Abstract

We calculate the initial non-equilibrium conditions from perturbative QCD (pQCD) within Glauber multiple scattering theory for \( \sqrt{s} = 200 \) AGeV and \( \sqrt{s} = 5.5 \) ATeV. At the soon available collider energies one will particularly test the small \( x \) region of the parton distributions entering the cross sections. Therefore shadowing effects, previously more or less unimportant, will lead to new effects on variables such as particle multiplicities \( dN/dy \), transverse energy production \( dE_T/dy \), and the initial temperature \( T_i \). In this paper we will have a closer look on the effects of shadowing by employing different parametrizations for the shadowing effect for valence quarks, sea quarks and gluons. Since the cross sections at midrapidity are dominated by processes involving gluons the amount of their depletion is particularly important. We will therefore have a closer look on the results for \( dN/dy \), \( dE_T/dy \), and \( T_i \) by using two different gluon shadowing ratios, differing strongly in size. As a matter of fact, the calculated quantities differ significantly.

\footnote{This work was supported by BMBF, DFG, and GSI}
1. Introduction

One of the challenging goals of heavy ion physics is the detection of the quark-gluon plasma, a state in which the partons are able to move freely within a distance larger than the typical confinement scale \( r_{\text{conf.}} \sim 1/\Lambda_{\text{QCD}} \sim 1/0.2 \text{ GeV} \sim 1 \text{ fm} \). The build-up of this state should happen early in a heavy ion reaction when the two streams of initially cold nuclear matter pass through each other. Thereby first virtual partons are transformed to real ones and later on in the expansion phase the fragmentation of the partons into colorless hadrons takes place. When separating pQCD from non-perturbative effects at some semi-hard scale \( p_0 = 2 \text{ GeV} \) the respective time scale of perturbative processes is thus of the order \( \tau \sim 1/p_0 \sim 0.1 \text{ fm/c} \) which approximately coincides with the lower bound of the initial formation time of the plasma in a local cell [1]. Therefore all further evolution of the system is significantly influenced by the initial conditions of pQCD since macroscopic parameters, as e.g. the initial temperature \( T_i \), directly enter into hydrodynamical calculations.

We here will focus on the very early phase of an ultrarelativistic heavy ion collision and use pQCD above the semi-hard scale \( p_{\text{sh.}} = p_0 = 2 \text{ GeV} \).

In a typical high energy \( pp \) or \( p\bar{p} \) event one measures distinct hadronic jets with a transverse momenta of several GeV \( (p_T \geq 5 \text{ GeV}) \) [2]. In contrast to the experimental very clean situation of hadronis jets at large \( p_T \) one encounters the problem of detectability of low transverse momentum jets in heavy ion collisions. These so-called minijets contribute significantly to the transverse energy produced in AB collisions due to their large multiplicity [3]. The major part of these set-free partons are gluons that strongly dominate the processes as their number is much larger for the relevant momentum fractions. In turn the shadowing effects are expected to be much larger for gluons than for the quark sea [4]. Therefore the relative contribution of the gluons should decrease but still dominate the cross sections. The shadowing of the gluons has the peculiarity of not being known exactly due to the neutrality of the mediators of the strong interaction which makes it impossible to access \( R_G(x, Q^2) \) directly in a deep inelastic \( e + A \) event. Therefore we
will here investigate two possible parametrizations of the shadowing ratio \( R_G = xG^A/A \cdot xG^N \) for gluons as will be described below in detail.

2. Minijets

As outlined above, we will here investigate the effects of shadowing on the minijet production cross sections. The production of a parton \( f = g, q, \bar{q} \) can in leading order be described as [3]

\[
\frac{d\sigma_f}{dy} = \int dp_T^2 \, dy_2 \sum_{ij,kl} x_1 f_i(x_1, Q^2) \, x_2 f_j(x_2, Q^2) \times \left[ \delta_{fk} \frac{d\hat{s}^{ij-kl}}{dt}(\hat{t}, \hat{u}) + \delta_{fl} \frac{d\hat{s}^{ij-kl}}{dt}(\hat{u}, \hat{t}) \right] \frac{1}{1 + \delta_{kl}}
\]

The factor \( 1/(1 + \delta_{kl}) \) enters due to the symmetry of processes with two identical partons in the final state. The exchange term \( d\hat{s}(\hat{t}, \hat{u}) \leftrightarrow d\hat{s}(\hat{u}, \hat{t}) \) accounts for the possible symmetries of e.g. having a quark from nucleon \( i \) and a gluon from nucleon \( j \) and vice versa, i.e. it handles the interchange of two of the propagators in the scattering process. The possible combinations of initial states are

\[
ij = gg, gq, qg, \bar{q}g, qq, q\bar{q}, \bar{q}g, \bar{q}q
\]

The momentum fractions of the partons in the initial state are

\[
x_1 = \frac{p_T}{\sqrt{s}} \left[ e^y + e^{y_2} \right], \quad x_2 = \frac{p_T}{\sqrt{s}} \left[ e^{-y} + e^{-y_2} \right]
\]

