They are nevertheless generated at the weak scale [12] and should be retained in the scalar potential at this scale:

$$V_{\text{soft}} = m^2_{\tilde{\nu}_i} |\tilde{\nu}_i|^2 + m^2_{H_1} |H^0_1|^2 + m^2_{H_2} |H^0_2|^2 + \left[m^2_{\nu,H_1} \tilde{\nu}_1 H^0_1 - \mu B_{\mu} H^0_1 H^0_2 - B_{\nu} \tilde{\nu}_1 H^0_2 + h.c\right] + \frac{1}{8} (g_1^2 + g_2^2)(|H^0_1|^2 - |H^0_2|^2)^2 + \ldots \ . \quad (3)$$

Where, we have retained only neutral fields and used standard notation with $B_{\nu}$ and $m^2_{\nu,H_1}$ representing the bilinear lepton number violating [15] soft terms. The weak scale value of the soft parameters is determined by the following [12] renormalization group equations (RGE):

$$\frac{dB_{\nu}}{dt} = B_{\nu} \left( -\frac{1}{2} Y^\tau - \frac{3}{2} Y^t + \frac{3}{2} \tilde{\alpha}_2 + \frac{3}{10} \tilde{\alpha}_1 \right) - \frac{3}{16 \pi^2} \mu \ h_k^D \ \lambda'_{kk} \left( \frac{1}{2} B_{\mu} + A^\nu_{kk} \right) .$$
\[
\frac{dm_{\nu H_1}^2}{dt} = m_{\nu H_1}^2 \left( -2Y^\tau - \frac{3}{2} Y^b \right) - \left( \frac{3}{32\pi^2} \right) h_k^D \lambda_{ikk} \left( m_{H_1}^2 + m_{L_i}^2 \right) + 2 m_{kk}^2 + 2 A_{ikk} A_{kk}^D + 2 m_{kk}^{D_p},
\]

and the standard RGE \[14\] for the parameters on the RHS. Since we allow only trilinear interactions in \( W_L \), \( m_{\nu H_1}^2 = B_{\nu} = 0 \) at high scale. As seen from the above equations, the presence of non-zero \( \lambda_{ikk} \) however generate non-zero values for \( m_{\nu H_1}^2 \) and \( B_{\nu} \). It is then convenient to parameterize them as,

\[
B_{\nu} = \lambda_{ipp} h_p^D \kappa_{ip}, \\
m_{\nu H_1}^2 = \lambda_{ipp} h_p^D \kappa'_{ip}.
\]

Here, \( p \) is summed over generations. The parameters \( \kappa \) and \( \kappa' \) represent the running of the parameters present in the RGE’s from the GUT scale to the weak scale.

The above soft potential would now give rise to sneutrino vevs,

\[
< \tilde{\nu}_i > = \frac{B_{\nu} v_2 - m_{\nu H_1}^2 v_1}{m_{L_i}^2 + \frac{1}{2} m_Z^2 \cos 2\beta}.
\]

The sneutrino vevs so generated will now mix the neutrinos with the neutralinos thus giving rise to a tree level neutrino mass matrix \[16\] :

\[
M^0_{ij} = \frac{\mu (c g^2 + g'^2)}{2(-c \mu M_2 + 2 M_w^2 c \beta s \beta (c + \tan^2 \theta_w))} < \tilde{\nu}_i > < \tilde{\nu}_j >.
\]

**B. Loop Level Mass**

The trilinear couplings in the superpotential would also give rise to a loop induced neutrino mass with the down squark and antisquark pairs being exchanged in the loops along with their ordinary partners \[4,17\]. This mass can be written as,

\[
M^l_{ij} = \frac{\lambda_{ilk} \lambda_{jkl}}{16 \pi^2} v_1 h_k^D \sin \phi_l \cos \phi_l \ln \frac{M^2_{ll}}{M^2_{D_i}}.
\]

In the above, \( \sin \phi_l \cos \phi_l \) determines the mixing of the squark-antisquark pairs and \( M^2_{ll} \) and \( M^2_{D_i} \) represent the eigenvalues of the standard 2×2 mass matrix of the down squark system \[14\]. The indices \( l \) and \( k \) are summed over. The \( v_1 \) stands for the vev of the Higgs field \( H_1^0 \). The mixing \( \sin \phi_l \cos \phi_l \) is proportional to \( h_l^D \) and thus one can write the loop mass as,
\[ m_\text{loop} \equiv \frac{v_1}{16\pi^2} \frac{\sin\phi_l \cos\phi_l}{h_l^D} \ln \frac{M^2_{2l}}{M^2_{1l}} \sim \frac{v_1^2}{16\pi^2} \frac{1}{M_{\text{SUSY}}}, \]  

(10)

with \( M_{\text{SUSY}} \sim 1 \text{ TeV} \) referring to the typical scale of SUSY breaking. Note that \( m_\text{loop} \) defined above is independent of the R violating couplings and is solely determined by the parameters of the minimal supersymmetric standard model (MSSM).

