Fermion Helicity Flip Induced by Torsion Field

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Abstract. – We show that in theories of gravitation with torsion the helicity of fermion particles is not conserved and we calculate the probability of spin flip, which is related to the anti-symmetric part of affine connection. Some cosmological consequences are discussed.

Introduction. – Attempts to conciliate General Relativity with Quantum Theory yielded to propose theories of gravitation including torsion fields, so that the natural arena is the space–time $U_4$ that is a generalization of Riemann manifold $V_4$.

The advantage to pass from $V_4$ to $U_4$ is due to the fact that the spin of a particle turns out to be related to the torsion just as its mass is responsible of the curvature. From this point of view, such a generalization tries to include the spin fields of matter into the same geometrical scheme of General Relativity.

One of the attempts in this direction is the Einstein–Cartan–Sciama–Kibble (ECSK) theory \cite{1}. However the torsion seems to play an important role in any fundamental theory. For instance: a torsion field appears in (super)string theory if we consider string fundamental modes; we need, at least, a scalar mode and two tensor modes: a symmetric and antisymmetric one. The latter, in the low energy limit for string effective action, gives the effects of a torsion field \cite{2}; any attempts of unification between gravity and electromagnetism require the inclusion torsion in four and in higher–dimensional theories as Kaluza–Klein ones \cite{3}; theories of gravity formulated in terms of twistors need the inclusion of torsion \cite{4}; in the supergravity theory torsion, curvature and matter fields are treated under the same standard \cite{5}; in cosmology torsion could have had a relevant role into dynamics of the early universe because it gives a

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repulsive contribution to the energy–momentum tensor so that cosmological models become singularity–free [6], and if the universe undergoes one or several phase transitions, torsion could give rise to topological defects (e.g. torsion walls [7]) which today can result as intrinsic angular momenta for cosmic structures as galaxies.

Some macroscopic observable effects of torsion in the framework of cosmology has been studied in Ref. [8] where it is shown that the presence of torsion into effective energy–momentum tensor alters the spectrum of cosmological perturbations giving characteristic lengths for large scale structures. As a final remark, we have to note that spacetime torsion, being related to the intrinsic spin degrees of freedom of matter [1], cannot be transformed away, so that we have to expect its remnants at any epoch of cosmological evolution.

All these arguments do not allow to neglect torsion in any comprehensive theory of gravity which takes into account non-gravitational counterpart of fundamental interactions.

The purpose of this paper is to show that, in presence of torsion, the helicity of fermion particles is not conserved. This effects could be important for testing some astrophysical consequences of torsion [9] because of smallness of coupling constant with respect to the other fundamental interactions.

Our starting point is to consider the Dirac equation in the space–time $U_4$. Due to the torsion, it acquires an additional coupling term of the form $(1/4)S_{\alpha\beta\sigma}^\gamma\gamma^\alpha\gamma^\beta\gamma^\sigma$, where $S_{\alpha\beta\sigma}$ is related to the antisymmetric part of the affine connection, $\Gamma_{[\alpha,\beta]}^\sigma = S_{\alpha\beta}^\sigma$. This term is, as we will see, responsible of the spin flipping of fermions.

It is worthwhile to note that helicity flips are induced also by gravitational fields, as consequence of coupling between spin and curvature [10].

The paper is organized as follows. In Section 2 we will shortly review the basic concepts leading to the Dirac equation in presence of torsion fied. In Section 3 we show that the helicity operator of a fermionic particle is not conserved. The probability that the flip helicity occurs is calculated in Section 4. Conclusions are discussed in Section 5.

**The Dirac Hamiltonian.** – The Dirac equation in curved space–time is written in terms of the vierbeins formalism [11]. One introduces the vierbein fields $e^a_\mu(x)$ where the Latin indices refer to the locally inertial frame and Greek indices to a generic non–inertial frame. The non–holonomic index $a$ labels the vierbein, while the holonomic index $\mu$ labels the components of a given vierbein. The connection in non–holonomic coordinates is given by [9]

$$\Gamma_{abc} = -\Omega_{abc} + \Omega_{cab} - S_{abc},$$  \hspace{1cm} (1)

where $\Omega_{a\beta}^\alpha = e^c_\alpha e^\beta_b e^\sigma_c \Omega_{a\beta}^\sigma$, and $S_{abc}$ is the anti-symmetric part of the affine connection. The covariant derivative is defined as

$$D_\mu \equiv \partial_\mu - \frac{1}{4}\Gamma_{\mu ab}\gamma^a\gamma^b,$$  \hspace{1cm} (2)

and the Dirac equation is given by

$$\gamma^\alpha D_\alpha \psi + \frac{imc}{\hbar} \psi = 0.$$  \hspace{1cm} (3)