The integration regions are

\[
p_0^2 \leq p_T^2 \leq \left( \frac{\sqrt{s}}{2 \cosh y} \right)^2, \quad -\ln \left( \frac{\sqrt{s}}{p_T} - e^{-y} \right) \leq y_2 \leq -\ln \left( \frac{\sqrt{s}}{p_T} - e^{-y} \right)
\]

with

\[
|y| \leq \ln \left( \frac{\sqrt{s}}{2p_0} + \sqrt{\frac{s}{4p_0^2} - 1} \right)
\]

The mandelstam variables are defined as

\[
\hat{s} = x_1 \cdot x_2 \cdot s, \quad \hat{t} = -p_T^2 \left[ 1 + e^{(y_2-y)} \right], \quad \hat{u} = -p_T^2 \left[ 1 + e^{(y-y_2)} \right]
\]
For the parton distributions entering the handbag graph we choose the GRV LO set [5] for RHIC. Since at LHC one probes smaller momentum fractions we there use the newer CTEQ4L parametrization [6] with $N_f = 4$ and $Q = p_T$. The normalization is done so that one has two outgoing partons in one collision, i.e.

$$\int dy \frac{d\sigma^f}{dy} = 2\sigma_{tot}$$  \hspace{1cm} (7)$$

In the calculations the boundaries for the calculations are either over the whole rapidity range or $|y| \leq 0.5$ for the central rapidity region.

To account for the higher order contributions at RHIC we choose a fixed $K$ factor of $K=2.5$ from comparison with experiment [3, 2]. In the range $5.5 \text{ GeV} \leq p_T \leq 25 \text{ GeV}$ a factor $K=2.5$ is needed to describe the UA1 data, and in the range $30 \text{ GeV} \leq p_T \leq 50 \text{ GeV}$ a factor of $K=1.6$ is needed. However the cross section has dropped so much at these large transverse momenta that we keep $K=2.5$ fixed for all $p_T$. For LHC energies the mean $p_T$ tends to be larger; so we choose $K=1.5$ for this case.

By applying Glauber theory we calculate the mean number of events per unit of rapidity:

$$\frac{dN^f}{dy} = T_{AA}(b)\frac{d\sigma^f}{dy}$$  \hspace{1cm} (8)$$

where the nuclear overlap function $T_{AA}(b)$ for central events is given by $T_{AA}(0) \approx A^2/\pi R_A^2$. For the nuclei in our calculation this gives $T_{AuAu}(0) = 29/mb$ and $T_{PbPb}(0) = 32/mb$. Again it should be emphasized that $dN^f/dy$ gives the number of collisions and that the number of partons is as twice as high in a $2 \rightarrow 2$ process. The necessary volume, needed to derive the densities from the absolute numbers, is calculated as

$$V_i = \pi R_A^2 \Delta y/p_0, \hspace{0.5cm} R_A = A^{1/3} \times 1.1 \hspace{0.1cm} fm$$  \hspace{1cm} (9)$$

Therefore we get $V_i(Au + Au) = 12.9 \hspace{0.1cm} fm^3$ and $V_i(Pb + Pb) = 13.4 \hspace{0.1cm} fm^3$.

For the energy density at midrapidity we need the first $E_T$ moment:

$$\sigma^f \langle E_T \rangle = \int dE_T \frac{d\sigma^f}{dE_T} \langle E_T \rangle$$
\begin{equation}
R_{F2} \text{ vs. } R_G \text{ at } Q^2 = 4 \text{ GeV}^2 \text{ for } ^{207}\text{Pb}.
\end{equation}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1}
\caption{\(R_{F2} \text{ vs. } R_G\) at \(Q^2 = 4 \text{ GeV}^2\) for \(^{207}\text{Pb}\).}
\end{figure}

\begin{equation}
= \int dp_T^2 dy_2 \sum_{ij,kl} x_1 f_i(x_1, Q^2) x_2 f_j(x_2, Q^2)
\times \left[ \delta_{fk} \frac{d\hat{\phi}^{ij}}{d\hat{u}}(\hat{t}, \hat{u}) + \delta_{fl} \frac{d\hat{\phi}^{ij}}{d\hat{u}}(\hat{u}, \hat{t}) \right] \frac{1}{1 + \delta_{kl}} p_T \epsilon(y)
\end{equation}

Here the acceptance function \(\epsilon(y)\) is \(\epsilon(y) = 1\) for \(|y| \leq 0.5\) and \(\epsilon(y) = 0\) otherwise.

3. Nuclear Shadowing

In heavy ion collisions one has to account for an effect that does not appear for processes involving two nucleons only: nuclear shadowing. In the lab frame the deep inelastic scattering at small Bjorken \(x \ (x \ll 0.1)\) proceeds via the vector mesons as described in the vector meson dominance model (VMD) where the handbag graph contribution becomes small. In VMD the interaction of the virtual photon with a nucleon or nucleus is described as a two step process: the photon fluctuation into a \(q\bar{q}\) pair (the \(\rho, \omega, \phi\) mesons at small \(Q^2\)) within the coherence time \(l_c\) and a subsequent strong interaction with the target [8]. The coherence time arises in this picture from the longitudinal momentum shift between the photon and the fluctuation: \(l_c \approx 1/\Delta k_z\) where \(\Delta k_z = k_1^z - k_2^z\). The cross section is:

