Exclusive Weak Radiative Decays of $B$ Mesons in the Covariant Oscillator Quark Model

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Exclusive weak radiative $B$ meson decays are studied using the covariant oscillator quark model. The branching ratios for the processes $B^0 \to K^{*0}_0 \gamma$ and $B^\pm \to K^{*\pm}_0 \gamma$ have been estimated. These are in reasonable agreement with the available experimental data. The calculation has been extended to the CKM-suppressed decay processes $B^- \to \rho^- \gamma$ and $B_s \to K^{*0}_0 \gamma$.

§1. Introduction

The study of the weak radiative decay $B \to K^{*}\gamma$ as a test of Standard Model (SM) has attracted considerable attention since the CLEO experiment \(^1\) gave the preliminary determination of the exclusive branching ratio $Br(B \to K^{*}\gamma) = (4.0 \pm 1.7 \pm 0.8) \times 10^{-5}$. The weak radiative decays of $B$ mesons (which proceed through a flavor changing neutral current, absent at the tree level in the SM) are remarkable for several reasons. The $B \to K^{*}\gamma$ decay arises from the quark level process $b \to s\gamma$ via penguin-type diagrams at the one loop level. Hence it is not only a significant test of Standard Model flavor-changing neutral current dynamics but also is sensitive to new physics appearing through virtual particles such as the top quark and $W$ boson in the internal loop. The study of this process provides valuable information concerning the Cabibbo-Kobayashi-Maskawa (CKM) parameters, $V_{td}$, $V_{ts}$ and $V_{tb}$. Furthermore, additional contributions in loop stemming from new bosons and fermions present in most of the extensions of the SM, suggests the possibility that this process provides a window to new physics.

The theoretical analysis of the decay process $B \to K^{*}\gamma$ requires long-distance QCD contributions which cannot be determined perturbatively. It is also not straightforward to calculate the exclusive decays by first principles of QCD, due to complications inherent in nonperturbative QCD. Therefore, one must resort to some phenomenological models to obtain reliable results. The heavy quark effective theory (HQET) \(^2\) is expected to be useful in this regard in so far as the $b$ quark is concerned. However the $s$ quark in the final hadron can neither be considered heavy enough to enable the use of HQET nor sufficiently light to permit the exploitation of the chiral perturbation theory in an unambiguous manner. There are several
methods available in the literature to study the exclusive \( B \to K^*\gamma \) decay process. Some of them include the QCD sum rule,\(^3\)-\(^5\) lattice QCD,\(^6\) nonrelativistic and relativistic quark models.\(^7\)-\(^11\) The HQET\(^{12}\) has also been applied to this decay process even though the s-quark mass is certainly not heavy enough, contrary to the requirement of HQET.

In this paper we study the rare radiative decay process \( B \to K^*\gamma \) using the covariant oscillator quark model (COQM).\(^{13}\) One of the most important motives for this model is to describe covariantly the center of mass motion of hadrons, while preserving the considerable success of the non-relativistic quark model regarding the static properties of hadrons. A keystone in COQM for doing this is treating directly the squared masses of hadrons in contrast to the mass itself, as done in conventional approaches. This makes the covariant treatment simple. The COQM has been applied to various problems\(^{14}\) with satisfactory results. Recently Ishida et al.\(^{15},^{16}\) have studied the weak decays of heavy hadrons using this model and derived the same relations of weak form factors for heavy-to-heavy transition as done in HQET.\(^2\) In addition, our model is also applicable to heavy-to-light transitions. As a consequence, this model does incorporate the features of heavy quark symmetry and can be used to compute the form factors for heavy-to-light transitions as well, which is beyond the scope of HQET. Actually, in previous papers we made analyses of the spectra of exclusive semi-leptonic\(^{16}\) decays of \( B \)-mesons and analyses of non-leptonic decays of \( B \) mesons\(^{17}\) and of hadronic weak decays of \( \Lambda_b \) baryons\(^{18}\) using this line of reasoning, leading to encouraging results. Keeping this success in mind, we extend its application to weak radiative decays of \( B \) mesons.

