HARD PHOTON CORRECTIONS FOR BHABHA SCATTERING

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A B S T R A C T

Numerical results for the α^3 differential cross-section for $e^+e^- \rightarrow e^+e^-$ are presented for the case of e^+e^- colliding beams. Special care is given to the hard photon corrections. Acoplanarity distributions for $e^+e^- \rightarrow e^+e^- \gamma$ are also presented. Attention is drawn to experimental conditions which require the calculation of still higher order corrections.

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1. INTRODUCTION

In previous papers, we discussed the radiative corrections for e^+e^- colliding beam experiments leading to $\mu^+\mu^-$ ^{1),2)} or $\gamma\gamma$ ³⁾ final states. In particular, the influence of undetected hard photons was taken into account.

In this paper, we present similar results for the case of Bhabha scattering:

$$e^+(p_+) + e^-(p_-) \rightarrow e^+(q_+) + e^-(q_-). \quad (1)$$

For colliding beam experiments, two points are of particular relevance when measuring this reaction. Firstly, one can measure the differential cross-section for reaction (1). When this is done at the percentage level, one has to know the theoretical cross-section up to order α^3 . This cross-section contains both virtual corrections and corrections due to the inclusion of events of the bremsstrahlung reaction

$$e^+(p_+) + e^-(p_-) \rightarrow e^+(q_+) + e^-(q_-) + \gamma(k), \quad (2)$$

which can simulate events of reaction (1).

Secondly, in an experiment, one can also obtain information on events of reaction (2) which are identified as such. It is thus useful to evaluate certain experimental distributions for this reaction.

In the following, we shall mainly concentrate on the results, since the procedure and kinematical details are given in extenso in Refs. 1-3.

The radiative corrections to the differential cross-section for reaction (1) can be written in the form

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^0}{d\Omega} (1 + \delta_T), \quad (3)$$

where $d\sigma/d\Omega$ is the α^3 cross-section, $d\sigma^0/d\Omega$ the lowest order one for Bhabha scattering, and δ_T the total radiative correction. This δ_T gets a contribution δ_V from the virtual corrections, i.e. from the interference between the lowest order amplitude and all e^4 amplitudes. It contains the vacuum polarizations, the vertex corrections, and the two photon exchange contributions. The complete analytical results for these contributions

were first given by Reahead⁴⁾ and Polovin⁵⁾. The infrared divergent character of δ_V is exhibited through the presence of a fictitious photon mass, λ .

The second contribution to δ_T comes from events of the bremsstrahlung process (2) which are experimentally undistinguishable from events of reaction (1). This contribution is dependent on the experimental criterion used for deciding whether an event is counted as one of reaction (1). So far, an often used criterion is the following:

- i) the electron and positron tracks must make an angle less than some prescribed acollinearity angle, ζ ;
- ii) the electron and positron must have an energy larger than some threshold energy, E_{th} , required to trigger the detection system.

The phase space as determined by these conditions is discussed in detail in Ref. 1, and represented by the shaded area in Fig. 1. The cross-section for reaction (2) must be integrated over this phase space. From Fig. 1, one can see furthermore that this phase space consists of an isotropic and an anisotropic part. The former lies above the line $k = k_1$, and the latter between the lines $k = k_1$ and $k = k_3$.

The integral over the isotropic part of the phase space is infrared divergent due to the divergence of the cross-section in the top C of the Dalitz plot where the photon momentum is zero. It is therefore profitable to separate the isotropic integral into two integrals, one of which has as integrand the soft photon cross-section, while the other one has the difference of the hard photon cross-section and the soft photon one as integrand. The second integral is then convergent and can be calculated numerically. The first one is infrared divergent, but can be evaluated analytically and depends on λ . The exact expressions can be obtained from Refs. 1 and 2. The cross-section for (2) has been given by Swanson⁶⁾ and by Hearn et al.⁷⁾.

Thus, the bremsstrahlung contribution, δ_B , consists of three parts, one analytical expression and two integrals which have to be evaluated numerically, one over the isotropic part of the phase space and one over the anisotropic part. The total correction, δ_T , can then be written as

$$\delta_T = \delta_A + \delta_N, \quad (4)$$

where the analytical expressions of δ_V and δ_B are combined in one analytical expression δ_A in which the infrared divergence has cancelled. Finally, δ_N is the contribution from the numerical integrals.

Stated differently, δ_A would be the virtual and the bremsstrahlung correction for an experiment where photons of an energy $k < k_1$ would not be detected (see Fig. 1) and where the bremsstrahlung cross-section would be taken in the soft photon approximation. Therefore, δ_N reflects both the fact that in the isotropic phase space the photon cannot be considered as soft anymore, and the fact that there is also an anisotropic part of phase space present where the photon is even harder.