\begin{equation}
\sigma(\gamma^*N) = \int_{0}^{1} dz \int d^2 r |\psi(z, r)|^2 \sigma_{q\bar{q}N}(r)
\end{equation}
where the Sudakov variable $z$ gives the momentum fraction carried by the quark or the antiquark. The interaction of the fluctuation with the nucleon can be described in the color transparency model as [9]

$$\sigma_{q\bar{q}N} = \frac{\pi^2}{3} r^2 \alpha_s(Q^2)x'g(x', Q^2)$$  \hspace{1cm} (12)

where $x' = M_{q\bar{q}}^2/(2m\nu)$, $r$ is the transverse separation of the pair and $Q^2 = 4/r^2$. For the interaction of the fluctuation with a nucleus one makes use of Glauber-Gribov multiple scattering theory [10] where the fluctuation interacts coherently with more than one nucleon in the nucleus when the coherence length exceeds the mean separation between two nucleons:

$$\sigma_{q\bar{q}A} = \int d^2b \left( 1 - e^{-\sigma_{q\bar{q}N}T_A(b)/2} \right)$$  \hspace{1cm} (13)

When expanding for large nuclei and taking the dominating double scattering term only one finds

$$\sigma_{hA} = A\sigma_{hN} \left[ 1 - A^{1/3} \frac{\sigma_{hN}}{8\pi a^2} + \ldots \right]$$  \hspace{1cm} (14)

with $a = 1.1 fm$.

Figures 1 and 2 show the results for $^{207}Pb$ and $^{40}Ca$ (for further details see [4]).
A very different scenario is employed in parton fusion models. Here the process of parton parton fusion in nuclei can be understood as an overlapping of quarks and gluons that yields a reduction of number densities at small $x$ and a creation of antishadowing for momentum conservation at larger $x$ [11]. The onset of this fusion process can be estimated to start at values of the momentum fraction where the longitudinal wavelength ($1/xP$) of a parton exceeds the size of a nucleon (or the inter-nucleon distance) inside the Lorentz contracted nucleus: $1/xP \approx 2R_nM_n/P$, corresponding to a value $x \approx 0.1$. Originally the idea of parton fusion was proposed in [12] and later proven in [13] to appear when the total transverse size $1/Q$ of the partons in a nucleon becomes larger than the proton radius to yield a transverse overlapping within a unit of rapidity, $xG(x) \geq Q^2R^2$. The usual gluon distribution in the nucleon on the light cone in light-cone gauge ($n \cdot A = A^+ = 0$) is given by

$$xG(x) = -(n^-)^2 \int \frac{d\lambda}{2\pi} \left\langle P \left| F^{+\mu}(0)F^{+\mu}(\lambda n) \right| P \right\rangle$$  \hspace{1cm} (15)

The recombination is then described as the fusion of two gluon ladders into a single vertex. One finally arrives at a modified Altarelli-Parisi equation where the fusion correction enters as a twist four light cone correlator. Typically the fusion correction in the free nucleon turns out to be significant only for unusually small values of $x$ or $Q^2$. As shown in [14] the situation changes dramatically in heavy nuclei. Here the strength of the fusion for ladders coming from independent constituents increases and is of the same order as the fusion from non-independent constituents. Therefore, parton recombination is strongly increased in heavy nuclei of $A \sim 200$.

Unfortunately the different models do not give the same results for the ratio $R_G(x, Q^2)$. We will therefore use two versions of parametrizations to investigate the effects of shadowing on the relevant variables. On the one hand we use a $Q^2$ dependent version (see figure 3) that tries to avoid any model dependence by using sum rules for baryon number and momentum [15] and on the other hand we use a modified version of a $Q^2$ independent parametrization (see figure 4) given in [16] which employs a much stronger
gluon shadowing in accordance with the results of [4]. Especially for RHIC, where the lower bound for the momentum fraction at midrapidity for $p_T = p_0 = 2 \text{ GeV}$ is given by $x = 2p_T/\sqrt{s} = 0.02$, the onset of the gluon shadowing, i.e. the transition region between shadowing and antishadowing, is of great importance. 

In [15] the onset of gluon shadowing ($R_G = 1$) is chosen at $x \approx 0.029$ for $Q = 2 \text{ GeV}$ motivated by the results found in [17] where the connection between the gluon distribution and the $Q^2$ dependence of $F_2$ via the DGLAP equations was employed:

$$\frac{\partial F_2}{\partial \ln Q^2} \sim \sum_i e_i^2 x G(2x, Q^2)$$ (16)

By using the NMC data [18] on deep inelastic scattering on a combination of Sn and C targets the ratio $G^{Sn}(x)/G^{C}(x)$ was derived in the range $0.011 \leq x \leq 0.18$. The cross over point can, despite the large errorbars, be guessed to be $x \approx 0.03$. However one should add here that the situation for $R_G^{Pb} = xG^{Pb}(x)/xG^{N}(x)$ can look rather different. Since this question of the onset of gluon shadowing is not yet settled we chose the same onset for quark and gluon shadowing in our modified parametrization to investigate

Figure 3: Shadowed parton distributions as parametrized by Eskola. Note that the gluon shadowing appears to be weaker than quark shadowing and that the onset happens for smaller $x$. 

