A Resolution to the Supersymmetric CP Problem with Large Soft Phases via D- branes

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Abstract

We examine the soft supersymmetry breaking parameters that result from various ways of embedding the Standard Model (SM) on D- branes within the Type I string picture, allowing the gaugino masses and $\mu$ to have large CP- violating phases. One embedding naturally provides the relations among soft parameters to satisfy the electron and neutron electric dipole moment constraints even with large phases, while with other embeddings large phases are not allowed. The string models provide some motivation for large phases in the soft breaking parameters. The results generally suggest how low energy data might teach us about Planck scale physics.

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The parameters of the Lagrangian of the Minimal Supersymmetric Standard Model (MSSM) include a number of CP-violating phases, which arise both in the soft breaking sector and in the phase of the supersymmetric higgsino mass parameter $\mu$ (for a careful count see [1]). The presence of these phases have typically been neglected in phenomenological analyses due to what traditionally has been called the supersymmetric CP problem: the electric dipole moments (EDM's) of the fermions receive one-loop contributions due to superpartner exchange which for large phases can exceed the experimental bounds. The current bounds for the electron [2] and neutron [3]

$$|d_e| < 4.3 \times 10^{-27} \text{ ecm (95\% c.l.)} \quad (1)$$
$$|d_n| < 6.3 \times 10^{-26} \text{ ecm (90\% c.l.)} \quad (2)$$

were thought to constrain the phases to be $O(10^{-2})$ for sparticle masses at the TeV scale [4–6]. However, the results of a recent reinvestigation of this issue [7,8] have demonstrated that cancellations between different contributions to the electric dipole moments can allow for phenomenologically viable regions of parameter space with phases of $O(1)$ and light sparticle masses, contrary to conventional wisdom. The phases, if nonnegligible, not only have significant phenomenological implications for CP-violating observables (such as in the K and B systems), but also have important consequences for the extraction of the MSSM parameters from experimental measurements of CP-conserving quantities, since almost none of the Lagrangian parameters are directly measured [9].

The results of [7,8] indicate that the cancellations can only occur if the soft breaking parameters satisfy certain approximate relations which are testable in future experiments; such relations may provide clues to the mechanism of supersymmetry breaking and the form of the underlying theory. As superstring theory is the only candidate for a fundamental theory which unifies gravitational and gauge interactions, a study of the patterns of CP-violating phases in soft supersymmetry breaking parameters derived in classes of four-dimensional superstring models is well motivated. In particular, we wish to determine if these models either allow for or predict phenomenologically viable large phase solutions. We find that in some models large phases naturally arise in a manner that leads to results consistent with the EDM constraints, while in others the constraints cannot be satisfied.

CP is a discrete gauge symmetry in string theory, and thus can only be broken spontaneously [10]. If this breaking occurs via the dynamics of compactification and/or supersymmetry breaking, then the four-dimensional effective field theory will exhibit explicit CP-violating phases. The origin of the nonperturbative dynamics of supersymmetry breaking in superstring theory remains unresolved, but progress can be made by utilizing a phenomenological approach first advocated by Brignole, Ibáñez, and Muñoz [11]. In their approach, it is assumed that supersymmetry breaking effects are communicated dominantly via the $F$-component vacuum expectation values (VEV’s) of the dilaton $S$ and moduli $T_i$, which are superfields uncharged under the SM gauge group and are generically present in four-dimensional string models. The effects of the unknown supersymmetry breaking dynamics are then encoded in a convenient parameterization of these $F$-component VEV’s [11,15]:

$$F^S = \sqrt{3}(S + S^*)m_{3/2}\sin \theta e^{i\alpha_s}$$
$$F^i = \sqrt{3}(T_i + T_i^*)m_{3/2}\cos \theta \Theta_i e^{i\alpha_i},$$  \quad (3)
in which $m_{3/2}$ is the gravitino mass and $\theta_i$, $\Theta_i$ are Goldstino angles (with $\sum_i \Theta_i^2 = 1$), which measure the relative contributions of $S$ and $T_i$ to the supersymmetry breaking. The $F$-component VEV’s are assumed to have arbitrary complex phases $\alpha_S$, $\alpha_i$, which provide sources for the CP-violating phases in the soft terms.