The paper is organized as follows. In §2 we present the expressions for the decay widths for the weak radiative decays of \( B \) mesons. In §3 we present a brief description of the covariant oscillator quark model. Using this model we have evaluated the required form factors. Section 4 contains our results and discussion.

§2. Methodology

The general amplitude of weak radiative decay with one real photon emission is given by

\[
\mathcal{M}(B(p) \to P^*(k)\gamma(q)) = i\epsilon_{\mu\nu\alpha\beta} \eta^\mu q^\nu k^\alpha f_1(q^2) \\
+ \eta^\mu \left[ \epsilon_{\mu}(M_B^2 - M_{P^*}^2) - (p+k)_\mu(\epsilon \cdot q) \right] f_2(q^2),
\]

(2.1)

where \( \eta \) and \( \epsilon \) are the polarization vectors of the photon and the vector meson \( P^* \), respectively. The first (second) term on the right-hand side of Eq. (2.1) is a parity conserving (violating) term, and because of the real photon we have \( q^2=0 \). The decay width implied by this amplitude is given as

\[
\Gamma(B \to P^*\gamma) = \frac{1}{32\pi} \left( \frac{M_B^2 - M_{P^*}^2}{M_B^2} \right)^3 \left[ |f_1(0)|^2 + 4|f_2(0)|^2 \right].
\]

(2.2)

Now, the exclusive decay \( B \to K^*\gamma \) is expected to be described well by the quark level process \( b \to s\gamma \) if the confinement effect is properly taken into account. Accord-
In the Standard Model, $B$ decays are described by the effective Hamiltonian obtained by integrating out the top quark and $W$ boson fields given as

$$\mathcal{H}_{\text{eff}}(b \rightarrow s\gamma) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{8} C_i(\mu) \, O_i(\mu),$$

(2.3)

where the $C_i(\mu)$ are the Wilson coefficients, arise from the renormalization group equation to provide the scaling down of the subtraction point appropriate to the problem i.e., $\mu \approx m_t$. The set $\{O_i\}$ is a complete set of renormalized, dimension-six operators which govern the $b \rightarrow s$ transition. They consist of two current operators $O_1$ and $O_2$ and four strong penguin operators $O_3$–$O_6$, which determine the nonleptonic decays, and the electromagnetic dipole operator $O_7$ and the chromomagnetic dipole operator $O_8$, which are responsible for rare $B$ decays, i.e., $b \rightarrow s + \gamma$ and $b \rightarrow s + g$, respectively. Out of the eight operators $O_i$, the one that contributes to $B \rightarrow K^*\gamma$ is

$$O_7 = \frac{e}{32\pi^2} F_{\mu\nu} [m_b \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) b + m_s \bar{s} \sigma^{\mu\nu} (1 - \gamma_5) b],$$

(2.4)

where $F_{\mu\nu}$ is the electromagnetic field strength tensor. The relevant Wilson coefficient is given by

$$C_7(\mu) = \left[ \frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right]^{16/23} \left\{ C_7(M_W) - \frac{8}{3} C_8(M_W) \right\} \times \left[ 1 - \left( \frac{\alpha_s(m_b)}{\alpha_s(M_W)} \right)^{2/23} \right] + \frac{232}{513} \left[ 1 - \left( \frac{\alpha_s(m_b)}{\alpha_s(M_W)} \right)^{19/23} \right],$$

(2.5)

with

$$C_7(M_W) = -\frac{x}{2} \left[ \frac{5}{3} x^2 + \frac{1}{12} x - \frac{7}{12} \right] - \frac{\left( \frac{3}{2} x^2 - x \ln x \right)}{(x - 1)^4},$$

(2.6)

and

$$C_8(M_W) = -\frac{x}{4} \left[ \frac{1}{2} x^2 - \frac{5}{2} x - 1 \right] + \frac{3x \ln x}{(x - 1)^4},$$