In the spirit to study only the effects due to the torsion, we will neglect gravitational effects to the spin flip (they have been analyzed in details in Ref. [10]). It means to neglect the $\Omega_{abc}$ terms in eq. (1) so that the Dirac equations assumes the form

$$\gamma^\alpha \psi,\alpha + \frac{imc}{\hbar} \psi = \frac{1}{4}S_{\alpha\beta\sigma}^\gamma\gamma^\alpha\gamma^\beta\gamma^\sigma \psi$$  \hspace{1cm} (4)
From it one derives the Hamiltonian

$$H = c\bar{\alpha} \cdot \vec{p} + mc^2\beta + \frac{i}{4} S_{\alpha\beta\sigma} \gamma^0 \gamma^\alpha \gamma^\beta \gamma^\sigma = H_0 + H',$$

(5)

where $H'$ is a perturbation of the unperturbed Hamiltonian $H_0 = c\bar{\alpha} \cdot \vec{p} + mc^2\beta$.

**Helicity flip of fermions.** In this section we will prove that the helicity of a fermion is not conserved in a space $U_4$. This follows by calculating the time variation of the helicity operator in the Heisenberg representation and showing that it does not vanish.

The helicity operator is defined as \[h = \vec{\Sigma} \cdot \vec{p},\]

(6)

where the spin matrix $\vec{\Sigma}$ and the versor $\vec{p}$ are

$$\Sigma_i^a = \left( \begin{array}{cc} \sigma^i & 0 \\ 0 & \sigma^i \end{array} \right), \quad \vec{p}^i = \frac{p^i}{|\vec{p}|}.$$

(7)

$\sigma^i, i = 1, 2, 3$ are the Pauli matrices and $p^\mu = (p^0, \vec{p})$ is the momentum. In the Heisenberg representation the dynamical evolution of the helicity operator is given by

$$ih\dot{h} = [h, H],$$

(8)

where $H$ is the Hamiltonian of the system under consideration. For the Hamiltonian (5) one gets

$$ih\dot{h} = cp^k \epsilon_{ijk} S_{\alpha\beta\sigma} \gamma^0 \left[ g^j \gamma^0 \gamma^\beta \gamma^\sigma + 2g^j \gamma^3 \right].$$

(9)

Eq. (9) implies that $\dot{h} \neq 0$ so that the helicity of fermion particle is not conserved.

**Probability of spin flipping.** In this Section, we will calculate the probability of the helicity flip induced by the torsion term in Eq. (5). We consider the totally anti-symmetric dual or a null vector, $S^\sigma = (|\vec{S}|, \vec{S})$. We also used the approximation $g_{\mu\nu} \approx \eta_{\mu\nu}$. Then the Hamiltonian (5) can be recast in the form

$$H' = -rac{3\hbar \epsilon |\vec{S}|}{2} \gamma^5 + \frac{3}{2} \vec{S} \cdot \left( \begin{array}{cc} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{array} \right).$$

(10)

where $\gamma^5$ is defined as \[\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3 = \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right).\]

(11)

The state of a fermion particle is described by spinor

$$\psi(x) = \left( \begin{array}{c} \psi_R \\ \psi_L \end{array} \right),$$

so that it can be rewritten as a superposition of states $|\psi_R>$ and $|\psi_L>$. For instance, at $t = 0$ one has

$$|\psi(0)> = a_0 |\psi_R> + b_0 |\psi_L>.$$

(12)
where \(a_0, b_0\) are constants, \(|\psi_R\rangle\) and \(|\psi_L\rangle\) are eigenkets of energy, i.e. \(H_0|\psi_{R/L}\rangle = E|\psi_{R/L}\rangle\). We choose the independent kets

\[|\psi_R\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |\psi_L\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}.\]

