Reconstruction of Stellar Orbits Close to Sagittarius A*: Possibilities for Testing General Relativity

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**ABSTRACT**

We have constructed families of possible orbits for a collection of stars located within 0.5" of Sgr A*. These are constrained by observed stellar positions and angular proper motions as listed in Ghez et al. (1998). Reconstructing these orbits allows us to make useful predictions for the likely positions of these stars in future observations. We also show that some of these stars may exhibit significant relativistic effects, allowing for interesting possible tests of general relativity around the $2.6 \times 10^6 M_\odot$ central object.

*Subject headings:* black hole physics — celestial mechanics, stellar dynamics — Galaxy: center — Galaxy: kinematics and dynamics — gravitation — relativity
1. Introduction

The generally accepted idea that supermassive, accreting black holes power the highly energetic phenomena in active galactic nuclei has motivated a great deal of effort to gather information about these extraordinary objects. Speculation about the presence of a black hole at the center of our own galaxy has been ongoing for over 20 years [see Genzel et al. (1996) and references therein for a recent summary]. Although the galactic center appears to be nearly radio dormant, there is ample evidence for the presence of a supermassive black hole. [See however Tsiklauri & Viollier (1998) and Munyaneza, Tsiklauri, & Viollier (1998) for a discussion of other possible interpretations.]

One of the most efficient ways to constrain the possible existence of a massive object in the galactic core is to observe the orbital motions of the stars and gas closest to the Galactic center. Progress toward this end has recently been made by Eckart et al. (1995; 1997), Genzel et al. (1996), and Ghez et al. (1998) who have obtained high angular resolution images of the Galaxy’s central stellar cluster.

Ghez et al. (1998) (hereafter referred to as GKMB) surveyed 90 stars with a brightness ranging from K = 9 to 17 mag using the W. M. Keck 10-m telescope. They reported on the RA and DEC separation from Sgr A* as well as the angular proper motions for these stars. Based upon a Keplerian fit to the data, they were able to deduce the mass of the central compact object to be $2.6(\pm 0.2) \times 10^6 M_\odot$.

Here we attempt to reconstruct general relativistic orbits of the innermost 8 stars in the GKMB survey. These are the stars for which the general relativistic effects should be greatest. The high velocities of these stars and the predicted, significant perigee precession could provide the groundwork for interesting tests of general relativity as future observations of these stars are made. We also discuss other detectable, relativistic effects as the stars pass near the massive object. Jaroszyński (1998) has presented Monte Carlo simulations to demonstrate the feasibility of measuring such relativistic effects. Jaroszyński (1999) and Salim & Gould (1998) also demonstrate how accurate determinations of the orbit parameters can be used to constrain distance and mass estimates for the Galactic center. Here, we assess what can and has been learned from the currently available data.

Fig. 1.— Keplerian fits of GKMB data assuming $\theta = \frac{\pi}{2}$ and $v_\theta = 0$.

2. Orbital Models

The published positions of the 8 stars studied here are only given in RA and DEC offsets. Furthermore, only angular velocities are known. Hence, it is not possible to fully constrain the angle of inclination of the orbit plane relative to an observer. As a first step, we calculated Keplerian orbits with the line of sight taken orthogonal to the orbital plane (i.e. the angle of inclination was assumed to be $\frac{\pi}{2}$). Later we relax this assumption and seek optimum angles based upon the data. Clearly, however, this analysis would be greatly aided by determinations of radial velocities. Hopefully these will be soon coming. The present analysis will therefore serve as a summary of possibilities until sufficiently accurate radial velocities are known. Results of the Keplerian fits are shown in Fig 1. These indicate which orbits are most likely to pass close to the central object, and hence, those which are likely to experience the largest relativistic effects.

Next, the orbits were fit using the equations of motion for a test object of negligible mass orbiting a Schwarzschild black hole. Given the extremely large mass of the central object ($2.6 \times 10^6 M_\odot$) the approx-
imation that the stars move in a fixed background metric is certainly acceptable. The restriction to a Schwarzschild metric is reasonable. It is likely that the black hole formed from the random accretion of stars and gas, and thus, accumulated relatively little net angular momentum. Furthermore, Jaroszyński (1998) has studied possible orbits in a Kerr background and concluded that any effects of black-hole rotation are negligible. Using the convention of Weinberg (1972), the equations of motion are

\[ \frac{d^2r}{d\tau^2} = -\frac{1}{2} \frac{d}{dr} \left( \frac{dr}{d\tau} \right)^2 + r \left( \frac{d\theta}{d\tau} \right)^2 + r \sin^2 \theta \left( \frac{d\phi}{d\tau} \right)^2 - \frac{1}{2A} \left( \frac{dr}{d\tau} \right)^2, \]  