(2.7)

where $x = m_t^2/M_W^2$. From the fact that $m_b \gg m_s$, only the term involving $m_b$ in the operator $O_7$ need be retained. Thus the matrix element of interest becomes

$$\langle K^*\gamma|O_7|B \rangle = \frac{ie}{16\pi^2} q_\mu \eta_\nu \, m_b \langle K^*|\bar{s} \sigma^{\mu\nu} (1 + \gamma_5) b|B \rangle,$$

(2.8)

where $q_\mu$ is the four momentum of the photon and $\eta_\mu$ is its polarization vector. It should be noted that here the matrix element of a tensor current between hadronic states for which not much information is available is involved. However, in the framework of HQET the associated heavy quark spin symmetry enables one to express the matrix element of the tensor operator in terms of the vector and axial vector form factors that also occur in semileptonic decays and may be estimated in different phenomenological models.
In the static limit of a heavy $b$ quark we may use the equation of motion $\gamma_0 b = b$ to derive the relations\textsuperscript{20}:

$$\langle K^*|\bar{s}\gamma_i (1 + \gamma_5) b|B\rangle = \langle K^*|\bar{s}\gamma_i (1 - \gamma_5) b|B\rangle.$$  \hspace{1cm} (2.9)

As a result, the form factors $f_1$ and $f_2$ in Eq. (2.1) can be related to the vector and axial vector form factors $V$ and $A_1$ appearing in the matrix element of the RHS of Eq. (2.9) defined by\textsuperscript{21}:

$$\langle K^*|\bar{s}\gamma_\mu b|B(p)\rangle = \frac{2i}{M_B + M_{K^*}} \epsilon_{\mu\nu\alpha\beta} \epsilon^\nu q^\alpha p^\beta V(q^2),$$  \hspace{1cm} (2.10)

$$\langle K^*|\bar{s}\gamma_5 b|B(p)\rangle = (M_B + M_{K^*})\epsilon_\mu A_1(q^2) - \frac{\epsilon \cdot q}{M_B + M_{K^*}} (p + k)_\mu A_2(q^2)$$
$$-2\frac{\epsilon \cdot q}{q^2} q_\mu M_{K^*} [A_3(q^2) - A_0(q^2)],$$  \hspace{1cm} (2.11)

with $q = p - k$. Here $A_3(q^2)$ is simply an abbreviation for:

$$A_3(q^2) = \frac{M_B + M_{K^*}}{2M_{K^*}} A_1(q^2) - \frac{M_B - M_{K^*}}{2M_{K^*}} A_2(q^2),$$  \hspace{1cm} (2.12)

and in order to cancel the singularity at $q^2 = 0$, we must have $A_3(q^2 = 0) = A_0(q^2 = 0)$. Thus with Eqs. (2.1) and (2.8)–(2.12), we obtain the following relations for $B \to K^*\gamma$ transition at $q^2 = 0$:\textsuperscript{22}

$$f_1(0) = \frac{G_F e}{\sqrt{2} 2\pi^2} C_7(\mu) V_{tb} V_{ts}^* m_b F(0)$$  \hspace{1cm} (2.13)

and

$$f_2(0) = \frac{1}{2} f_1(0),$$  \hspace{1cm} (2.14)

where

$$F(0) = \frac{M_B - M_{K^*}}{2M_B} V(0) + \frac{M_B + M_{K^*}}{2M_B} A_1(0).$$  \hspace{1cm} (2.15)

Substituting the above values of $f_1$ and $f_2$ into Eq. (2.2), we obtain the decay width for $B \to K^*\gamma$ as

$$\Gamma(B \to K^*\gamma) = \frac{\alpha G_F^2 m_b^2}{128\pi^4 M_B^3} |V_{tb} V_{td}|^2 |C_7(\mu)|^2 (M_B^2 - M_{K^*}^2)^3$$
$$\times \left[ (M_B + M_{K^*}) A_1(0) + (M_B - M_{K^*}) V(0) \right]^2. \hspace{1cm} (2.16)$$

Similarly, the decay width for the CKM suppressed FCNC radiative transition $b \to d\gamma$, which is responsible for $B^- \to \rho^-\gamma$ and $B_s \to K^*\gamma$, is obtained from Eq. (2.16) with $M_B$ and $M_{K^*}$ replaced by the corresponding initial and final meson masses. The relevant CKM factor in this case is $|V_{td}|^2$. 
Now to evaluate the form factors $A_1(0)$ and $V(0)$, we use the covariant oscillator quark model, which is presented in the next section.

