SA
PAUL SCHERRER INSTITUT

PSI-PR-98-19
August 1998

SW 9916

On the Variation of Pulsar Periods in Close Binary Systems

Philippe Jetzer^{a,b}, Marcus Strässle^b and Norbert Straumann^b

a. Paul Scherrer Institut, Laboratory for Astrophysics, CH-5232 Villigen PSI

b. Institut für Theoretische Physik der Universität Zürich, Winterthurerstrasse 190,
CH-8057 Zürich, Switzerland

Paul Scherrer Institut
CH - 5232 Villigen PSI
Telefon 056 310 21 11
Telefax 056 310 21 99

On the variation of pulsar periods in close binary systems

Philippe Jetzer,

*Paul Scherrer Institut, Laboratory for Astrophysics, CH-5232 Villigen
PSI and Institut für Theoretische Physik der Universität Zürich,
Winterthurerstrasse 190, CH-8057 Zürich*

Marcus Strässle and Norbert Straumann

*Institut für Theoretische Physik der Universität Zürich, Winterthurerstrasse 190,
CH-8057 Zürich*

X-ray pulsars, which form together with a main sequence star or a giant a close binary system, show strong variations in their pulse period. Epochs of spin-up and spin-down interchange on timescales of some tens up to some thousand days.

We study this phenomenon in the framework of the disk-fed model by Ghosh and Lamb, which we generalize by allowing for an inclination angle χ between the magnetic dipole moment $\vec{\mu}$ and the rotation axis. Moreover, we take into account the relatively small magnetic field induced by currents in the magnetosphere flowing along the dipole field lines. This component, which we calculate numerically, leads to a torque perpendicular to the rotation axis and induces a precession of it. Due to this additional degree of freedom, the calculated pulse period history is much smoother than in the classical Ghosh and Lamb model.

Within this extended model we compute, as an example, the pulse period history of Cen X-3 using the observed X-ray flux as a measure for the mass accretion rate. Qualitative accordance with the measured pulse period data (see e.g. Nelson et al. 1997) is found to be good.

Key words: X-ray pulsar – binary pulsar – spin-up spin-down

1 Introduction

Since the first discovery of X-ray pulsars by the Uhuru satellite, more than 300 X-ray pulsars have been detected. Detailed observations have shown dramatic

torque reversals between spin-up and spin-down on timescales of some tens up to some thousand days (cf. Figs. 1–2). The variation in the pulse period is explained by accretion onto a magnetic neutron star. In a series of papers, Ghosh and Lamb (Ghosh & Lamb 1978), (Ghosh & Lamb 1979a), (Ghosh & Lamb 1979b) discussed an aligned rotator model of a disk-fed neutron star. They pointed out that the magnetic field of the neutron star can penetrate the disk plasma, due to plasma instabilities. As a consequence, the magnetic field lines become twisted, a toroidal magnetic field component inside the disk is established and a corresponding stress is exerted on the neutron star. Depending on the relative size of the rotation frequency of the inner disk and the pulsar rotation frequency, this torque can be either positive or negative, this way explaining the spin-up or spin-down periods of X-ray pulsars.

Ghosh and Lamb applied their results to several X-ray pulsars. In general, their model gives satisfactory agreement with the observations in a static limit. For most X-ray pulsars it is sufficient to assume magnetic moments of the order of $\sim 10^{30}$ gauss cm³, however, for some of them, they had to assume much higher values. This may be due to the fact that the pulsars in question are wind-fed rather than disk-fed, or that some assumptions of the model are not fulfilled.

Recently, new models have been proposed to explain the observations (see e.g. van Kerkwijk et al. 1998, Nelson et al. 1997). However, they are all based on phenomena like retrograde or warped disks for which there is only weak evidence. Here, we discuss a possible extension of the Ghosh and Lamb model, which relies only on well established facts of the pulsar model. To that purpose, we introduce an inclination angle χ between the magnetic dipole moment $\vec{\mu}$ and the rotation axis; this way we gain an additional degree of freedom. Furthermore, we take into account the magnetic field induced by the currents flowing in the magnetosphere along the dipole field lines. This additional magnetic field can be computed numerically in a first order approximation, since it is small compared to the dipole field. It leads then to a small torque perpendicular to the rotation axis, which induces a precession of the pulsar rotation axis. As a consequence of the precession, the variation of the spin period gets smoother on timescales, which agree well with the observed ones.

The paper is organized as follows: in Sect. 2 we present a generalization of the Ghosh and Lamb model by allowing for an inclination angle χ between the magnetic moment and the rotation axis and by calculating the additional component of the magnetic field due to currents in the inner magnetosphere along the magnetic field lines of the dipole. In Sect. 3 we derive the equations for the torque exerted on the pulsar, and in Sect. 4 we discuss the magnetic field configuration inside the disk. Moreover, by assuming that the pulsar rotates near its equilibrium spin period, we can estimate its magnetic moment. In Sect. 5, we then solve the equations of motion for the star, which we treat as

Fig. 1. Pulse period history for Cen X-3. Data is taken from the review of Nagase (Nagase 1989). The dashed line is a linear fit through the data points.

a rigid rotator, slightly oblate due to its fast rotation. We apply our results to Cen X-3, using the X-ray flux as measured by the Ariel-V satellite and the BATSE detector on CGRO as input data. Sect. 6 concludes the paper with a short summary.