Within particular classes of four-dimensional string models the couplings of the dilaton, moduli, and MSSM matter fields are calculable (at least at the tree-level), leading in turn to specific patterns of the soft breaking parameters. We have analyzed the phase structure of the soft terms arising in three classes of four-dimensional string models: (i) orbifold compactifications of perturbative heterotic string theory, (ii) Hořava-Witten type M theory compactifications, and (iii) the Type IIB orientifold models, which are examples within the general Type I string picture. In our analysis [28], we take the predictions for the soft breaking parameters at the string scale, and evolve the parameters to the electroweak scale using renormalization group equations (RGE’s). Our results indicate that the patterns of CP-violating phases consistent with the EDM constraints strongly depend on the type of string model under consideration.

First, we note that the general results of [8] demonstrate that sufficient cancellations among the various contributions to the EDM’s are difficult to achieve unless there are large relative phases in the soft masses of the gaugino sector. This feature is due to the approximate $U(1)_R$ symmetry of the Lagrangian of the MSSM [16], which allows one of the phases of the gaugino masses to be set to zero at the electroweak scale without loss of generality [16,8,17,18]. Furthermore, the phases of the gaugino mass parameters do not run at one-loop order, and thus at the electroweak scale only deviate from the string-scale values by small two-loop corrections. Therefore, if the phases of the gaugino masses are universal at the string scale, they will be approximately zero at the electroweak scale (after the $U(1)_R$ rotation). Cancellations among the chargino and neutralino contributions to the electron EDM are then necessarily due to the interplay between the phases of $A_e$ and $\mu$ ($\varphi_{A_e}$ and $\varphi_\mu$, respectively). The analysis of [8] demonstrates that cancellations are then difficult to achieve as the pure gaugino part of the neutralino diagram adds destructively with the contribution from the gaugino-higgsino mixing, which in turn has to cancel against the chargino diagram. As a result, the cancellation mechanism is generally insufficient, and hence in this case the phases of the other soft breaking parameters as well as the $\mu$ parameter must naturally be $\lesssim 10^{-2}$ (the traditional bound) [5].

This feature is predicted [19] in perturbative heterotic models at tree level, due to the universal coupling of the dilaton to all gauge groups in the tree-level gauge kinetic function $f_a = k_a S$ (in which $k_a$ is the Kač-Moody level of the gauge group). In these models, nonuniversal gaugino masses do occur at the loop level due to moduli-dependent threshold corrections. Hence, nontrivial CP effects require both moduli dominance and large threshold effects in order to overcome the tendency of the dilaton $F$-term to enforce universal gaugino masses [28,20]. Similar statements apply to the soft breaking parameters derived in the Hořava-Witten M theory [23] type scenarios [24,25]. In these scenarios, the gaugino mass parameters are universal because the observable sector gauge groups all arise from one of the ten-dimensional boundaries. Therefore, only a very small fraction of the $\varphi_\mu - \varphi_{A_e}$ parameter space leads to models allowed by the electron EDM [28].

However, within the more general Type I string picture, there is the possibility of nonuniversal gaugino masses at tree level, which has important implications for the possible CP-
violating effects. We focus on examples within the four-dimensional Type IIB orientifold models, in which the Type IIB theory is compactified on orientifolds (which are orbifold compactifications accompanied by the worldsheet parity projection) [12–15]. In these models, consistency conditions (tadpole cancellation) require the addition of open string (Type I) sectors and Dirichlet branes, upon which the open strings must end. It is important to note that orientifolds are illustrative of a much larger class of models in the Type I picture, containing more general configurations of nonperturbative objects (e.g. D-brane bound states) in more general singular backgrounds (e.g. conifolds [29]).

While the number and type of D-branes required in a given model depends on the details of the orientifold group, we consider the general situation with one set of nine-branes and three sets of five-branes (5\textsubscript{i}), in which the index \(i\) labels the complex coordinate of the internal space included in the world-volume of the five-brane. Each set of coincident D-branes gives rise to a (generically non-Abelian) gauge group. Chiral matter fields also arise from the open string sectors, and can be classified into two categories. The first category consists of open strings which start and end on D-branes of the same sector, for which the corresponding matter fields are charged under the gauge group (typically in the fundamental or antisymmetric tensor representations) of that set of branes. The second category consists of open strings which start and end on different sets of branes; in this case, the states are bifundamental representations under the two gauge groups from the two D-brane sectors.

Model-building techniques within this framework are at an early stage and there is as yet no “standard” model; furthermore this framework does not provide any generic solution to the related problems of the runaway dilaton, supersymmetry breaking, and the cosmological constant. On the other hand, recent investigations [15] have uncovered the generic structure of the tree-level couplings of this class of models. The results indicate that the phenomenological implications of these models crucially depend on the embedding of the SM gauge group into the different D-brane sectors, and may have distinctive properties from those of the perturbative heterotic models traditionally considered in studies of superstring phenomenology.

