Gaugino Condensation in $\mathcal{N}=1$ Supergravity Models with Multiple Dilaton-Like Fields

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Abstract

We study supersymmetry breaking by hidden-sector gaugino condensation in $\mathcal{N}=1$ D=4 supergravity models with multiple dilaton-like moduli fields. Our work is motivated by Type I string theory, in which the low-energy effective Lagrangian can have different dilaton-like fields coupling to different sectors of the theory. We construct the effective Lagrangian for gaugino condensation and use it to compute the visible-sector gaugino masses. We find that the gaugino masses can be of order the gravitino mass, in stark contrast to heterotic string models with a single dilaton field.

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1 Introduction

Before the recent string duality revolution, most string phenomenology centered on perturbative $\mathcal{N}=1$ D=4 heterotic string theories. The discovery of string duality and D-branes, however, opened a variety of new approaches to string phenomenology based on Type I and Type II string theories. For example, much recent work has focussed on the intriguing possibility that our four-dimensional world lies at the intersection of a set of $Dp$-branes ($3 \leq p \leq 9$) embedded in 9 + 1-dimensional spacetime.\(^1\)

A well-known problem with the usual perturbative heterotic string phenomenology is that hidden-sector gaugino condensation \([4]\) gives rise to visible-sector gaugino masses that are much smaller than the scale of supersymmetry breaking \([5][6]\). This is a consequence of the fact that a single dilaton couples to all gauge and matter fields. Gaugino condensation induces a small $F$ term for the dilaton field, and the dilaton couplings then give small masses to the gauginos.

In Type I models, however, the situation can be very different. In these models, the hidden and visible sectors can live on different D-branes. Each sector has its own dilaton-like fields \([2]\). The hidden-sector dilatons receive small $F$ terms from gaugino condensation. However, these $F$ terms are not responsible for the visible-sector gaugino masses, and therefore the gaugino masses are not forced to be small.

Inspired by this possibility, in this paper we study the question of supersymmetry breaking by hidden-sector gaugino condensation in $\mathcal{N}=1$ D=4 supergravity models with multiple dilaton-like fields. We compute the visible-sector gaugino masses and find that they can indeed be of order the supersymmetry breaking scale. We see that Type I models offer an appealing solution to the gaugino mass problem associated with heterotic string theories.

We approach this problem in the spirit of effective field theory. We take our visible sector to be composed of multiple pure $\mathcal{N}=1$ super Yang-Mills theories, each coupled to gravity, and each to its own dilaton-like field. We take our hidden sector to be composed of gaugino condensate fields, one on each set of branes. The condensate fields are also coupled to gravity, and to their associated dilatons.\(^2\) By construction, this theory describes the low-energy limit of a Type I string theory, where each super Yang-Mills theory lives on its own set of D-branes. Of course, our analysis also applies to other string/M theory vacua with multiple dilaton-like fields.

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\(^1\)See \([1][2][3]\) for recent reviews.

\(^2\)To simplify our presentation, we ignore all charged chiral superfields; this restriction does not affect the results of our analysis.
Various $\mathcal{N}=1$ D=4 Type I models have been proposed in the literature [7]–[19]. Particularly simple examples can be constructed from Type I models with D9-branes and D5-branes compactified on $T^2 \times T^2 \times T^2$, where $R_i$ is the radius of the $i$th two-torus, $i = 1, 2, 3$. (D5-branes that wrap on the $i$th two-torus are denoted as D5$_i$-branes.) These models have one dilaton field $S$, three untwisted moduli fields $T_i$ ($i = 1, 2, 3$) and additional twisted moduli fields from the closed string sector. Other D-brane configurations can be obtained from these by T-duality.

One such example is a Type I model with two sectors, built from D9-branes and D5$_1$-branes. The gauge bosons arise from open strings ending on the D9- and D5$_1$-branes.

The low-energy effective Lagrangian is as follows\(^3\) [1][15],

$$
\mathcal{L} = \frac{1}{8} \int \! d^4 \theta \frac{E}{R} S(W^a W_a)_{D9} + \frac{1}{8} \int \! d^4 \theta \frac{E}{R^i} \bar{S}(W_a W^a)_{D9} \\
+ \frac{1}{8} \int \! d^4 \theta \frac{E}{R} T_i(W^a W_a)_{D5_i} + \frac{1}{8} \int \! d^4 \theta \frac{E}{R^i} \bar{T}_i(W_a W^a)_{D5_i} + \cdots
$$

where

$$
K = - \ln \left( S + \bar{S} \right) - \sum_{i=1}^3 \ln \left( T_i + \bar{T}_i \right) + \cdots
$$

is the Kähler function. The dots denote possible contributions from other neutral chiral superfields, and

$$
\langle S + \bar{S} \rangle = \frac{R_1^2 R_2^2 R_3^2}{\pi \lambda_I \alpha'^3}, \quad \langle T_i + \bar{T}_i \rangle = \frac{R_i^2}{\pi \lambda_i \alpha'}, \quad i = 1, 2, 3.
$$

In these expressions, $\lambda_I$ is the string coupling, $\alpha' = M_I^{-2}$, and $M_I$ is the string scale. The field strengths $(W^a W_a)_{D9}$ and $(W^a W_a)_{D5_i}$ contain gauge fields living on D9- and D5$_i$-branes, respectively. Note that the moduli $S, T_i$ are dilaton-like fields, while $T_2, T_3$ are not.\(^4\)