The numerical integrations are complicated due to the fact that rapid changes in the size of the cross-section occur over the phase space. The regions where strong peaking occurs (whenever a photon is emitted within a small solid angle with respect to one of the charged particles) are computed separately. In addition to the peaking regions of Ref. 1, we now also have peaking along the edges of the Dalitz plot. In every sub-region, the integration is done by means of the integration routine RIWIAD⁸⁾, which distributes the integration points in an optimal way.

Besides the above radiative corrections to reaction (1), it is also of interest to compare with the theoretical predictions the events which clearly belong to reaction (2).

If we take a reference frame with $\vec{p}_+$ as z-axis and $\vec{p}_+ \wedge \vec{q}_+$ as y-axis, the outgoing positron has a polar angle θ and a vanishing azimuthal angle. The outgoing electron then has a polar angle θ' and an azimuthal angle ϕ' . For reaction (1), one necessarily has $\theta' = \pi - \theta$ and $\phi' = \pi$. The angle $\psi = \pi - \phi'$ thus measures the acoplanarity of the event, which now in lowest order comes from reaction (2). Results will be given for the acoplanarity distribution $d\sigma/d\psi$, where all outgoing electron-positron pairs are restricted by the conditions

$$\begin{aligned} q_{+0}, q_{-0} &\geq E_{th}, \\ \theta_{min} &\leq \theta, \theta' \leq \theta_{max}. \end{aligned} \quad (5)$$

Total cross-sections will also be given for all events having an acoplanarity larger than some prescribed angle and satisfying conditions (5).

For a somewhat different situation, an acoplanarity distribution has also been given by Calva-Tellez⁹⁾.

In the next Section, we present the results for the various quantities mentioned above. In an Appendix, we give explicitly a formula for the relativistic limit of δ_A .

2. RESULTS

The lowest order differential cross-section is tabulated for convenience in Table I for a set of scattering angles and energies. The strong peaking due to photon exchange in the t-channel is evident.

In Table II, the results for δ_T are given for the same set of angles and energies such that by means of Eq. (3) the relevant differential cross-section up to order α^3 can be obtained.

In order to exhibit the importance of hard photon corrections (δ_N), we also tabulated the analytical expression δ_A , which only takes into account the virtual and the soft photons. By far the largest contribution to δ_N comes from the bremsstrahlung integral over the anisotropic phase space. This, incidentally, makes δ_A a fairly good approximation for the total correction, δ_T , in an experiment, where the selection criterion for events is that the photon energy is smaller than k_1 . In Table III, we list a representative set of values for k_1 for different values of the beam energy and the acollinearity angle.

For a scattering angle around 90° , the region where the matrix element peaks due to the emission of a photon along the initial electron or positron direction lies largely outside the anisotropic phase-space. For scattering angles away from 90° , the region of this peaking moves into the anisotropic phase space. This is the reason why towards 5° and 175° δ_N becomes larger. That δ_N is much larger for 5° than for 175° is understandable through the enhancement coming from the one photon exchange in the t-channel. This is important both for process (1) and (2) at small scattering angles. If the photon in reaction (2) is emitted along the incoming positron direction, the cross-section for (2) is large, not only because of the usual peaking in this direction, but also because $t = (p_- - q_-)^2$ is decreased with respect to the value it has in process (1) for a fixed scattering angle θ .

It should be realized that in practise scattering angles of 5° or 175° can only be measured with maximum acollinearity angle ζ of less than 5° . The points are nevertheless given in order to establish the trend of the results as a function of the scattering angle.

In Table IV, the variation of the corrections with ζ is illustrated. From the results at $\theta = 5^\circ$, it is clear that δ_T remains large, even for a smaller maximum acollinearity angle. Since the available phase space increases with ζ , δ_T also increases. The influence of the anisotropic phase space is demonstrated by the increase of E_{th} : δ_T becomes smaller since the anisotropic phase space is decreased.

The corrections for small scattering angles (and $E_{th} = 0.2$ GeV) are so large that still higher order corrections are likely to be of importance in this case. As discussed above, the size of these corrections is due to the hard bremsstrahlung cross-section which is large in the anisotropic region. This should be contrasted with an often occurring case where radiative corrections become large (and negative) because the maximum photon energy becomes small, in which case a part of the first order correction is often exponentiated in order to include higher order corrections. In our case, this maximum photon energy is not at all small as can be seen from Table III.