As evident from eqs.(7,8), the tree as well as loop induced masses have very similar structure. Both involve down-quark Yukawa couplings for the reason explained in the introduction. The tree level contribution involves diagonal couplings \( \lambda'_{ikk} \) while the \( \mathcal{M}^l \) contains off-diagonal \( \lambda'_{ikl} \) as well [18]. If all \( \lambda'_{ikl} \) are assumed to be of similar magnitude then the tree level mass is seen to dominate over the loop mass as we will discuss in the next section.

3. Neutrino Masses and Mixing

We now make a simplifying approximation which allows us to discuss neutrino masses and mixing analytically. It is seen from the RG eqs.(4) that the parameters \( \kappa_{ik}, \kappa'_{ik} \), defined in eqn. (5) are independent of generation structure in the limit in which generation dependence of the scalar masses \( m^2_{Li}, m^2_{Qi} \) and soft parameters \( A^\nu_{ikk} \) and \( A^D_{ikk} \) is neglected. Since we are assuming the universal boundary conditions, this is true in the leading order in which the \( Q^2 \) dependence of the parameters multiplying \( \lambda'_{ikk} h_k^D \) in eq.(4) is neglected. \( Q^2 \) dependence in these parameters generated through the gauge couplings will also be flavor blind though Yukawa couplings will lead to some generation dependent corrections. But their impact on the the conclusions based on the analytic approximation below is not expected to be significant. The neglect of the generation dependence of \( \kappa_{ik}, \kappa'_{ik} \) allows us to rewrite eq.(7) as,

\[ \mathcal{M}^0_{ij} \equiv m_0 a_i a_j, \]  

(11)

where,

\[ a_i \equiv \lambda'_{ikk} h_k^D \]  

(12)
$m_0$ is now completely determined by the standard MSSM parameters and the dependence of the R-violating parameters gets factored out as in eq.(9). $m_0$ can be determined by solving the RGE (4). Roughly, $m_0$ is given by,

$$m_0 \sim \left( \frac{3}{4\pi^2} \right)^2 \frac{v^2}{M_{\text{SUSY}}} \left( \ln \frac{M_Z^2}{M_Z^2} \right)^2.$$  

Let us rewrite the loop induced mass matrix as,

$$M'_{ij} = m_{\text{loop}} \lambda'_{ik} \lambda'_{kl} h^D_k h^D_l$$

$$= m_{\text{loop}} a_i a_j + m_{\text{loop}} h^D_2 h^D_3 A_{ij} + O(h^D_1, h^D_2),$$  

where,

$$A_{ij} = \lambda'_{i23} \lambda'_{j32} + \lambda'_{i32} \lambda'_{j23} - \lambda'_{i22} \lambda'_{j33} - \lambda'_{i33} \lambda'_{j22}.$$  

Neglecting $O(h^D_1, h^D_2)$ corrections to the loop induced mass matrix, the total mass matrix is given by,

$$M''_{ij} \approx (m_0 + m_{\text{loop}}) a_i a_j + m_{\text{loop}} h^D_2 h^D_3 A_{ij}$$

$$\approx M'_{ij} + m_{\text{loop}} h^D_2 h^D_3 A_{ij}.$$  

The matrix $M'$ has a special structure. It has only one eigenvalue. It can be easily diagonalised using a unitary transformation,

$$U^T M' U = \text{diag} \ (0, 0, m_3),$$  

where,

$$m_3 \approx (m_0 + m_{\text{loop}}) \ (a_1^2 + a_2^2 + a_3^2)$$

$$\sim (m_0 + m_{\text{loop}}) \ (\lambda^2_{333} + \lambda^2_{233} + \lambda^2_{133}) \ h^D_3.$$  

The matrix U is determined as,

$$U = \begin{pmatrix}
    c_2 & s_2 c_3 & s_2 s_3 \\
    -s_2 & c_2 c_3 & c_2 s_3 \\
    0 & -s_3 & c_3
\end{pmatrix},$$  

where,

$$c_i = \cos \theta_i, s_i = \sin \theta_i.$$
with,
\[ s_2 = \frac{a_1}{\sqrt{a_1^2 + a_2^2}}, \quad s_3 = \left(\frac{a_1^2 + a_2^2}{a_1^2 + a_2^2 + a_3^2}\right)^{\frac{1}{2}}. \]  