Here it should be noted that the relations (2.13) and (2.14) are also derivable\textsuperscript{15} directly from Eq. (2.8) in COQM, by using the covariant spin wave functions, that is, the Bargmann-Wigner spinor functions. Accordingly, the expression of $F(0)$ in terms of a space-time wave function (Eq. (3.10) given later) is also derived directly.

§3. Model framework and the hadronic form factors

The general treatment of COQM may be called the “boosted LS-coupling scheme,” and the wave-functions, being tensors in $U(4) \times O(3,1)$-space, reduce to those in $SU(2)_{\text{spin}} \times O(3)_{\text{orbit}}$-space in the nonrelativistic quark model in the hadron rest frame. The spinor and space-time portion of the wave functions satisfy separately the respective covariant equations, the Bargmann-Wigner (BW) equation for the former, and the covariant oscillator equation for the latter. The form of the meson wave function has been determined completely through the analysis of mass spectra.

In COQM the meson states are described by bi-local fields $\Phi^B_A(x_{1\mu}, x_{2\mu})$, where $x_{1\mu}(x_{2\mu})$ is the space time coordinate of the constituent quark (antiquark), $A = (a, \alpha)$ ($B = (b, \beta)$), describing its flavor and covariant spinor. Here we write only the positive frequency part of the relevant ground state fields:

$$
\Phi^B_A(x_{1\mu}, x_{2\mu}) = e^{iP \cdot X} U(P)^B_A f_{ab}(x_{\mu}; P),
$$

where $U$ and $f$ are the covariant spinor and internal space-time wave functions, respectively, satisfying the Bargmann-Wigner and oscillator wave equations. The quantity $x_{\mu}$ ($X_{\mu}$) is the relative (CM) coordinate, $x_{\mu} \equiv x_{1\mu} - x_{2\mu}$ ($X_{\mu} \equiv m_1 x_{1\mu} + m_2 x_{2\mu}/(m_1 + m_2)$, with $m_i$ the quark masses). The function $U$ is given by

$$
U(P) = \frac{1}{2\sqrt{2}} \left[ (-\gamma_5 P_s(V) + i\gamma_\mu V_\mu(V))(1 + iv \cdot \gamma) \right],
$$

where $P_s(V_\mu)$ represents the pseudoscalar (vector) meson field, and $v_\mu \equiv P_\mu/M$ ($P_\mu(M)$ is the four momentum (mass) of the meson). The function $U$, being represented by the direct product of a quark and antiquark Dirac spinor with the meson velocity, is reduced to the non-relativistic Pauli-spin function in the meson rest frame. The function $f$ is given by

$$
f(x_{\mu}; P) = \frac{\beta}{\pi} \exp \left( -\frac{\beta}{2} \left( x_{\mu}^2 + \frac{(x \cdot P)^2}{M^2} \right) \right). \quad (3.3)
$$

The value of the parameter $\beta$ is determined from the mass spectra\textsuperscript{23} as $\beta_\rho = 0.14$, $\beta_{K^*} = 0.142$, $\beta_B = 0.151$ and $\beta_{B_s} = 0.160$ (in units of GeV$^2$).

The effective action for weak interactions of mesons with $W$-bosons is given by

$$
S_W = \int d^4x_1 d^4x_2 (\Phi_{F,P}(x_1, x_2) i\gamma_\mu (1 + \gamma_5) \Phi_{F,P}(x_1, x_2)) W_{\mu\alpha}(x_1),
$$

