Inclusive J/ψ and ψ' production in π^- -nucleus interactions at $\sqrt{s} \simeq 26$ GeV

The BEATRICE Collaboration

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Abstract

Charmonium production by 350 GeV/c negative pions incident on tungsten, copper and silicon has been studied in the WA92 experiment. J/ψ and ψ' mesons are identified through their decays into muon pairs and are selected in the Feynman- x range $x_F > 0$. Inclusive J/ψ and ψ' cross-sections and J/ψ differential cross-sections as functions of x_F and p_T^2 are measured. Assuming the relation $\sigma = \sigma_0 A^\alpha$ between the nuclear cross-section, σ , and the atomic mass, A , of a target nucleus, a fit to J/ψ data for different targets yields $\alpha = 0.87 \pm 0.05 \pm 0.04$. Taking $\alpha = 0.87$, the cross-sections per nucleon for J/ψ and ψ' are $\sigma_0(J/\psi) = 216 \pm 5 \pm 13$ nb and $\sigma_0(\psi') = 28 \pm 8 \pm 11$ nb, and their ratio is $\sigma_0(\psi')/\sigma_0(J/\psi) = 0.13 \pm 0.05 \pm 0.01$. The measurements of charmonium production are compared with results on open-charm production previously obtained in the same experiment. The ratio between the J/ψ cross-section and the charmed-meson cross-section, relating to the same experimental conditions, is found to be $\sigma_0(J/\psi)/\sigma_0(D\bar{D}) = (1.75 \pm 0.05 \pm 0.15) \times 10^{-2}$.

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1 Introduction

Recent interest in the theory of quarkonium production has been motivated by the anomalously large cross-section for ψ' and direct J/ψ production observed in the CDF experiment at the Tevatron [1]. In an attempt to explain this cross-section excess, several theoretical models have been revisited and new production mechanisms have been proposed. The first theoretical descriptions of charmonium hadroproduction were the Colour-Evaporation Model [2] and the Colour-Singlet Model [3]. In these models, the dominant contribution to charmonium production comes from quark-antiquark annihilation or gluon-gluon fusion giving rise to a bound $c\bar{c}$ pair. In a recent extension of the Colour-Singlet Model, the so called Factorization Approach [4], a quark-antiquark pair can initially form either a colour-singlet state or a colour-octet state. The latter evolves non-perturbatively into a charmonium state through the emission of soft gluons. In the Colour-Evaporation Model, by contrast, colour is not treated, but is assumed to be neutralized by soft gluon emission before the $c\bar{c}$ pair hadronizes as a J/ψ or ψ' . In this model, charmonium states are taken to derive from $c\bar{c}$ pairs created below the threshold for open charm production. Different charmonium states are predicted to be produced, independently of the centre-of-mass energy, as a constant fraction of the total charmonium cross-section.

As well as “hard” (parton) processes, charmonium production can involve diffractive processes. These have been found to give a negligible contribution to most data collected in fixed-target experiments.

Some features of charmonium data for fixed-target production by protons and pions (centre-of-mass energies up to 50 GeV), can be reproduced using both the Factorization Approach [5, 6] and the Colour-Evaporation Model [7, 8, 9]. However, the theoretical predictions can suffer from large uncertainties associated with QCD calculations, and depend on parameters that are assumed to be process independent. This process independence is an essential ingredient both in the Factorization Approach and in the Colour-Evaporation Model, and must be verified by considering charmonium production in different interactions. Hadroproduction data, taken at the Tevatron or in fixed-target experiments, photoproduction data and data from e^+e^- annihilations provide complementary cross-checks. Experimental measurements for π -nucleon interactions are rather scattered, and additional measurements are important for testing the predictive capability of the different theoretical approaches.

In this paper, results from the WA92 experiment on J/ψ and ψ' production in π^- -nucleus interactions at $\sqrt{s} \simeq 26$ GeV are presented. Interactions occurred in tungsten and copper targets, and in the silicon of some of the detectors used. J/ψ and ψ' mesons are identified through their decays into muon pairs. In total $1096 \pm 34 \pm 5$ J/ψ and $19.6 \pm 6.6 \pm 3.4$ ψ' signal events are reconstructed. Results for inclusive and differential cross-sections are presented, and comparisons are made with open-charm measurements relating to the same experimental conditions and data samples.