2 The model

2.1 The inclined rotator

Following Ghosh and Lamb (Ghosh & Lamb 1978), we introduce here the main characteristics of a model describing disk-accretion by a magnetized neutron star. In leading order, we assume that the magnetic field of the pulsar is described by a dipole field. The axis of the magnetic moment $\vec{\mu}$ is inclined with respect to the rotation axis by an angle χ . In cylindrical coordinates (r, φ, z) , where the z-axis coincides with the rotation axis, the oblique dipole field is parametrized by

$$B_r = \mu \frac{3rz \cos(\chi) + (2r^2 - z^2) \sin(\chi) \sin(\varphi)}{\sqrt{r^2 + z^2}^5}, \quad (1)$$

$$B_\varphi = -\mu \frac{\sin(\chi) \cos(\varphi)}{\sqrt{r^2 + z^2}^3} \quad (2)$$

Fig. 2. Pulse period history for Vela X-1. Data is taken from the review of Nagase (Nagase 1989). The dashed line is a linear fit through the data points. For this case the data set was split into two independent sets, separated by the turning point from spin-up to spin-down.

and

$$B_z = \mu \frac{(2z^2 - r^2) \cos(\chi) + 3rz \sin(\chi) \sin(\varphi)}{\sqrt{r^2 + z^2}^5}. \quad (3)$$

The neutron star forms a close binary system together with a main sequence star or a giant, which loses mass to the compact object, either by Roche-lobe overflow, or by a stellar wind. In the present study, we consider only situations, where the accreted matter has enough angular momentum to form an accretion disk in the equatorial plane of the pulsar.

A thin accretion disk can be described by the standard model of Shakura and Sunyaev (Shakura & Sunyaev 1973), if we neglect the influence of the magnetic field. Hence, the standard α -disk model will be valid only at large distances from the neutron star. The velocity field $\vec{v}$ of the matter inside the disk is assumed to correspond to the Keplerian circular motion with only minor corrections. Viscous shear inside the disk gives rise to a velocity component v_r in radial direction towards the star, which is, however, small compared to the Keplerian velocity. With some additional assumptions on the viscosity of the plasma in the disk, Shakura and Sunyaev (Shakura & Sunyaev 1973) derived a parametrization for this simple model. In the following we will always adopt their disk parameters.

According to a detailed study of Ghosh, Lamb et al. (Ghosh et al. 1977), (Ghosh & Lamb 1978), (Ghosh & Lamb 1979a), (Ghosh & Lamb 1979b), (Lamb et al. 1973) the surroundings of a magnetic neutron star can be divided into four regions (I–IV). A schematic illustration is given in Fig. 3.

I) At large distances from the star ($r > r_{out}$) the influence of the magnetic field is weak, due to the fact that currents in the disk screen off the pulsar field, and to a good approximation the disk-model of Shakura and Sunyaev (Shakura & Sunyaev 1973) applies.

II) In an outer transition zone, bounded by r_{out} and the inner radius r_o , a fraction – typically 20% – of the pulsar magnetic flux penetrates the disk plasma due to its finite conductivity. Thus, the influence of the pulsar magnetic field becomes increasingly important. Nevertheless, the azimuthal velocity is still to a very good approximation Keplerian. In the model of Ghosh and Lamb (Ghosh & Lamb 1978), (Ghosh & Lamb 1979a) the slippage of field lines through the disk plasma is parametrized by an effective conductivity

$$\sigma_{eff}(r) = (c^2/4\pi)h^{-1}r^{-1}(\Omega_K - \Omega_S)^{-1}\gamma_\varphi,$$

where c is the light velocity, $2h$ is the thickness of the disk, $\Omega_K = \sqrt{GM/r^3}$ the angular velocity of the disk at the radial distance r from the neutron star with mass M , Ω_S the pulsar angular velocity and γ_φ the average azimuthal pitch defined by

$$\gamma_\varphi(r) = -(B_\varphi/B_z)|_{z=h}.$$

Shear in the toroidal velocity field generates an additional toroidal magnetic field from the existing poloidal field. Ghosh and Lamb (Ghosh & Lamb 1978), (Ghosh & Lamb 1979a) assume a balance between the growth and the decay of this field component due to reconnection. Thus, they find the following expression for the average azimuthal pitch:

$$\gamma_\varphi(r) = \left(\frac{\gamma_a}{\xi}\right) 2h\sqrt{4\pi\rho} \left(\frac{\Omega_K - \Omega_S}{B_z}\right).$$

The ratio γ_a/ξ is assumed to be constant throughout the outer transition zone. Its value is fixed by requiring that γ_φ joins smoothly the value γ_o at the inner boundary r_o . We only need the ratio γ_a/ξ and we refer for a definition of γ_a and ξ to the paper of Ghosh and Lamb (Ghosh & Lamb 1979a). The matter density $\rho = \rho(r, M, \dot{M})$ is again taken from the Shakura and Sunyaev (Shakura & Sunyaev 1973) disk model, for which we get, once we insert all the constants $\rho = 5.1 \times 10^{-16} \dot{M}^{2/5} M^{11/20} r^{-33/20}$. As usual, the value for $\dot{M}$ is obtained by the relation

$$\dot{M} = \frac{L_X R_{Star}}{GM},$$

where L_X denotes the measured X-ray luminosity.