It is of course also possible to get a much smaller maximum photon energy by decreasing the acollinearity angle. If in addition the outgoing electron and positron are also restricted to large values (large E_{th}), then the size of the anisotropic phase space will also be reduced. In this situation, the correction δ_T may become large and negative, which also necessitates the inclusion of still higher order corrections. E.g., for $\zeta = 1^\circ$ and $p_{+0} = 2$ GeV, δ_A becomes equal to -50% for $\theta = 90^\circ$.

All the quoted results apply for the case when one identifies electrons and positrons in the final states. In the case where the charges are not detected, the cross-sections (3) for θ and $\pi - \theta$ should be combined.

The acoplanarity distribution, $d\sigma/d\psi$, is given in Table V. Towards small acoplanarity angles this distribution peaks, since then, photons of lower energy are included (e.g., the minimum value for the photon energy is 30 MeV for $p_{+0} = 0.5$ GeV and $\psi = 5^\circ$).

Also in the backward region, $\psi \sim 180^\circ$, another peaking occurs. This can be understood by realizing that it is then possible to have electron and positron in the same direction, while the photon goes the opposite way. In this situation, the s-channel photon propagator enhances the cross-section.

If a detailed behaviour of $d\sigma/d\psi$ for very small angles ψ is required, the presence of very soft photons makes a discussion of higher order corrections necessary. Experimentally, one may e.g. measure all the events with $-\epsilon < \psi < \epsilon$, where ϵ is a very small angle. The events belong to both reactions (1) and (2). Since the maximum photon energy for isotropic photon emission may become very small, the correction to what one would expect from the lowest order cross-section becomes large and negative, which then requires the calculation of higher order corrections.

It is also of interest to know the total cross-section when ψ is larger than some prescribed value ψ_0 . It gives the total number of bremsstrahlung events under the restrictions (5) for which $\psi_0 < \psi < 180^\circ$. Table VI lists these cross-sections.

When the charge of the particles in the final state is not known, the acoplanarity distribution of Table V should be multiplied by two, and similarly the values in Table VI.

In conclusion, when measuring Bhabha scattering at the percentage level, radiative corrections are to be taken into account. In particular, hard photon corrections cannot be ignored. If experiments can be done at small scattering angles, the first order correction becomes large and still higher order corrections will be required. Similarly, if small acollinearity or acoplanarity angles can be reached, the experiments may be able to test corrections of orders higher than α^3 .

APPENDIX

The α^3 differential cross-section for Bhabha scattering, including virtual corrections and soft photon bremsstrahlung, is given in the relativistic limit ($E \gg m$, $\sin \theta \gg m/E$) by

$$\begin{aligned} \frac{d\sigma}{d\Omega} = & \frac{\alpha^2}{16E^2} \frac{1}{(1-x)^2} \\ & \left\{ \left[9 + 6x^2 + x^4 \right] \left[1 + \frac{\alpha}{\pi} \left(4(1-u+v-w) \log \frac{E}{k_1} \right. \right. \right. \\ & \left. \left. + u^2 - v^2 + w^2 + 2 \operatorname{Li}_2 \left(\frac{1+x}{2} \right) - 2 \operatorname{Li}_2 \left(\frac{1-x}{2} \right) - \frac{2}{3} \pi^2 \right) \right] \right. \\ & + \frac{\alpha}{\pi} \left[\frac{u}{3} (3 - 42x + 42x^2 - 14x^3 + 11x^4) \right. \\ & \left. - v(5 - 7x + 3x^2 - x^3) + \frac{w}{3} (111 + 21x + 33x^2 + 11x^3) \right. \\ & + \frac{u^2}{2} (3 + 7x - 5x^2 - 3x^3 - 2x^4) + v^2 (3 - 3x + x^2 - x^3) \\ & \left. - \frac{w^2}{2} (9 + 7x + 11x^2 + 5x^3) - 2uvx(2 - x - x^3) \right. \\ & \left. - uw(21 + 3x + 9x^2 - 3x^3 + 2x^4) \right. \\ & \left. + 2vw(6 + 5x + 4x^2 + x^3) - \frac{46}{9} (9 + 6x^2 + x^4) \right. \\ & \left. + \frac{\pi^2}{3} (18 - 15x + 12x^2 - 3x^3 + 4x^4) \right] \left. \right\}. \end{aligned}$$

In this formula, we have introduced E , the beam energy, k_1 , the maximum photon energy, and

$$x = \cos \theta,$$

$$u = \log (4E^2/m^2),$$

$$v = \log (2 E^2 (1+x) / m^2) ,$$

$$w = \log (2 E^2 (1-x) / m^2) .$$

Also, the dilogarithm is defined by

$$Li_2 (x) = - \int_0^x \frac{\log (1-z)}{z} dz .$$

This formula for the cross-section reproduces all the values for δ_A in Tables II and IV, calculated with the exact formulae, whereas the expression given by Calva-Tellez⁹⁾ does not.