The time evolution of the state (12)

\[|\psi(t)\rangle = a(t)|\psi_R\rangle + b(t)|\psi_L\rangle,\]  

(13)

is given by recasting Dirac’s equation as a Schrödinger like one

\[i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = (H_0 + H')|\psi(t)\rangle.\]  

(14)

Inserting Eq. (13) into Eq. (14), at first order of perturbative calculation, one gets

\[i\hbar \frac{\partial}{\partial t} \begin{pmatrix} a \\ b \end{pmatrix} = M \begin{pmatrix} a \\ b \end{pmatrix}.\]  

(15)

where \(M\) is the matrix

\[M = \begin{pmatrix} \langle \psi_R|H_0 + H'|\psi_R\rangle & \langle \psi_R|H'|\psi_L\rangle \\ \langle \psi_L|H'|\psi_R\rangle & \langle \psi_L|H_0 + H'|\psi_L\rangle \end{pmatrix}.\]  

(16)

Explicit calculation of the matrix elements yields

\[M = \begin{pmatrix} E + i3\hbar S/4 & \hbar c|\vec{S}| \\ \hbar c|\vec{S}| & E + i3\hbar c|\vec{S}|^2 \end{pmatrix}.\]  

(17)

By diagonalizing the matrix (17), one derives the eigenvalues

\[\lambda_{\pm} = E + i3|\vec{S}|^2/2 \pm \frac{3hc}{2}|\vec{S}|,\]  

(18)

and the corresponding normalized eigenkets

\[|\lambda_{\pm}\rangle = \frac{1}{\sqrt{2}}[|\psi_R\rangle \pm \eta_{\pm}|\psi_L\rangle].\]  

(19)

In Eq. (19), \(\eta_{\pm}\) are the phase factors that we choose to be equal to one. It implies that at \(t = 0\), \(|\psi(0)\rangle = |\psi_R\rangle\), i.e. \(a_0 = 1, b_0 = 0\) in the Eq. (12). Then, the evolution of the state \(|\psi(t)\rangle >\) can be written as

\[|\psi(t)\rangle = \frac{1}{\sqrt{2}}[e^{-i\lambda_+ t/\hbar}|\lambda_+\rangle + e^{-i\lambda_- t/\hbar}|\lambda_-\rangle] = e^{-i(E/h + 3c|\vec{S}|/4)t} e^{-3c|\vec{S}|^2/2 t}[\cos \frac{c|\vec{S}|}{2} t|\psi_R\rangle + \sin \frac{c|\vec{S}|}{2} t|\psi_L\rangle].\]  

(20)

Eq. (20) describes the state of a fermion at time \(t\) if it starts as \(|\psi_R\rangle\). The probability to find it in state \(|\psi_R\rangle\) at time \(t\) is \(P_R(t) \sim \cos^2(3c|\vec{S}|/4)/t\), while the probability that the spin flip occurs is \(P_L(t) \sim \sin^2(3c|\vec{S}|/4)/t\).

The frequency of spin flipping is \(\omega = 3c|\vec{S}|/4\), from which follows the characteristic length \(L = 8\pi/3|\vec{S}|\).

Due to the dissipative term, the state decreases exponentially. This fact has important consequences in the very early universe.
Conclusions. – In this paper, we calculate the probability that a background torsion source induces a spin flip on fermion particles moving in it. The torsion field is described by a null vector. We are dealing with high energy fermion particles, so that helicity can be identified with spin; this method can work both for fermion massive and massless particles.

This phenomenology can occur in a regime where the effects of torsion become of the same order of magnitude or bigger than those due to energy momentum tensor at extremely high densities and at sufficiently high polarization of fermion particles. Such a scenario could realize at early cosmological epoch where particle density becomes similar to the critical cosmological density; for example, this happens if electrons are taken into account and \( kT \simeq 10^{11}\text{GeV} \) [13]. It means that, at this epoch, the probability \( P_L(t) \) has to be different from zero. In this sense, torsion and spin density can assume relevant roles in the today observed astrophysical structures, resulting, for example, as intrinsic macroscopic angular momenta [7].

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