A second example contains D9-branes as well as D5$_i$-branes compactified on all three tori ($i = 1, 2, 3$). It has four sectors, and four dilaton-like fields, $S, T_1, T_2, T_3$, as defined in (1.3). These fields couple to the gauge fields on the D9- and D5$_i$-branes and give rise to the effective Lagrangian

$$
\mathcal{L} = \frac{1}{8} \int \! d^4 \theta \frac{E}{R} S(W^a W_a)_{D9} + \frac{1}{8} \int \! d^4 \theta \frac{E}{R^i} \bar{S}(W_a W^a)_{D9} \\
+ \sum_{i=1}^3 \frac{1}{8} \int \! d^4 \theta \frac{E}{R} T_i(W^a W_a)_{D5_i} + \sum_{i=1}^3 \frac{1}{8} \int \! d^4 \theta \frac{E}{R^i} \bar{T}_i(W_a W^a)_{D5_i} + \cdots
$$

\(^3\)For convenience, we use the Kähler superspace formulation throughout this paper [20].

\(^4\)Our definition of the $S$ and $T_i$ moduli agrees with [20], but differs from [1].

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The plan of this paper is as follows. In Section 2, we define our $\mathcal{N}=1$ $D=4$ supergravity model with multiple dilaton-like fields. In Section 3, we argue that the extra dilaton-like fields allow gaugino masses to be as large as the gravitino mass. We also present an explicit example of this scenario.

In Appendix A, we exhibit the linear–chiral duality for $\mathcal{N}=1$ $D=4$ supergravity models with multiple linear supermultiplets. In Appendix B we extract the relevant pieces of the component supergravity Lagrangian. Finally, in Appendix C we check our results by comparing with the chiral supermultiplet formulation.

2 Supergravity with Multiple Dilaton-Like Fields

In this section we define the $\mathcal{N}=1$ $D=4$ supergravity model that we will study. We start by considering a system with $N$ different types of D-branes. Of these, we take $\tilde{N}$ to be $Dp\tilde{A}$-branes ($\tilde{A}=1,\cdots,\tilde{N}$) with weakly coupled visible-sector fields on their world volumes. We take the remaining $N-\tilde{N}$ to be $DpA$-branes ($A=1,\cdots,N-\tilde{N}$) with strongly coupled hidden-sector fields in the condensation phase.

Let us first construct the effective theory of the visible sector. For each value of $\tilde{A}$, let $W_{\tilde{A}}$ denote the super Yang-Mills field strength on the $\tilde{A}$-th set of branes, and let $\tilde{V}_{\tilde{A}}$ be the real superfield which contains the $\tilde{A}$-th dilaton-like field [21]. The field $\tilde{V}_{\tilde{A}}$ obeys the constraint

$$-(\bar{D}^2 - 8R)\tilde{V}_{\tilde{A}} = W_{\tilde{A}} W_{\tilde{A}}.$$  (2.1)

This constraint couples the dilatons to the super Yang-Mills fields.

In Kähler superspace [20], the supergravity kinetic terms are given by a Lagrangian $L$ and a Kähler function $K$. For the visible-sector fields, we take them to be

$$L = \int d^4\theta E \left\{ -3 + \tilde{N} + \sum_{\tilde{A}=1}^{\tilde{N}} \tilde{f}_{\tilde{A}}(\tilde{V}_{\tilde{A}}) \right\},$$

$$K = \sum_{\tilde{A}=1}^{\tilde{N}} \left\{ \ln \tilde{V}_{\tilde{A}} + \tilde{g}_{\tilde{A}}(\tilde{V}_{\tilde{A}}) \right\},$$  (2.2)

where

$$\tilde{V}_{\tilde{A}} \frac{d\tilde{g}_{\tilde{A}}}{d\tilde{V}_{\tilde{A}}} = \tilde{f}_{\tilde{A}} - \tilde{V}_{\tilde{A}} \frac{d\tilde{f}_{\tilde{A}}}{d\tilde{V}_{\tilde{A}}}.$$  (2.3)

In these expressions, the leading terms describe the tree-level couplings of the gauge and dilaton-like fields; the functions $\tilde{g}_{\tilde{A}}(\tilde{V}_{\tilde{A}})$ and $\tilde{f}_{\tilde{A}}(\tilde{V}_{\tilde{A}})$ contain corrections beyond tree level. The condition (2.3) guarantees that the Einstein gravity term is canonically
normalized. (For simplicity, we do not include mixings between the different $\tilde{V}_A$'s. Such mixings arise at the loop level, but they do not change our conclusions.\textsuperscript{5})

In the hidden sector, the effective Lagrangian is different because the super Yang-Mills fields are in a strongly-coupled condensation phase. This leads us to replace the Yang-Mills fields by chiral condensate superfields $U_A$\textsuperscript{22}. These fields are contained within the real superfields $V_A$\textsuperscript{23}\textsuperscript{24}, where

$$-(\bar{D}^2 - 8R)V_A = U_A.$$ \hfill (2.4)

The hidden-sector dilaton-like fields are also contained in the fields $V_A$.