Fig. 3. Schematic illustration of the situation around an accreting neutron star. The star rotates around the z -axis with angular velocity $\vec{\omega}$. Its magnetic dipole moment $\vec{\mu}$ is inclined by the angle χ to $\vec{\omega}$. Plasma flowing in the accretion disk—indicated by the arrows—deforms the field lines and is forced to follow the magnetic field lines at distances r smaller than r_o . S_1 , S_2 and S_3 denote the surfaces of integration, as defined within the text. For further details we refer to Fig. 1 in Ghosh and Lamb (Ghosh & Lamb 1978).

III) At the radial distance r_o the Ghosh and Lamb model predicts the existence of a narrow inner transition zone. The radial dimension δ of this boundary layer is typically about h . Here the velocity of the disk plasma changes from the Keplerian value to the corotation value. The dominant stresses are no longer viscous, but magnetic.

IV) Finally, at radial distances smaller than $r_o - \delta$ the magnetic field completely dominates the behaviour of matter. Viscous stresses are negligible. Plasma, which enters this magnetosphere in the inner transition zone via Rayleigh-Taylor instabilities, is assumed to flow along the magnetic field lines.

Since it is our aim to calculate the accretion torque exerted on a non-aligned disk-fed pulsar, we have to derive a generalized expression for the Alfvén-radius r_A and thus—as we will see—for r_o . The Alfvén-radius is defined as the distance at which the magnetic field of the neutron star begins to dominate the behaviour of matter flowing towards the star. This distance is crucial in order to determine the geometry of the accretion disk, to evaluate the dimensions of the accretion columns above the stellar magnetic poles and the accreted torque. Since there is no accurate description of the flow of matter towards an oblique neutron star, the Alfvén-radius will be model dependent. Lamb and Pethick (Lamb & Pethick 1974) give a short survey of different proposals.

To obtain r_A , one possible choice is to argue that the pulsar magnetic field will be able to control the accreting plasma once the energy density of the pulsar

magnetic field exceeds the energy density of the inflowing plasma. Assuming spherical accretion, this condition can be written as

$$\frac{B^2(r_A)}{8\pi} = \frac{1}{2}\rho(r_A)v_{ff}^2(r_A). \quad (4)$$

Here $v_{ff}(r) = \sqrt{2GM/r}$ is the free-fall velocity and the matter density is given by

$$\rho = \frac{\dot{M}}{4\pi v_{ff} r_A^2}, \quad (5)$$

$\dot{M}$ being the mass accretion rate. For B^2 we take the value of the undistorted dipole field in the disk plane, i.e.

$$B^2 = \frac{\mu^2}{r^6} [1 + 3(\sin(\chi)\sin(\varphi))^2],$$

and find for the Alfvén-surface

$$r_A(\chi, \varphi) = \left(\frac{\mu^4 (1 + 3(\sin(\chi)\sin(\varphi))^2)^2}{\dot{M}^2 2GM} \right)^{\frac{1}{7}}. \quad (6)$$

For the case of disk accretion and $\chi = 0$, Ghosh, Lamb and Pethick (Ghosh et al. 1977) showed that Eq. (6) is an upper bound for the pulsar magnetosphere. For the location of the inner transition zone Ghosh and Lamb (Ghosh & Lamb 1979a) found the expression

$$r_o \approx 0.41(\gamma_\varphi^{11/27} C_b^{16/27} C_\omega^{-25/27} C_p^{8/27})^{2/7} r_A. \quad (7)$$

The boundary layer constants γ_φ , C_b and C_p are given in Ghosh and Lamb (Ghosh & Lamb 1979a). The constant C_ω depends on the other constants. The procedure how to obtain a consistent value for C_ω can be found in Ghosh and Lamb (Ghosh & Lamb 1979a). One typically finds $r_o \approx 0.5 r_A$, which we also used in our calculation.

As discussed in Ghosh et al. (Ghosh et al. 1977), the numerical factor depends on the dimension of the inner transition zone and the processes by which the magnetic field penetrates the plasma. If the stellar field threads the inflowing plasma rapidly, then one expects $r_o \approx r_A$, whereas if it threads it slowly the value for r_o can be well below $0.5r_A$.

r_A as defined above in Eq. (6) depends on the magnetic inclination angle χ and also on the azimuthal angle φ . Thus, in the disk plane the Alfvén-points

are no longer located on a circle. The timescale for the plasma to flow from the Alfvén-radius onto the pulsar surface is of the order of the free-fall time $\tau = \frac{r_A}{v_{ff}} < 10^{-2}$ s, which is much shorter than the rotation period for all considered pulsars. Thus, plasma once trapped by the field experiences a local magnetic field and not an azimuthally averaged field.

