Measuring $V_{ub}$ in Inclusive $B$ Decays to Charm

Adam F. Falk and Alexey A. Petrov

Department of Physics and Astronomy, The Johns Hopkins University, 3400 North Charles Street, Baltimore, Maryland 21218 USA


We propose a novel method of measuring the CKM matrix element $V_{ub}$, via inclusive $B$ decays to “wrong sign” charm. This process is mediated by the quark level decay $b \to u\bar{s}s'$. When normalized to the inclusive semileptonic decay rate, the theoretical expression is very well behaved, with small uncertainties from unknown quark masses and no leading renormalon ambiguity. We compute the decay rate for this process, including the leading perturbative and nonperturbative corrections.

The accurate measurement of the Cabibbo-Kobayashi-Maskawa parameter $V_{ub}$ remains an important outstanding problem in $B$ physics. The magnitude of $V_{ub}$ corresponds to one of the sides of the Unitarity Triangle, the angles of which will be constrained or extracted from experiments to be performed soon by the CDF, CLEO-III, BaBar and BELLE collaborations. Measuring $|V_{ub}|$ with precision would provide a vital complementary check on the adequacy and consistency of the CKM framework for the physics of flavor in the Standard Model [1].

The methods which are currently available for probing $V_{ub}$ are unfortunately plagued by dependence on phenomenological models whose uncertainties are difficult to quantify reliably. As a result, despite heroic experimental efforts present constraints on this parameter are unacceptably weak. The analyses which have been used fall into two classes, inclusive decays of the form $B \to X_u l\nu$, and exclusive transitions such as $B \to (\pi, \rho) l\nu$.

The inclusive decay rate has the advantage that it can be predicted in the form of a systematic expansion in powers of $1/m_b$ [2]. However, the measurement of this process is complicated by the overwhelming background of inclusive $B \to X_c l\nu$ transitions. At present, this background may be suppressed only by confining oneself to kinematic regions in which only charmless final states can contribute, such as $E_l > 2.3$ GeV or $M(X) < 1.9$ GeV. Unfortunately, the same operator product expansion techniques which allow one to calculate reliably the total inclusive rate break down when the phase space is restricted in this way. Phenomenological models must then be used to reconstruct the rate in the unobserved kinematic regions, and the model independence of the analysis is lost [3,4].

Exclusive transitions such as $B \to (\pi, \rho) l\nu$ are easier to study experimentally. On the other hand theoretical predictions of exclusive decay channels are polluted by our ignorance of the physics of quark hadronization. Various approaches to this problem have been proposed (for example, heavy quark symmetry, chiral expansions in the soft pion limit, dispersion relations, QCD sum rules, and lattice calculations), but in many of these cases significant model dependence remains. It is therefore desirable to continue to look for a model independent method for the extraction of $V_{ub}$.

We propose that $V_{ub}$ may be extracted from the inclusive nonleptonic transition $b \to u\bar{s}s'$, where $s' = s \cos \theta_C - d \sin \theta_C$ is the flavor eigenstate which couples to $c$ and we take $m_s = m_d = 0$. Here $\theta_C$ is the Cabibbo mixing angle, with $\sin \theta_C \approx 0.2205$. This process stands out for its theoretical simplicity. Because the transition involves four distinct quark flavors, only diagrams proportional to $V_{ub}$ enter the theoretical expression, with no complications from penguins or strong rescattering contributions. (For similar reasons, it also has been proposed recently that $V_{ub}$ may be extracted from exclusive processes mediated by this quark transition [5].) Moreover, as we show below, the ratio of this rate to that for $b \to cl\nu$ receives well controlled radiative corrections, with no leading infrared renormalon ambiguity. We will compute the largest nonperturbative corrections, which are spectator-dependent and arise at order $1/m_b^2$. We warn the reader that our analysis will rely implicitly on the use of parton-hadron duality. This assumption is common to all extractions of $V_{ub}$ from inclusive $B$ decays, and while it is not unreasonable to expect it to hold in this case, it has not been proven rigorously to do so.