The effective Lagrangian for this sector contains kinetic and superpotential terms. The kinetic terms are as above,

$$\mathcal{L} = \int d^4\theta E \left\{ -3 + N - \tilde{N} + \sum_{A=1}^{N-\tilde{N}} f_A(V_A) \right\},$$

$$K = \sum_{A=1}^{N-\tilde{N}} \{ \ln V_A + g_A(V_A) \},$$ \hfill (2.5)

where

$$V_A \frac{dg_A}{dV_A} = f_A - V_A \frac{df_A}{dV_A}. \hfill (2.6)$$

The leading terms describe the tree-level couplings, while $g_A(V_A)$ and $f_A(V_A)$ contain corrections beyond tree level.

The superpotential terms are generated by nonperturbative effects associated with gaugino condensation. In what follows, we take the superpotential to be given by

$$\mathcal{L}_A = \int d^4\theta E \frac{1}{R} \frac{b_A}{8} U_A \ln(e^{-K/2} U_A) + \text{h.c.}$$

$$= \int d^4\theta E b_A V_A \ln \left( e^{-K} U_A U_A \right),$$ \hfill (2.7)

where $b_A = 2b_A'/3$, and $b_A'$ is the one-loop $\beta$-function coefficient of $Dp_A$-sector. The form of this term is dictated by the anomalies of the underlying super Yang-Mills theory\textsuperscript{[4][22][25][26][27]}.

In rigid supersymmetry, the superpotential is fixed by the chiral and conformal anomalies\textsuperscript{[22]}. In local supersymmetry, the Kähler anomaly also comes into play\textsuperscript{[28][29]}. In particular, it fixes the explicit $K$ dependence of the superpotential. Under\textsuperscript{5}Type I models typically contain chiral superfields charged under the $Dp_{\tilde{A}}$ and $Dp_{\tilde{A}'}$ gauge groups ($\tilde{A} \neq \tilde{A}')$. These superfields mix the different dilaton-like fields. Our study ignores charged chiral superfields, so it is consistent to ignore mixings between different $\tilde{V}_{\tilde{A}}$'s.
an arbitrary Kähler transformation, \( K \to K + F + \bar{F} \), the fields \( V_A \to \tilde{V}_A \) and \( U_A \to U_A e^{(\bar{F} - F)/2} \). The superpotential then transforms as follows,

\[
\mathcal{L}_A \equiv \int d^4\theta E b_A V_A \ln(e^{-K} \tilde{U}_A U_A)
\]

\[
= \mathcal{L}_A - \int d^4\theta \frac{E}{8} b_A U_A F - \int d^4\theta \frac{1}{8} b_A \tilde{U}_A F.
\]

This is precisely the right transformation to match the Kähler anomaly of the underlying super Yang-Mills theory [4][25][26][27].

The full supergravity Lagrangian contains contributions from both of these sectors. It can be written as follows,\(^6\)

\[
\mathcal{L} = \int d^4\theta E \left\{-3 + N + \sum_{A=1}^{\tilde{N}} \tilde{f}_A(\tilde{V}_A) + \sum_{A=1}^{N-\tilde{N}} f_A(V_A) \right\}
\]

\[
+ \sum_{A=1}^{N-\tilde{N}} \int d^4\theta E b_A V_A \ln \left(e^{-K} \tilde{U}_A U_A\right),
\]

where the Kähler function is given by

\[
K = \sum_{A=1}^{\tilde{N}} \left\{ \ln \tilde{V}_A + \tilde{g}_A(\tilde{V}_A) \right\} + \sum_{A=1}^{N-\tilde{N}} \left\{ \ln V_A + g_A(V_A) \right\}
\]

\[
+ G(\Phi_1, \bar{\Phi}_1, \ldots, \Phi_n, \bar{\Phi}_n),
\]

and

\[
\tilde{V}_A \frac{d\tilde{g}_A}{d\tilde{V}_A} = \tilde{f}_A - \tilde{V}_A \frac{d\tilde{f}_A}{d\tilde{V}_A}, \quad \tilde{A} = 1, \ldots, \tilde{N},
\]

\[
V_A \frac{dg_A}{dV_A} = f_A - V_A \frac{df_A}{dV_A}, \quad A = 1, \ldots, N-\tilde{N}.
\]

The hidden-sector superpotential couples all sectors through its explicit \( K \) dependence; it gives rise to gaugino masses in the visible sector.

### 3 Gaugino Masses

To find the gaugino masses, we need certain terms from the component-field Lagrangian. These terms are computed in Appendix B.

