This kernel is quite wide in the redshift direction, scaling as $D_L D_{LS}/D_S$ where $D_L$ is the distance from the observer to the lens, $D_S$ from the observer to the source and $D_{LS}$ from the lens to the source\footnote{In a distribution of source distances, one takes an appropriate integral of this expression.} (Mellier 1999). Under the assumption that the cluster is the most massive object along the line-of-sight and is well localized in space (the thin-lens approximation), the convergence map is proportional to the projected surface density map of the lensing cluster itself.

Any additional mass located near the cluster and along the line-of-sight will also contribute to the lensing signal. Since the kernel is such a slowly varying function of distance, material even large distances from the cluster will contribute within the thin lens approximation. For a source at $z = 1$ and a cluster at $z = 0.5$ the kernel changes by only 1\% within $\pm 40\ h^{-1}$ Mpc of the cluster in a universe with $\Omega_m = 0.3 = 1 - \Omega_L$, with similar results in other cosmologies. As a result, weak lensing observations will probe the projected density of a cluster plus all of the material in its vicinity. Note that this “nearby” material is essentially “at” the redshift of the cluster for the purposes of lensing, and so cannot be distinguished by using extra information such as source redshifts.

To study the effect of the mass upon the projected mass inferred from lensing observations of the simulated clusters, we have examined the mass distribution around several clusters of galaxies extracted from a large cosmological simulation. The simulated clusters were taken from the X-Ray Cluster Data Archive of the Laboratory for Computational Astrophysics of the National Center for Supercomputing Applications (NCSA), and the Missouri Astrophysics Research Group of the University of Missouri. To produce these clusters, a particle-mesh N-body simulation incorporating adaptive mesh refinement was performed in a volume $256 h^{-1}$ Mpc on a side. Regions where clusters formed were identified; for each cluster, the simulation was then re-run (including a baryonic fluid) with finer resolution grids centered upon the cluster of interest. In the adaptive mesh refinement technique, the mesh resolution dynamically improves as needed in high-density regions. The “final” mesh scale at the highest resolution was $15.8 h^{-1}$ kpc, allowing good resolution of the filamentary structure around the cluster. Inside the cluster, the characteristic separation between the smallest mass particles, given by $d = (4\pi \rho_\text{crit}^3 / 3N)^{1/3}$, with $N$ the number of particles inside the region, was approximately $86$ kpc for all three clusters examined here. The code is described in detail in Norman & Bryan (1999).

The clusters used here were extracted at $z = 0$ from simulations of a CDM model, with parameters $\Omega_m = 0.3$, $\Omega_{\Lambda} = 0.7$, $\Omega_b = 0.26$, $h = 0.7$, and $\sigma_8 = 0.928$. In this Letter we observe these clusters as if they were at $z = 0.5$. In future work we plan to investigate the dependence of these results on cosmology and on cluster redshift.

Since the number density of rich clusters is approximately $\phi_\ast \sim 10^{-5} \text{Mpc}^{-3}$, the typical separation between them is $\rho_\ast^{-1/3} \sim 40 \ h^{-1}$ Mpc. This is a characteristic scale for filaments: volumes containing a cluster and with one-dimensional extent $\sim 40 \ h^{-1}$ Mpc should also contain much of the nearby filamentary structure. Three such volumes, containing a rich cluster (Clusters 0, 2 and 4) as well as satellites and filaments, were extracted from the archive. We selected clusters that did not appear to be mergers or have a large secondary mass concentration nearby. Such systems might be excluded observationally from studying, for instance, the galaxy line-of-sight velocity distribution. For each volume, the “center” of our cluster was determined using a maximum-density algorithm. As the extracted volumes were not spherical, it was possible that some lines of sight could contain more mass than others simply by geometry. To avoid such biases we restrict our analysis to particles that lay within the largest sphere, centered on the cluster, which was contained entirely within the extracted volume. The radii of these spheres, $R_{\text{sphere}}$, are listed in Table 1, along with other properties of the clusters. Note that these radii are large compared to the projected values of $r_{200}$ obtained for each cluster; thus no significant radial surface density gradient is introduced by a decreasing chord length through the sphere with radius. It is also important to note that since our volumes are by necessity limited, our results should be interpreted as a lower limit to the size of the effect; the magnitude of the lensing kernel is still significant at the edge of our spherical volume.