Is such an analysis feasible experimentally? What is required is to find “wrong-sign” charmed states in flavor-tagged $B$ decays. This is certainly a challenging measurement. However, some of the necessary techniques already exist, as can be seen in Ref. [6]. In addition, the combination of moving pairs of $B$ mesons and the excellent vertexing capabilities of the the BaBar and BELLE detectors should certainly improve the prospects for the measurement of this transition [7]. Of course, the largest source of “wrong-sign” charm is from decays mediated by the transition $b \to c\bar{s}s'$, which are enhanced roughly by a factor of 100 over those which we propose to study. Whether such final states can be rejected at this level is an experimental question which we are certainly not equipped to answer. However, we note that while numerically the problem would seem to be similar to the quite intractable one of digging $b \to ul\nu$ out of the $b \to cl\nu$...
background, it is actually quite different in its details. In particular, in the case of nonleptonic decays there is no missing neutrino, and perhaps the enormous samples of $b \to c\bar{c}s'$ decays which will be available to BaBar, BELLE and CLEO-III will allow one to understand the kinematic features of those decays well enough that they may be subtracted reliably. In any case, our purpose in this Letter is simply to argue that the theoretical situation is so attractive that the feasibility of the experiment is worth investigating.

We begin with the transition $b \to \bar{c}X$ at lowest order. The decay is mediated by the local $\Delta B = 1$ Lagrangian,

$$\mathcal{L} = \frac{4G_F}{\sqrt{2}} V_{ub} \left[ c_1(\mu) \bar{u}_c \gamma_\mu P_L b_\alpha s^\alpha_\gamma \gamma^\mu P_L c_\beta + c_2(\mu) \bar{u}_c \gamma_\mu P_L b_\alpha s^\alpha_\gamma \gamma^\mu P_L c_\beta \right] + \text{h.c.}$$

$$= \frac{4G_F}{\sqrt{2}} V_{ub} \left[ (c_1(\mu) + \frac{1}{N_c} c_2(\mu)) \bar{u}_c \gamma_\mu P_L b s^\gamma \gamma^\mu P_L c + 2c_2(\mu) \bar{u}_c \gamma_\mu T^a P_L b s^\gamma \gamma^\mu T^a P_L c \right] + \text{h.c.}, \quad (1)$$

where $P_L$ is the left-handed projector and $N_c = 3$ is the number of colors. At tree level, $c_1 = 1$ and $c_2 = 0$; to leading logarithmic order, taking $\alpha_s(m_Z) = 0.117$, the Wilson coefficients at the scale $m_b$ are given by $c_1(m_b) \simeq 1.11$ and $c_2(m_b) \simeq -0.25$ [8].

The inclusive decay rate in the channel of interest may be written as the imaginary part of the matrix element of the forward transition operator $T$,

$$\Gamma(b \to \bar{c}X) = |\langle B| T \rangle|^2 \frac{1}{m_b} \text{Im} \left\{ i \int d^4 x T \left[ \mathcal{L}^1(x), \mathcal{L}(0) \right] \right\} |B|. \quad (2)$$

At lowest order, for which the quark level process is $b \to u\bar{c}s'$, we parameterize the $\bar{c}s'$ loop by

$$\Pi_{\mu\nu}^{\ell}(q) = \frac{N_c}{8\pi} \left[ A(q^2) (q_\mu q_\nu - q^2 g_{\mu\nu}) + B(q^2) q_\mu q_\nu \right], \quad (3)$$

where $q^\mu$ is the momentum carried by the $\bar{c}s'$ pair. Then the decay rate may be written as [9]

$$\Gamma(b \to u\bar{c}s') = 3N_c \Gamma_0 |V_{ub}|^2 x_c^2 \int_{1/(1-x_c)}^{1/x_c} dz (1/1-x_c - z)^2 \times |A(z)(2z + 1/x_c) + B(z)/x_c|, \quad (4)$$