Of particular importance for the purposes of this study is that the dilaton no longer plays a universal role as it did in the perturbative heterotic case, as can be seen from the form of the (tree-level) gauge kinetic functions determined in [15] using T-duality and the form of the Type I low energy effective action:

\[
\begin{align*}
    f_9 &= S \\
    f_{5i} &= T_i.
\end{align*}
\]  

(4)

This result illustrates a distinctive feature of this class of models, which is that in a sense there is a different “dilaton” for each type of brane. This fact has important implications both for gauge coupling unification [15] and the patterns of gaugino masses obtained in this class of models, which strongly depend on the details of the SM embedding into the five-brane and nine-brane sectors. For example, in the case in which the SM gauge group is associated with a single D-brane sector, the pattern of the gaugino masses resembles that of the tree-level gaugino masses in the weakly coupled heterotic models [28], as can be seen from the similarity between (4) and the corresponding tree-level expression for \(f\) in the perturbative heterotic case.
However, an alternate possibility is that the SM gauge group is not associated with a single set of branes, but rather is embedded within multiple D-brane sectors. We consider the case in which $SU(3)$ and $SU(2)$ originate from the $5_1$ and $5_2$ sectors, respectively [30]. In this case, the quark doublet states necessarily arise from open strings connecting the two D-brane sectors; as these states have a nontrivial hypercharge assignment, their presence restricts $U(1)_Y$ to originate from the $5_1$ and/or $5_2$ sectors as well. We consider two models of the resulting soft terms corresponding to the two simplest possibilities for the hypercharge embedding, which are to have $U(1)_Y$ in either the $5_1$ or $5_2$ sector. Depending on the details of the hypercharge embedding, the remaining MSSM states may either be states which (in analogy with the quark doublets) are trapped on the intersection of these two sets of branes, or states associated with the single $5_i$ sector which contains $U(1)_Y$. In any event the natural starting point for constructing models with these features are orientifolds which realize identical GUT gauge groups and massless matter on two sets of intersecting 5-branes. The existence of such symmetrical arrangements is often guaranteed by T-duality. For example, Shiu and Tye [14] have exhibited an explicit model which realizes the Pati-Salam gauge fields of $SU(4) \times SU(2)_L \times SU(2)_R$ and identical chiral matter content on two sets of 5-branes. Additional Higgsing and modding by discrete symmetries could then in principle produce the asymmetrical structures outlined above.

In the case with $U(1)_Y$ and $SU(3)$ from the $5_1$ sector, the gaugino masses and $A$ terms take the form (see the general formulae in [15]):

$$M_1 = \sqrt{3} m_{3/2} \cos \theta \Theta_1 e^{-i\alpha_1} = M_3 = -A_{t,e,u,d}$$
$$M_2 = \sqrt{3} m_{3/2} \cos \theta \Theta_2 e^{-i\alpha_2}.$$  \hspace{1cm} (5)

Note that in this case the phases $\varphi_1$ and $\varphi_3$ of the mass parameters $M_1$ and $M_3$ are equal and distinct from that of the $SU(2)$ gaugino mass parameter $M_2$. The soft mass-squares are given by

$$m_{5_15_1}^2 = m_{3/2}^2 (1 - \frac{3}{2} (\sin^2 \theta + \cos^2 \theta \Theta_3^2))$$
$$m_{5_1}^2 = m_{3/2}^2 (1 - 3 \sin^2 \theta).$$  \hspace{1cm} (6)

The $SU(2)$ doublets of the MSSM are necessarily states which arise from open strings which connect the $5_1$ and $5_2$ brane sectors, and hence have mass-squares given by the above expression for $m_{5_15_2}^2$. The $SU(2)$ singlets may either be fields of the same type or states which originate from the $5_1$ sector, although we note that requiring the presence of the MSSM Yukawa couplings indicates that these states should be of the latter type [15]. Similar expressions apply for the case in which $U(1)_Y$ and $SU(2)$ are associated with the same five-brane sector, although in this case the relations among the phases are $\varphi_1 = \varphi_2 \neq \varphi_3$. In each of these models, $\mu$ and $B$ are in principle free complex parameters in the analysis (although their phases are related by the approximate PQ symmetry of the MSSM Lagrangian; see [16,8,28] for further details).

