Gauge coupling instability and dynamical mass generation in $\mathcal{N}=1$ three-dimensional supersymmetric QED

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Using superfield Dyson-Schwinger equations, we compute the infrared dynamics of the semi-amputated full vertex, corresponding to the effective running gauge coupling, in N -flavor $\mathcal{N}=1$ three-dimensional supersymmetric QED. It is shown that the presence of a supersymmetry-preserving mass for the matter multiplet stabilizes the infrared gauge coupling against oscillations present in the massless case, and we therefore infer that the massive vacuum is thus selected at the level of the (quantum) effective action. We further demonstrate that such a mass can indeed be generated dynamically in a self-consistent way by appealing to the superfield Dyson-Schwinger gap equation for the full matter propagator. [S0556-2821(99)06118-4]

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I. INTRODUCTION

Three dimensional U(1) gauge field theories, whether as toy models of four dimensional physics or as interesting physical theories in their own right, have attracted considerable attention [1–8]. In particular, they have been shown to exhibit a rich and nontrivial infrared structure, exhibiting dynamical mass generation and critical behavior; the study of these phenomena requires nonperturbative techniques. The question of how the incorporation of supersymmetry affects the infrared physics of these theories has also been addressed [1,9,10] and it has been found that when $\mathcal{N}=1$ mass can be generated dynamically without breaking the supersymmetry, while mass generation in the $\mathcal{N}=2$ model is forbidden. For the $\mathcal{N}=1$ model an important question remains unanswered: whether the massive or massless solutions are selected by the theory. In nonsupersymmetric theories this question can be addressed by appealing to the effective potential and energetic favorability [1,2]; in supersymmetric theories the effective potential vanishes and this argument fails. It has been suggested [1] that the issue can be resolved in supersymmetric theories at the level of the (quantum) effective action (which does not vanish) and the Ward identities arising from supersymmetry. The Ward identities have been found to play a crucial role in this selection in models which have additional constraints [11,12], but do not yield any extra information for the unconstrained U(1) gauge field theory [10]. In this paper we propose a resolution of this problem for the basic N -flavor $\mathcal{N}=1$ U(1) theory at the level of the effective action: by considering an appropriate set of Dyson-Schwinger equations we show that a mass can be self-consistently generated without breaking supersymmetry, and that this mass stabilizes the infrared effective gauge coupling which in the massless case oscillates without definite sign.

We adopt a superfield formalism throughout this paper, and work with a large number N of matter flavors. We consider the superfield Dyson-Schwinger equation for the semi-

amputated full three-point vertex [13–15] which corresponds to the effective (running) gauge coupling [16], and compute the infrared dynamics of this vertex in the presence of vanishing and nonvanishing masses for the matter multiplet. In the massless case the dynamics are described by a nonlinear differential equation of Emden-Fowler type [17,18], which admits only oscillatory solutions in the infrared; we interpret this as indicating instabilities in the (quantum) effective action. Repeating the computation with the insertion (by hand) of a finite mass for the matter multiplet we find that these instabilities disappear, and the solution shows the existence of a nontrivial infrared fixed point for the running coupling. We then demonstrate that a mass for the matter superfield can be dynamically generated in a self-consistent way in this approach by appealing to the Dyson-Schwinger equation for the full matter propagator cast in terms of the semi-amputated full vertex. In agreement with Ref. [10] we find no evidence for a critical flavor number, above which dynamical mass generation does not occur.

In Ref. [10] the gap equation for the full matter propagator was studied incorporating a full vertex which by construction satisfied the U(1) Ward identity; instead, in this work, the form of the full vertex in the deep infrared will be determined self-consistently from the Dyson-Schwinger equation. The truncated form of the Dyson-Schwinger equation is not manifestly gauge invariant, i.e. it does not respect the Ward identity for general incoming momenta and there is a residual explicit dependence on the covariant gauge fixing parameter. However in the physical, on-shell limit of vanishing incoming momenta we find that the Ward identity is satisfied and that the dependence on the gauge fixing parameter drops out. Since in the present work we are interested only in obtaining and analyzing the structure of the vertex in the deep infrared, the lack of gauge invariance away from vanishing incoming momentum will not adversely affect our conclusions. The superfield formalism we adopt keeps supersymmetry manifest and thereby radically simplifies the sys-

tem of equations one would obtain for the full vertices in a component computation. The disadvantages of the superfield formalism [19,20] lie in the propagation of gauge artifacts in the connection superfield which result in spurious infrared divergences. By working in a general (covariant) gauge we will have enough flexibility to remove these divergences by an appropriate gauge choice.