2 The experiment

The WA92 experimental setup is described in detail elsewhere [10] and is only outlined here. Use was made of a 350 GeV/c π^- beam, directed towards a 2-mm-thick target of copper or tungsten. Data recording took place in 1992 and, with a slightly modified setup, in 1993. The coordinate reference system employed is a right-handed (x, y, z) Cartesian system, with the origin in the target centre and the x -axis positively oriented along the beam direction. A view of the entire apparatus and enlarged views of the interaction

region are shown in Fig. 1.

The experiment's tracking system consisted of silicon-microstrip detectors planes arranged as a Beam Hodoscope, a Decay Detector [11] and a Vertex Detector [12], followed by multiwire proportional chambers and drift chambers [13]. The multiwire proportional chambers were positioned in a 1.8 Tesla magnetic field, directed along the z -axis. The full tracking system gave a momentum resolution $\sigma_p/p^2 \simeq 1.6 \times 10^{-4} (\text{GeV}/c)^{-1}$. A silicon-microstrip counter termed the In-Target detector, 300 μm thick and placed 100 μm downstream of the target, was used at the trigger level to select interactions in the target and to confirm the position of the primary interaction given by the Beam Hodoscope.

A lead-glass electromagnetic calorimeter and a muon filter were placed downstream of the magnet. The muon filter[14] comprised two hodoscopes formed of Resistive-Plate Chambers (RPCs), the first placed behind a 2-m-thick absorber, predominantly of iron but with a tungsten core; the second separated from the first by a further 1.2 m thickness of iron. Each hodoscope consisted of three detector planes divided into upper and lower halves. Two detector planes per hodoscope had horizontal read-out strips, and were used to reconstruct muon tracks in the non-bending projection. The remaining planes had vertical read-out strips, and served to remove ambiguities when forming associations with tracks measured in the spectrometer. The two hodoscopes accepted muons that had momentum $p_\mu \geq 7 \text{ GeV}/c$, and were within the angular region: 0 to ± 180 mrad in the horizontal plane and ± 8 mrad to ± 70 mrad in the vertical plane.

The experiment's trigger system was designed to provide a large acceptance for beauty particles [15]. Interactions were selected by a coincidence between three beam scintillator counters, two additional scintillators placed downstream of the target and the In-Target detector. The energy deposit in at least one of the strips of the In-Target detector was required to be above the mean energy deposition for five minimum-ionizing particles. The coincidence with the In-Target detector rejected about 4% of the interactions occurring in the target and around 90% of the interactions in the silicon of the In-Target detector, Decay Detector and Vertex Detector. The fraction of accepted events with primary interactions in silicon, most being in the In-Target detector, was reduced to 4%. The on-line event selection additionally made use of a trigger on secondary vertices, a trigger for muons, a trigger for high- p_T particles, and a trigger for electromagnetic energy. There was some variation of the exact trigger requests with target material and year of data taking [16]. A trigger requiring an interaction in coincidence with at least two muons, the trigger most relevant for the present analysis, selected about 3% of the events recorded on tape. The muon trigger [17] involved the processing of signals read from the RPCs of the muon filter. Coincidences, in the non-bending plane, between at least one strip (of 3 cm width) of the first RPC hodoscope, and a strip from a group three-strips wide in the second hodoscope, and consistent with a track having an origin in the target, were identified as muons. For muons with momentum larger than 7 GeV/c and within the geometrical acceptance of the hodoscopes, the identification efficiency of the trigger algorithm was measured to be about 97%.

3 Charmonium selection

3.1 Event samples

The present analysis has been performed using the full data set of the WA92 experiment. The data set consists of about 37×10^6 events collected with a tungsten target and about 100×10^6 events collected with a copper target. About 4% of the recorded events are expected to have the primary interaction in silicon. The effective luminosities

for interactions on tungsten, copper and silicon nuclei were respectively $(1.35 \pm 0.11) \text{ nb}^{-1}$, $(7.78 \pm 0.39) \text{ nb}^{-1}$ and $(12.80 \pm 0.87) \text{ nb}^{-1}$, where the errors are mainly systematic and are dominated by uncertainties on inelastic π^- -nucleus cross-sections [18] and on the interaction-trigger acceptance [16]. The luminosity value relative to silicon includes interactions in the In-Target detector and in the planes of the Decay detector, the material of these detectors having a total thickness of about 4.5 mm.