In our numerical analysis of these models, we impose the boundary conditions (5) and (6) at the GUT scale $M_G = 3 \times 10^{16}$ GeV (where we assume the couplings unify), and evolve the parameters to the electroweak scale via the renormalization group equations [31]. The sparticle masses and the CP-violating phases depend on the free parameters $m_{3/2}$,
$\theta, \Theta_i, i = 1, 2, 3,$ which are related by $\Theta_1^2 + \Theta_2^2 + \Theta_3^2 = 1,$ as well as the two phases $\alpha_1$ and $\alpha_2$ (physical results only depend on $\alpha_1 - \alpha_2$). To avoid negative scalar mass-squares we restrict our consideration to values of $\theta$ which satisfy $\sin^2 \theta < \frac{1}{3}$, and also assume that $\Theta_3 = 0$ (indicating that the modulus $T_3$ associated with the $5_3$ brane sector plays no role in supersymmetry breaking, and thus is essentially decoupled from the observable sector). $B$ and $\mu$ are in principle free parameters, as they are not determined by this embedding. However, we explore the phenomenologically motivated scenario in which the electroweak symmetry is broken radiatively as a result of RGE evolution of the Higgs masses $m_{H_1}^2$ and $m_{H_2}^2$. As the minimization conditions are imposed at the electroweak scale, the values of $B_H$ and $|\mu|^2$ can be expressed in terms of $\tan \beta$ and $M_Z$ [27]. We note that even under these assumptions $\varphi_\mu$ remains an independent parameter, and thus the model depends on two phases at the GUT scale: $\alpha_1 - \alpha_2$ and $\varphi_\mu$ (though due to RG running the phases of the $A$ terms at the electroweak scale will deviate from their string-scale values). Also, the squark and slepton masses from (6) are of the same order as the gaugino masses, so they would allow very large EDM’s if there were no relations among the soft masses and phases.

We find, remarkably, that in order to satisfy the experimental constraints on the electron and neutron EDM’s in this model, the large individual contributions from chargino, neutralino and gluino loops do not have to be suppressed by small CP phases. A cancellation between the chargino and neutralino loop contributions naturally causes the electron EDM to be acceptably small. As emphasized in [8], the contributions to chargino and neutralino diagrams from gaugino-higgsino mixing naturally have opposite signs and the additional $\varphi_1$ dependence of the neutralino exchange contribution can provide for a match in size between the chargino and neutralino contributions. In the neutron case, the contribution of the chargino loop is offset by the gluino loop contributions to the electric dipole operator $O_1$ and the chromoelectric dipole operator $O_2$. Since $\varphi_1 = \varphi_3$ in this scenario, the sign of the gluino contribution is fixed and it automatically has the correct sign to balance the chargino contribution in the same region of gaugino phases which ensures cancellation in the electron case. This simple and effective mechanism therefore provides extensive regions of parameter space where the electron and neutron EDM constraints are satisfied simultaneously while allowing for $O(1)$ CP-violating phases.

In order to demonstrate the coincidence of the regions allowed by the experimental constraints on the EDM’s, we consider a specific set of relevant parameters. We choose $m_{3/2} = 150 \text{ GeV}$, $\theta = 0.2$ and $\tan \beta = 2$ which leads to a reasonably light spectrum of the superpartners and require that the EW symmetry is to be broken radiatively. In Fig. 1, we plot the allowed regions for both electron and neutron EDM depending on the values of $\Theta_1$ and $\Theta_2 = \sqrt{1 - \Theta_1^2}$ while $\Theta_3$ is set to zero. Frame a), where $\Theta_1 = 0.85$, shows a very precise overlap between the electron and neutron EDM allowed regions resulting from the fact that $\varphi_1 = \varphi_3$ and the cancellation mechanism works similarly in both cases. In frame b), we consider the situation in which the magnitudes of all three gaugino masses are unified in magnitude while still allowing for different phases due to different origin of $M_1$ and $M_3$ compared to $M_2$. Finally, in frame c) we set $\Theta_1 = 0.55$ and study the situation when the magnitude of $M_2$ is significantly larger than that of $M_1 = M_3$. We find that in this scenario the alignment between the EDM allowed regions is spoiled and only small CP-violating phases are allowed by the experimental constraints. The behavior illustrated in
FIG. 1. Illustration of the overlap between the regions allowed by the electron and neutron EDM constraints. We choose $m_{3/2} = 150$ GeV, $\theta = 0.2$ and $\tan \beta = 2$, and assume that the EW symmetry is broken radiatively. Allowed points are shown for (a) $\Theta_1 = 0.85$, (b) $\Theta_1 = \sqrt{1/2}$, and (c) $\Theta_1 = 0.55$. The black circles are the points allowed for the eEDM and grey blocks for the nEDM. In (a), the overlap is essentially total so only the grey blocks show up; in (b) and (c) the overlap becomes less good. The sensitivity is encouraging, suggesting that the experimental data can determine the Goldstino angles.