with $x_c = m_c^2/m_b^2$ and $\Gamma_0 = G_F^2 m_b^3/192\pi^3$. At tree level, with $q^2 = z m_c^2$, the form factors $A_0(z)$ and $B_0(z)$ are

$$A_0(z) = \frac{2}{3} \chi (1 - 1/z)^2 (1 + 1/2z),$$

$$B_0(z) = \chi b_0(z) = \chi (1 - 1/z)^2 / z \quad (5)$$

with $\chi = (c_1 + c_2/N_c)^2 + 2c_2^2/N_c \simeq 1.09$. The two terms in $\chi$ come respectively from the squares of the color singlet and color octet operators in $\mathcal{L}$. Neglecting the octet operator would induce only a four percent error in $\chi$. Integrating over $z$ and normalizing the result to the semileptonic decay rate $\Gamma(b \to cl\nu) = \Gamma_0 |V_{cb}|^2 f(x_c)$, where

$$f(x_c) = 1 - 8x_c + 8x_c^3 - x_c^4 - 12x_c^2 \ln x_c, \quad (6)$$

yields a very simple expression at leading order,

$$R = \frac{\Gamma(b \to \bar{c}X)}{\Gamma(b \to cl\nu)} = \chi |V_{ub}/V_{cb}|^2 \left\{ 1 + \mathcal{O}(\alpha_s, 1/m_b^2) \right\}. \quad (7)$$

Note that at this order the ratio $R$ is independent of the heavy quark masses. With no corrections included, the only distinction between these two processes is in the kinematics of the final state products. However, the phase space function $f(x_c)$ turns out to be the same in the two cases and cancels in the ratio. The only dependence on $x_c$ will be in the higher order corrections to $R$. Hence the effect of uncertainties and ambiguities in the definition and extraction of $m_c$, $m_b$ and $x_c$ will be much smaller in $R$ than in the individual rates.

We now turn to the effects of perturbative QCD. The radiative correction to $\Gamma(b \to cl\nu)$ comes from the finite renormalization of the $c\bar{c}\gamma^*bL$ current, plus gluon bremsstrahlung. It has been computed analytically by Nir [10], and the result can be put into the form $\Gamma(b \to cl\nu) = \Gamma_0 |V_{cb}|^2 \left[f(x_c) + a_{bc}(x_c) \alpha_s/\pi \right]$. The radiative correction to $\Gamma(b \to u\bar{c}s')$ is more complicated, and we will only consider the corrections to the color singlet transition. In this case we can neglect gluons which propagate from the $bn$ line to the $\bar{c}s'$ line, simplifying the calculation enormously. As noted above, at tree level the color octet transition only contributes four percent to the total decay rate, so the error in omitting the radiative correction to this tiny contribution should be small. Of course, this is a phenomenological approximation, which is not formally consistent in terms of an expansion in leading logarithms. Here we sacrifice formal consistency in favor of identifying and keeping the terms which are numerically the largest. A more complete analysis is presented in Ref. [11].

In this framework, then, there remain two classes of radiative corrections to $\Gamma(b \to u\bar{c}s')$. The first, $a_{bc\ell}$, comes from finite renormalization of the $u_L\gamma^*bL$ current. This correction, which can be extracted from an analysis of $\Gamma(b \to u\bar{c}\nu)$ by Czarnecki, Ježabek and Kühn [12], also depends on $x_c$ through the phase space of the $\bar{c}s'$ pair. The second contribution, $a_{S\ell\mu\nu}$, comes from the radiative corrections to the $\bar{c}s'$ loop [9,13]. The effect, in the case of the color singlet operators, is to add corrections to the form factors in $\Pi_{\mu\nu}^{\ell}(q)$,

$$A_1(z) = \chi a_0(z) \frac{4\alpha_s}{3\pi} \left[f_1(z) + \frac{2z}{1 + 2z} f_2(z) \right],$$