This paper is organized as follows: in Sec. II we construct the action functional for $\mathcal{N}=1$ three-dimensional supersymmetric QED (QED₃) in superfield formalism and give the form of the dressed propagators for the matter and connection superfields. In Sec. III we introduce the superfield semi-amputated vertex and construct its Dyson-Schwinger equation; we then solve the Dyson-Schwinger equation in the presence of vanishing and nonvanishing masses for the matter superfield and interpret the results. In Sec. IV we discuss the infrared properties of the matter and connection superfield propagators, in association with approximations made in Sec. III; we show in Sec. V that when the previously computed vertex is incorporated in the gap equation for the matter propagator, a mass is dynamically generated self-consistently. Finally we present our conclusions in Sec. VI.

II. THE ACTION

We consider a model with $\mathcal{N}=1$ supersymmetry, N matter flavors and local U(1) gauge invariance. The required action functional then comprises three parts: the gauge invariant classical field strength term for the (spinor) connection Γ_α , a (Lorentz) gauge fixing term, and a locally U(1) invariant kinetic term for the matter superfields Φ and Φ^* :

$$S = S_g^{\text{class}} + S_g^{\text{GF}} + S_m; \quad (2.1)$$

$$\begin{aligned} S_g^{\text{class}} &= \int d^3x d^2\theta \Gamma_\alpha \left(-\frac{1}{8} D^\eta D^\alpha D^\beta D_\eta \right) \Gamma_\beta, \\ S_g^{\text{GF}} &= \int d^3x d^2\theta \Gamma_\alpha \left(\frac{1}{4\xi} D^\alpha D^2 D^\beta \right) \Gamma_\beta, \\ S_m &= \int d^3x d^2\theta \left(-\frac{1}{2} \right) [\nabla^\alpha \Phi]^* [\nabla_\alpha \Phi]. \end{aligned} \quad (2.2)$$

We have included in the matter part an implicit sum over N flavors, which do not interact with each other directly, but interact with the same connection superfield. The U(1) covariant derivative ∇^α is given by

$$\nabla^\alpha \doteq D^\alpha - ie\Gamma^\alpha,$$

where e is the (dimensionful) gauge coupling. We work within large- N , in which the quantity

$$\frac{e^2 N}{4} = \alpha$$

is kept fixed (but large) as N becomes large. From the action (2.1) it is easy to derive the dressed propagators for the matter and connection superfields:

FIG. 1. Structure and momentum assignments of the dressed three-point vertex; solid lines represent matter superfields and wavy lines the connection superfield.

$$\begin{aligned} \Delta(p; 12) &= \frac{i}{A(p)} \frac{D^2(p) - M(p)}{p^2 + M^2(p)} \delta^2(12), \\ \Delta_{\alpha\beta}(p; 12) &= -i \frac{1}{p^4} \frac{1}{B(p)} [(1 + \xi) p_{\alpha\beta} D^2 \\ &\quad - (1 - \xi) C_{\alpha\beta} p^2] \delta^2(12). \end{aligned} \quad (2.3)$$

The matter superfield contains a possible dynamically generated mass function $M(p)$ and the scalar functions A and B parametrize the dressing of the matter and connection superfields respectively. The unknown functions A , B , and M can at least in principle be determined from the appropriate Dyson-Schwinger equations; we return to these in Secs. IV and V. The dressed three-point vertex derived from the action above is shown in Fig. 1.

In this paper we use a convenient spinor index notation for three-vectors, in which they are represented as symmetric second rank spinors; spinor indices are raised and lowered by the antisymmetric metric C . We collect here some useful identities which will be used in what follows [21] ($\{\partial_\mu, \theta^\nu\} = \delta_\mu^\nu$):

$$\begin{aligned} A^\alpha &= C^{\alpha\beta} A_\beta, \\ A_\beta &= A^\alpha C_{\alpha\beta} \\ A^2 &\doteq \frac{1}{2} A^\alpha A_\alpha, \\ C_{\mu\nu} C^{\alpha\beta} &= \delta_{[\mu}^\alpha \delta_{\nu]}^\beta, \\ p_{\mu\nu} q^{\nu\alpha} &= \delta_\mu^\alpha p \cdot q, \\ D^\mu(q) &= \partial^\mu + \theta^\nu q_{\mu\nu}, \\ D^2(q) &= \partial^2 + \theta^\mu q_{\mu\nu} \partial^\nu + q^2 \theta^2, \\ D^\mu(q) D^2(q) &= q^{\mu\lambda} D_\lambda(q), \\ D^\mu(q) D^\nu(q) &= q^{\mu\nu} + C^{\nu\mu} D^2(q). \end{aligned} \quad (2.4)$$

In the next section we construct and analyze the truncated Dyson-Schwinger equation for the full three-point vertex shown in Fig. 1 with and without a mass for the matter multiplet and demonstrate that oscillations present in the massless case disappear when a mass for the matter multiplet is included.