Each of the clusters we examined was surrounded by a large amount of mass. Most of this material appeared by eye to be collapsed into “heads” along a filamentary structure, although a small number of clumps could be found outside the filaments. We show a projection of a fraction of the points from the simulation of cluster 4 in Fig. 1. The filamentary structure and satellites are easily evident. Note that this filamentary structure extends well beyond our radius $R_{\text{sphere}}$. No single projection can show the full 3D nature of the structure, in which the filamentarity is even more apparent. Since much of this mass is at low density it is unlikely it would be a site for galaxy formation or otherwise emit light. Thus this structure would not be easy to constrain by observations of redshifts near the cluster.

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### Table 1

<table>
<thead>
<tr>
<th>Number</th>
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<tr>
<td>$R_{\text{sphere}}$ (h$^{-1}$Mpc)</td>
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<tr>
<td>$r_{200}$ (h$^{-1}$Mpc)</td>
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<tr>
<td>$M_{200}$ (h$^{-1}10^{15} M_\odot$)</td>
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<tr>
<td>$M_{500}$ (h$^{-1}10^{15} M_\odot$)</td>
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<tr>
<td>$M_{510}$ (h$^{-1}10^{15} M_\odot$)</td>
</tr>
<tr>
<td>$M_{\text{tot}}$ (h$^{-1}10^{15} M_\odot$)</td>
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Parameters for the 3 simulated clusters discussed in this Letter. $R_{\text{sphere}}$ is the size of the sphere, centered on the cluster, which we consider in this work; $r_{200}$ is the 3D radius defined in Eq. 1 and $M_{200}$ the mass enclosed. $M_{500}$ and $M_{510}$ are masses cutting out particles above thresholds of 70 and 10 times local density respectively (see text). $M_{\text{tot}}$ is the total mass in the sphere.
We observed each of the three selected clusters from 10,000 randomly chosen viewing angles. For each cluster and viewing angle, the projected surface density map was constructed and used to estimate \( r_{200} \), the radius within which the mean interior density contrast is 200. In three dimensions, this radius is defined in terms of the enclosed mass by

\[
M(< r_{200}) = 200 \times \left( \frac{4\pi}{3} \right) \Omega_m \rho_{\text{crit}} r_{200}^3.
\]

The projected estimate of \( r_{200} \) was extracted from the surface density map by considering the radius of the circle, centered on the cluster, which contained the amount of mass given by Eq. (1) above, i.e.

\[
\int_0^{2\pi} d\theta \int_0^{r_{200}} R dR \Sigma(R, \theta) = M(< r_{200})
\]

with \( \Sigma(R, \theta) \) the surface density on the map in terms of a two-dimensional radius \( R \). This radius was compared to the cluster's true \( r_{200} \), extracted from the three-dimensional mass distribution. The ratio of the projected mass to true mass is given simply by the cube of the ratio of the estimated value of \( r_{200} \) to the true value. For each cluster, a value of this ratio was obtained for each viewing angle.

3. RESULTS

Our main result is displayed in Fig. 2, where we show the distribution of projected vs. “true” cluster mass in each of the three simulated clusters. We have checked that the features in the histogram do not come from shot noise due to discrete particles in the simulation. However, the “spikiness” is due to a discrete number of objects in the neighborhood of the cluster. A small lump of matter near the cluster will project entirely within \( r_{200} \) for a fraction of the lines of sight. For any such line of sight, the effect on the projected value of \( r_{200} \) is identical.

We expect the ratio \( M_{\text{proj}}/M_{\text{true}} \) to be greater than unity since only additional mass can be included in the projection. The size of the smallest offset from unity for clusters 0 and 4 is approximately twice what would be expected for material uniformly distributed at the mean density. This suggests that matter near the cluster is itself clustered and at higher than mean density. The width of the histogram in Fig. 2, as a fraction of the true mass, depends on the true mass of the cluster. Though we have only a few clusters in this study, it appears that the mass in nearby material is not proportional to the cluster mass, thus the relative effect of this material is smaller the larger the cluster. The total mass in the sphere, \( M_{\text{tot}} \), is also listed in Table 1 for reference.