Figure 4: Our variation of the parametrization given in [16] with much stronger gluon shadowing as found in the VMD calculation in [4]. The stronger shadowing is also motivated by the fact that we calculate central collisions where the shadowing effect is stronger than for b-averaged collisions.

4. Results

In the following we will give the results for the different parton species $f = g, q, \bar{q}$ at RHIC and LHC including the different shadowing parametrizations or none shadowing, respectively. The results for the number of partons $f dN^f/dy$ can easily be derived from $f dy d\sigma^f/dy$ by the relation $dN^f/dy = 2T_{AA}(0) d\sigma^f/dy$. All results include a K-factor of $K=2.5$ for RHIC and $K=1.5$ for LHC. On the one hand we give the results for the whole $y$-range and on the other hand we give the result for the central rapidity region which is of special interest, not only from the experimental setup point of view but also since it is the region where highest parton densities and the strongest
shadowing effects are expected. Let us start by giving the results without shadowing corrections for RHIC. The first three tables give the unshadowed multiplicities integrated over the whole rapidity range and over the central region, respectively. Tables 4 through 6 give the first $E_T$ moments for the respective parton species. The rapidity distributions for the cross sections are depicted in figure 5.

**Table 1:** $\int dy \, dN^{g}/dy$ for $\sqrt{s} = 200$ AGeV

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gg \to gg$</th>
<th>$gq \to gq + g\bar{q} \to g\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>all $y$</td>
<td>1841.5</td>
<td>768.5</td>
<td>2610.0</td>
</tr>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>385.8</td>
</tr>
</tbody>
</table>

**Table 2:** $\int dy \, dN^{q}/dy$ for $\sqrt{s} = 200$ AGeV

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gq \to gq$</th>
<th>$qq \to qq$</th>
<th>$gg \to q\bar{q}$</th>
<th>$q\bar{q} \to q\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>all $y$</td>
<td>620.5</td>
<td>114.5</td>
<td>13.0</td>
<td>45.5</td>
<td>793.5</td>
</tr>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>42.0</td>
<td>14.75</td>
<td>3.0</td>
</tr>
</tbody>
</table>

**Table 3:** $\int dy \, dN^{\bar{q}}/dy$ for $\sqrt{s} = 200$ AGeV

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$g\bar{q} \to g\bar{q}$</th>
<th>$g\bar{q} \to q\bar{q}$</th>
<th>$gg \to q\bar{q}$</th>
<th>$q\bar{q} \to q\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>all $y$</td>
<td>148.3</td>
<td>45.5</td>
<td>13.0</td>
<td>4.3</td>
<td>169.8</td>
</tr>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>25.0</td>
<td>10.3</td>
<td>3.0</td>
</tr>
</tbody>
</table>
Figure 5: Unshadowed rapidity distributions of gluons, quarks, and anti-quarks.
Table 4: $\sigma_g \langle E_T \rangle$ [mb GeV]

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gg \to gg$</th>
<th>$gq \to gq + g\bar{q} \to g\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>18.02</td>
</tr>
</tbody>
</table>

Table 5: $\sigma_q \langle E_T \rangle$ [mb GeV]

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gq \to gq$</th>
<th>$qg \to qg$</th>
<th>$gg \to q\bar{q}$</th>
<th>$q\bar{q} \to q\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>2.065</td>
<td>0.786</td>
<td>0.1398</td>
</tr>
</tbody>
</table>

Table 6: $\sigma_{\bar{q}} \langle E_T \rangle$ [mb GeV]

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gq \to \bar{q}q$</th>
<th>$qg \to q\bar{q}$</th>
<th>$gg \to q\bar{q}$</th>
<th>$q\bar{q} \to q\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>1.206</td>
<td>0.51</td>
<td>0.139</td>
</tr>
</tbody>
</table>
Figure 6: Rapidity distributions of gluons, quarks, and antiquarks with our modified shadowing parametrization shown in figure 4.
For the strong gluon shadowing shown in figure 4 one finds the following multiplicities for the different parton species (the rapidity distributions are shown in figure 6):

**Table 7:** $\int dy \, dN^g/dy$ for $\sqrt{s} = 200$ AGeV

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gg \rightarrow gg$</th>
<th>$gq \rightarrow gq + g\bar{q} \rightarrow g\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>all $y$</td>
<td>1162.9</td>
<td>498.8</td>
<td>1661.7</td>
</tr>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>245.1</td>
</tr>
</tbody>
</table>

**Table 8:** $\int dy \, dN^q/dy$ for $\sqrt{s} = 200$ AGeV

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gq \rightarrow gq$</th>
<th>$qq \rightarrow qq$</th>
<th>$gg \rightarrow q\bar{q}$</th>
<th>$q\bar{q} \rightarrow q\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>all $y$</td>
<td>392.95</td>
<td>97.15</td>
<td>7.25</td>
<td>36.25</td>
<td>533.6</td>
</tr>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>31.9</td>
<td>11.7</td>
<td>1.73</td>
</tr>
</tbody>
</table>