The total mass matrix is now given by,

\[ U^T M' U \approx m_3 \text{diag}(0, 0, 1) + m_{\text{loop}} h_2^D h_3^D A' \]

\[ \approx m_3 \begin{pmatrix} \epsilon A'_{11} & \epsilon A'_{12} & \epsilon A'_{13} \\ \epsilon A'_{12} & \epsilon A'_{22} & \epsilon A'_{23} \\ \epsilon A'_{13} & \epsilon A'_{23} & 1 \end{pmatrix}, \]  

where,

\[ A' = U^T A U \]  

and

\[ \epsilon A'_{ij} \approx \frac{m_{\text{loop}}}{m_3} h_2^D h_3^D A'_{ij} \approx \frac{m_{\text{loop}}}{m_0} \frac{h_2^D}{h_3^D}. \]

The last equality follows under the assumption that \( \lambda'_{ijk} \) are similar in magnitude and \( m_{\text{loop}} \ll m_0 \).

Choosing,

\[ U' = \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]  

with, \( s_1, c_1 \) defined by,

\[ \tan 2\theta_1 = \frac{2A'_{12}}{A'_{22} - A'_{11}}, \]  

we have,

\[ U'^T U'^T M_{\nu} U U' \approx m_3 \begin{pmatrix} \epsilon \delta_1 & 0 & c_1 \epsilon A'_{13} - s_1 \epsilon A'_{23} \\ 0 & \epsilon \delta_2 & s_1 \epsilon A'_{13} - c_1 \epsilon A'_{23} \\ c_1 \epsilon A'_{13} - s_1 \epsilon A'_{23} & s_1 \epsilon A'_{13} - c_1 \epsilon A'_{23} & 1 \end{pmatrix}, \]

\[ \approx m_3 \text{diag}(\delta_1, \delta_2, 1). \]
The off-diagonal elements will generate additional mixing in the model. But, as $\epsilon A' \ll 1$, we can neglect these off-diagonal elements. The eigenvalues in this approximation are given as,

$$m_{\nu_1} \sim \epsilon m_3 \delta_1 \quad ; \quad m_{\nu_2} \sim \epsilon m_3 \delta_2 \quad ; \quad m_{\nu_3} \sim m_3 ,$$  

(26)

where,

$$\delta_1 = (c_1^2 A'_{11} - 2c_1 s_1 A'_{12} + s_1^2 A'_{22}) \quad ,$$

$$\delta_2 = (s_1^2 A'_{11} + 2c_1 s_1 A'_{12} + c_1^2 A'_{22}) \quad .$$

(27)

Note that both $\delta_1$ and $\delta_2$ are generically of 0 ($\lambda'^2$) when all $\lambda'_{ijk}$ are assumed to be similar in magnitude. As a consequence, the neutrino masses follow the hierarchy,

$$m_{\nu_1} \sim m_{\nu_2} \ll m_{\nu_3}$$

With,

$$\frac{m_{\nu_2}}{m_{\nu_3}} \sim \frac{m_s m_{\text{loop}}}{m_b m_0} \left( \frac{\delta_2}{\Sigma_i \lambda'^2_{i33}} \right)$$

(28)

The last factor in the above is of O(1) and the remaining part is controlled completely by the standard parameters of the MSSM.

Eq.(28) may be regarded as a generic prediction of the model. It is seen from eqs. (10,13) that typically,

$$\frac{m_{\text{loop}}}{m_0} \sim \frac{\pi^2}{9 \ln \left( \frac{M^2}{M^2_Z} \right)^2} \sim 10^{-4}$$

(29)

Thus the neutrino mass ratio in eq.(28) is suppressed considerably compared to eq.(2) obtained when sneutrino vev contribution is completely neglected. The exact value of this suppression factor is dependent on MSSM parameters and we will calculate it in the next section.

The mixing among neutrinos is governed by,
The angles are determined by the ratios of the trilinear couplings and hence can be naturally large. Thus, as in supersymmetric model with purely bilinear $R$ violation [6] one gets hierarchical masses and large mixing without fine tuning in any parameters.