Acceptances have been estimated using simulated J/ψ and ψ' events, produced for each experimental configuration and target material. Events were generated according to Feynman- x (x_F) and transverse momentum (p_T) distributions that reproduce the shapes observed in the WA92 data. This means that J/ψ mesons coming from χ_c and ψ' decays, measured to be about 40% of all J/ψ mesons [19], were implicitly included. The contribution to charmonium production from B particle decays is negligible at current fixed-target energies. Both J/ψ and ψ' were made to decay isotropically into $\mu^+\mu^-$, possible polarization effects being taken into account separately (see section 3.4).

Generated events were processed through the WA92 detector simulation, based on GEANT 3.21 [20]. The fraction of the beam-particle momentum not carried away by the $\mu^+\mu^-$ system was assigned to a π^- particle, and the interaction of this particle with a target nucleus was then simulated using FLUKA [21]. Detector inefficiencies, random electronic noise, and a detailed emulation of trigger responses were introduced in order to reproduce experimental conditions. Comparisons of topological and kinematical event properties, including primary and secondary track multiplicities, number of secondary vertices, track momenta and transverse momenta, show good agreement between experiment and simulation. To ensure that errors associated with simulation statistics were negligible, each sample of simulated events contained at least 30 times the relevant number of J/ψ or ψ' decays found in the data.

3.2 Charmonium candidates

Reconstruction, of both experimental interactions and simulated events has been performed using the TRIDENT program[10]. In the reconstruction a track is associated to a (primary or secondary) vertex when the track's closest approach to the vertex is less than twice the uncertainty on the approach distance. Muon identification involves the definition of space points using both projections of the two muon hodoscopes. Tracks reconstructed in the magnetic spectrometer are extrapolated to the hodoscopes to find matching space points. A track is classed as belonging to a muon if it matches with at least one space point in each hodoscope. From simulation studies, the probability of making correct associations for muons with momentum above 10 GeV/ c is about 97%. When several tracks are compatible with the same space points, an ambiguity is flagged. The fraction of true muon hits ambiguously assigned is about 7%.

In the present analysis, events found to contain at least two muons of opposite charge are selected. Background resulting from hadron tracks mis-associated with hits in the RPCs is reduced by requiring that at least one of the two muons be identified unambiguously. To reduce background from muons that originate in hadron showers (initiated in the iron absorbers), it is required that at least one muon of a pair be associated with the primary vertex.

Among the selected events, interactions in the target, in the In-Target detector and in the Decay Detector are all considered. The material in which an interaction occurs is determined from the the x -coordinate of an event's primary vertex, measured with a resolution, σ_x , of about 115 μm . Interactions in the target material are selected by

requiring that the signed x -distances of the primary vertex from the target's upstream and downstream edges, distances Δx_U and Δx_D respectively, satisfy $\Delta x_U/\sigma_x > -3$ and $\Delta x_D/\sigma_x < -0.5$. Conversely, interactions in the silicon planes are selected by requiring $\Delta x_D/\sigma_x > 0.5$ (primary vertex downstream of the target edge). From a study of simulated J/ψ and ψ' events, interactions in the target are selected with negligible contamination from interactions in silicon, while about 9% of interactions selected as being in silicon are, in reality, interactions in the target.

Requiring that the selected $\mu^+\mu^-$ pairs have $x_F > 0$, which has little effect on the total acceptance, allows direct comparison with most other experiments. The $\mu^+\mu^-$ invariant-mass distributions relative to tungsten, copper and silicon, with all selection criteria applied, are shown in Fig. 2. For all materials, a clear peak is present around the J/ψ mass of 3.097 GeV/ c^2 [22]. The peak's full width at half maximum depends on the amount of multiple scattering in the production material, and ranges, from 40 MeV/ c^2 for silicon to 60 MeV/ c^2 for tungsten. A peak around the ψ' mass of 3.686 GeV/ c^2 [22] is also visible. Muon pairs having an invariant mass in the range 3.0–3.2 GeV/ c^2 or in the range 3.58–3.78 GeV/ c^2 (signal regions) are selected as J/ψ candidates or as ψ' candidates respectively. The mass regions chosen contain about 93% of the charmonium signal. In the case of J/ψ candidates, the signal-to-noise ratio is about 15:1 for tungsten and copper, and about 6:1 for silicon. The numbers of selected J/ψ and ψ' candidates relative to the different production materials are reported in Table 1.