\(^6\)Note also that these expressions contain contributions from extra moduli fields \( \Phi_i, \ i = 1, \ldots, n. \)

In the context of Type I models, the \( \Phi_i \) are twisted or non-dilaton-like untwisted moduli fields.
From (B.4) we find that the scalar potential takes the following form,

\[ V = \sum_{A=1}^{N-\hat{N}} \frac{1}{16\ell_A^2} \left( 1 + f^A - \ell_A f^A \right) \bar{u}_A u_A \]

\[ + \frac{1}{16} \left\{ \sum_{A=1}^{N-\hat{N}} \left( \frac{1 + f^A - \ell_A f^A}{\ell_A} u_A \right) \left( \sum_{B=1}^{\hat{N}} b_B \bar{u}_B \right) + \text{h.c.} \right\} \]

\[ + \frac{1}{16} \left\{ -3 + \sum_{i,j} G_i G_j^{-1} G_i \right. \]

\[ + \sum_{A=1}^{N-\hat{N}} \left( 1 + \tilde{f}^A - \tilde{\ell}_A \tilde{f}^A \right) + \sum_{A=1}^{N-\hat{N}} \left( 1 + f^A - \ell_A f^A \right) \left[ \sum_{A=1}^{N-\hat{N}} b_A \bar{u}_A \right] \]

\[ \left. \left\{ \sum_{A=1}^{N-\hat{N}} b_A \bar{u}_A \right\}^2 \right\} \right] \right| \right| \mid}_{\theta=\bar{\theta}=0}. \]

(3.1)

In this expression, \( \tilde{\ell}_A = \tilde{V}_A \mid_{\theta=\bar{\theta}=0} \) and \( \ell_A = V_A \mid_{\theta=\bar{\theta}=0} \) are the dilaton-like scalar fields which couple to the visible and hidden sectors, respectively. Moreover, \( u_A = U_A \mid_{\theta=\bar{\theta}=0} \) is the gaugino condensate field on each of the \( D_{p,A} \)-branes; it depends on \( \ell_A \) according to (B.7). Other notation is as follows,

\[ g^A_\ell = \frac{dg_A(V_A)}{d\bar{V}_A} \mid_{\theta=\bar{\theta}=0}, \quad \tilde{g}^A_\ell = \frac{d\tilde{g}_A(\tilde{V}_A)}{d\tilde{V}_A} \mid_{\theta=\bar{\theta}=0}, \]

\[ f^A_\ell = \frac{df_A(V_A)}{d\bar{V}_A} \mid_{\theta=\bar{\theta}=0}, \quad \tilde{f}^A_\ell = \frac{d\tilde{f}_A(\tilde{V}_A)}{d\tilde{V}_A} \mid_{\theta=\bar{\theta}=0}, \]

\[ G_i = \frac{\partial G}{\partial \Phi_i} \mid_{\theta=\bar{\theta}=0}, \quad G_j = \frac{\partial G}{\partial \bar{\Phi}_j} \mid_{\theta=\bar{\theta}=0}, \]

\[ G_{ij} = \frac{\partial^2 G}{\partial \Phi_i \partial \bar{\Phi}_j} \mid_{\theta=\bar{\theta}=0}. \]

(3.2)

The gaugino mass has its origin in the explicit \( K \) dependence of the hidden-sector superpotential. We find (B.5)

\[ m_{\lambda} = \left( 1 + \tilde{f}^\hat{A} - \tilde{\ell}_A \tilde{f}^\hat{A} \right) \sum_{A=1}^{N-\hat{N}} \frac{1}{4} b_A u_A \right), \]

\[ = \left( 1 + \tilde{f}^\hat{A} - \tilde{\ell}_A \tilde{f}^\hat{A} \right) m_{\tilde{G}}, \quad \hat{A} = 1, \cdots, \hat{N}. \]

(3.3)

The gravitino mass is simply

\[ m_{\tilde{G}} = \left( \sum_{A=1}^{N-\hat{N}} \frac{1}{4} b_A u_A \right), \]

(3.4)

To understand these formulae, let us first examine the gaugino mass in a theory with just one dilaton [6][30][31]. In this case the gaugino mass is as follows [6],

\[ m_{\lambda} = \left( \frac{1 + b\ell}{b\ell} \right) \left( \frac{1 + f - \ell f_\ell}{1 + f} \right) m_{\tilde{G}}. \]

(3.5)
All scalar fields are evaluated at the minimum of the potential,

\[ V = \frac{1}{16\ell^2} \left\{ (1 + f - \ell f_\ell)(1 + b\ell)^2 - 3b^2\ell^2 \right\} \bar{u}u. \] (3.6)

From this we see that any vacuum with zero cosmological constant satisfies \( \langle 1 + f - \ell f_\ell \rangle = 3b^2(\ell^2) + \mathcal{O}(b^3) \). Since \( \langle 1 + f \rangle \) and \( \langle \ell \rangle \) are numbers of order one, equation (3.5) implies that \( m_\lambda \) is smaller than \( m_\tilde{G} \) by a factor of \( b \).

Let us compare this to a model in which the visible and hidden sectors couple to different dilaton-like fields. The scalar potential of (3.1) receives contributions from both sectors, proportional to \( \langle 1 + \tilde{f}_A - \tilde{\ell}_A \tilde{f}_A^\dagger \rangle \) and \( \langle 1 + f_A - \ell_A f_A^\dagger \rangle \). However, to leading order, only \( \langle 1 + \tilde{f}_A - \tilde{\ell}_A \tilde{f}_A^\dagger \rangle \) contributes to the gaugino masses. As above, minimizing the potential forces \( \langle 1 + f_A - \ell_A f_A^\dagger \rangle \) to be \( \mathcal{O}(b_A^2) \), but it does not constrain \( \langle 1 + \tilde{f}_A - \tilde{\ell}_A \tilde{f}_A^\dagger \rangle \). This suggests that the gaugino masses can indeed be as large as the gravitino mass.