To compute the accretion torque we also need an upper bound r_{out} , from where on the magnetic field inside the disk is completely screened. Following Ghosh and Lamb (Ghosh & Lamb 1979a) we take

$$r_{out} = 103 r_o \alpha^{-\frac{32}{73}} \gamma_o^{-\frac{40}{73}} (1 - \omega_s)^{\frac{32}{73}}, \quad (8)$$

where the constant α is given by the standard α -disk model of Shakura and Sunyaev (Shakura & Sunyaev 1973), $\gamma_o = \gamma_\varphi(r_o)$ and ω_s is the so called fastness parameter defined by

$$\omega_s = \frac{\Omega_S}{\sqrt{GM r_o^{-3}}}.$$

We notice that the calculated value for the torque is not very sensitive to this upper bound. In addition, since the typical Keplerian orbital period at r_{out} is about 100-times larger than the pulse period, it is sufficient to use an azimuthally averaged value for r_{out} .

2.2 The magnetic field induced by currents in the magnetosphere

As already mentioned, we extend the Ghosh and Lamb model in two ways. Having studied the effects of an inclination angle χ in the previous subsection, we now investigate the appearance of deviations from the pure magnetic dipole field due to the presence of currents along the magnetic field lines. In first order approximation, we assume that the accreted matter follows the dipole field lines. Our Ansatz is justified, if the magnetic field induced by currents in the magnetosphere is small compared to the original dipole field. The numerical calculations show, that this condition is fulfilled, even for an accretion rate above the Eddington limit, if the magnetic field strength equals 10^{12} gauss at the pulsar surface. For Cen X-3 we find, as we will see below, that the magnitude of the induced magnetic field in the boundary layer is only about 8% of the dipole value.

Due to the different drift velocities of electrons and protons, the inflowing plasma causes a net current and thus generates an additional magnetic field component. The problem was studied by Mestel, (Mestel 1975), (Mestel 1968) in the context of magnetic braking by a stellar wind, which is essentially the

inverse of our problem. Since we assume that the current has the direction of the magnetic field, we set

$$\vec{j} = \kappa \vec{B}, \quad (9)$$

$\kappa(r, \vartheta, \varphi)$ being a function to be discussed. Every vector field can be decomposed into a curl-free and a divergence-free part. Hence, there are two potentials U and $\vec{V}$ such that $\vec{B} = \vec{B}_{\parallel} + \vec{B}_{\perp}$ with $\vec{B}_{\parallel} = \vec{\nabla}U$ and $\vec{B}_{\perp} = \vec{\nabla} \wedge \vec{V}$. Inserting this into the corresponding Maxwell-equation we find:

$$\Delta \vec{V} = -\frac{4\pi}{c} \kappa \vec{B}, \quad (10)$$

and thus

$$\vec{V}(\vec{x}) = \frac{1}{c} \int \frac{\kappa(\vec{x}') \vec{B}(\vec{x}') d^3 x'}{|\vec{x} - \vec{x}'|}. \quad (11)$$

From this we obtain the integral relation

$$\vec{B}_{\perp}(\vec{x}) = -\frac{1}{c} \int \frac{\kappa(\vec{x}')}{|\vec{x} - \vec{x}'|^3} (\vec{x} - \vec{x}') \wedge \vec{B}(\vec{x}') d^3 x'. \quad (12)$$

Since we expect the induced magnetic field to be small as compared to the dipole field, we can use Eq. (12) to obtain by iteration approximate solutions. To this purpose, we split $\vec{B}$ into two components, a large one $\vec{B}_0$, which corresponds to the undistorted dipole field and a small one $\vec{B}_1$. $\vec{B}_{1\perp}$ induced by currents flowing along the $\vec{B}_0$ field lines is then given up to first order by:

$$\vec{B}_{1\perp}(\vec{x}) = -\frac{1}{c} \int \frac{\kappa(\vec{x}')}{|\vec{x} - \vec{x}'|^3} (\vec{x} - \vec{x}') \wedge \vec{B}_0(\vec{x}') d^3 x'. \quad (13)$$

Since the dipole field has no $\vec{B}_{0\perp}$ component, $\vec{B}_{1\parallel}$ vanishes.