Both for the experiment and for the simulation, the fraction of J/ψ candidates that satisfied the dimuon trigger is about 98%. The remaining 2% of J/ψ candidates were accepted by a trigger including a request for a single muon (a second muon being recovered in the reconstruction).

3.3 Background subtraction

Background arises mainly from cases where at least one of the two particles interpreted as a muon pair is either a hadron misidentified as a muon, or a muon from the in-flight decay of a hadron (usually a pion or kaon). In these cases the particles taken as a pair have uncorrelated charge, so it should be possible to model the background using same-sign muons ($\mu^\pm\mu^\pm$). Experimentally, the invariant-mass distributions for $\mu^\pm\mu^\pm$ pairs, shown in Fig. 2, accurately reproduce the shape of the background but underestimate the total background by a few percent. The small amount of background not represented by same-sign muons is due to $\mu^+\mu^-$ pairs from the Drell-Yan process and from semi-leptonic decays of charmed hadrons. The latter, studied with simulated $c\bar{c}$ events, are found to contribute about 1% of the total background. Background due to semi-leptonic decays of beauty hadrons is negligible [15].

To estimate the number of background events, the $\mu^+\mu^-$ invariant-mass distribution is fitted over the range 2.2–4.2 GeV/ c^2 . The fitting function with best χ^2 is found to be the sum of two Gaussians centred around the J/ψ peak, two Gaussians centred around the ψ' peak and two exponentials representing the background. For the two Gaussians describing the ψ' signal, central values and widths are fixed at the values expected from the simulation. Backgrounds under the peaks for J/ψ and ψ' are reported in Table 1, where uncertainties are obtained from the fit. Signals after background subtraction are also listed in Table 1, where the first error is statistical and the second represents the uncertainty on the background. For interactions with the primary vertex reconstructed in silicon only J/ψ candidates are considered, and the fraction of J/ψ where the true production vertex is in the target constitutes an additional background, evaluated using

the simulation.

The x_F and p_T^2 distributions for background events are estimated from entries in the side bands of the signal regions ($2.3\text{--}2.8\text{ GeV}/c^2$, $3.35\text{--}3.55\text{ GeV}/c^2$, $3.8\text{--}4.2\text{ GeV}/c^2$), with appropriate normalization. Since, within the experimental sensitivity, no dependence on nuclear effects is observed in the data (as discussed in section 4.2), distributions for different targets have been added together. Throughout the x_F and p_T^2 intervals considered the number of J/ψ events after background subtraction is about ten times the number of background events, as shown in Fig. 3.

3.4 Acceptances and systematic uncertainties

Using simulated charmonium events, J/ψ acceptances have been studied as a function of x_F and p_T^2 for different materials, as shown in Fig. 4. Acceptance curves relative to tungsten and copper are very similar, as expected. Acceptances for silicon are much lower, because of the In-target trigger condition, but curves have essentially the same shape as for the other materials. The acceptance is almost independent of p_T^2 , but depends quite strongly on x_F , decreasing at high x_F values and at $x_F \approx 0$. A similar behaviour (not shown) is observed for ψ' events. The total J/ψ and ψ' acceptance is close to 20% for tungsten and copper, and is about 2% for silicon.

The following systematic uncertainties on the total J/ψ and ψ' acceptance are considered (Table 2).

- Simulated events have been generated so as to reproduce the x_F and p_T^2 distributions observed in the data (see Fig. 3). The systematic uncertainty associated with the statistical fluctuations in the x_F and p_T^2 distributions of the data sample has been estimated to be of the order of 2% for both J/ψ and ψ' . Most of this uncertainty is due to an imprecise knowledge of the x_F spectrum, since the acceptance varies little as a function of p_T^2 .
- Several detector effects have been investigated and the resulting overall error on the total acceptance is about 2.5%. This is dominated by a contribution of 2.3% from the uncertainty in the reconstruction of the primary-vertex position. Uncertainties in the invariant-mass resolution give a systematic error of about 0.7%. The muon-identification efficiency has been checked, from data, to a high level of accuracy, and the associated small uncertainty has negligible effect on the total acceptance.
- The acceptance has been calculated assuming that J/ψ and ψ' decay isotropically, which is consistent with the absence of polarization observed in WA92 (see next section). The uncertainty on the value of the polarization corresponds to an uncertainty on the J/ψ and ψ' total acceptances of 2.2%.