FIG. 2. Schematic form of the Dyson-Schwinger equation for the full vertex. Solid lines represent matter superfields, and wavy lines the connection superfield. Blobs indicate full nonperturbative quantities.

III. DYSON-SCHWINGER EQUATION FOR THE VERTEX

Throughout this paper we use a truncation to leading order in $1/N$; the truncated Dyson-Schwinger equation for the full vertex is shown schematically in Fig. 2:

The graph on the left hand side of Fig. 2 is written as follows:

$$\left(-\frac{e}{2} G(p, p-q, q) \right) \int d^2\theta \Phi(-p, \theta) \Gamma_\alpha(p-q, \theta) D^\alpha(q) \Phi^*(q, \theta), \quad (3.1)$$

where we have assumed that we can perform the factorization into a scalar function G multiplying the superspace structure shown above; the first graph on the right of Fig. 2 is of the same form but with $G \mapsto 1$. This factorization is similar in effect to the approach taken in nonsupersymmetric QED₃ in which the vertex is assumed to be a scalar function multiplying the usual Dirac matrix. This approximation is computationally convenient, but does not satisfy the Ward identity except at vanishing incoming momentum, where the full vertex becomes on-shell. Given this limitation we will only be interested in the behavior of the theory in this regime; as we will show, even with this restriction we can obtain interesting information about the infrared fixed point structure. As we will see in Sec. V, the present approach gives rise to the same qualitative picture as that obtained in Ref. [10], namely that a mass can be generated dynamically for the matter multiplet without breaking supersymmetry.

Following Refs. [13–15] we define the semiamputed full nonperturbative vertex $\hat{G}$ as

$$\hat{G}(p_1, p_2, p_3) \doteq Z(p_1, p_2, p_3) G(p_1, p_2, p_3), \quad (3.2)$$

where Z is defined in terms of the functions A and B appearing in the dressed propagators (2.3) as

$$Z(p_1, p_2, p_3) \doteq A^{-1/2}(p_1) B^{-1/2}(p_2) A^{-1/2}(p_3) \geq 0. \quad (3.3)$$

The quantity $e\hat{G}$ is the appropriate and natural generalization of the running charge in super-renormalizable gauge field theories [16]. This definition is the same as the generalization of the running charge in nonsupersymmetric QED₃ [15]. As we will show, this definition also simplifies the structure of the integral equation for the vertex function considerably.

A. Vertex with vanishing mass

The one-loop graph on the right of Fig. 2 reads as follows:

$$\begin{aligned} & -\frac{e}{2} \left(-i \frac{e^2}{4} \right) \int \bar{d}^3 k d^2 \theta_1 d^2 \theta_2 d^2 \theta_3 G^3 \\ & \times \left[\frac{D^\mu(p+k) D^2(p+k)}{A(p+k)(p+k)^2} \delta^2(12) \right] \\ & \times \left[\frac{D^\alpha(q+k) D^2(q+k)}{A(q+k)(q+k)^2} \delta^2(23) \right] \\ & \times \left[\frac{(1+\xi) k_{\mu\nu} - (1-\xi) C_{\mu\nu} k^2}{B(k) k^4} \delta^2(31) \right] \\ & \times \Phi(-p, \theta_1) \Gamma_\alpha(p-q, \theta_2) D^\nu(q) \Phi^*(q, \theta_3), \end{aligned} \quad (3.4)$$

where

$$G^3 = G(p, -k, p+k) G(p+k, p-q, q+k) G(q+k, k, q). \quad (3.5)$$

Following [15] we consider the external connection superfield momentum to be vanishingly small, leaving one external scale p in the problem; computing the superspace part of Eq. (3.4) using the identities (2.4) we obtain an integral equation for the full vertex G . Multiplying this equation through by $Z(p)$ we obtain the desired integral equation for the semiamputed vertex $\hat{G}$ from the Dyson-Schwinger equation in Fig. 2:

$$\begin{aligned} \hat{G}(p) &= Z(p) + \frac{1}{4} e^2 \int \bar{d}^3 k \hat{G}^3(k) \\ & \times \frac{(1+\xi) k \cdot (p+k) + (1-\xi) k^2}{(p+k)^2 k^4}. \end{aligned} \quad (3.6)$$