**Table 9:** $\int dy \, dN^{\bar{q}}/dy$ for $\sqrt{s} = 200$ AGeV

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$\bar{q} \bar{q} \rightarrow \bar{q} \bar{q}$</th>
<th>$q \bar{q} \rightarrow q \bar{q}$</th>
<th>$gg \rightarrow q \bar{q}$</th>
<th>$\bar{q} \bar{q} \rightarrow \bar{q} \bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>all $y$</td>
<td>105.85</td>
<td>36.25</td>
<td>7.25</td>
<td>3.63</td>
<td>152.98</td>
</tr>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>18.85</td>
<td>8.13</td>
<td>1.89</td>
</tr>
</tbody>
</table>

The first $E_T$ moments for the reactions including our modified strong gluon shadowing are given by:

**Table 10:** $\sigma^g \langle E_T \rangle$ [mb GeV]

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gg \rightarrow gg$</th>
<th>$gq \rightarrow gq + g\bar{q} \rightarrow g\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>11.87</td>
</tr>
</tbody>
</table>
We also calculated the multiplicities and first $E_T$ moments by employing the newest available shadowing parametrization of Eskola et al shown in figure 3. As emphasized above one should note that the shadowing of gluons in this parametrization is smaller than the quark shadowing since it was tried to stay away from any model dependence and just stick to sum rules expressing the momentum and baryon number conservation but still assuming that at small $x$ ($x \approx 10^{-4}$) the gluon ratio should coincide with the sea quark ratio. By employing this version we find the results listet in the following tables and shown in figure 7:

<table>
<thead>
<tr>
<th>Table 11: $\sigma^q \langle E_T \rangle$ [mb GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>range of $y$</td>
</tr>
<tr>
<td>$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 12: $\sigma^\bar{q} \langle E_T \rangle$ [mb GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>range of $y$</td>
</tr>
<tr>
<td>$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 13: $\int dy , dN^g/dy$ for $\sqrt{s} = 200$ AGeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>range of $y$</td>
</tr>
<tr>
<td>all $y$</td>
</tr>
<tr>
<td>$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 14: $\int dy , dN^q/dy$ for $\sqrt{s} = 200$ AGeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>range of $y$</td>
</tr>
<tr>
<td>all $y$</td>
</tr>
<tr>
<td>$</td>
</tr>
</tbody>
</table>
Figure 7: Rapidity distributions of gluons, quarks, and antiquarks with the ’98 version of the shadowing parametrization shown in figure 3.

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gq \rightarrow gq$</th>
<th>$qq \rightarrow qq$</th>
<th>$gg \rightarrow q\bar{q}$</th>
<th>$q\bar{q} \rightarrow q\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>all $y$</td>
<td>133.5</td>
<td>37.7</td>
<td>130.5</td>
<td>33.3</td>
<td>334.8</td>
</tr>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>21.75</td>
<td>8.28</td>
<td>3.05</td>
</tr>
</tbody>
</table>
Figure 8: Comparison of the rapidity distributions with strong gluon shadowing (left figure) and weak shadowing (right figure) to unshadowed distribution for RHIC (see text).

In figure 8 we directly compared the strong gluon shadowed distributions (left figure) with the unshadowed one. The same was done for the comparison of the $Q^2$ dependent '98 shadowing version with the unshadowed one (right figure). The solid lines give the total contribution, the dotted ones the contribution from the $gg$ subprocess and the dashed lines give the $gq + g\bar{q}$ contribution. The thick lines denote the unshadowed distributions and the thin ones the two shadowed ones. Note that due to the onset of gluon shadowing in the '98 version at such small values of $x$ one even gets an enhancement for the $gg \to gg$ subprocess at RHIC. We also calculated the $p_T$ distribution without and with the two shadowing versions at midrapidity (figure 9). Unlike the strong shadowing case the cross over point of the curves already happens at $p_T \approx 2.5$ GeV for the '98 gluon shadowing version which immediately explains the enhancement in the rapidity distribution. For the first $E_T$ moment of the transverse energy we find with Eskola's shadowing parametrization

**Table 16: $\sigma^g \langle E_T \rangle$ [mb GeV]**

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gg \to gg$</th>
<th>$gq \to gq + g\bar{q} \to g\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>19.2</td>
</tr>
</tbody>
</table>
Figure 9: $p_T$ distributions for the two shadowing parametrizations at midrapidity.

Table 17: $\sigma^{g}\langle E_T \rangle$ [mb GeV]

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gq \to gq$</th>
<th>$gq \to qq$</th>
<th>$gg \to q\bar{q}$</th>
<th>$q\bar{q} \to q\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>2.03</td>
<td>0.67</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 18: $\sigma^{\bar{q}}\langle E_T \rangle$ [mb GeV]

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gq \to g\bar{q}$</th>
<th>$q\bar{q} \to q\bar{q}$</th>
<th>$gg \to q\bar{q}$</th>
<th>$q\bar{q} \to q\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>1.123</td>
<td>0.032</td>
<td>0.148</td>
</tr>
</tbody>
</table>

From the results above we can calculate the total transverse energy $E_T = \sigma < E_T > T_{AA}(0)$ carried by the partons, the number and energy densities $n_f$ and $\varepsilon_f$, and also derive the initial temperature $T_i$ if we assume the behavior of an ideal gas of partons. To do so we need the initial volume. With $R_A = A^{1/3} \cdot 1.1 \text{ fm}, T_{AuAu}(0) = 29/mb$, and $R_{Au} = 6.4 \text{ fm}$ we find $V_i = \pi R_A^2 \Delta y \tau = 12.9 \text{ fm}^3$.