To see this explicitly, let us consider an example with just two types of D-branes (\( \tilde{N}=1 \) and \( N=2 \)). Each has its own dilaton-like field, \( \tilde{\ell}_A \) and \( \ell_A \). The scalar potential follows from (3.1), with no summation over the indices \( A \) and \( \tilde{A} \):

\[ V = \frac{1}{16\ell_A^2} \left\{ (1 + f_A - \ell_A f_A^\dagger)(1 + b_A\ell_A)^2 + b_A^2\ell_A^2 \left[ (1 + \tilde{f}_A - \tilde{\ell}_A \tilde{f}_A^\dagger) - 3 \right] \right\} \bar{u}_A u_A. \] (3.7)

The conditions for a nontrivial vacuum with vanishing cosmological constant are

\[
\left\langle \frac{\partial}{\partial \ell_A} \left( 1 + \tilde{f}_A - \tilde{\ell}_A \tilde{f}_A^\dagger \right) \right\rangle = 0,
\left\langle \frac{\partial}{\partial \ell_A} \left( 1 + f_A - \ell_A f_A^\dagger \right) \right\rangle = 2b_A^2\langle \ell_A \rangle \left[ 3 - \langle 1 + \tilde{f}_A - \tilde{\ell}_A \tilde{f}_A^\dagger \rangle \right] + \mathcal{O}(b_A^3),
\left\langle 1 + f_A - \ell_A f_A^\dagger \right\rangle = b_A^2\langle \ell_A \rangle \left[ 3 - \langle 1 + \tilde{f}_A - \tilde{\ell}_A \tilde{f}_A^\dagger \rangle \right] + \mathcal{O}(b_A^3),
\] (3.8)

where all \( \mathcal{O}(b_A^3) \) terms are suppressed. These equations have a consistent solution if \( \langle 1 + f_A - \ell_A f_A^\dagger \rangle = \mathcal{O}(b_A^2) \) and \( \langle 1 + \tilde{f}_A - \tilde{\ell}_A \tilde{f}_A^\dagger \rangle \) is \( \mathcal{O}(1) \). Since this latter term fixes the visible-sector gaugino mass, we expect \( m_A^4 \) to be of order \( m_\tilde{G} \).

To actually compute the gaugino mass, we need to specify the functions \( f_A^3 \) and \( \tilde{f}_A^3 \).

(The functions \( g_A \) and \( \tilde{g}_A \) are determined via (2.11)). We choose them to stabilize the runaway vacuum typically associated with dilaton-like fields.\(^7\) The general procedure is described in [30][34]. For now, we consider the following simple choice,\(^8\)

\[ f_A = \mathcal{P} \cdot e^{-\mathcal{Q}/\ell_A}, \quad \tilde{f}_A = \tilde{\mathcal{P}} \cdot e^{-\tilde{\mathcal{Q}}/\ell_A}, \quad \mathcal{P}, \mathcal{Q}, \tilde{\mathcal{P}}, \tilde{\mathcal{Q}} > 0. \] (3.9)

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\(^7\)See [32][33] for recent reviews.

\(^8\)This choice is motivated by the Type I string theory discussed in [35].
Substituting (3.9) into (3.8), we find that the potential has an extremum, located at

\[
\langle \hat{\ell}_A \rangle = \frac{\hat{Q}}{2},
\]

\[
\langle \ell_A \rangle = \frac{Q}{2} + O(b_A^2),
\]

\[
\mathcal{P} = e^2 + O(b_A^2),
\]

(3.10)

for any value of \(\hat{\mathcal{P}}\). It is, in fact, a global minimum of the potential with vanishing cosmological constant, as illustrated in Figures 1 and 2.

Let us now evaluate the gaugino mass in this vacuum. We find

\[
m_\lambda^\lambda = \frac{e^2 - \hat{\mathcal{P}}}{e^2 + \mathcal{P}} m_{\tilde{G}},
\]

(3.11)

where

\[
0.2 m_{\tilde{G}} < m_\lambda^\lambda < 0.8 m_{\tilde{G}} \quad \text{for} \quad 4.9 > \hat{\mathcal{P}} > 0.8.
\]

(3.12)

We see that, for reasonable values of \(\hat{\mathcal{P}}\), the gaugino mass is of order the gravitino mass.
4 Conclusion

The presence of multiple dilaton-like moduli fields is a very important feature of $\mathcal{N}=1$ D=4 Type I string models. The extra dilaton-like fields can change the resulting phenomenology in many ways. In this paper we examined their effect on supersymmetry breaking by hidden-sector gaugino condensation.

We studied a scenario in which different dilaton-like fields couple to the hidden and visible sectors. We assumed that supersymmetry is broken by gaugino condensation in the hidden sector. We found that the visible-sector gaugino masses can be as large as gravitino mass because of the extra dilaton-like fields. Our results stand in contrast to the usual heterotic string phenomenology, where the gaugino masses are typically much smaller.