We have made the approximation that $\hat{G}^3$ is the cube of one scalar function dependent on the scale k only [15]: this is justified by the self-consistency of our results. Note that in the absence of the inhomogeneous term Z on the right hand side of Eq. (3.6) one obtains a integral equation involving only $\hat{G}$, which can in principle be solved. In the following we will drop this inhomogeneous term, returning in Sec. IV to a discussion of why this may be done with safety.

The angular integration is easy to perform, leaving the following equation, which we have recast in the dimensionless variables $x \doteq p/\alpha$, $y \doteq k/\alpha$:

$$\begin{aligned} \hat{G}(x) &= \frac{1}{4\pi^2 N} \int dy \frac{\hat{G}^3}{y^2} \left\{ (1+\xi) \left(1 + \frac{y^2 - x^2}{2xy} \ln \left| \frac{y+x}{y-x} \right| \right) \right. \\ & \left. + (1-\xi) \frac{y}{x} \ln \left| \frac{y+x}{y-x} \right| \right\}. \end{aligned} \quad (3.7)$$

Since we are interested in the deep infrared we consider the limit $x \ll 1$, expand the logarithms in the above equation to second order and obtain

$$x^2 \hat{G} \simeq \frac{(2-\xi)}{3\pi^2 N} \int_0^x dy \hat{G}^3(y) + \frac{x^2}{\pi^2 N} \int_x^\infty dy \frac{\hat{G}^3(y)}{y^2} + \frac{x^4 \xi}{3\pi^2 N} \int_x^\infty dy \frac{\hat{G}^3(y)}{y^4}. \quad (3.8)$$

We drop the last term in the limit $x \ll 1$ and after appropriate differentiations with respect to x we arrive at the equivalent differential equation:

$$x^3 \hat{G}'' + \left[3x^2 + \frac{x(1+\xi)}{\pi^2 N} \hat{G}^2 \right] \hat{G}' + \frac{5-\xi}{3\pi^2 N} \hat{G}^3 = 0. \quad (3.9)$$

In the gauge $\xi = -1$ and changing variables $x \mapsto w = x^{-2}$ the above equation can be recast as a differential equation of Emden-Fowler type [17,18]:

$$\frac{d^2}{dw^2} \hat{G} + \frac{1}{2\pi^2 N} w^{-3/2} \hat{G}^3 = 0; \quad (3.10)$$

in the limit $w \rightarrow \infty$ it has been shown [18] that the only (real and nondivergent) solutions of this equation are oscillatory. We interpret this as indicating that in the absence of a mass for the matter multiplet the gauge coupling is subject to instabilities which render it unphysical. In the next subsection we study the effects of a mass introduced by hand and show that these instabilities are removed, and that the coupling is driven to a nontrivial infrared fixed point.

B. Vertex with nonvanishing mass

In this subsection we repeat the computation above retaining the effects of a constant mass $M(p) \simeq M(0) = M \neq 0$ in the propagators (2.3). Here we will put in the mass by hand, in Sec. V we will show that the semi-amputated full vertex we consider admits the dynamical generation of a mass. The one-loop graph on the right of Fig. 2 now reads

$$-\frac{e}{2} \left(-i \frac{e^2}{4} \right) \int \bar{d}^3 k d^2 \theta_1 d^2 \theta_2 d^2 \theta_3 G^3 \times \left[\frac{D^\mu(p+k)(D^2(p+k)-M)}{A(p+k)((p+k)^2+M^2)} \delta^2(12) \right] \times \left[\frac{D^\alpha(q+k)(D^2(q+k)-M)}{A(q+k)((q+k)^2+M^2)} \delta^2(23) \right] \times \left[\frac{(1+\xi)k_{\mu\nu} - (1-\xi)C_{\mu\nu}k^2}{B(k)k^4} \delta^2(31) \right] \times \Phi(-p, \theta_1) \Gamma_\alpha(p-q, \theta_2) D^\nu(q) \Phi^*(q, \theta_3), \quad (3.11)$$

where as before G^3 is given by Eq. (3.5). Again the super-space parts of this equation can be evaluated using the identities (2.4); in contrast to the case of nonsupersymmetric QED₃, where the inclusion of a mass for the fermions significantly alters the structure of the integral equation, here the mass terms only appear in the denominators of the matter