Therefore at RHIC without any shadowing and with $K=2.5$ we have at midrapidity a total number of 567 gluons, 64 quarks, and 39 antiquarks. These carry a transverse energy of 774 GeV (gluons), 93 GeV (quarks), and 55 GeV (antiquarks).

It is then straightforward to derive the number densities by dividing by the initial volume to yield: $n_g = 44 \text{ fm}^{-3}$, $n_q = 4.96 \text{ fm}^{-3}$, $n_{\bar{q}} = 3.02 \text{ fm}^{-3}$. 18
The energy densities can be derived in an analogous way to give:

\[ \varepsilon_g = 60 \text{ GeV/fm}^{-3}, \quad \varepsilon_q = 7.2 \text{ GeV/fm}^{-3}, \quad \text{and} \quad \varepsilon_{\bar{q}} = 4.3 \text{ GeV/fm}^{-3}. \]

If we assume total equilibrium we can derive the initial temperature from these numbers as

\[ \varepsilon_{\text{ideal}} = 16\pi^2 \frac{3}{90} T_{eq}^4 \quad (17) \]

At this point some comments are appropriate: one could wonder whether the system can be in equilibrium since one has only hard 2 \rightarrow 2 parton scatterings in this Glauber approach. Also one often assumes global equilibrium to be established after, say 1 fm/c. Now here we are mainly interested in local equilibrium as it is required for example for hydrodynamical calculations. The equilibration of partons in a local cell happens to be much faster for the following reasons. The high \( Q^2 \) hard scatterings among the partons are absolutely unimportant for the equipartition of longitudinal and transverse degrees of freedom. It are the soft interactions that are responsible for this feature and there is a huge resource of soft partons available in the nucleons, even when assuming the parton distributions to be shadowed in heavy nuclei. The link to the short equilibration time is the fact that even though the nucleus is Lorentz contracted to \( L/cosh y \), the partons obey the uncertainty principle and are therefore smeared out to distances \( 1/\xi P \) in the infinite momentum frame and so the major part of the partons is outside the Lorentz contracted disk. Based on some basic principles and by using the Fokker-Planck equation [1, 19] the time it takes to establish local equilibrium in a cell was estimated to have a lower bound of \( \tau_0 \approx 0.15 \text{ fm/c} \). As noted above we introduced a lower momentum cut-off \( p_0 = 2 \text{ GeV} \) corresponding to a proper time of about 0.1 \( \text{fm/c} \). So therefore we may not be far from local equilibration and the calculation on the initial temperature from the initial energy density could be rather justified.

For the temperature we take into account only the gluons due to their large multiplicity and energy density that dominates the respective values for the quarks. We then find \( T_i = 549.52 \text{ MeV} \) for RHIC. If we neglect all higher orders, i.e. take a K-factor of \( K=1 \) (which of course is wrong, but it is instructive to see the impact on \( T_i \)), we get \( T_i^{K=1} = 437 \text{ MeV} \).
The same quantities were then calculated for the two different shadowing scenarios. For the calculations employing the strong gluon shadowing we found that there are 365 gluons, 49 quarks, and 30 antiquarks carrying transverse energies of 516 GeV (gluons), 29 GeV (quarks), and 17 GeV (antiquarks). The resulting number and energy densities are found to be $n_g = 28.3 \text{ fm}^{-3}$, $n_q = 3.79 \text{ fm}^{-3}$, $n_{\bar{q}} = 2.33 \text{ fm}^{-3}$, $\varepsilon_g = 40 \text{ GeV/fm}^{-3}$, $\varepsilon_q = 2.3 \text{ GeV/fm}^{-3}$, and $\varepsilon_{\bar{q}} = 1.3 \text{ GeV/fm}^{-3}$. When we calculate the initial temperature for an ideal parton gas from these numbers we find that the initial temperature decreases due to the reduced number and energy densities having their origin in the shadowing of the parton distributions. We find $T_{i,\text{shad}} = 496.5 \text{ MeV}$ for a K factor of 2.5 and when neglecting all higher order contributions we derive $T_{i,\text{shad}}^K = 394.9 \text{ MeV}$. So what we can learn here ist the following: due to the reduced number of partons involved in the hard processes a reduction in the number densities and therefore in the energy densities entering the formula for the temperature of a thermalized parton gas results. One should note that the onset of shadowing in our modified shadowing parametrization was chosen same for quarks and gluons in accordance with the onset of coherent scattering of a quark antiquark or gluon gluon pair, respectively off a nucleus. Now in the second shadowing parametrization we employed one finds that the onset of shadowing for gluons starts at smaller momentum fractions from $xG^{Sn}(x)/xG^C(x)$ data. With a momentum cut-off $p_0 = 2 \text{ GeV}$ the momentum fractions involved in processes at midrapidity are bound from below at $x = 0.02$. Therefore one is right on the edge of the onset of shadowing of the parametrizations and one should expect the very interesting case that one is on the edge to the antishadowing region for gluons in the parametrization of Eskola et al but not so for the parametrization employing the strong gluon shadowing. This behavior is immediately reflected in the number and energy densities. We found that for this specific shadowing parametrization one has 567 gluons, 58 quarks, and 34 antiquarks carrying transverse energies of 790 GeV (gluons), 83 GeV (quarks), and 50 GeV (antiquarks). We found the following densities: $n_g = 44 \text{ fm}^{-3}$, $n_q = 4.49 \text{ fm}^{-3}$, $n_{\bar{q}} = 2.64 \text{ fm}^{-3}$, $\varepsilon_g = 61.2 \text{ GeV/fm}^{-3}$, $\varepsilon_q = 6.43 \text{ GeV/fm}^{-3}$, and $\varepsilon_{\bar{q}} = 3.88 \text{ GeV/fm}^{-3}$. 