As mentioned above, the plasma velocity inside the magnetosphere is about the free-fall velocity. Hence, we take for the unknown function κ , as a rough approximation,

$$\kappa = \eta \frac{\rho_c v_{ff}}{|\vec{B}_0|} \quad (14)$$

for every field line, where field channeled flux occurs and zero elsewhere. ρ_c

denotes the charge density and η an unknown parameter $\lesssim 1$. In good approximation, the charge density is connected with the matter density by

$$\rho_c = \frac{e}{m_p} \rho. \quad (15)$$

One can now use Eq. (13) to calculate numerically in lowest order the additional magnetic field component. However, this is not a trivial task. Besides the numerical effort to calculate the above vector valued integral, one has to take care of the nature of the current. In fact, there is only a small transition region, where plasma enters the magnetosphere, hence, matter will be accreted only on field lines that penetrate the transition zone. Using the data for Cen X-3 as an example, we find that the induced magnetic field in the boundary layer is indeed small, being at most 8% of the undistorted dipole value. The parameters we used for the calculation can be found in Table 1.

3 Evaluation of the accretion torque

A dipole field is curl-free and thus also force free. The additional toroidal field component, as introduced by Ghosh and Lamb (Ghosh & Lamb 1978), causes a magnetic torque along the pulsar rotation axis. In our model, we calculate the relatively small deviations of the magnetic field from the pure dipole and, since these are not curl-free, the pulsar gains also a relatively small torque (corresponding to about 1 – 5% of the torque along the rotation axis) perpendicular to the rotation axis. These components induce a precession of the pulsar rotation axis.

We now calculate all three components of the torque exerted on the pulsar. Following the arguments in Ghosh and Lamb (Ghosh & Lamb 1979b), we neglect the contribution due to viscous stress right from the beginning, since it is always small compared to the contributions due to the magnetic field and the matter flow.

The magnetic torque density d_k can be written as a total divergence

$$d_k = [\vec{r} \wedge \left(\frac{1}{4\pi} (\nabla \wedge \vec{B}) \wedge \vec{B} \right)]_k = -\frac{\partial}{\partial x_p} D_{kp}, \quad (16)$$

with

$$D_{kp} = \varepsilon_{kij} x_i T_{jp} \quad (17)$$

and the Maxwell stress tensor T_{jp} given by

$$T_{jp} = -\frac{B^2}{8\pi}\delta_{jp} + \frac{B_j B_p}{4\pi}. \quad (18)$$

Also the total torque density n_k can be written as a total divergence. We get

$$n_k = -\frac{\partial}{\partial x_p} (\varepsilon_{kij} x_i P_{jp}) \equiv \frac{\partial}{\partial x_p} M_{kp}, \quad (19)$$

with the symmetric tensor

$$P_{jp} = \rho v_j v_p - T_{jp}. \quad (20)$$

The last identity can be easily derived using the continuity equation. The total torque is obtained by integrating Eq. (19) over the whole volume outside the pulsar. By Gauss' law, the volume integral can be transformed into a surface integral. As a convenient starting point for computing the accretion torque, we thus have

$$N_k = -\oint_S M_{kp} \hat{n}_p dS. \quad (21)$$

$\hat{n}_p$ is the unit outward normal of the integration surface S . S splits into three parts, S_1 being the border of the boundary layer, S_2 the upper and lower edge of the accretion disk and the remaining part S_3 closing the surface in the matter free region (see Fig. 3). In cylindrical coordinates Eq. (21) reads

$$\begin{aligned} N_r = & -\oint_S \left(z\rho v_\varphi v_r - z\frac{B_\varphi B_r}{4\pi} \right) \hat{n}_r dS \\ & -\oint_S \left(z\rho v_\varphi^2 + z\frac{B^2}{8\pi} - z\frac{B_\varphi^2}{4\pi} \right) \hat{n}_\varphi dS \\ & -\oint_S \left(z\rho v_\varphi v_z - z\frac{B_\varphi B_z}{4\pi} \right) \hat{n}_z dS, \end{aligned} \quad (22)$$

$$\begin{aligned} N_\varphi = & -\oint_S \left(z\rho v_r^2 + z\frac{B^2}{8\pi} - z\frac{B_r^2}{4\pi} - r\rho v_\varphi v_r + r\frac{B_\varphi B_r}{4\pi} \right) \hat{n}_r dS \\ & -\oint_S \left(z\rho v_r v_\varphi - z\frac{B_\varphi B_r}{4\pi} - r\rho v_\varphi^2 - r\frac{B^2}{8\pi} + r\frac{B_\varphi^2}{4\pi} \right) \hat{n}_\varphi dS \end{aligned}$$

$$- \oint_S \left(z \rho v_r v_z - z \frac{B_r B_z}{4\pi} - r \rho v_\varphi v_z + r \frac{B_\varphi B_z}{4\pi} \right) \hat{n}_z dS, \quad (23)$$

$$\begin{aligned} N_z = & - \oint_S \left(r \rho v_\varphi v_r - r \frac{B_\varphi B_r}{4\pi} \right) \hat{n}_r dS \\ & - \oint_S \left(r \rho v_\varphi^2 + r \frac{B^2}{8\pi} - r \frac{B_\varphi^2}{4\pi} \right) \hat{n}_\varphi dS \\ & - \oint_S \left(r \rho v_\varphi v_z - r \frac{B_\varphi B_z}{4\pi} \right) \hat{n}_z dS. \end{aligned} \quad (24)$$

These formulae are of general validity with no restrictions on the symmetry of the magnetic field. Equations (22)–(24) have to be integrated over the surfaces S_1 – S_3 which are depicted in Fig. 3. The outward normal vector in the disk plane can be obtained from Eq. (6).