(1)

\[ \frac{d^2\theta}{d\tau^2} = -2 \frac{d\theta}{dr} \frac{dr}{d\tau} + \sin \theta \cos \theta \left( \frac{d\phi}{d\tau} \right)^2, \]  

(2)

\[ \frac{d^2\phi}{d\tau^2} = -2 \frac{d\phi}{dr} \frac{dr}{d\tau} - 2 \cot \theta \left( \frac{d\phi}{d\tau} \right)^2, \]  

(3)

and

\[ \frac{d^2t}{d\tau^2} = -\frac{d}{dr} \left( \frac{dt}{d\tau} \right) \left( \frac{dr}{d\tau} \right), \]  

(4)

where the Schwarzschild metric parameters are

\[ A(r) = \left[ 1 - \frac{2M}{r} \right]^{-1}, \]  

(5)

and

\[ B(r) = \left[ 1 - \frac{2M}{r} \right]. \]  

(6)

In order to solve the above equations, a complete set of initial conditions \((r, v_r, \theta, v_{\theta}, \phi, \text{ and } v_\phi)\) must be known. (We choose \(t = 0\) to correspond to the June 1995 observations.) Without radial velocity data, we cannot determine all 6 parameters directly, but by using a \(\chi^2\) minimization scheme, we are able to constrain them. To do this, we evolved the 3-dimensional orbit over each time interval using a fourth-order Runge-Kutta method and projected this orbit onto a 2-dimensional plane corresponding to the observational plane. We then minimized the RA and DEC offsets by matching the 3 available data points for each star. Because this problem is under constrained (only 3 data points for 6 free parameters), the \(\chi^2\) for most of these fits is quite low and the corresponding uncertainties in orbit parameters in most cases are quite large. Future observations will obviously significantly improve the statistical analysis of the data.

The code was checked by evolving Newtonian orbits through 100 revolutions. This test indicated in no noticeable deviation (e.g. precession) of the orbits in the Newtonian limit as expected.

3. Results

As discussed above, we have limited this study to the 8 stars closest to the galactic center (S0-1 through S0-8 in GKM8). (In this paper we follow the naming convention of GKM8.) Throughout this paper we have used a coordinate system with the origin fixed at Sgr A*. The z-axis is then defined along the line of sight from this origin, \(\theta\) is the angle from the z-axis, and \(\phi\) is measured from the positive RA-axis. Also, we adopt 8 kpc as the distance to the Galactic center (Reid, 1993).

3.1. Orbit Parameters

Optimum orbit parameters from the \(\chi^2\) minimization are summarized in Table 1. We have given the best fit initial conditions \((t = 0\) corresponding to June 1995) for the orbits of these 8 stars. The 1σ errors are also listed. Where no error is given it is because the velocities could take on any value between 0 and \(c\) within the 1σ error bar.

Table 2 summarizes the Keplerian orbit parameters for 4 of these stars. Included in the table are the semi-major axis length, the eccentricity of the orbits, the orbital period, the precession of perigee, and the distance of closest approach. The perigee precession is given both as the angular shift per revolution in the orbital plane and as an angular displacement of apogee on the sky per revolution. The stars S0-1, S0-4, S0-7, and S0-8 are not listed in Table 2 because their optimum parameters do not correspond to bound orbits around the central object. For these stars, the ratios of their speeds to their respective escape velocities \(\frac{|v_a|}{v_{esc}}\) are 1.18(\(+0.41\) \(-0.79\)), 4.13(\(+4.59\) \(-3.94\)), 1.74(\(+0.14\) \(-1.01\)), and 9.75(\(+10.8\) \(-8.78\)). Thus, the 1σ lower limits on these ratios, (0.39, 0.19, 0.71, and 0.97, respectively) are all consistent with bound stellar orbits. This is an important issue for future study. Most authors have assumed closed orbits when using the motions of these stars to determine the mass of the central object. If a significant fraction of the stars are unbound, then a revision of the mass estimate may be
The uncertainties in the parameters for this orbit allow for the possibility that it is not in a closed orbit, so only lower bounds
are given.

Perigee precession is the angular displacement of perigee in the orbit plane per revolution.