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This paper contains several misprints. In the first line of their Eq. (2.19), one should replace $p_1 \cdot p_2 \ p_2 \cdot p_4$ by $p_1 \cdot p_2 \ p_1 \cdot p_4$. The complete expression for A_2 should be multiplied by 2, and A_3 by -2. Also, in the last line of A_3 , $p_4 \cdot k_2$ should be replaced by $p_4 \cdot k^2$. The expressions then agree with those of Ref. 6. We have in addition checked the $m \rightarrow 0$ limit of their formulae.
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Table 1

The lowest order cross-section (in nb) for different values of the beam energy, p_{+0} (in GeV), and the scattering angle, θ .

$\theta \backslash p_{+0}$	0.5	1.0	2.0	3.0	5.0
5°	5706 10^3	1427 10^3	3566 10^2	1585 10^2	5706 10^1
20°	2149 10^1	5373	1343	597.0	214.9
40°	1218	304.6	76.15	33.85	12.18
60°	219.0	54.75	13.69	6.08	2.19
90°	46.65	11.66	2.92	1.30	0.47
120°	24.34	6.08	1.52	0.68	0.24
140°	21.38	5.35	1.34	0.59	0.21
160°	20.77	5.19	1.30	0.58	0.21
175°	20.74	5.18	1.30	0.58	0.21

Table 2

The radiative corrections (in %) to the lowest order cross-section for different values of the beam energy, p_{+0} (in GeV), and the scattering angle, θ . The acollinearity angle $\zeta = 10^\circ$, and the threshold energy $E_{th} = 0.2$ GeV.

$\begin{array}{c} p_{+0} \\ \theta \end{array}$	0.5		1.0		2.0		3.0		5.0	
	δ_A	δ_T	δ_A	δ_T	δ_A	δ_T	δ_A	δ_T	δ_A	δ_T
5°	-7.0	2.6 ± 0.1	-8.2	9.7 ± 0.6	-9.4	22.2 ± 1.5	-10.1	33.9 ± 3.6	-10.9	46.1 ± 3.4
20°	-9.5	0.2 ± 0.1	-10.6	1.2 ± 0.1	-11.8	1.7 ± 0.1	-12.5	2.0 ± 0.1	-13.4	2.2 ± 0.1
40°	-10.8	-4.1 ± 0.1	-12.0	-3.6 ± 0.1	-13.2	-3.6 ± 0.1	-13.9	-3.8 ± 0.1	-14.8	-3.9 ± 0.1
60°	-11.8	-6.5 ± 0.1	-13.0	-6.3 ± 0.1	-14.1	-6.5 ± 0.1	-14.8	-6.7 ± 0.1	-15.7	-6.8 ± 0.1
90°	-12.8	-8.4 ± 0.1	-14.0	-8.3 ± 0.1	-15.2	-8.6 ± 0.1	-15.8	-8.8 ± 0.1	-16.7	-9.0 ± 0.1
120°	-14.1	-9.0 ± 0.1	-15.3	-8.9 ± 0.1	-16.4	-9.1 ± 0.1	-17.1	-9.3 ± 0.1	-18.0	-9.4 ± 0.1
140°	-15.6	-9.0 ± 0.1	-16.8	-8.7 ± 0.1	-18.0	-8.7 ± 0.1	-18.7	-8.7 ± 0.1	-19.5	-8.8 ± 0.1
160°	-18.6	-8.5 ± 0.1	-19.8	-7.9 ± 0.1	-21.0	-7.5 ± 0.1	-21.7	-7.4 ± 0.1	-22.6	-7.3 ± 0.1
175°	-25.9	-10.1 ± 0.1	-27.1	-7.3 ± 0.1	-28.3	-5.9 ± 0.1	-29.0	-5.4 ± 0.1	-29.8	-4.5 ± 0.1

Table 3

The maximum energy, k_1 (in MeV), of the isotropically emitted photons for different values of the beam energy, p_{+0} (in GeV), and acollinearity angle, ζ .

$\zeta \backslash p_{+0}$	0.5	1.0	2.0	3.0	5.0
5°	42	84	167	251	418
10°	80	160	321	481	802
15°	115	231	462	693	1155

Table 4

The radiative corrections (in %) to the lowest order cross-section as a function of the scattering angle, θ , for different values of the acollinearity angle, ζ , and the threshold energy, E_{th} . The beam energy $p_{+0} = 3$ GeV.