We would like to thank Mary K. Gaillard and Gary Shiu for useful discussions. This work was supported in part by NSF grant No. PHY-9404057.
Appendix

A Linear–Chiral Duality

In this Appendix we prove that a supergravity model with $N$ linear supermultiplets is dual to another supergravity model with $N$ chiral supermultiplets.\textsuperscript{9} We will show that both models can be obtained from the following Lagrangian,

$$\mathcal{L} = \int d^4 \theta E \left\{ F(V_1, \cdots, V_N) + \sum_{A=1}^{N} (S_A + \bar{S}_A)(\Omega_A - V_A) \right\} \quad (A.1)$$

with

$$K = K(V_1, \cdots, V_N) \quad (A.2)$$

and

$$-3 + \sum_{A=1}^{N} V_A \frac{\partial K}{\partial V_A} = F - \sum_{A=1}^{N} V_A \frac{\partial F}{\partial V_A}. \quad (A.3)$$

In these expressions, the $V_A$’s are unconstrained real superfields and the $S_A$’s are ordinary chiral supermultiplets, for $A = 1, \cdots, N$. The field $\Omega_A$ is a Chern-Simons superform; it obeys

$$-(\bar{D}^2 - 8R)\Omega_A = W_A W_A, \quad (A.4)$$

for $A = 1, \cdots, N$.

Let us first integrate out the $S_A$’s in $\mathcal{L}$. Their equations of motion are as follows,

$$-(\bar{D}^2 - 8R)V_A = W_A W_A. \quad (A.5)$$

This equation can be used to eliminate the second term in (A.1), reducing $\mathcal{L}$ to a model with $N$ linear supermultiplets. Note that the Einstein gravity term is canonically normalized because of (A.3).

Let us now return to $\mathcal{L}$ and integrate out the $V_A$’s. Their equations of motion are as follows,

$$S_A + \bar{S}_A = \frac{\partial F}{\partial V_A} - \frac{1}{3} \frac{\partial K}{\partial V_A} \left\{ F - \sum_{B=1}^{N} V_B(S_B + \bar{S}_B) \right\}. \quad (A.6)$$

If we multiply (A.6) by $V_A$ and sum over $A = 1, \cdots, N$, we find\textsuperscript{10}

$$F(V_1, \cdots, V_N) - \sum_{A=1}^{N} V_A(S_A + \bar{S}_A) = -3. \quad (A.7)$$

\textsuperscript{9}This duality was briefly discussed in [36].

\textsuperscript{10}Equation (A.7) is obtained by assuming $\sum_{A=1}^{N} V_A \frac{\partial K}{\partial V_A} \neq 3$, as is true for the models considered in this paper.
It is now trivial to use (A.7) to rewrite (A.6) in a very simple form:

\[ S_A + \bar{S}_A = \frac{\partial (K + F)}{\partial V_A}. \] (A.8)

Using these relations, we find

\[ \mathcal{L} = -3 \int d^4 \theta E + \left\{ \sum_{A=1}^{N} \frac{1}{8} \int d^4 \theta \frac{E}{R} S_A (\mathcal{W}_m \mathcal{W}_n)_A + \text{h.c.} \right\}, \] (A.9)

with

\[ K = K(S_1 + \bar{S}_1, \ldots, S_N + \bar{S}_N). \] (A.10)

This completes the proof of linear–chiral duality.

**B Component-Field Lagrangian**

In this Appendix, we compute the necessary elements of the component-field Lagrangian corresponding to the superfield Lagrangian (2.10)–(2.11). We use the chiral density method [30][37].

We start by enumerating the definitions of bosonic component fields. In the hidden sector, we have

\[ \ell_A = V_A|_{\theta = \bar{\theta} = 0}, \]

\[ 2\sigma_{a\dot{a}}^m B_m^A - \frac{4}{3} \ell^A \sigma_{a\dot{a}}^a b_a = \left[ D_\alpha, D_{\dot{\alpha}} \right] V_A|_{\theta = \bar{\theta} = 0}, \]

\[ u_A = U_A|_{\theta = \bar{\theta} = 0} \equiv -(\bar{D}^2 - 8R)V_A|_{\theta = \bar{\theta} = 0}, \]

\[ -4F_A = -D^2(\bar{D}^2 - 8R)V_A|_{\theta = \bar{\theta} = 0}. \] (B.1)

In these expressions, the \( \ell_A \) are dilaton-like scalar fields, and the \( B_m^A \) are axionic degrees of freedom in the same supermultiplets. The fields \( u_A \) are the gaugino condensate fields of the hidden sector.

The visible-sector fields are defined in a similar way:\( ^{11} \)

\[ \ell_{\tilde{A}} = \tilde{V}_{\tilde{A}}|_{\theta = \bar{\theta} = 0}, \]

\[ 2\sigma_{a\dot{a}}^m \tilde{B}_m^{\tilde{A}} - \frac{4}{3} \ell^{\tilde{A}} \sigma_{a\dot{a}}^a b_a + 2\text{Tr}(\lambda_{\tilde{A}}^{\dot{\alpha}} \lambda_{a\dot{a}}^a) = \left[ D_\alpha, D_{\dot{\alpha}} \right] \tilde{V}_{\tilde{A}}|_{\theta = \bar{\theta} = 0}, \]

\[ -\text{Tr}(\lambda^{\dot{A}} \lambda_{\tilde{A}}) = -(\bar{D}^2 - 8R)\tilde{V}_{\tilde{A}}|_{\theta = \bar{\theta} = 0}, \]

\[ 8i\text{Tr}(\lambda^{\dot{A}} \sigma^m D_{m\dot{A}}) + 4\text{Tr}(\lambda^{\dot{A}} \lambda_{\tilde{A}}) \tilde{M} + 2\text{Tr}(\tilde{F}_{mn} \tilde{F}_{mn}) + i\epsilon_{mpq} \text{Tr}(\tilde{F}_{mn}^{\tilde{A}} \tilde{F}_{mpq}^{\tilde{A}}) - 4\text{Tr}(\tilde{D}^A \tilde{D}^A) = -D^2(\bar{D}^2 - 8R)\tilde{V}_{\tilde{A}}|_{\theta = \bar{\theta} = 0}. \] (B.2)