Fig. 1 for a particular choice of parameters is quite general provided the gaugino masses at the unification scale are close in magnitude. We considered only small and moderate values of $\tan \beta$ since for models with large $\tan \beta$ new types of contributions can become important.

The cancellation mechanism in this scenario provides a remarkably large range of allowed CP-violating soft phases and requires a specific correlation between $\varphi_\mu$ and $\varphi_1 = \varphi_3$ as shown in Fig. 1. It is also interesting to observe that the actual values of the electron and neutron EDM’s for the allowed points in the phase parameter space are typically slightly below the experimental limit and should be within the reach of the next generation of EDM measuring experiments. In Fig. 2 we plot the EDM values for the allowed points in the case
FIG. 2. Range of the electron and neutron EDM values vs. $\varphi_1 = \varphi_3$ predicted by Eqs. (5) and (6) for the parameters of Figure 1 a). All of the points are allowed by the experimental bounds on the EDM’s (note the different scales for the eEDM and nEDM).

of $\Theta_1 = 0.85$ with all the other parameters set to the same values as in previous discussion of Fig. 1. This indicates that if the CP- violating phases indeed originate from this type of D- brane configuration, non-zero measured values for both EDM’s much bigger than the SM prediction can be expected.

Equally remarkably, if we modify the way the SM is embedded in the D- brane sectors we are unable to satisfy the EDM constraints with nonnegligible phases. For example, the other possibility of arranging the SM gauge groups, such that $U(1)_Y$ is instead on the 5$_2$ brane with $SU(2)$, does not allow for large phase solutions. The reasons for this behavior are similar to that of the Hořava-Witten scenario: we can use the $U(1)_R$ symmetry of the soft terms to put $\varphi_2 = \varphi_1 = 0$, which severely limits the possibility of cancellation between the chargino and neutralino contributions to the electron EDM. The effect of $\varphi_A e$ alone is not enough to offset the potentially large chargino contribution and only a very narrow range of values of $\varphi_\mu$ (close to 0, $\pi$, . . . ) passes the electron EDM constraint.

There are a number of interesting implications of these results:

- They show explicitly how relations among soft parameters such as Eqs. (5) and (6) can naturally give small EDM’s even with large phases.
- They illustrate how we are able to learn about (even nonperturbative) Planck scale
physics using low energy data. If the soft phases are measured in (say) collider super-partner data, or at B factories, and found to be large, we have seen that they may provide guidance as to how the SM is to be embedded on branes.

- They illustrate very simply that large soft phases are at least consistent with, and perhaps motivated by, some string models. In particular, the requirement that the phases (but not necessarily the magnitudes) of the gaugino mass parameters are nonuniversal for viable large phase solutions can naturally be realized in Type I models in which the SM gauge group is split among different brane sectors.

- They suggest that $d_\alpha$ and $d_e$ are not much smaller than the current limits.

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REFERENCES

The corresponding analysis of the dilaton-dominated scenario was recently reported in [21]; their results demonstrate this behavior explicitly. We refer the reader also to [22] and references therein for the analysis of the $CP$-violating phases in the gaugino condensation approach.


[28] M. Brhlik, L. Everett, G. L. Kane, and J. Lykken, in progress.


[30] We choose the $5_i$ sectors for the sake of definiteness. We could also have assumed that one of the gauge groups arises from the nine-brane sector and obtain similar results for the $CP$-violating effects. However, in this case [15] it may be more difficult to obtain consistent unification of the gauge couplings at the GUT scale (although this point is not crucial for the purposes of this study).

[31] For the sake of simplicity, we utilize the freedom in Type I string models to choose the string scale to coincide with the GUT scale; this possibility is not valid in the perturbative heterotic case, which fixes $M_{\text{String}} \sim 5 \times 10^{17}$ GeV [26]. It is beyond the scope of this paper to consider all possibilities, and we refer the reader to a comprehensive discussion of this issue and its implications for gauge coupling unification in [15].