4 The magnetic field

4.1 Field configuration inside the disk

We now evaluate the general formulae derived in the last section for some configurations of the magnetic field inside the disk. To that purpose we follow again the papers by Ghosh and Lamb (Ghosh et al. 1977), (Ghosh & Lamb 1978), (Ghosh & Lamb 1979a), (Ghosh & Lamb 1979b).

As only about 20% of the pulsar magnetic flux threads the disk outside its inner edge r_o , we set for the poloidal field components (Eqs. (1)–(3)) inside the disk

$$B_r = 0.2 \mu \frac{3rz \cos(\chi) + (2r^2 - z^2) \sin(\chi) \sin(\varphi)}{\sqrt{r^2 + z^2}^5} \quad (25)$$

and

$$B_z = 0.2 \mu \frac{(2z^2 - r^2) \cos(\chi) + 3rz \sin(\chi) \sin(\varphi)}{\sqrt{r^2 + z^2}^5}. \quad (26)$$

Shear in the toroidal velocity field generates a toroidal magnetic field from the

existing poloidal field. At the upper edge of the disk ($z = h$) this additional field component can be parametrized as

$$b_\varphi = \left(\frac{\gamma_a}{\xi} \right) 2h\sqrt{4\pi\rho}(\Omega_K - \Omega_S) , \quad (27)$$

where the constant γ_a/ξ is fixed by requiring that $\gamma_\varphi = -B_\varphi/B_z|_{z=h}$ joins smoothly its value γ_o at r_o . Due to magnetic-flux reconnection one expects $\gamma_o \approx 1$ in the boundary layer. We use this value for γ_o throughout the paper. The component b_φ changes sign on a length-scale of $2h$. For the toroidal field component we thus have

$$B_\varphi = -0.2\mu \frac{\sin(\chi)\cos(\varphi)}{\sqrt{r^2+z^2}^3} + \frac{z}{h} \left(\frac{\gamma_a}{\xi} \right) 2h\sqrt{4\pi\rho}(\Omega_K - \Omega_S) . \quad (28)$$

To these dominant components we have to add the small deviations $\vec{B}_{1\perp}$ discussed in Sect. 2. These cause a torque in the plane perpendicular to the rotation axis. The velocity field is the standard one with $v_\varphi = \sqrt{GM/r}$ and v_r as given by the disk-model of Shakura and Sunyaev (Shakura & Sunyaev 1973), from where we also take the value for the matter density ρ . We can now integrate Eqs. (22)-(24) to get all three components of the torque exerted on the pulsar. This integration has to be done numerically. We neglect all terms involving products of components of $\vec{B}_{1\perp}$, since they are of higher order.

4.2 Estimate of the magnetic moment

The magnetic moment μ of pulsars is usually unknown, but in some cases, like for Her X-1 and 4U 1626-67, there is sufficient information to arrive at a reliable estimate; see the review of Nagase (Nagase 1989). For other pulsars one can estimate the magnetic moment μ assuming that the pulsar rotates near to its equilibrium spin period (Elsner & Lamb 1977).

From Eq. (27) we see that b_φ changes sign at the corotation radius, i.e. where $\Omega_K = \Omega_S$. A slow rotator gains a net positive torque and tends thus to spin up, whereas a fast rotator acquires a total negative torque and, therefore, spins down. For every neutron star with steady disk accretion we find an “equilibrium spin period”, where N_z vanishes exactly. In practice, since mass accretion varies with time, the spin period will oscillate around the equilibrium value. An estimate for the magnetic moment can then be obtained by adjusting μ such that N_z as calculated by Eq. (24) agrees with the simple formula

$$-\frac{\dot{P}}{P} = \frac{N_z}{I\Omega_S} , \quad (29)$$

where P is the average pulse period $\dot{P}$, its variation with time, I the moment of inertia and Ω_S the angular velocity of the pulsar.

5 Computation of a pulse period history

5.1 Solution of the equation of motion

Knowing the torque acting upon the pulsar, we can solve the equations of motion for the star, which we treat as a rigid rotator, slightly oblate due to its fast rotation. The Euler equations in the body frame in cylindrical coordinates with $I_\varphi = I_r$ read:

$$\begin{aligned} I_r \frac{d\Omega_r}{dt} - (I_r - I_z) \Omega_\varphi \Omega_z &= N_r(\vec{\Omega}; \chi), \\ I_r \frac{d\Omega_\varphi}{dt} + (I_r - I_z) \Omega_r \Omega_z &= N_\varphi(\vec{\Omega}; \chi), \\ I_z \frac{d\Omega_z}{dt} &= N_z(\vec{\Omega}; \chi). \end{aligned} \quad (30)$$

Here $I := I_z \approx I_r$ is the moment of inertia of the pulsar. A measure for the oblateness ϵ of the pulsar is given by

$$\epsilon := \frac{I_r - I_z}{I_r},$$

for which we take the expression derived in the textbook of Shapiro and Teukolsky (Shapiro & Teukolsky 1983), namely

$$\epsilon \approx \frac{25 R_{Star} I \Omega_S^2}{12 G M^2}. \quad (31)$$

Notice that the precession induced by the torque perpendicular to the rotation axis is also present if ϵ vanishes. We have checked numerically that in fact the calculated pulse period history depends, as expected, very little on ϵ . However, if one had very precise data, one could in principle also get information about ϵ .