The signal shown in Fig. 2 comes from (primarily filamentary) material outside the cluster and is not the well known projection effect arising from cluster asphericity. To verify this, we repeated the procedure described above for a subset of particles aimed at selecting the cluster alone. This was done by first selecting out particles with a local density contrast of greater than 70 (chosen because density profiles near \( r^{-2} \) reach a local density contrast near 70 at a mean interior density contrast of 200); a small sphere containing the cluster but little nearby material was then cut out of this subset. The histogram produced by viewing the clearly protlate cluster at a large number of randomly chosen viewing angles produced a much narrower distribution, with a maximum offset of less than 10%
in the mass ratio and a mean offset of approximately half that value.

While it is beyond the scope of this Letter to perform a detailed modelling of any observational weak lensing strategy, we show in Fig. 3 two sample profiles obtained from aperture densitometry on our noiseless projected mass maps. Specifically, for Cluster 4, we show the profile along the lines of sight giving the largest and smallest mass for comparison. The $\chi$ statistic is the mean value of the convergence $\kappa$, within a disk of radius $r_1$ minus the mean value within an annulus $r_1 \leq r \leq r_2$ (Fahlman et al. 1994, Kaiser 1995). Here we calculate $\kappa$ directly from the projected mass, though observationally it would be computed from the tangential shear. We have taken $r_2 = 800\,''$. Such a large radius is not (currently) achievable observationally, but it minimizes the impact of objects near the cluster and provides a lower limit on the size of the projection effect. We have explicitly checked that reducing the radius to half this value does not change our result.

In calculating the convergence $\kappa$, the cluster was again assumed to be at a redshift of 0.5, with the lensed sources at a redshift of 1.0. In any real observation, of course, the lensed sources will span a range of redshifts. For material very close to the cluster, such as here, this will not affect our conclusions, and the error introduced by incorrectly modelling the redshift distribution of the foreground sources is not the subject of this work. In addition to being easy to estimate, the $\chi$ statistic is robust and minimizes contamination by foreground mass (Mellier 1999) because it is insensitive to the sheet mass degeneracy. This does not, however, remove the sensitivity to clustered material, as can be seen in Fig. 3. The mass which would be inferred from Fig. 3 along two lines of sight differ by a factor of 1.7.

While the distribution of the projection effect varies from cluster to cluster, it seems clear that positive biases in projected mass of 28% are typical, with biases above 30% not uncommon. Furthermore, we emphasize again that these estimates are in fact lower limits; some lines of sight through the untruncated volume produced overestimates as large as 80% or more. While appropriate modeling of a mean density profile outside $r_1$ (drawn perhaps from simulations such as these) can be used to produce an unbiased estimator, the width of these histograms implies a large amount of scatter around such an estimator of the true (unprojected) mass. It is clear that this effect can be quite significant and must be taken into account when attempting to understand the results of mass reconstruction analyses.

4. CONCLUSIONS

Clusters of galaxies are part of the large-scale structure of the universe and observations of them should be considered within this context. The filamentary structure near a cluster can contain a reasonable fraction of the mass of a cluster in tenuous material. Should a filament lie close to the line-of-sight to a cluster it will contribute to the weak lensing signal and positively bias the projected mass. We have shown that such a bias could easily be 30% (see Fig. 2).

Weak lensing remains one of the best methods for determining the mass of clusters of galaxies. However methods which obtain the mass from estimates of the projected surface density must consider the effect outlined in this Letter. This is clearly of particular significance for attempts to determine the mass function of clusters directly through surveys of weak lensing-determined masses.

We have not attempted to address the detailed question of how much this filamentary signal would affect a particular reconstruction algorithm; the answer is no doubt algorithm dependent. We hope to return to this issue in future work, as well as to consider the effect of cosmological model and evolution with cluster redshift.

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