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These numbers result in an initial temperature of $T_{i,\text{shad}} = 552.3 \text{ MeV}$ and $T_{i,\text{shad}}^{K=1} = 439.2 \text{ MeV}$, respectively.

We also went through the same program to investigate the impact of the different shadowing parametrizations at the higher LHC energy of $\sqrt{s} = 5.5 \text{ TeV}$. We here used the newer parton distributions of CTEQ4L since the involved momentum fractions are so small that any new information at small $x$ are valuable. When comparing GRV '94 and CTEQ4L one finds a difference of about a factor of two at $x \approx 10^{-5}$. At LHC energies the effect of shadowing should be much more relevant than at RHIC due to the region of smaller $x$ that gets probed. Because of the strong dominance of the gluon component in the nucleon we restricted ourself to the calculation of $\sigma^g$, $\bar{N}^g$, and therefore on the transverse energy and temperature produced by the final state gluons only. Let us first begin with the unshadowed results.

\begin{table}[h]
\centering
\caption{\(\int dy\,dN^g/dy\) for \(\sqrt{s} = 5.5 \text{ ATeV}\)}
\begin{tabular}{|l|c|c|c|}
\hline
range of $y$ & $gg \to gg$ & $gq \to gq + g\bar{q} \to g\bar{q}$ & TOTAL \\
\hline
all $y$ & 73645.44 & 12012.16 & 85666.6 \\
$|y| \leq 0.5$ & 12478.08 & 2104.96 & 14583.04 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{$\sigma^g \langle E_T \rangle$ [mb GeV]} 
\begin{tabular}{|l|c|c|c|}
\hline
range of $y$ & $gg \to gg$ & $gq \to gq + g\bar{q} \to g\bar{q}$ & TOTAL \\
\hline
$|y| \leq 0.5$ & 438.09 & 74.92 & 513.01 \\
\hline
\end{tabular}
\end{table}

The rapidity distributions for unshadowed and shadowed gluons at LHC is depicted in figure 10.
Figure 10: Rapidity distributions of unshadowed (upper figure) and shadowed
gluons (lower two figures) at LHC. The figure in the middle was derived by
employing the strong gluon shadowing, whereas the bottom figure employed
the $Q^2$ dependent '98 version. Note the change in shape when the strong
gluon shadowing is employed.
For the strong gluon shadowing we find the following results

**Table 21:** $\int dy \, dN^g/\,dy$ for $\sqrt{s} = 5.5$ ATeV

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gg \rightarrow gg$</th>
<th>$gq \rightarrow gq + g\bar{q} \rightarrow g\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>all $y$</td>
<td>11937.91</td>
<td>3117.44</td>
<td>15055.35</td>
</tr>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>1009.92</td>
</tr>
</tbody>
</table>

**Table 22:** $\sigma^g \langle E_T \rangle$ [mb GeV]

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gg \rightarrow gg$</th>
<th>$gq \rightarrow gq + g\bar{q} \rightarrow g\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>48.17</td>
</tr>
</tbody>
</table>

With the weaker shadowing one finds

**Table 23:** $\int dy \, dN^g/\,dy$ for $\sqrt{s} = 5.5$ ATeV

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gg \rightarrow gg$</th>
<th>$gq \rightarrow gq + g\bar{q} \rightarrow g\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>all $y$</td>
<td>49838.1</td>
<td>7735.68</td>
<td>57573.78</td>
</tr>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>6333.44</td>
</tr>
</tbody>
</table>

**Table 24:** $\sigma^g \langle E_T \rangle$ [mb GeV]

<table>
<thead>
<tr>
<th>range of $y$</th>
<th>$gg \rightarrow gg$</th>
<th>$gq \rightarrow gq + g\bar{q} \rightarrow g\bar{q}$</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>y</td>
<td>\leq 0.5$</td>
<td>245.92</td>
</tr>
</tbody>
</table>

A direct comparison between the results for shadowed and unshadowed parton distribution functions is shown in figure 11 and the $p_T$ distributions for LHC are shown in figure 12.
Figure 11: Comparison of the rapidity distributions with strong gluon shadowing (left figure) and weak shadowing (right figure) to unshadowed distribution for LHC. The solid lines give the total contribution, the dotted ones depict the $gg \rightarrow gg$ process and the dashed ones stand for the $gq \rightarrow gq + g\bar{q} \rightarrow g\bar{q}$ processes. The thick lines again give the unshadowed results.