\( ^{11} \)We include the gaugino fields for the reader's convenience.
where

\[
\begin{align*}
-i\lambda^A_a &= \mathcal{W}_a^A|_{\theta=\bar{\theta}=0}, \\
-2\tilde{D}^A &= D^a\mathcal{W}_a^A|_{\theta=\bar{\theta}=0} = D_a\mathcal{W}_a^A|_{\theta=\bar{\theta}=0}, \\
\tilde{B}^m_A &= \frac{1}{2}\epsilon^{mpq}\{\partial_q\tilde{b}_{pq}^A + \frac{1}{6}\text{Tr}((\tilde{a}_{[q}\partial_{p]}\tilde{a}_{m]} - \frac{2i}{3}a_{[q}\partial_{p]}\tilde{a}_{m]})^A\}.
\end{align*}
\]

(B.3)

In these expressions, the \(\tilde{\ell}_A\) are dilaton-like scalar fields. The \(\tilde{B}_m^A\) are dual field strengths of the antisymmetric tensors \(\tilde{b}_{pq}^A\). The \(\tilde{F}_{mn}^A\) are the Yang-Mills field strengths, while the \(\tilde{a}_m^A\) are the corresponding gauge fields. The fields \(M\), \(M\), and \(b_a\) are the auxiliary fields of the supergravity multiplet [38]. The bosonic components of the \(\Phi_i\) are \(\phi_i = \Phi_i|_{\theta=\bar{\theta}=0}\) and \(-4F_i = D^2\Phi_i|_{\theta=\bar{\theta}=0}\).

Using these definitions, we find the following bosonic component-field Lagrangian:

\[
\frac{1}{\sqrt{-g}}\mathcal{L}_B = -\frac{1}{4}R - \sum_{A=1}^{\tilde{N}} \frac{1}{8\ell^2_A} \left( 1 + \tilde{\ell}_A g_{\ell}^A \right) \nabla^m \tilde{\ell}_A \nabla_m \tilde{\ell}_A
\]

\[
+ \sum_{A=1}^{\tilde{N}} \frac{1}{8\ell^2_A} \left( 1 + \tilde{\ell}_A g_{\ell}^A \right) \tilde{B}_A^m \tilde{B}_m^A - \sum_{A=1}^{\tilde{N}} \frac{1}{16\ell_A} \text{Tr}(\tilde{F}_{mn} \tilde{F}^{mn})
\]

\[
- \sum_{A=1}^{N-N} \frac{1}{8\ell^2_A} \left( 1 + \ell_A g_{\ell}^A \right) \nabla^m \ell_A \nabla_m \ell_A + \sum_{A=1}^{N-N} \frac{1}{8\ell^2_A} \left( 1 + \ell_A g_{\ell}^A \right) B_m^A \tilde{B}_m^A
\]

\[
+ \sum_{A=1}^{N-N} \frac{i}{4} b_A B_m^A \nabla_m \ln \left( \frac{u_A}{\tilde{u}_A} \right) - \frac{1}{2} \sum_{i,j} G_{ij} \nabla^m \phi_i \nabla_m \phi_j
\]

\[
- \frac{1}{18} \left( N - 3 + \sum_{A=1}^{N} \tilde{\ell}_A g_{\ell}^A + \sum_{A=1}^{N-N} \ell_A g_{\ell}^A \right) b^a b_a + \sum_{A=1}^{\tilde{N}} \frac{1}{8\ell_A} \text{Tr}(\tilde{D}^A \tilde{D}^A)
\]

\[
+ \frac{1}{2} \sum_{i,j} G_{ij} F_i \tilde{F}_j - \frac{1}{4} \left( \sum_{A=1}^{N-N} b_A u_A \right) \sum_i G_i F_i
\]

\[
+ \sum_{A=1}^{N-N} \frac{1}{8\ell_A} \left( 1 + f_a + b_a \ell_A \ln(e^{-K} \tilde{u}_A u_A) + 2b_A \ell_A \right) F_A
\]

\[
+ \frac{1}{18} \left\{ N - 3 + \sum_{A=1}^{N} (\tilde{f}_A - \tilde{\ell}_A \tilde{f}_A^A) + \sum_{A=1}^{N-N} (f^A - \ell_A f^A) \right\} M M
\]

\[
- \sum_{A=1}^{N-N} \frac{1}{8\ell_A} \left( 1 + f_A + b_A \ell_A \ln(e^{-K} \tilde{u}_A u_A) \right) u_A \tilde{M}
\]

\[
- \frac{1}{12} \sum_{B=1}^{N-N} b_B \tilde{u}_B \left\{ N + \sum_{A=1}^{\tilde{N}} (\tilde{f}_A - \tilde{\ell}_A \tilde{f}_A^A) + \sum_{A=1}^{N-N} (f^A - \ell_A f^A) \right\} M
\]