4 Results

4.1 J/ψ polarization

Since the polarization magnitude, λ , for charmonium is characteristic of the production process, its measurement is of particular value for comparisons with theoretical models. In the Factorization Approach J/ψ and ψ' are produced with a large transverse polarization, expected to be in the ranges $0.31 < \lambda(J/\psi) < 0.63$ and $0.15 < \lambda(\psi') < 0.44$ [5]. In contrast, the Colour-Evaporation Model predicts an absence of polarization in J/ψ and ψ' production [9], the initial polarization of $c\bar{c}$ pairs being destroyed by multiple exchanges of soft gluons.

Fig. 5 shows the J/ψ acceptance as a function of the μ^+ emission angle, θ^* , calculated in the J/ψ rest frame with respect to the J/ψ direction in the laboratory, and the μ^+

angular distribution obtained after background subtraction and acceptance correction. Assuming the shape of the angular distribution to be described by $(1 + \lambda \cdot \cos^2 \theta^*)$, the value measured is $\lambda(J/\psi) = -0.01 \pm 0.19$, consistent with the choice of no polarization in simulated events. This result for $\lambda(J/\psi)$ is in agreement with measurements made in other π -nucleus experiments, for J/ψ production in the x_F range 0–0.8 [19, 23, 24]. On the other hand, a significant deviation from $\lambda(J/\psi) \approx 0$, has been observed in experiment E615 for $x_F > 0.95$ [25]. In one experiment [26], ψ' polarization has also been measured, and the result obtained, $\lambda(\psi') = 0.02 \pm 0.14$, is again consistent with a flat angular distribution.

4.2 Inclusive cross-sections and A -dependence

A value of the $x_F > 0$ nuclear cross-section, σ , for J/ψ and ψ' production has been calculated for each data sample using the expression $\sigma = N/(B L \epsilon)$, where N is the number of J/ψ or ψ' events after background subtraction, B is the relevant muonic branching fraction, L is the effective integrated luminosity for the data sample considered and ϵ is the signal acceptance. The J/ψ muonic branching fraction is $B(J/\psi \rightarrow \mu^+ \mu^-) = (6.01 \pm 0.19)\%$ [22]. For the ψ' , given that $B(\psi' \rightarrow \mu^+ \mu^-) = (0.77 \pm 0.17)\%$ [22] is poorly measured, use has been made of the weighted average of electronic and muonic branching fractions [22]: $B(\psi' \rightarrow \ell^+ \ell^-) = (0.84 \pm 0.06)\%$. Results obtained for the various target nuclei are reported in Table 1, and the associated statistical and systematic uncertainties are detailed in Table 2. Errors on the J/ψ cross-sections for interactions in tungsten and copper are dominated by the systematic uncertainty on the luminosity, whereas the precision of the J/ψ cross-section for silicon is limited by statistics. Errors on ψ' cross-sections are dominated by statistical uncertainties.

The dependence of the nuclear cross-section, σ , on the atomic mass number A , is usually expressed via the relation $\sigma = \sigma_0 A^\alpha$, where σ_0 is the cross-section per nucleon and α is a parameter that is almost independent of the centre-of-mass energy of the interaction [18]. In the present analysis, the α parameter for inclusive J/ψ production is determined from a fit to the J/ψ production cross-sections measured for silicon, copper and tungsten. The curve fitted to the data is shown in Fig. 6, and yields

$$\alpha(J/\psi)\{Si, Cu, W\} = 0.87 \pm 0.05 \pm 0.04 ,$$

where the first error is statistical and the second is systematic. This result is consistent with results from other experiments [27, 28, 29, 30], performed at different energies, and using a variety of target materials, not including hydrogen. The mean of the α value from these experiments is 0.87. Experiments using a hydrogen target have given α to be higher, in the range 0.95–0.98 [31, 32]. If the processes involved in J/ψ and ψ' production are the same, the values of $\alpha(\psi')$ and $\alpha(J/\psi)$ should be similar. Although the large statistical uncertainties on ψ' cross-sections mean that the result has low significance, a fit to the data points for copper and tungsten (Fig. 6) gives $\alpha(\psi')\{Cu, W\} = 0.86 \pm 0.66 \pm 0.26$. For the remainder of this section, both J/ψ and ψ' are taken to have an A -dependence characterised by $\alpha = 0.87$.