propagators, for the terms linear in M in the numerator cancel. In this respect, a mass for the matter superfield behaves like a trivial infrared regulator, similar to the effect of a photon mass in the nonsupersymmetric model [15]. On considering the external connexion momentum vanishingly small and multiplying through by $Z(p)$ as before, the integral equation to be compared with Eq. (3.6) reads

$$\hat{G}(p) = Z(p) + \frac{1}{4} e^2 \int \bar{d}^3 k \hat{G}^3(k) \frac{(1+\xi)k \cdot (p+k) + (1-\xi)k^2}{((p+k)^2 + M^2)k^4}. \quad (3.12)$$

As before, it is easy to perform the angular integration, the result of which reads (in dimensionless variables $x \doteq p/\alpha$, $y \doteq k/\alpha$ and $m \doteq M/\alpha$)

$$\hat{G}(x) = \frac{1}{4\pi^2 N} \int dy \frac{\hat{G}^3}{y^2} \left\{ (1+\xi) \left(1 + \frac{y^2 - x^2 - m^2}{4xy} \times \ln \left[\frac{(y+x)^2 + m^2}{(y-x)^2 + m^2} \right] \right) + (1-\xi) \frac{y}{2x} \ln \left[\frac{(y+x)^2 + m^2}{(y-x)^2 + m^2} \right] \right\}. \quad (3.13)$$

We have again dropped the inhomogeneous term Z (see Sec. IV); considering the deep infrared limit $x \ll 1$, and now also $x \ll m$, we can expand the logarithms above to second order to obtain the approximate form:

$$\hat{G} \simeq \frac{(3-\xi)}{4\pi^2 N} \frac{1}{m^2} \int_0^x dy \hat{G}^3(y) - \frac{(1+\xi)}{4\pi^2 N} \frac{x^2}{m^2} \int_0^x dy \frac{\hat{G}^3(y)}{y^2} + \frac{1}{\pi^2 N} \int_x^\infty dy \frac{\hat{G}^3(y)}{y^2 + m^2} - \frac{(1+\xi)}{4\pi^2 N} x^2 \int_x^\infty dy \frac{\hat{G}^3(y)}{y^2(y^2 + m^2)}. \quad (3.14)$$

Differentiation with respect to x yields the integral-differential equation

$$\hat{G}'(x) = \frac{(3-\xi)}{4\pi^2 N} \frac{\hat{G}^3(x)}{m^2} - \frac{1}{\pi^2 N} \frac{\hat{G}^3(x)}{x^2 + m^2} - \frac{(1+\xi)}{4\pi^2 N} \frac{\hat{G}^3(x)}{m^2} + \frac{(1+\xi)}{4\pi^2 N} \frac{\hat{G}^3(x)}{x^2 + m^2} - \frac{(1+\xi)}{2\pi^2 N} \frac{x}{m^2} \int_0^x dy \frac{\hat{G}^3(y)}{y^2} - \frac{(1+\xi)}{2\pi^2 N} x \int_x^\infty dy \frac{\hat{G}^3(y)}{y^2(y^2 + x^2)}. \quad (3.15)$$

By the convenient choice of gauge $\xi = -1$ this can be reduced to a first order nonlinear differential equation, which can be integrated with ease to yield

$$\hat{G}(x) = \frac{1}{\left[c + \frac{2}{\pi^2 N m^2} \left(m \arctan\left(\frac{x}{m}\right) - x \right) \right]^{1/2}}, \quad (3.16)$$

where c is an integration constant to be determined from the boundary condition obtained from the original integral equation. In the limit $x \rightarrow 0$ the renormalization group β function vanishes:

$$\lim_{x \rightarrow 0} x \frac{d\hat{G}(x)}{dx} = \lim_{x \rightarrow 0} \frac{1}{\pi^2 N} \frac{\hat{G}^3(x) x^3}{m^2 (x^2 + m^2)} \rightarrow 0, \quad (3.17)$$

and hence there is a nontrivial (N -independent) fixed point at $x=0$ given by

$$\hat{G}(0) = c^{-1/2}. \quad (3.18)$$

Returning to the integral equation (3.12) and taking the limit $x \rightarrow 0$ we can investigate the constraints on the integration constant c :

$$\hat{G}(0) = \frac{1}{\pi^2 N} \int_0^\infty dy \frac{\hat{G}^3(y)}{y^2 + m^2}. \quad (3.19)$$