These Keplerian parameters are based upon the orbital data given in Table 1. Uncertainties are based upon the 1σ errors based on a χ² minimization of the deviations from the 1995, 1996, and 1997 observations of GKMB. The parameters given correspond to the June 1995 orbit parameters. In cases where errors are not given, velocities could have any value between 0 and c within the 1σ error.

All values of θ are in the range 0 ≤ θ ≤ π/2, which assumes the stars are located on the near side of Sgr A*. Any stars located on the far side would instead have a value θ′ = π − θ, where obviously π/2 ≤ θ′ ≤ π.

### Table 1

#### June 1995 Orbit Parameters

<table>
<thead>
<tr>
<th>Star ID</th>
<th>r(σ_e) (10^3AU)</th>
<th>v_r(σ_{v_r}) (10^3km/s)</th>
<th>θ_0(σ_θ) (deg)</th>
<th>v_φ(σ_{v_φ}) (deg/yr)</th>
<th>φ(σ_φ) (deg)</th>
<th>v_φ(σ_{v_φ}) (deg/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0-1</td>
<td>16.3(37.5)</td>
<td>-16.3(16.3)</td>
<td>33.5(15.5)</td>
<td>0(8.8)</td>
<td>160(10.5)</td>
<td>14.6(5.9)</td>
</tr>
<tr>
<td>S0-2</td>
<td>11.9(9.8)</td>
<td>-1.68(-)</td>
<td>90(22)</td>
<td>0(59(-)</td>
<td>87.3(12.2)</td>
<td>2.86(-4.2)</td>
</tr>
<tr>
<td>S0-3</td>
<td>17.1(2.4)</td>
<td>5.73(4.02)</td>
<td>90(27)</td>
<td>0(-)</td>
<td>24.4(+3.0)</td>
<td>0.46(+0.78)</td>
</tr>
<tr>
<td>S0-4</td>
<td>93.3(+7.4)</td>
<td>29.0(+31.8)</td>
<td>16.5(+1.5)</td>
<td>0.072(+1.19)</td>
<td>333(+2.0)</td>
<td>0.60(+1.01)</td>
</tr>
<tr>
<td>S0-5</td>
<td>27.6(17.0)</td>
<td>5.76(4.02)</td>
<td>90(36)</td>
<td>0(-)</td>
<td>307(+8.8)</td>
<td>1.25(+12.5)</td>
</tr>
<tr>
<td>S0-6</td>
<td>34.6(+1.0)</td>
<td>-4.11(3.69)</td>
<td>89(+15)</td>
<td>0(-)</td>
<td>290(+2.5)</td>
<td>0.96(+4.83)</td>
</tr>
<tr>
<td>S0-7</td>
<td>241(+26.6)</td>
<td>6.51(-)</td>
<td>8.65(+1.50)</td>
<td>0.20(-)</td>
<td>9.6(+1.0)</td>
<td>0.02(+0.62)</td>
</tr>
<tr>
<td>S0-8</td>
<td>133(+22.5)</td>
<td>-56.7(+51.0)</td>
<td>16.2(+3.5)</td>
<td>0.039(+2.87)</td>
<td>127(+7.0)</td>
<td>2.87(+11.4)</td>
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</table>

### Table 2

#### Keplerian Orbit Parameters

<table>
<thead>
<tr>
<th>Star ID</th>
<th>Semi-major axis (10^3AU)</th>
<th>Eccentricity</th>
<th>Period (years)</th>
<th>Perigee Precession a (deg/yr)</th>
<th>Apogee Shift b (10^-3arcsec/rev)</th>
<th>Perigee Distance (AU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0-2</td>
<td>6.14(±0.54)</td>
<td>0.9573(±0.0113)</td>
<td>9.5(±1.3)</td>
<td>0.5(±0.3)</td>
<td>0.7(±1.14)</td>
<td>26.2(±0.6)</td>
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<tr>
<td>S0-3</td>
<td>9.81(±3.59)</td>
<td>0.9972(±0.0010)</td>
<td>17(±4)</td>
<td>5.1(±1.1)</td>
<td>9.6(±1.79)</td>
<td>2.72(±0.01)</td>
</tr>
<tr>
<td>S0-5c</td>
<td>18.3(-3.5)</td>
<td>0.9233(-0.0192)</td>
<td>49(-4)</td>
<td>0.1(-0.01)</td>
<td>0.7(-1.2)</td>
<td>141(-28)</td>
</tr>
<tr>
<td>S0-6c</td>
<td>21.2(-1.0)</td>
<td>0.8951(-0.0086)</td>
<td>61(-4)</td>
<td>0.07(-0.01)</td>
<td>0.5(-0.05)</td>
<td>224(-27)</td>
</tr>
</tbody>
</table>

*a These Keplerian parameters are based upon the orbital data given in Table 1. Uncertainties are based upon the 1σ errors for the orbit parameters.