$$B_1(z) = \chi b_0(z) \frac{4\alpha_s}{3\pi} \left[f_1(z) - 1 \right]. \quad (8)$$
where \( f_1(z) \) and \( f_2(z) \) are functions of the charm quark mass which have been computed by Voloshin [9]. We find a radiative correction to \( R \) of the form

\[
R = \chi |V_{ub}/V_{cb}|^2 \left\{ 1 + g(x_c) \frac{\alpha_s}{\pi} + \ldots \right\},
\]

with \( g(x_c) = a_{ub}(x_c) - a_{bc}(x_c) + a_{cs}(x_c) \). The function \( g(x_c) \), along with each individual term, is plotted in Fig. 1 for 4.5 GeV \( \leq m_b \leq 4.9 \) GeV, which via the relation \( m_c = m_B - m_D = 3.34 \) GeV corresponds to the range 0.06 \( \leq x_c \leq 0.12 \). For the “central value” \( x_c = 0.09 \), and with \( \alpha_s(m_b) = 0.22 \), one has \( g(x_c)\alpha_s/\pi = 0.21 \). We see that the one loop radiative correction is fairly large.

In principle, we could also include corrections of order \( \alpha_s^2/\lambda_0 \). Such terms, and all those of the form \( \alpha_s^3/\lambda_0^2 \), play an important role in the resolution of infrared renormalon ambiguities associated with dependence on the pole mass \( m_b \) [14]. In particular, if \( m_b \) (or equivalently, the Heavy Quark Effective Theory (HQET) parameter \( \Lambda \)) appears in an expression at leading order, then the associated perturbative series has a renormalon ambiguity at order \( \Lambda_{QCD}/m_b \), and the term proportional to \( \alpha_s^2/\lambda_0 \) typically is large and must be included. In the present case, however, the leading dependence \( m^2_b f(x_c) \) cancels in the ratio \( R \), and therefore the leading infrared renormalon cancels as well. As a result, there is no reason to expect two loop corrections to be enhanced, even those proportional to \( \alpha_s^2/\lambda_0 \). It is another attractive feature of the process \( b \to ucs' \) that, when normalized to \( b \to c\tau\nu \), its perturbative series has no leading renormalon and is well behaved.

We now turn to the nonperturbative corrections to \( R \). Because \( R \) is independent of \( m_b \) and \( m_c \), at leading order, the first corrections appear at order \( 1/m_c^4 \). They are proportional to the HQET parameters \( \lambda_1 \) and \( \lambda_2 \), which parameterize bound state effects (kinetic energy and hyperfine interactions) of the b quark in the initial B meson [15]. These corrections were calculated for the process \( b \to c\tau\nu \) in Ref. [16]. The results of that calculation apply to \( b \to ucs' \) with the identifications \( m_c \to 0 \) and \( m_\tau \to m_c; \) they apply to \( b \to c\nu \) in the limit \( m_\tau \to 0 \). However, inspection of Eq. (2.15) of Ref. [16] reveals that the correction is actually symmetric in \( m_c \) and \( m_\tau \), and hence the corrections at order \( 1/m_c^4 \) cancel exactly in \( R \).

The leading nonperturbative corrections arise, then, at order \( 1/m_b^3 \). Two types of contribution appear at this order. First, there are additional bound state effects, which may be written in terms of six new HQET parameters \( \rho_1 \), \( \rho_2 \), and \( T_1, \ldots, T_4 \) [17]. We have no reason to suppose that these contributions should be enhanced over their usual size, typically at the percent level in total rates. On the contrary, we might expect somewhat of a cancellation in \( R \) as obtained for \( \lambda_1 \) and \( \lambda_2 \). We will not include such terms here. The second type of effect comes from annihilation processes such as \( b\bar{u} \to \bar{c}s' \). These diagrams contribute only to nonleptonic decays, hence only to the numerator of \( R \). In addition, they are expected to be enhanced over other terms at order \( 1/m_b^3 \) by a phase space factor of \( 16\pi^2 \), corresponding to a two-body instead of a three-body final state. Finally, they contribute differently to \( B^- \) and \( \bar{B}^0 \) decays, and can be probed by comparing \( R \) in the two cases. (Similar processes are responsible for the lifetime difference \( \tau(B^-) \neq \tau(B^0) \) [18–20].)