Since the right hand side of this equation is manifestly positive definite, the trivial solution $\hat{G}(0) = 1/\sqrt{c} = 0$ is ruled out, unless $\hat{G}$ is trivially zero. Noting that in the small x limit $\hat{G}$ differs from its fixed point value by a quantity of order $\mathcal{O}(x^3)$, we can crudely approximate the integral in Eq. (3.19) as follows:

$$\hat{G}(0) = \frac{1}{\pi^2 N} \int_0^m dy \frac{\hat{G}^3(0)}{y^2 + m^2} + \frac{1}{\pi^2 N} \int_m^\infty dy \frac{1}{y^2 + m^2}, \quad (3.20)$$

where we have set $\hat{G}$ in the second integral to its ultraviolet asymptote of unity: both of these approximations are underestimates, and therefore after performing the integrations we have

$$4\pi m \hat{G}(0) > \frac{1}{N} (1 + \hat{G}^3(0)). \quad (3.21)$$

In principle for a given m this leads to a (small) critical coupling, below which there is no mass generation. Note that even on restoring the inhomogeneous term Z (see Sec. IV) the inequality is only modified to

$$4\pi m (\hat{G}(0) - Z(0)) > \frac{1}{N} (1 + \hat{G}^3(0)). \quad (3.22)$$

Since at the level of the original integral equation (3.12) in the gauge $\xi = -1$ it is obvious that $\hat{G} \geq Z$, we obtain again a (small) critical coupling. Note that in the above analysis there is no evidence for a critical flavor number.

To conclude this section we should check our assertion that the value of $\hat{G}$ at vanishing x is indeed ξ -independent.

To accomplish this we return to the integral equation (3.14) and retain the general gauge dependence. The equation can then readily be converted to a differential equation:

$$x\hat{G}'' + \left[-1 - \frac{3}{2m^2\pi^2 N} (1 - \xi)x\hat{G}^2 + \frac{3}{4\pi^2 N} (3 - \xi) \frac{x}{x^2 + m^2} \hat{G}^2 \right] \hat{G}' + \frac{1}{\pi^2 N} \left[\frac{1}{m^2} - \frac{(5 + \xi)}{4} \frac{1}{x^2 + m^2} - \frac{2x^2}{(x^2 + m^2)^2} \right] \hat{G}^3 = 0. \quad (3.23)$$

Considering the limit $x \rightarrow 0$ this reduces to a first order differential equation (in which the term $x\hat{G}''$ has been dropped as subleading in x)

$$\hat{G}' + \frac{(1 + \xi)}{4\pi^2 m^2} \hat{G}^3 = 0$$

which can be integrated easily to give for small x

$$\hat{G}(x) \simeq \frac{1}{\left(c + \frac{(1 + \xi)}{4\pi^2 N m^2} x \right)^{1/2}}, \quad (3.24)$$

demonstrating that the term in $\hat{G}''$ is indeed subleading and also that all the ξ dependence vanishes at $x=0$, so information about the fixed point is ξ independent as expected from the on-shell nature of $\hat{G}(0)$.

We have shown in this section that in the presence of a mass for the matter multiplet the full vertex is stabilized and driven to a nontrivial infrared fixed point. In the above analysis the mass for the matter multiplet has been included by hand; however we will show in Sec. V that a mass can be dynamically generated self-consistently by coupling the vertex equation to the corresponding Dyson-Schwinger equation for the matter propagator. In the next section we turn to a discussion of the inhomogeneous term Z and why it can be safely omitted.

IV. THE INHOMOGENEOUS TERM Z AND THE FUNCTIONS A AND B

In this section we analyze the Dyson-Schwinger equations for the functions A and B with which we have respectively dressed the matter and connection superfield propagators. First we consider the Dyson-Schwinger equation for the matter propagator, shown schematically in Fig. 3.

The difference between the graphs on the left of Fig. 3 is easily computed to be

$$-i(A(p) - 1) \int d^2\theta \Phi(-p, \theta) D^2(p) \Phi^*(p, \theta) - i \frac{M(p)}{A(p)} \int d^2\theta \Phi(-p, \theta) \Phi^*(p, \theta). \quad (4.1)$$

FIG. 3. Schematic form of the Dyson-Schwinger equation for the full matter propagator. Solid lines represent matter superfield propagators, and wavy lines gauge superfield propagators; blobs indicate full nonperturbative quantities.