(3.4)
where we have denoted the interacting (spectator) quarks as $1\ (2)$ (the KM matrix elements and the coupling constants are omitted). This is obtained from consideration of Lorentz covariance, assuming a quark current with the standard $V-A$ form. In Eq. (3.4), $\Phi_{I,P}\ (\Phi_{F,P'})$ denotes the initial (final) meson with definite four momentum $P_\mu\ (P'_\mu)$, and $q_\mu$ is the momentum of $W$-boson. The function $\Phi$ is the Pauli-conjugate of $\Phi$ defined by $\bar{\Phi} = -\gamma^4 \Phi^\dagger \gamma^4$, and $\langle \rangle$ represents the trace of Dirac spinor indices. Our relevant effective currents $J_\mu(X)_{P',P}$ are obtained\(^\text{15}\) by identifying the above action with

$$S_W = \int d^4X J_\mu(X)_{P',P} W_\mu(X)_q,$$

(3.5)

$$J_\mu = I^{qh}(w) \sqrt{MM'} \times (\bar{P}_s(v')P_s(v)(v + v')_\mu$$

$$+ \bar{V}_\lambda(v')P_s(v)(\epsilon_\mu\lambda\alpha\beta v'_\alpha v_\beta - \delta_\lambda\mu(w + 1) - v_\lambda v'_\mu),$$

(3.6)

where $M(M')$ denotes the physical mass of the initial (final) meson and $w = -v \cdot v'$. The quantity $I^{qh}(w)$, the overlapping of the initial and final wave functions, represents the universal form factor function.\(^\text{*}\) It describes the confinement effects of quarks, and is given by

$$I^{qh}(w) = \frac{4\beta\beta'}{\beta + \beta'} \frac{1}{\sqrt{C(w)}} \exp(-G(w)); \quad C(w) = (\beta - \beta')^2 + 4\beta\beta'w^2,$$

(3.7)

and

$$G(w) = \frac{m_n^2}{2C(w)} \left[ (\beta + \beta') \left( \frac{M}{M_s} \right)^2 + \left( \frac{M'}{M'_s} \right)^2 - 2 \frac{MM'}{M_sM'_s}w \right]$$

$$+ 2 \left[ \beta' \left( \frac{M}{M_s} \right)^2 + \beta \left( \frac{M'}{M'_s} \right)^2 \right] (w^2 - 1),$$

(3.8)

where $M_s(M'_s)$ represents the sum of the constituent quark masses of the initial (final) meson, and $m_n$ is the spectator quark mass.

Comparing our effective current (3.6) with Eqs. (2.10) and (2.11), we obtain\(^\text{**}\) the relations between the invariant form factors $V$ and $A_1$ with the form factor function $I(w)$ for $B \rightarrow K^*$ transitions as

$$V(0) = A_1(0) = F(0),$$

(3.9)

$$F(0) = \frac{M_B + M_{K^*}}{2\sqrt{M_BM_{K^*}}} I^{qb}(w|q^2=0),$$

(3.10)

and similarly for $B^- \rightarrow \rho^-$ and $B_s \rightarrow K^*$ transitions with $I^{qb}(w)$ replacing $I^{sb}(w)$.

\(^\text{*) In this paper we apply the pure-confining approximation, neglecting the effect of the one-gluon-exchange potential. This approximation is expected to be good for the heavy/light-quark meson systems.\(^\text{**}) Here, note the remark given at the end of \S 2.
§4. Results and conclusion

To estimate the branching ratios for weak radiative decays of $B$ mesons, we use the following values for various quantities. The quark masses (in GeV) are taken as $m_u = m_d = 0.4$, $m_s = 0.55$ and $m_b = 5$. The particle masses and their lifetimes are taken from Ref. 24). The relevant CKM parameters required for all the processes considered here are taken from Ref. 24) as $V_{td} = 0.0085$, $V_{ts} = 0.0385$ and $V_{tb} = 0.99925$. The renormalized Wilson coefficient $C_7(m_b)$ used in the estimation of branching ratios is taken to be $|C_7(m_b)| = 0.311477$.25) With the above values we first evaluate the form factor $F(0)$ using Eqs. (2.15) and (3.7)–(3.9). Our results for the form factor values given in Table I compare well with the recent calculations within the framework of the relativistic quark model,10) light cone QCD sum rule,4) and hybrid sum rule.5) With the calculated values of the form factors, the branching ratios for different channels are estimated using Eq. (2.16) as well as the mean life values of the appropriate decaying meson, which are given in Table II. The branching ratios $Br(B^0 \to K^{*0} \gamma)$ and $Br(B^{\pm} \to K^{*\pm} \gamma)$ are found to be in very good agreement with the available data. Here it may be worthwhile to note that the relevant process is relativistic, and the form factor function plays a significant role, such that $I = 0.235$. Since there are no data as yet available for the branching ratios in the case $B \to \rho \gamma$ and $B_s \to K^{*\gamma}$, the model predictions must be compared with other theoretical predictions available in the literature. In fact there are few theoretical attempts made so far in this sector. Recently, Singer26) predicted that in the relativistic quark model,