By solving the system of differential equations (30) we obtain the time evolution of the angular velocity vector $\vec{\Omega}$ as well as of the angle χ between $\vec{\Omega}$ and the unit vector $\vec{P}$ parallel to the magnetic moment. Thus

Fig. 4. The X-ray flux for Cen X-3 as measured by the Ariel-V satellite. We assume that the variation in the measured energy band (3 – 6 keV) is a good approximation for the variation of the total luminosity and the accretion rate. Data is taken from the HEASARC data restoration project.

$$\cos(\chi) = \frac{\langle \vec{\Omega}, \vec{P} \rangle}{|\vec{\Omega}| |\vec{P}|}, \quad (32)$$

where $\vec{P} = (\sin \chi_0, 0, \cos \chi_0)$, χ_0 being the initial value of the inclination angle at $t = 0$. Since the accretion torque is also a function of χ , we have to solve the differential equations (30) iteratively together with Eqs. (22)–(24).

For a constant torque Eq. (30) can be solved in terms of Fresnel integrals (Abramowitz & Stegun 1970). The solution for the z-component of the angular velocity is

$$\Omega_z = \Omega_o \left(1 + \frac{N_z}{\Omega_o I_z} t \right), \quad (33)$$

where Ω_o is the initial angular velocity at $t = 0$. For convenience we set

$$\beta := \frac{N_z}{\Omega_o I_z}.$$

The remaining two equations form a system of coupled differential equations, for which a fundamental system of solutions of the homogeneous system is

Fig. 5. The torque in z-direction for Cen X-3 as calculated by Eqs. (24) and (30) using the Ariel-V data as input.

Fig. 6. Time evolution of the inclination angle χ for Cen X-3 using the Ariel-V data as input. The gaps are due to a lack of data points. The parameters used for the calculation are given in Table 1, η was chosen to be 1.

given by

$$\Phi = \begin{pmatrix} \cos(\epsilon \Omega_o(t + \frac{\beta}{2}t^2)) & \sin(\epsilon \Omega_o(t + \frac{\beta}{2}t^2)) \\ -\sin(\epsilon \Omega_o(t + \frac{\beta}{2}t^2)) & \cos(\epsilon \Omega_o(t + \frac{\beta}{2}t^2)) \end{pmatrix},$$

which provides the particular solution

Fig. 7. Time evolution of the inclination angle χ for Cen X-3 using the BATSE data as input. The parameters used for the calculation are given in Table 1, η was chosen to be 1.

Fig. 8. Calculated pulse period history for Cen X-3 using the Ariel-V data as input. The gaps are due to a lack of data points. The parameters used for the calculation are given in Table 1, η was chosen to be 1.

$$\begin{pmatrix} p_r \\ p_\varphi \end{pmatrix} = \Phi \int_0^t \Phi^{-1} \begin{pmatrix} N_r/I_r \\ N_\varphi/I_r \end{pmatrix} d\tilde{t} + const. \quad (34)$$

Fig. 9. Calculated pulse period history for Cen X-3 using the BATSE data as input. The parameters used for the calculation are given in Table 1, η was chosen to be 1.

Fig. 10. The pulse period history for Cen X-3 as measured by the BATSE detector on CGRO. Data is kindly provided by the BATSE team.

The above integral can be expressed in terms of Fresnel integrals. Since the complete expressions for p_r and p_φ are a bit lengthy –but nevertheless easy to obtain– we do not give them in full form. The general solution of Eq. (30)

thus reads

$$\Omega_r = c_1 \cos(\epsilon \Omega_o(t + \frac{\beta}{2}t^2)) + c_2 \sin(\epsilon \Omega_o(t + \frac{\beta}{2}t^2)) + p_r \quad (35)$$

and

$$\Omega_\varphi = -c_1 \sin(\epsilon \Omega_o(t + \frac{\beta}{2}t^2)) + c_2 \cos(\epsilon \Omega_o(t + \frac{\beta}{2}t^2)) + p_\varphi. \quad (36)$$

For the numerical computation we assume the torque to be constant over limited time intervals, which are determined by the available data set of the X-ray luminosity. After each such time interval we obtain a new value for $\vec{\Omega}$ with the help of Eqs. (33), (35) and (36). Equation (32) yields the new value of χ . These new values are then used to calculate the accretion torque during the next time step, and so on. As a result we arrive at the evolution of the inclination angle χ , as well as the theoretical pulse period history. We show, as an example, in Figs. 4–9 the results for Cen X-3. The parameters used for the calculation are given in Table 1.