We adopt the one-loop dressed approximation to the Dyson-Schwinger equation in which both vertices in Fig. 3 are fully dressed and all two-particle irreducible corrections are dropped [22]. In doing so the resulting integral equation can be cast in terms of $\hat{G}$ alone.

The graphs on the right hand side of Fig. 3 can be manipulated to obtain functions multiplying the two superspace structures

$$\int d^2\theta \Phi(-p, \theta) D^2(p) \Phi^*(p, \theta),$$

$$\int d^2\theta \Phi(-p, \theta) \Phi^*(p, \theta); \quad (4.2)$$

comparison with Eq. (4.1) shows that the function multiplying the first of these structures is to be identified with the contribution to the wave function renormalization $A(p)$ and the function which multiplies the second corresponds to the self energy $M(p)/A(p)$, to which we will return in the next section. The first “seagull” graph on the right hand side of the figure contributes only an irrelevant p -independent contribution to the wave function renormalization while the last graph on the right hand side of the figure contributes both to the wave function renormalization and the self energy. Evaluating the superspace parts of the last graph leads to the following integral equation for $A(p)$:

$$A(p) = 1 + (1 + \xi)A(p) \frac{ie^2}{2} \int d^3k \frac{\hat{G}^2(k)}{k^2 + M^2(k)} \frac{k \cdot (k-p)}{(k-p)^4}. \quad (4.3)$$

In the gauge $\xi = -1$ which we adopt throughout this paper, this reduces to unity, leaving a constant $A(p)$. Note that this is in line with the result of Ref. [10], where the wave function renormalization was computed to be

$$A(p) = \left(\frac{p}{\alpha} \right)^{2(1+\xi)/N\pi^2}. \quad (4.4)$$

We turn now to the connection superfield and the function $B(p)$. In standard large- N treatments [9,10] the function B includes the effects of massless matter loops resummed to leading order in $1/N$, giving

$$B(p) \sim 1 + \alpha/p.$$

Therefore in the deep infrared we find

$$Z(x) = A^{-1}(x) B^{-1/2}(x) \sim x^{1/2} \quad (4.5)$$

FIG. 4. Schematic form of the Dyson-Schwinger equation for the full connection propagator. Solid lines represent matter superfields and wavy lines represent connection superfields; blobs indicate full nonperturbative quantities.

and the inhomogeneous term goes to zero and can be safely dropped compared with a nonvanishing $\hat{G}(0)$ in a large- N framework.

For completeness it is instructive to consider the function B outside this large- N resummation, and consider the Dyson-Schwinger equation shown schematically in Fig. 4. Again we adopt the one-loop dressed approximation, which enables us to recast the equation in terms of $\hat{G}$ alone. The difference between the graphs on the left of Fig. 4 is easily computed (in the gauge $\xi = -1$) to be

$$(B(p) - 1) = \frac{1}{2} C^{\alpha\beta} p^2 \int d^2\theta \Gamma_{\alpha}(-p, \theta) \Gamma_{\beta}(p, \theta). \quad (4.6)$$

In computing the graph on the right of Fig. 4 the superspace structure in Eq. (4.6) naturally appears, and the resulting integral equation for B is (in which we have already performed the simple angular integration)

$$p^3(B(p) - 1) = \frac{e^2 N}{16\pi^2} B(p) \int dk k \hat{G}^2(k) \ln \left(\frac{(k+p)^2 + M^2}{(k-p)^2 + M^2} \right). \quad (4.7)$$

Converting to dimensionless variables, considering constant M and expanding the logarithm as usual we can convert to an equivalent differential equation in which we have approximated $\hat{G}$ by its constant value at $x=0$, noting from Eq. (3.16) that in doing so we have only dropped terms of order $\mathcal{O}(x^3)$:

$$\frac{d}{dx} (x^2 B^{-1}(x)) = 2x - \frac{\hat{G}^2(0)}{\pi^2} \frac{x^2}{m^2} + \frac{\hat{G}^2(0)}{\pi^2} \frac{x^2}{x^2 + m^2}. \quad (4.8)$$

This can be integrated easily with result

$$B^{-1}(x) = 1 - \frac{1}{3} \frac{\hat{G}^2(0)}{\pi^2} \frac{x}{m^2} + \frac{\hat{G}^2(0)}{\pi^2} \frac{1}{x} - \frac{\hat{G}^2(0)}{\pi^2} \frac{m}{x^2} \arctan \left(\frac{x}{m} \right); \quad (4.9)$$

for finite solutions the constant of integration must vanish. In the limit of small x there are cancellations to order $\mathcal{O}(x^3)$ and B reduces to

$$B(x) \simeq \frac{1}{1 + \mathcal{O}(x^3)}. \quad (4.10)$$

From this analysis it is clear that even when the connection propagator is not resummed to leading order in $1/N$, the inhomogeneous term Z reduces to a finite constant in the infrared and can still be dropped with safety, justifying fully our assumption in Sec. III.