Annihilation diagrams are associated with four quark operators of dimension six. Performing the operator product expansion to this order, we find a contribution to the transition operator for \( B^- \) and \( \bar{B}^0 \) decays,

\[
T_{sp} = \frac{G_F^2 m_b}{3\pi} |V_{ub}/V_{cb}|^2 (1 - x_c)^2 \left\{ N_c(c_1 + c_2/N_c)^2 \right. \\
\times \left[ (1 + 2x_c)O_{qsp}^q - (1 + x_c/2)O_{V-A}^q \right] \\
+ 2c_2^2 \left[ (1 + 2x_c)T_{S-P}^{q} - (1 + x_c/2)T_{V-A}^{q} \right] \\
\left. + \sin^2 \theta_C N_c(c_2 + c_1/N_c)^2 \right\} \\
\times \left[ (1 + 2x_c)O_{S-P}^{\bar{q}} - (1 + x_c/2)O_{V-A}^{\bar{q}} \right] \\
+ 2\sin^2 \theta_C c_1^2 \left[ (1 + 2x_c)T_{S-P}^{\bar{q}} - (1 + x_c/2)T_{V-A}^{\bar{q}} \right],
\]

where we define [20]

\[
O_{V-A}^q = \bar{b}L_{\gamma^0}\gamma^0 qL_{\gamma^0}bL, \\
O_{S-P}^{\bar{q}} = bR_{\gamma^0}qL_{\gamma^0}bL, \\
T_{V-A}^q = \bar{b}R_{\gamma^0}T^{\gamma^0}qL_{\gamma^0}T^{\gamma^0}bL, \\
T_{S-P}^{\bar{q}} = bR_{\gamma^0}T^{\gamma^0}qL_{\gamma^0}T^{\gamma^0}bL.
\]

The operators \( O_{V-A}^q \) and \( O_{S-P}^{\bar{q}} \) are color singlet, while \( T_{V-A}^q \) and \( T_{S-P}^{\bar{q}} \) are color octet. Also, note that the terms mediating \( \bar{B}^0 \) decays are suppressed by an additional factor of \( \sin^2 \theta_C \), because annihilating the initial state requires an operator with \( q = d \) rather than \( q = s' \).

The contribution of \( T_{sp} \) to \( B \) decays is obtained by evaluating the forward matrix elements \( \langle B_0 | T_{sp} | B_0 \rangle \). The matrix elements of the four quark operators depend on nonperturbative QCD and have not been computed in a controlled way. Although models such as quenched lattice QCD and QCD sum rules can yield estimates of
their values, reliable information will probably have to wait until full unquenched lattice analyses are available. It is convenient to parameterize the matrix elements in terms of “bag parameters” $B_i$ and $\epsilon_i$ [20],

\[
\begin{align*}
\langle B_q | O_{q-A}^i | B_q \rangle &= \frac{1}{N_c} f_B^2 m_B^2 B_1, \\
\langle B_q | O_{q-p}^i | B_q \rangle &= \frac{1}{N_c} f_B^2 m_B^2 B_2, \\
\langle B_q | T_{q-A}^i | B_q \rangle &= \frac{1}{4} f_B^2 m_B^2 \epsilon_1, \\
\langle B_q | T_{q-p}^i | B_q \rangle &= \frac{1}{4} f_B^2 m_B^2 \epsilon_2.
\end{align*}
\]

(12)