Within our model there are three free parameters η , μ and χ_o , which have to be chosen. We found, that for a wide range, there is roughly a linear dependence on η for the variation of the pulse period. For instance, choosing $\eta = 0.5$ results in an enhancement of the variation of the pulse period by $\sim 40\%$ as compared with the result shown in Figure 8. As mentioned at the end of Sect. 4, we assume that the pulsar rotates near to its equilibrium spin period. This requirement leads then to a relation between μ and χ_o , which can be calculated numerically. For the simulation we chose the initial value of χ such that it corresponds roughly to the smallest magnetic moment.

6 Summary

Comparing Fig. 8 with Fig. 1, respectively Fig. 9 with Fig. 10, one sees that we are not able to reproduce the observations quantitatively. This is, however, not surprising in view of the many model assumptions, in particular since three important parameters (η , $\vec{\mu}$ and χ_o) are not known and we had to make some rough guesses for them. In fact, we just use the inclination angle as an additional degree of freedom.

On the other hand, we are able to explain at least qualitatively the observations. In fact, within our model we see that the pulse period shows changes from spin-up to spin-down with the right order of magnitude on the correct timescale. The qualitative agreement with the measured pulse period variations of Cen X-3 as given by Fig. 10 is remarkably good. For comparison,

Pulse Period P [s] 4.8317	[JD-2440000] 4687.0	L_X [erg/s] 10^{38}
Mass M [$M_\odot$] 1.06	Moment of Inertia I [$g\text{ cm}^2$] 7.54×10^{44}	Radius R [cm] 1.037×10^6
Accretion rate $\dot{M}$ [g/s] 7.3×10^{17}	Magnetic moment μ [gauss cm ³] 2×10^{30}	γ_a/ξ 0.5
Inclination angle $\chi_\circ$ 40°	Radius of boundary layer $\langle r_\circ \rangle$ [cm] 3.12×10^8	Oblateness ϵ 9.3×10^{-9}

Table 1

Parameters used for the computation of the pulse period history of Cen X-3. Most data is taken from the review of Nagase (Nagase 1989), with the exception of I and R_{Star} which are taken from Wiringa et al. (Wiringa et al. 1988). μ , γ_a/ξ , $\chi_\circ$ and $r_\circ$ are fixed in the text.

we also studied the evolution of the pulse period assuming a fixed inclination angle. For this case, the amplitude of the pulse period variations turns out to be typically one order of magnitude larger than observed. Such a result is expected, since the system with an additional degree of freedom can react much more smoothly to any variation in the matter inflow.

We conclude, therefore, that a small precession of the pulsar inclination angle χ can explain the observed long term variations in the period of X-ray pulsars. In our treatment, there is no need to introduce new phenomena, contrary to recent suggestions of e.g. van Kerkwijk et al. (van Kerkwijk et al. 1998), rather we just exploit the consequences of the disk-fed oblique rotator model.

Acknowledgement: This work has been partially supported by the Swiss National Science Foundation. We thank the referee for his suggestion to include the new BATSE data into our work. We are especially obliged to G. Fishman, M. Finger and the whole BATSE team for their interest in the work and their kindness to provide us with the BATSE measurements of Cen X-3 before public release of the data.

References

- Abramowitz, M., Stegun, I.A. 1970, Handbook of mathematical functions, Dover Publications Inc.
- Elsner, R.F., Lamb, F.K. 1977, ApJ, **215**, 897.
- Ghosh, P., Lamb, F.K., Pethick, C.J. 1977, ApJ, **217**, 578.

- Ghosh, P., Lamb, F.K. 1978, ApJ, **223**, L83.
- Ghosh, P., Lamb, F.K. 1979a, ApJ, **232**, 259.
- Ghosh, P., Lamb, F.K. 1979b, ApJ, **234**, 296.
- Lamb, F.K., Pethick, C.J. 1974, in: Astrophysics and gravitation, Proceedings of the 16th Solvay Congress, 135.
- Lamb, F.K., Pethick, C.J., Pines, D. 1973, ApJ, **184**, 271.
- Mestel, L. 1968, MNRAS, **140**, 177.
- Mestel, L. 1975, in: Magnetohydrodynamics, Saas-Fee Lectures 1974, eds. L. Mestel and N.O. Weiss (Swiss Society of Astronomy and Astrophysics).
- Nagase, F. 1989, in: Accretion-powered X-ray pulsars, PASJ, **41**, 1.
- Nelson, R.W. et al. 1997, ApJ, **488**, L117.
- Shakura, N.I., Sunyaev, R.A. 1973, A&A, **24**, 337.
- Shapiro, S.L., Teukolsky, S.A. 1983, Black Holes, White Dwarfs and Neutron Stars, John Wiley & Sons.
- van Kerkwijk, M.H., et al., 1998, astro-ph 9802162
- Wiringa, R.B., et al., 1988, Phys. Rev., **C38**, 1010.