Figure 12: $p_T$ distributions for the two shadowing parametrizations at midrapidity for LHC.
Therefore we find the following numbers at LHC: for unshadowed parton distributions one has at midrapidity 14583 gluons that carry a transverse energy of 16.4 TeV. The number density thus is \( n_g = 1092.4 \, \text{fm}^{-3} \) and the energy density is given by \( \varepsilon_g = 1229.7 \, \text{GeV/fm}^{-3} \). The initial temperature of an ideal gas derived with these numbers is \( T_1 = 1169 \, \text{MeV} \) and \( T_1^{K=1.0} = 1056.5 \, \text{MeV} \) for \( K=1 \). With the strong gluon shadowing we find 1268 gluons carrying a transverse energy of 1.93 TeV. We therefore have \( n_g = 94.9 \, \text{fm}^{-3} \) and \( \varepsilon_g = 144.8 \, \text{GeV/fm}^{-3} \) resulting in \( T_1 = 684.9 \, \text{MeV} \) for \( K=1.5 \) and \( T_1^{K=1.0} = 618.9 \, \text{MeV} \). With Eskola’s newest shadowing version we find 7382 gluons which carry a total transverse energy of 9.18 TeV, \( n_g = 9552.9 \, \text{fm}^{-3} \), and \( \varepsilon_g = 678.6 \, \text{GeV/fm}^{-3} \) which results in a temperature \( T_1 = 1011.08 \, \text{MeV} \) for \( K=1.5 \) and \( T_1^{K=1.0} = 913.62 \, \text{MeV} \) for \( K=1 \).

5. Entropy production and \( \pi \) multiplicities

As is known, total entropy and entropy density, respectively, play a very important role in the formation of a quark-gluon plasma. Total entropy reaches its final value when the system equilibrates and can, if assuming an adiabatic further evolution, be related to the effective number of degrees of freedom in the quark-gluon and in a pure pion plasma via \([20, 21]\)

\[
r = \frac{s^\pi(T_c)}{s^{\mathrm{qg}}(T_c)} \approx 0.7 \pm 0.2
\]

(18)

where \( s^\pi \) and \( s^{\mathrm{qg}} \) are the entropy densities in the pion and quark-gluon plasma. The total entropy can then be related to the pion multiplicity as

\[
\frac{dS}{dy} = c^{\mathrm{qg}} \left( \frac{dN^{\mathrm{qg}}}{dy} \right)_{b=0} \approx \frac{c^\pi}{r} \left( \frac{dN^\pi}{dy} \right)_{b=0}
\]

(19)

where \( c^{\mathrm{qg}} = 4.02 \) for \( N_f = 4 \) and \( c^\pi \approx 3.6 \).

A note on the separation between hard and soft processes is appropriate at this point. As emphasized above we introduced a cut-off at \( p_0 = 2 \, \text{GeV} \) to ensure the applicability of perturbative QCD. Nevertheless there is always a soft component contributing to the production of transverse energy neglected in our studies so far. In \([14]\) it was shown that with \( p_0 = 2 \, \text{GeV} \) at SPS the hard partons only carry about 4% of the total transverse energy \( E_T \). At RHIC energies they carry \( \approx 50\% \) and for \( \sqrt{s} = 2 \, \text{TeV} \) the hard partons already
carry \approx 80\% of the total transverse energy. Since we here solely want to investigate the role of shadowing in hard reactions we will not calculate the pion multiplicity for RHIC where the soft contribution still is significant but restrict ourselves to the pion number at \( y \approx 0 \) for LHC energies.

If we employ the numbers for the entropy densities in the different plasmas and use our findings on the contributions of shadowing to the number of minijets we find that at \( y = 0 \) one has

\[
\begin{align*}
\left( \frac{dN_{\pi}}{dy} \right)_{b=0} & \approx 8309, \\
\left( \frac{dN_{\pi}}{dy} \right)_{b=0} & \approx 4818, \\
\left( \frac{dN_{\pi}}{dy} \right)_{b=0} & \approx 989,
\end{align*}
\]

(20)

when employing no shadowing, the ’98 version of Eskola, and the strong gluon shadowing parametrization.

### 6. Conclusions

In this paper we investigated the influence of nuclear shadowing on rapidity spectra, transverse energy production and on macroscopic quantities such as the initial temperature. We employed two different versions of parametrizations for the shadowing: one with a strong initial gluon shadowing and a model independent one recently published by Eskola et al [15]. We found that the latter one gives an enhancement of minijet production at RHIC in contrast to the other case were a reduction to \( \approx 65\% \) results. This difference directly manifests itself in the initial temperature \( T_i \) which happens to be smaller only for the strong gluon shadowing. At LHC the situation changes since there also the weakly shadowed gluons finally result in lower spectra and \( T_i \). Since the two shadowing parametrizations differ so drastically one finds a large difference in the results for the number of minijets at midrapidity: for the strong shadowing one has \( \approx 1300 \) gluons whereas for the weaker shadowing one finds \( \approx 7000 \) gluons. Since there are so few gluons for the strong gluon shadowing we find that the initial temperature at LHC is not dramatically higher than at RHIC!

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References


[10] V. N. Gribov, JETP 30 (1970) 709,
    R. Glauber in Lectures in theoretical physics, ed. W.E. Brittin et al. (1959)