In the vacuum insertion ansatz, we have $B_1 = B_2 = 1$ and $\epsilon_1 = \epsilon_2 = 0$, and only the color singlet operators contribute to the decay. In an expansion in powers of $1/N_c$, one finds $B_i = O(1)$, while $\epsilon_i = O(1/N_c)$; in this more general case, the color octet matrix elements are suppressed but nonvanishing. For example, one QCD sum rules estimate gives $\epsilon_1 \approx -0.15$ and $\epsilon_2 \approx 0$ [21]. In terms of these parameters, the contributions to $R(B^-, \bar{B}^0) = \langle V_{ub}/V_{cb} \rangle^2 [1 + g(x_c)\alpha_s/\pi + \delta_3(B^-, \bar{B}^0)]$ are given by

\[
\begin{align*}
\delta_3(B^-) &= \frac{16\alpha_s^2 f_B^2 (1 - x_c^2)}{\chi N_c m_B^2 f(x_c)} \left\{ N_c (c_1 + c_2/N_c) \times [(1 + 2x_c)B_2 - (1 + x_c/2)B_1] \\
& \quad + 2c_2 [(1 + 2x_c)e_2 - (1 + x_c/2)e_1] \right\}, \\
\delta_3(\bar{B}^0) &= \frac{16\alpha_s^2 f_B^2 (1 - x_c^2)}{\chi N_c m_B^2 f(x_c)} \frac{\sin^2 \theta_c}{2} \left\{ N_c (c_2 + c_1/N_c) \times [(1 + 2x_c)B_2 - (1 + x_c/2)B_1] \\
& \quad + 2c_1 [(1 + 2x_c)e_2 - (1 + x_c/2)e_1] \right\}.
\end{align*}
\]

(13)

With $x_c = 0.09$ and $f_B = 200$ MeV, we find

\[
\begin{align*}
\delta_3(B^-) &= -0.46B_1 + 0.52B_2 - 0.018e_1 + 0.020e_2, \\
\delta_3(\bar{B}^0) &= -0.00032B_1 + 0.00037B_2 - 0.017e_1 + 0.020e_2.
\end{align*}
\]

(14)

We see that the annihilation process is much larger in charged than in neutral B decays. Using factorization, with $B_1 = 1$ and $\epsilon_i = 0$, we have $\langle \delta_3(B^-, \bar{B}^0) \rangle = (0.059, 4.2 \times 10^{-3})$; taking instead the sum rules estimate $\epsilon_2 = -0.15$ gives $\langle \delta_3(B^-, \bar{B}^0) \rangle = (0.062, 2.6 \times 10^{-3})$. Since it depends dominantly on the largely unconstrained octet matrix element $\epsilon_2$, there is huge uncertainty in $\delta_3(\bar{B}^0)$; on the other hand, for any reasonable values of the bag parameters the annihilation contribution to $\bar{B}^0$ decay is below the percent level and is safely negligible. By contrast, $\delta_3(B^-)$ is much larger, but since it depends mostly on the singlet matrix elements $B_1$, it has a smaller fractional uncertainty.

To summarize, we have computed the inclusive branching fraction for “wrong sign” charm production in B decays, normalized to inclusive semileptonic decay. The leading perturbative correction $g(x_c)\alpha_s/\pi$ is analytically calculable, independent of the spectator quark, and approximately 20%. The annihilation contribution $\delta_3$ is smaller but more uncertain; it enters at the 5% level for $B^-$ decays and is negligible for $\bar{B}^0$ decays. There is no leading renormalon ambiguity in $R$, and dependence on quark masses appears only in the radiative and nonperturbative corrections themselves. As a result, this process is an excellent place to extract $|V_{ub}|$ from a theoretical point of view. This measurement will be challenging experimentally, but given the crucial role of $V_{ub}$ in constraining the Unitary Triangle and the present lack of a viable alternative, it certainly is worth pursuing.

After this letter was submitted, we became aware that the attractiveness of this process for probing $V_{ub}$ has been noted previously [22], although not discussed in any detail.

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