The magnitude of α is dependent on the production mechanisms involved. If partonic processes dominate, α is expected to be close to unity; if production is mainly diffractive, an α value of $2/3$ should be observed. Since diffractive processes favour particle production at high x_F and low p_T , it is interesting to study α as a function of the kinematic variables.

The measured variation of $\alpha(J/\psi)$ with x_F and p_T^2 , is shown in Fig. 7. Within the large uncertainties, there is no evident dependence of α on either x_F or p_T^2 . This suggests that any contribution from diffractive processes is small. For the remainder of this paper, the x_F and p_T^2 distributions of J/ψ and ψ' are assumed to be independent of the target material, and the differential cross-sections are obtained as the weighted average of the distributions for the different event samples.

Other experiments measuring $\alpha(J/\psi)$ for π^- interactions on nuclei other than hydrogen show α to be essentially independent of x_F [27, 28, 29], while some indicate an increase of α with p_T^2 . In experiments WA39 [31] and NA3 [32], performed using H_2 -W and H_2 -Pt targets respectively, α was found to decrease at high x_F and to increase at high p_T^2 . The NA3 collaboration concluded that a significant part (18%) of the cross-section must be due to diffractively produced J/ψ , but it should be noted that the A^α dependence does not hold for hydrogen targets [30].

Using the measured nuclear cross-sections reported in Table 1, and $\alpha = 0.87$, the cross-sections per nucleon for J/ψ and ψ' at $x_F > 0$, and their ratio, are

$$\sigma_0(J/\psi) = 216 \pm 5 \pm 13 \text{ nb/nucleon},$$

$$\sigma_0(\psi') = 28 \pm 8 \pm 11 \text{ nb/nucleon},$$

$$\sigma_0(\psi')/\sigma_0(J/\psi) = 0.13 \pm 0.05 \pm 0.01 ,$$

where correlations in systematic uncertainties have been taken into account.

The ratio of the ψ' cross-section to the J/ψ cross-section agrees with measurements from other experiments [19, 23, 33], performed at various energies. It is consistent with the value of 0.16 predicted by the Factorization Approach [5], but is significantly lower than the value of 0.48 predicted by the Colour-Evaporation Model [9].

In Fig. 8, the cross-section measurement for J/ψ production is compared with values from other experiments [34], for π^- particles of 15–530 GeV/c, interacting in different target nuclei. Data for interactions in hydrogen are excluded. All cross-sections shown are for $\alpha = 0.87$, $B(J/\psi \rightarrow \mu^+\mu^-) = (6.01 \pm 0.19)\%$ and $x_F > 0$. As far as possible, uncertainties include both statistical and systematic contributions.

A second-order polynomial is fitted to all cross-section data for $\sqrt{s} > 8.5$ GeV/c, excepting the WA92 measurement. The fit predicts $\sigma_0(J/\psi) = 230.1$ nb/nucleon for a centre-of-mass energy of 25.6 GeV, consistent with the WA92 result.

Predictions of the energy dependence of the J/ψ cross-section as calculated using the Factorization Approach [5], and as calculated using the Colour-Evaporation Model [35] are shown in Fig. 8. For each model, two choices of the factorization scale, μ , are considered: $\mu = m_c$ and $\mu = 0.8 m_c$, with m_c the charmed-quark mass. All calculations are performed to next-to-leading order [36]. The parton distribution functions used are the CTEQ3 leading-order set [37] for protons and the GRV leading-order set [38] for pions. In the Factorization Approach, which includes both colour-singlet processes and colour-octet processes, and takes into account J/ψ production in the decays of χ_c and ψ' , as well as direct J/ψ production, m_c is arbitrarily fixed at 1.5 GeV/c² [39]. Coefficients for the colour-octet processes are determined partly from fits to CDF data [40] and partly from a fit to proton-nucleon data [39].