$$Br(B_s \to K^{*\gamma}) = (1.4 \pm 0.6) \times 10^{-6}, \quad (4.1)$$

and in the relativistic independent quark model, Barik et al.11) obtained

$$Br(B \to \rho \gamma) = 1.24 \times 10^{-6},$$
$$Br(B_s \to K^{*\gamma}) = 9.28 \times 10^{-7}. \quad (4.2)$$

Finally, the hadronization ratio $R$ defined as $\Gamma(B \to K^{*\gamma})/\Gamma(b \to s \gamma)$, where

$$\Gamma(b \to s \gamma) = \frac{G_F^2 \alpha}{32 \pi^4} |V_{ts}V_{tb}|^2 \left| C_7(m_b) \right|^2 m_b^5,$$  

is found to be 0.12, which agrees well with the experimentally observed value $R = 0.19 \pm 0.09$.27)  

In view of the consistency of our predictions, obtained with no free parameters, with the large number of theoretical predictions as well as the experimental data, it may be concluded that the framework of COQM provides a suitable scheme to estimate the confined effect of quarks, and so far as the relevant flavor changing neutral current is concerned, the Standard Model is valid.
Table I. Prediction for the rare radiative decay form factor $F(0)$ in comparison with various theoretical predictions.

<table>
<thead>
<tr>
<th>Decay process</th>
<th>Present work</th>
<th>Ref. 10)</th>
<th>Ref. 4)</th>
<th>Ref. 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm \to K^{\star\pm} \gamma$</td>
<td>0.328</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$B^0 \to K^{\star0} \gamma$</td>
<td>0.329</td>
<td>0.32 ± 0.03</td>
<td>0.32 ± 0.05</td>
<td>0.308 ± 0.013 ± 0.036 ± 0.006</td>
</tr>
<tr>
<td>$B^- \to \rho^- \gamma$</td>
<td>0.295</td>
<td>0.26 ± 0.03</td>
<td>0.24 ± 0.04</td>
<td>0.27 ± 0.011 ± 0.032</td>
</tr>
<tr>
<td>$B_s \to K^{\star0} \gamma$</td>
<td>0.250</td>
<td>0.23 ± 0.02</td>
<td>0.20 ± 0.04</td>
<td>-</td>
</tr>
</tbody>
</table>

Table II. Prediction for the branching ratios of the exclusive rare decays of $B$ mesons along with their experimental values.

<table>
<thead>
<tr>
<th>Decay process</th>
<th>Br ratio</th>
<th>Br ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>This work</td>
<td>Expt. 24)</td>
</tr>
<tr>
<td>$B^0 \to K^{\star0} \gamma$</td>
<td>$3.96 \times 10^{-5}$</td>
<td>$(4.0 \pm 1.9) \times 10^{-5}$</td>
</tr>
<tr>
<td>$B^\pm \to K^{\star\pm} \gamma$</td>
<td>$4.168 \times 10^{-5}$</td>
<td>$(5.7 \pm 3.3) \times 10^{-5}$</td>
</tr>
<tr>
<td>$B^- \to \rho^- \gamma$</td>
<td>$1.68 \times 10^{-6}$</td>
<td>-</td>
</tr>
<tr>
<td>$B_s \to K^{\star0} \gamma$</td>
<td>$1.158 \times 10^{-6}$</td>
<td>-</td>
</tr>
</tbody>
</table>

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References

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