4. Neutrino Anomalies

We now discuss the phenomenological implications of neutrino masses, eq.(26) and mixing, eq.(30). Due to hierarchy in masses, one could simultaneously solve the solar and atmospheric $\nu$ problems provided, $m_{\nu_1} \sim m_{\nu_2} \sim 10^{-5}$ eV and $m_{\nu_3} \sim 10^{-2}$ eV.

In order to determine these masses exactly, we have numerically integrated eqs.(4) along with similar equations for the parameters appearing in them. We have imposed the standard universal boundary condition and required radiative breaking of the $SU(2) \times U(1)$ symmetry. Solution of the RGE determines both $m_{\text{loop}}$ (eq.(9)) and $m_0$ (eq.(11)). We display these in figs. (1a,1b) as a function of $\mu$ for $\tan\beta = 2.1$, $M_2 = 400$ and 200 GeV respectively. The ratio $\frac{m_{\text{loop}}}{m_0}$ is quite sensitive to the sign of $\mu$. For $\mu > 0$, this ratio is rather small, typically, $\sim 10^{-3}$, while it can be much larger for $\mu < 0$. There exists a region with negative $\mu$ in which $\frac{m_{\text{loop}}}{m_0} \geq 1$. In this region, two contributions to the sneutrino vev in eq. (6) cancel and $m_0$ gets suppressed. Barring this region, the $\frac{m_{\text{loop}}}{m_0}$ is seen to be around $\sim 10^{-1} - 10^{-2}$ for negative $\mu$ leading to

$$\frac{m_{\nu_2}}{m_{\nu_3}} \sim \frac{m_h m_{\text{loop}}}{m_b m_0} \sim 2 (10^{-3} - 10^{-4})$$

(31)

For $m_{\nu_3} \sim 10^{-1} - 10^{-2}$ eV, one thus obtains $m_{\nu_2} \sim m_{\nu_3} \sim 2 (10^{-4} - 10^{-6})$ eV which is in the right range required to solve the solar neutrino problem through vacuum oscillations. The typical value of $m_0 \sim \text{GeV}$ found in Fig.(1b) implies through eq.(18),

$$\lambda' \sim 10^{-4}$$
Thus one needs to choose all $\lambda'_{ijk}$ of this order. Once this is done, one automatically obtains solar neutrino scale for some range in the MSSM parameters.

While, hierarchy needed for the vacuum solution follows more naturally, one could also obtain scales relevant to the MSW conversion. This happens for very specific region of parameters with negative $\mu$ in which two contributions to sneutrino vev, eq. (6), cancel. As already mentioned, $\frac{m_{\nu e}}{m_0}$ can be 1 in this region. One then recovers the result of [7], namely, eq.(2) which allows MSW solution for the solar neutrino problem. The reference [8] which used hierarchical $\lambda'_{ijk}$ also concentrated on this region in order to obtain the MSW solution.

We showed in Fig. 1 neutrino mass ratio for specific value of $M_2$ and $\tan \beta$. Qualitatively similar results follow for other values of these parameters. We have displayed in Table.1 values for the MSSM parameters and what they imply for $\frac{m_{\nu e}}{m_{\nu s}}$. We have shown illustrative values of the parameters which lead to the vacuum as well as MSW solution. The latter arise only for limited parameter range corresponding to cancellations in eq.(6). The former is a more generic possibility which arise for larger region with both positive and negative values of $\mu$. The MSW solution in the present context will have to be restricted to the large angle solution if one does not want to impose any discrete symmetries or fine tune $\lambda$‘s.

The constraints on mixing matrix $K$, eq.(30), implied by the experimental results are also easy to satisfy keeping all the $\lambda'_{ijk}$ similar in magnitude. Hierarchy in masses, $m_{\nu 2}, m_{\nu 3}$ lead to the following survival probabilities for the solar and atmospheric neutrinos after undergoing vacuum oscillations:

$$P_e = 1 - 4 K_{e1}^2 K_{e2}^2 \sin^2 \left( \frac{\Delta_{21}^2 t}{4E} \right) - 2 K_{e3}^2 (1 - K_{e3}^2)$$

$$P_\mu = 1 - 4 K_{\mu 3}^2 (1 - K_{\mu 3}^2) \sin^2 \left( \frac{\Delta_{31}^2 t}{4E} \right) .$$

Where, $\Delta m_{ij}^2 = m_{\nu i}^2 - m_{\nu j}^2$ and $\Delta m_{31}^2 \sim \Delta m_{32}^2$.