In the case of the Colour-Evaporation Model, m_c is determined from a fit to the experimental data of Fig. 8, this giving $m_c = 1.41$ GeV/c²; the fraction of charmonium

states produced as a J/ψ is assumed to be 0.50, the value suggested by photoproduction data [35].

The Factorization Approach and the Colour-Evaporation Model reproduce well the form of the energy dependence of the J/ψ cross-section observed experimentally. However, the absolute normalization of the predictions is affected by large uncertainties, because of the dependence of the QCD calculations: the choice of m_c , renormalization scale and factorization scale, and because the parton-density functions are not well determined.

4.3 Differential cross-sections

The J/ψ invariant differential cross-sections with respect to x_F and p_T^2 are shown in Fig. 9.

The x_F spectrum is fitted with the usual parametric function, $d\sigma_0/dx_F \propto (1 - |x_F|)^n$, and also using two other functions. One of these, previously used by E705 [33], is derived from the parton-distribution functions of pions and nucleons, and has the form $d\sigma_0/dx_F \propto (1 - x_1)^{n_1}(1 - x_2)^{n_2}/(x_1 + x_2)$, where $x_{1,2} = (x^* \pm x_F)/2$, $x^* = (x_F^2 + 4M_\psi^2/s)^{1/2}$ and $n_{1,2}$ are fit parameters. The other function, similar in form and used by E706 [19], is $d\sigma_0/dx_F \propto x_1^a(1 - x_1)^k(1 - x_2)^{k+2}/(x_1 + x_2)$. Here, the two parton-distribution functions are related to a single parameter, k , and the empirical factor x_1^a improves the fit at low x_F . Fitted parameters, and values for the fit χ^2 per degree of freedom, are calculated for $x_F > 0$ (Table 3). All fitted curves accurately describe the experimental data, but a lower χ^2 per degree of freedom is found for the two fit functions derived from parton distributions. The parameter values obtained with these two fit functions are consistent with the values obtained by E705 and E706.

The p_T^2 spectrum is also fitted using different functions (Table 3). With the usual parameterization $d\sigma_0/dp_T^2 \propto e^{-bp_T^2}$, the data are well fitted only for $p_T^2 < 6$ (GeV/c)². Over the full p_T^2 range of 0–16 (GeV/c)², a better fit is obtained using the function $d\sigma_0/dp_T^2 \propto (1 + p_T^2/\beta^2)^{-c}$. The same fit functions, yielding similar parameter values, have been used to describe data obtained in other experiments at various energies [19, 23, 33].

The integral of the cross-section over the interval $0.5 < x_F < 0.8$ represents about 6% of the integral over the interval $0 < x_F < 0.8$. This result, like the absence of a clear dependence of α on x_F and p_T^2 , supports the conclusion that diffractive processes do not contribute strongly to J/ψ production.

4.4 Comparison of J/ψ production with open-charm production

The present study offers the chance of comparison, within the same experiment, of charmonium production and open-charm production, measurements of the latter having already been reported elsewhere [16, 41]. Given the multi-component trigger used, the charmonium measurements and the open-charm measurements relate to the same event samples, and so share some systematic uncertainties, for example uncertainties on luminosity values.

The acceptance ranges in x_F and p_T^2 for charmonium states and for charmed mesons are similar, so that comparison of α measurements is straightforward. The value of $\alpha(J/\psi) = 0.87 \pm 0.05 \pm 0.04$ and $\alpha(D\text{-mesons}) = 0.95 \pm 0.06 \pm 0.03$, measured at $x_F > 0$ for $D^0, \bar{D}^0, D^+, D^-, D_s^+, D_s^-$, are compatible with the experimental uncertainties. Neither for J/ψ nor for charmed mesons is evidence found that α has a dependence on either x_F or p_T^2 (Fig. 7), suggesting that in no case is there significant diffractive production.

The comparison of charmonium production with open-charm production is of particular interest as a test of the Colour-Evaporation Model. According to this model, the inclusive

J/ψ cross-section as a fraction of the open-charm cross-section is independent of the centre-of-mass energy. Taking the WA92 measurement of $\sigma_0(D\bar{D}) = 12.37 \pm 0.19 \pm 0.86$ $\mu\text{b}/\text{nucleon}$ (obtained for $D^0, \bar{D}^0, D^+, D^-, D_s^+$ and D_s^- , at positive x_F , and assuming $\alpha(D\text{-mesons}) = 1$), leads to

$$\frac{\sigma_0(J/\psi)}{\sigma_0(D\bar{D})} = (1.75 \pm 0.05 \pm 0.15) \times 10^{-2},$$

where correlated systematic uncertainties cancel. The value found is close to the value of 2×10^{-2} estimated from data of other experiments at various energies [9].