*b Perigee precession is the angular displacement of perigee in the orbit plane per revolution.

*b Apogee shift is the apparent motion of apogee on the sky as seen by an observer on Earth.

*c The uncertainties in the parameters for this orbit allow for the possibility that it is not in a closed orbit, so only lower bounds are given.
Fig. 2.— Projected orbits of stars S0-2 and S0-3 using different initial values of $\theta$ (the angle between the line of sight to Sgr A* and a radial vector from Sgr A* to the star). Displayed for S0-2 are $\theta = \frac{\pi}{2}$ (solid line), $\theta = \frac{\pi}{4}$ (dotted line), and $\theta = \frac{\pi}{8}$ (dashed line). Displayed for S0-3 are $\theta = \frac{\pi}{2}$ (solid line) and $\theta = \frac{\pi}{4}$. Initially, $v_{\theta} = 0$ for all of these plots.

required.

The lack of radial velocity data is the most severe shortcoming of the present analysis. Using the coordinate system we have chosen, this lack of radial velocity information translates into uncertainties in $r$, $v_r$, $\theta$, and $v_{\theta}$. Fig’s 2 and 3 show plots using various initial values of $\theta$. Because of the rapid motion of the star S0-1, the data actually allows us to constrain the value to $\theta = \frac{\pi}{6} \pm \frac{\pi}{12}$ or $\frac{5\pi}{6} \pm \frac{\pi}{12}$ depending on whether the star is on the near or far side of Sgr A*.

Other stars are even better constrained in $\theta$. Fig 4 shows the similar impact of the uncertainty in $v_{\theta}$ on the projected orbits.

The GKMB data also has some rather large uncertainties in the $x$ and $y$ velocities of the stars. Fig 5 illustrates the dramatic impact of these large uncertainties on the orbits for stars S0-5 and S0-7. Obviously future observations will help to narrow down the possibilities.

Fig. 3.— Projected orbits of the star S0-1 using four different initial values of $\theta$. Displayed are $\theta = \frac{\pi}{2}$ (solid line), $\theta = \frac{\pi}{4}$ (long dashed line), $\theta = \frac{\pi}{8}$ (short dashed line), and $\theta = \frac{5\pi}{8}$ (dotted line). Initially, $v_{\theta} = 0$ for all of these plots.

3.2. General Relativistic effects

Fig 6 shows the significant anticipated perigee precession for the stars S0-2 and S0-3 over a multi-orbit cycle. If we look, for example, at the case of S0-3, we see that the perigee precession is, in principle, measurable. The apogee shift is about 5 deg/rev corresponding to an angular displacement on the sky of about 0.01 arcsec over a period of approximately 17 years.

It is also interesting to look at the gravitational redshift ($\Delta \nu/\nu = GM/c^2 R$) of the light emitted from these stars as they proceed through their orbits. The star S0-3 has the largest redshift, $\Delta \nu/\nu = 0.009$ at perigee, or about a 1% shift in the frequency of the light. At apogee, the redshift for this star is only $\Delta \nu/\nu = 0.00001$, almost 3 orders of magnitude smaller. The possibility of gravitational lensing of stars near the the Galactic center has been discussed elsewhere (Alexander & Sternberg 1998; Salim & Gould 1998; Jaroszyński 1999). It has been concluded
that this effect is probably negligible (Jaroszyński 1999).