θ	$\zeta=5^\circ, E_{th}=0.2$ GeV		$\zeta=15^\circ, E_{th}=0.2$ GeV		$\zeta=10^\circ, E_{th}=0.8$ GeV	
	δ_A	δ_T	δ_A	δ_T	δ_A	δ_T
5°	-17.0	30.4 ± 2.0	-6.2	36.0 ± 2.3	-10.1	7.9 ± 0.2
20°	-21.1	-3.9 ± 0.1	-7.7	8.1 ± 0.1	-12.5	1.3 ± 0.1
40°	-23.4	-9.5 ± 0.1	-8.6	-0.2 ± 0.1	-13.9	-4.4 ± 0.1
60°	-24.9	-12.9 ± 0.1	-9.2	-3.1 ± 0.1	-14.8	-7.3 ± 0.1
90°	-26.6	-15.7 ± 0.1	-9.8	-5.1 ± 0.1	-15.8	-9.4 ± 0.1
120°	-28.5	-16.6 ± 0.1	-10.7	-5.2 ± 0.1	-17.1	-9.9 ± 0.1
140°	-30.6	-16.5 ± 0.1	-12.0	-4.6 ± 0.1	-18.7	-9.4 ± 0.1
160°	-34.5	-15.6 ± 0.1	-14.5	-2.9 ± 0.1	-21.7	-8.0 ± 0.1
175°	-43.5	-14.1 ± 0.1	-20.8	-0.2 ± 0.1	-29.0	-6.6 ± 0.1

Table 5

The acoplanarity distribution, $d\sigma/d\psi$ (in nb), for different values of the beam energy, P_{+0} (in GeV), and acoplanarity angle, ψ . The threshold energy $E_{th} = 0.2$ GeV, and $\theta_{min} = 45^\circ$, $\theta_{max} = 135^\circ$.

P_{+0} ψ	0.5	1.0	2.0	3.0	5.0
5°	137.9 ± 1.7	36.40 ± 0.54	9.32 ± 0.16	4.16 ± 0.08	1.50 ± 0.02
10°	43.07 ± 0.30	11.66 ± 0.08	2.99 ± 0.03	1.34 ± 0.01	0.48 ± 0.01
20°	11.59 ± 0.04	3.33 ± 0.01	0.86 ± 0.01	0.39 ± 0.01	0.14 ± 0.01
40°	2.53 ± 0.01	0.83 ± 0.01	0.22 ± 0.01	0.10 ± 0.01	0.04 ± 0.01
60°	0.93 ± 0.01	0.35 ± 0.01	0.10 ± 0.01	0.05 ± 0.01	0.02 ± 0.01
90°	0.33 ± 0.01	0.16 ± 0.01	0.05 ± 0.01	0.02 ± 0.01	0.01 ± 0.01
120°	0.18 ± 0.01	0.10 ± 0.01	0.03 ± 0.01	0.02 ± 0.01	0.01 ± 0.01
150°	0.15 ± 0.01	0.10 ± 0.01	0.03 ± 0.01	0.02 ± 0.01	0.01 ± 0.01
180°	10.17 ± 0.08	14.00 ± 0.15	8.46 ± 0.08	5.89 ± 0.03	3.61 ± 0.03

Table 6

The total cross-section for $e^+e^- \rightarrow e^+e^-\gamma$ (in nb) with the acoplanarity angle ψ larger than 5° , 10° , or 20° for various beam energies, p_{+0} (in GeV). Again, $E_{th} = 0.2$ GeV, $\theta_{min} = 45^\circ$, and $\theta_{max} = 135^\circ$.

p_{+0} ψ	0.5	1.0	2.0	3.0	5.0
$> 5^\circ$	13.82 ± 0.13	4.12 ± 0.06	1.08 ± 0.01	0.49 ± 0.01	0.18 ± 0.01
$> 10^\circ$	7.19 ± 0.04	2.31 ± 0.04	0.64 ± 0.01	0.29 ± 0.01	0.11 ± 0.01
$> 20^\circ$	3.22 ± 0.02	1.20 ± 0.02	0.35 ± 0.01	0.16 ± 0.01	0.06 ± 0.01

Figure caption

Fig. 1 : Dalitz plot for the $e^+e^-\gamma$ events. Along curve 1, the e^+ and e^- make an angle $\pi - \zeta$. The shaded area determines the experimental phase space.

Fig. 1