These survival probabilities assume the standard two generation form when $K_{e3} = 0$ and one could utilize existing constraints on mixing angles. In practice, the $K_{e3}$ may not be zero but is constrained to be small from the non-observation [19] of $\nu_e$ oscillations at CHOOZ. To the extent it is small, one could use the two generation constraints for the solar and atmospheric analysis. The combined constraints which are needed [1,20] to be satisfied are:
\[
0.6 \leq 4 K_{\mu 3}^2 (1 - K_{\mu 3}^2) = s_3^4 \sin^2 2\theta_2 + c_2^2 \sin^2 2\theta_3 \leq 1.
\]
\[
K_{e 3} \leq 0.18
\]
\[
0.8 \leq 4 K_{e 1}^2 K_{e 2}^2 = 4(c_1 c_2 - s_1 s_2 c_3)^2 (s_1 c_2 + s_2 c_1 c_3)^2 \leq 1.
\]

It is possible to satisfy all these constraints by choosing for example,
\[
c_3 = s_3 = s_1 = c_1 = \frac{1}{\sqrt{2}}; \quad s_2 = 0.28
\]

The relative smallness of \(s_2\) required here does not imply significant fine tuning and can be easily obtained, e.g. by choosing,
\[
\frac{\lambda'_{133}}{\lambda'_{233}} \sim \frac{1}{3}
\]

We have so far concentrated on the \(\lambda'_{ijk}\) couplings alone. The analogous discussion can be carried out for \(\lambda_{ijk}\) couplings appearing in the eq.(1). Here also, the tree level contribution to neutrino masses will dominate over the loop contribution although the structure of mixing matrix will differ slightly due to the anti-symmetry of the couplings \(\lambda_{ijk}\) in indices \(i\) and \(j\).

5. Discussion :

We have discussed in detail the structure of neutrino masses and mixing in MSSM in the presence of trilinear R-violating couplings, specifically \(\lambda'_{ijk}\). Noteworthy feature of the present analysis is that it is possible to obtain required neutrino mass pattern under fairly general assumption of all the \(\lambda'_{ijk}\) being of equal magnitudes. This is to be contrasted with the recent analysis [7–9] which had to make very specific choice of the trilinear couplings in order to reproduce neutrino mass pattern. It is quite interesting that hierarchy among neutrino masses is controlled by few parameters in MSSM and is largely independent of the trilinear \(R\) violating couplings. Thus one could understand the required neutrino mass ratio without being specific about the exact values of large number of the trilinear couplings. This 'model-independence' is an attractive feature of the scenario discussed here.

The key difference of the present work compared to many of the other works is proper inclusion of the sneutrino vev contribution. While we had to resort to specific case of the
minimal supergravity model for calculational purpose, the sneutrino vev contribution would arise in any other scheme with $\lambda'_{ikk} \neq 0$ at a high scale such as $M_{GUT}$. Such contribution thus cannot be neglected a priori. On the contrary, the inclusion of this contribution makes the model more interesting and fairly predictive in spite of the presence of large number of unknown couplings.


[15] It should be noted here that the RGE running can also give rise to flavor violating terms of the type $m^2_{\nu_i \nu_j}$ at the weak scale, see, Y. Grossman and H. E. Haber, Phys. Rev. Lett. 78 (1997) 3438; hep-ph/9810536. But, these terms being second order in sneutrino field contribute negligibly to the sneutrino vev as long we restrict ourselves to leading order in the sneutrino vev.


[18] It should be noted that models considered in [7] contain diagonal $\chi^\prime_{\ell \ell}$ even after imposition of the discrete symmetries and hence contain the tree level neutrino mass.


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<th>$M_2$ (GeV)</th>
<th>$\mu$ (GeV)</th>
<th>$m_0$ (GeV)</th>
<th>$m_{loop}$ (GeV)</th>
<th>Ratio $\frac{m_{\nu_2}}{m_{\nu_3}}$</th>
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**Table 1:** The values of $m_0$, $m_{loop}$ and ratio of the eigenvalues, $\frac{m_{\nu_2}}{m_{\nu_3}}$ for various values of the standard MSSM parameters $m$, $M_2$ and $\mu$ for $\tan \beta = 2.1$, $A=0$. 


Figure 1a. The absolute values of tree level contribution, \( m_0 \), the loop level contribution, \( m_{loop} \) and their ratio \( \frac{m_{loop}}{m_0} \) are plotted with respect to \( \mu \) (positive) for \( M_2 = 400 \text{ GeV}, A=0 \) and \( \tan \beta = 2.1 \). The \( m_0 \) and \( m_{loop} \) are defined in the text.
Figure 1b. Same as in Fig. (1a) but for $M_2 = 200$ GeV and $\mu$ (negative).