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\[- \sum_{A=1}^{N-\tilde{N}} \frac{1}{32\ell_A^2} \left( 1 + f^A - \ell_A f^A \right) \bar{u}_A u_A - \frac{1}{16} \left( \sum_{B=1}^{N-\tilde{N}} b_B \bar{u}_B \right) \left\{ \sum_{A=1}^{N-\tilde{N}} \frac{1}{\ell_A} \left( 1 + f^A - \ell_A f^A \right) u_A \right\} + \text{h.c.} \]

We also find the following kinetic and mass terms for the gauginos and gravitino,

\[
\frac{1}{\sqrt{-g}} \mathcal{L}_{m_\lambda} = - \sum_{A=1}^{N-\tilde{N}} \frac{1 + \tilde{f}^A}{4\ell_A} \text{Tr} \left( i \lambda^\tilde{A} \sigma^m \nabla_m \lambda^A \right) + \frac{1}{16} \left( \sum_{A=1}^{N-\tilde{N}} b_A \bar{u}_A \right) \left\{ \sum_{\tilde{A}=1}^{\tilde{N}} \frac{1 + \tilde{f}^\tilde{A} - \tilde{\ell}_{\tilde{A}} \tilde{f}^\tilde{A}}{\tilde{\ell}_{\tilde{A}}} \text{Tr} \left( \lambda^\tilde{A} \lambda^\tilde{A} \right) \right\} + \text{h.c.} \]

\[
\frac{1}{\sqrt{-g}} \mathcal{L}_{m_{\tilde{G}}} = \frac{1}{2} \epsilon^{mnpq} \bar{\psi}_m \sigma_n \nabla_p \psi_q - \sum_{A=1}^{N-\tilde{N}} \frac{1}{8\ell_A} \left\{ 1 + f_A + b_A \ell_A \ln \left( e^{-K} \bar{u}_A u_A \right) \right\} \bar{u}_A (\psi_m \sigma^{mn} \psi_n) + \text{h.c.} \]

To extract the potential (3.1), we must eliminate the auxiliary fields. We will not do that here, except to note that the equations of motion for the auxiliary fields \((F_A + \bar{F}_A)\) are

\[
1 + f_A + b_A \ell_A \ln \left( e^{-K} \bar{u}_A u_A \right) + 2b_A \ell_A = 0. \tag{B.6}
\]

This fixes the modulus of the condensate field \(u_A\) to be

\[
u_A \bar{u}_A = \exp \left[ \left( K - 2 \right) - \left( \frac{1 + f_A}{b_A \ell_A} \right) \right]. \tag{B.7}
\]

The modulus has the correct dependence on the gauge group and gauge coupling, as expected from the usual renormalization group arguments [6][31].

**C Gaugino Masses in the Chiral Supermultiplet Formulation**

In this Appendix we compute the gaugino masses using the chiral supermultiplet formulation. By linear–chiral duality, the result must be identical to the one obtained in Section 3.
Following Appendix A, it is straightforward to write down the chiral supermultiplet formulation of the model defined by \((2.10)–(2.11)\). The real superfields \(\tilde{V}_A\) and \(V_A\) dualize to chiral superfields\(^{12}\) \(\tilde{S}_A\) and \(S_A\). The linear–chiral duality relations include \((A.5)\) and
\[
\tilde{S}_A + \bar{\tilde{S}}_A = \frac{1 + \tilde{f}_A(\bar{V}_A)}{V_A}, \tag{C.1}
\]
This gives rise to the following identities:
\[
\tilde{s}_A + \bar{\tilde{s}}_A = \frac{1 + \tilde{f}_A}{\ell_A},
\]
\[
\langle \tilde{F}_{\tilde{S}_A} \rangle = -\frac{\langle M \rangle}{3} \left( \frac{1 + \tilde{f}_A}{\ell_A} - \frac{\bar{\tilde{f}}_A}{\ell_A} \right). \tag{C.2}
\]
The first is the \(\theta=\bar{\theta}=0\) component of \((C.1)\). The second is obtained by acting on both sides of \((C.1)\) with the operator \(\bar{D}^2\), and then taking the vacuum expectation value of the lowest component.

The expression for the gaugino masses is standard,
\[
m_A^\lambda = -\frac{\langle \tilde{F}_{\tilde{S}_A} \rangle}{\langle \tilde{s}_A + \bar{\tilde{s}}_A \rangle}, \tag{C.3}
\]
where \(\tilde{s}_A = \tilde{S}_A|_{\theta=\bar{\theta}=0}\) and \(F_{\tilde{S}_A} = -\frac{1}{4}D^2\tilde{S}_A|_{\theta=\bar{\theta}=0}\). Using \((C.2)\) and \(\langle M \rangle = 3m_G\), we find a final result for the gaugino masses that is identical to \((3.3)\), obtained from the linear supermultiplet formalism.

References


\(^{12}\)It can be shown that the linear–chiral duality established in Appendix A also applies to the effective theory description of Veneziano and Yankielowicz [22]. In particular, the linear–chiral duality relations remain the same.