As a final point of interest, however, we consider the hydrodynamic effects of these stars passing close to the central object. In particular, we have in mind possible tidal distortion or disruption of the stars. Because stars S0-2 and S0-3 may be in short period orbits close to the central object, these are the most likely candidates for tidal effects. Given the apparent K magnitudes of these stars (K=14.1 for S0-2 and K=14.7 for S0-3) and an approximate K-band extinction of $A_K \approx 3$ (Blum, Sellgren, & DePoy 1996), we conclude they are most likely either early-type, main-sequence (B0 V) or late-type, giant stars (K3 III) [see for example Eckart et al. (1995)]. Typical stellar radii for such stars are $8.4R_\odot$ and $20R_\odot$, respectively. For a star of mass $m \ll M$ the Roche-lobe radius $R$ is given by

$$\frac{R}{r} = 0.49 \left( \frac{m}{M} \right)^{1/3},$$

(7)

where $r$ is the radial distance of the star from a central object of mass $M$. For S0-2, the Roche-lobe radius at perigee is $52(\pm 14)R_\odot$ in the case of a B0 V star and $21(\pm 7)R_\odot$ in the case of a K3 III star, indicating possible Roche-lobe overflow if this star is a K3 III star. Fig 7 shows a comparison between the Roche-lobe for the star S0-2 at perigee and the typical radius of a K3 III star. For S0-3, the effect is even more dramatic. The Roche-lobe radius at perigee is $5.4(0.1)R_\odot$ in the case of a B0 V star and $2.2(0.1)R_\odot$ in the case of a K3 III star, indicating possible Roche-overflow in either case. Since the dynamical timescale for both types of stars is on the order of a few days, any tidal distortion near perigee will have ample time to affect the entire star. The long-term effects of these distortions on the thermonuclear burning of these stars is a topic under current investigation.
4. Conclusion

This study has allowed us to make predictions for the possible positions and velocities of S0-1 through S0-8 for future observations. As an illustration, Fig 8 and Table 3 summarize possible positions for these stars in June 1999 and June 2000.

We have shown here that continued study of the dynamics of at least two of the stars orbiting nearest the galactic center may provide interesting opportunities to test general relativity. These stars should exhibit detectable perihelion precession and gravitational redshifts as they orbit the $2.6 \times 10^6 M_\odot$ central black hole. These studies will enhance our understanding the nature of the central object that resides at our Galactic core. This environment may also provide an interesting laboratory in which to explore effects of intermittent tidal distortion/disruption on the orbiting stars.

The authors wish to acknowledge useful discussion with A. Ghez. This work was supported by the National Science Foundation under grant PHY-

Fig. 6.— Multi-orbit projection of stars S0-2 and S0-3 to demonstrate noticeable perigee shift. The years when apogee will be reached are listed for S0-3.

Fig. 7.— Comparison of the Roche-lobe for the star S0-2 at perigee with the typical radius of a K3 III star. The axes are in units of solar radii ($R_\odot = 6.96 \times 10^{10}$ cm).

REFERENCES


97-22086.
### Table 3

**June 1999 and June 2000 observation predictions**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>S0-1</td>
<td>-0.005(±0.025)</td>
<td>-0.073(±0.088)</td>
<td>0.040(±0.041)</td>
<td>-0.055(±0.162)</td>
</tr>
<tr>
<td>S0-2</td>
<td>-0.016(±0.030)</td>
<td>0.030(±0.019)</td>
<td>0.026(±0.061)</td>
<td>0.065(±0.024)</td>
</tr>
<tr>
<td>S0-3</td>
<td>0.223(±0.043)</td>
<td>0.109(±0.020)</td>
<td>0.221(±0.0057)</td>
<td>0.110(±0.026)</td>
</tr>
<tr>
<td>S0-4</td>
<td>0.376(±0.086)</td>
<td>-0.208(±0.045)</td>
<td>0.396(±0.107)</td>
<td>-0.223(±0.055)</td>
</tr>
<tr>
<td>S0-5</td>
<td>0.218(±0.024)</td>
<td>-0.336(±0.017)</td>
<td>0.217(±0.030)</td>
<td>-0.346(±0.021)</td>
</tr>
<tr>
<td>S0-6</td>
<td>0.103(±0.138)</td>
<td>-0.347(±0.025)</td>
<td>0.090(±0.172)</td>
<td>-0.361(±0.063)</td>
</tr>
<tr>
<td>S0-7</td>
<td>0.504(±0.054)</td>
<td>0.085(±0.019)</td>
<td>0.517(±0.055)</td>
<td>0.087(±0.025)</td>
</tr>
<tr>
<td>S0-8</td>
<td>-0.251(±0.301)</td>
<td>0.183(±0.077)</td>
<td>-0.244(±0.376)</td>
<td>0.133(±0.278)</td>
</tr>
</tbody>
</table>

*All values given are measured in arcsec. Uncertainties given in parenthesis are derived from the 1σ errors in the orbit parameters.*


Fig. 8.— Eight year orbit projections with positions marked for June 2000 observations.