V. MASS

In this section we demonstrate that a mass for the matter multiplet can be generated dynamically when we feed the solution for the full vertex (3.16) into the Dyson-Schwinger equation for the matter propagator. The latter is shown schematically in Fig. 3, and now we evaluate the terms in the graph on the right hand side which multiply the second of the superspace structures in Eq. (4.2). Evaluating the superspace integrals leaves the following integral equation for the mass function:

$$M(p) = \frac{ie^2}{2} \int \bar{d}^3k \frac{\hat{G}^2(k)M(k)}{(k^2 + M^2(k))(k-p)^4} [(1+\xi)k \cdot (k-p) + (1-\xi)(k-p)^2]. \quad (5.1)$$

Choosing again the gauge $\xi = -1$, we see that it removes the spurious infrared divergences and simplifies the equation considerably. After performing the angular integration and expanding the resulting logarithms, we obtain in dimensionless variables

$$m(x) = \frac{2}{\pi^2 N x^2} \int_0^x dy \frac{y^2 m(y) \hat{G}^2(y)}{y^2 + m^2(y)} + \frac{2}{\pi^2 N} \int_x^\infty dy \frac{m(y) \hat{G}^2(y)}{y^2 + m^2(y)}. \quad (5.2)$$

Differentiating with respect to x this can be converted to an equivalent differential equation,

$$x m'' + 3 m' + \frac{4}{\pi^2 N} \frac{m \hat{G}^2}{x^2 + m^2} = 0. \quad (5.3)$$

In the deep infrared $x \ll m$, and approximating $\hat{G}(x)$ by $\hat{G}(0)$, noting again that the corrections to this approximation are of order $\mathcal{O}(x^3)$ [see Eq. (3.16)]; Eq. (5.3) then admits the following approximate solution:

$$m(x) \approx m(0)(1 - ax - \mathcal{O}(x^2)), \quad a > 0 \quad (5.4)$$

which exhibits a constant dynamical mass $m(0)$ and has the correct (decreasing) behavior away from $x=0$; note that this solution is similar to the small x expansion of the mass found using the same method for nonsupersymmetric QED₃ in Ref. [15]. The solution above is to be compared with the case

when the vertex is chosen to satisfy the U(1) Ward identity for general momenta [10], where the solution for the mass behaves as

$$m(x) = m(0)e^{-x^2} \approx m(0)(1 - x^2 + \mathcal{O}(x^4)).$$

The semiamputated vertex considered here only satisfies the Ward identity at $x=0$ where the vertex is on-shell, and therefore the small- x dependence may differ from the more complete solution, though the qualitative (decreasing) behavior is retained.

VI. CONCLUSION

In this paper we have proposed that the dynamical generation of a mass in $\mathcal{N}=1$ supersymmetric QED₃ is selected by the dynamics over the massless alternative, for it stabilizes the running gauge coupling against oscillations. In particular the picture which has emerged from our analysis is that in the absence of a mass for the matter multiplet, the full vertex oscillates in the infrared, leading to instability in the effective action. When a finite dynamical mass is added, the infrared physics is stabilized and there exists a nontrivial infrared fixed point. We have demonstrated that the incorporation of our full vertex solution into the Dyson-Schwinger equation for the matter propagator leads self-consistently to the dynamical generation of a mass.

Supersymmetry has been kept manifest throughout by adopting a superfield formalism which has the additional advantage of simplifying considerably the system of equations and Ward identities which would have to be solved in a component calculation. By working in a general gauge we have retained enough freedom to remove infrared divergences which arise from the propagation of spurious degrees of freedom in the connection superfield [19,20], and we have been able to simplify the analysis significantly. We have employed a large- N framework consistently throughout this paper; this in combination with the concept of a semiamputated full vertex allowed us to decouple the Dyson-Schwinger equations so that the functions A and B used to dress the full propagators do not appear in the analysis of the vertex.

There is the interesting possibility that the techniques employed in this paper and in Ref. [15] could be extended to models in higher dimensions and with non-Abelian gauge groups.

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