Within the framework of the Colour-Evaporation Model, charmonium production and $D\bar{D}$ production are described by the same dynamics [7, 9]. For the limiting case $m_c \approx m_D$, distributions of the kinematic variables for charmonium states and for $D\bar{D}$ are expected to be very similar. The x_F and p_T^2 spectra for $D\bar{D}$ pairs [41], from an analysis of 475 such pairs, at $x_F > 0$, is shown in Fig. 9. For comparison, the inclusive charmed-meson spectra are also shown. At high x_F , the differential cross-section with respect to x_F decreases more slowly for $D\bar{D}$ pairs than for J/ψ . The situation for the differential cross-section with respect to p_T^2 is less clear, as the data for $D\bar{D}$ pairs are rather scattered. The x_F and p_T^2 spectra for charmed mesons are softer than the corresponding spectra for J/ψ . Quantitative comparisons of the x_F and p_T^2 spectra are obtained by fitting the data for $D\bar{D}$ pairs and for charmed mesons using the same functions adopted for the J/ψ , and as explained in section 4.3. The fitted curves give a reasonable description of the data for $D\bar{D}$ pairs, but are a poor representation of the charmed-meson spectra, with fitted parameter values quite different from those for the J/ψ . All fit results, and the mean values of x_F and p_T^2 , are reported in Table 3.

Overall, the kinematical properties of $D\bar{D}$ pairs and of J/ψ mesons are less similar than would be expected from the Colour-Evaporation Model. However, the Model should be corrected for the use of the approximation $m_c \approx m_D$, and to take into account the energy carried away in the soft gluon emission in the final-state particles.

5 Conclusions

Charmonium production by 350 GeV/ c negative pions incident on tungsten, copper and silicon nuclei has been studied in the WA92 experiment. In total, $(1096 \pm 34 \pm 5)$ J/ψ and $(19.6 \pm 6.6 \pm 3.4)$ ψ' signal events are reconstructed in the range $x_F > 0$. Inclusive J/ψ and ψ' cross-sections and differential J/ψ cross-sections are measured as functions of x_F and p_T^2 . For J/ψ events, the cross-section dependence on the nuclear mass has been measured from a fit to the data for different materials. The result obtained is $\alpha(J/\psi)\{Si, Cu, W\} = 0.87 \pm 0.05 \pm 0.04$. Fixing α at 0.87, the inclusive cross-section per nucleon for J/ψ production is $\sigma_0(J/\psi) = 216 \pm 5 \pm 13$ nb. This value of $\sigma_0(J/\psi)$ is consistent, within errors, with values obtained in other experiments at similar energies. No indication of diffractive processes contributing substantially to J/ψ production has been observed. The ratio of the ψ' cross-section to the J/ψ cross-sections is measured to be $\sigma_0(\psi')/\sigma_0(J/\psi) = 0.13 \pm 0.05 \pm 0.01$, consistent with the value expected from the Factorization Approach, and much lower than the value predicted by the Colour-Evaporation Model. However, the non-observation of polarization in the J/ψ angular distribution, $\lambda(J/\psi) = -0.01 \pm 0.19$ being measured, is in agreement with the Colour-Evaporation Model but is incompatible with the predictions of the Factorization Approach. Similarities have been observed between J/ψ kinematics and the kinematics of

$D\bar{D}$ pairs, but results are not completely consistent with the expectations of the Colour-Evaporation Model. The ratio of the J/ψ cross-section to the charmed-meson cross-section, measured under the same experimental conditions and using the same data samples, is $\sigma_0(J/\psi)/\sigma_0(D\bar{D}) = (1.75 \pm 0.05 \pm 0.15) \times 10^{-2}$. This result, which is consistent with the prediction of the Colour-Evaporation Model, can be relevant for testing the validity of other charmonium production models.

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Table 1: Luminosities, numbers of events, total efficiencies and cross-sections for J/ψ and ψ' , for interactions on tungsten, copper and silicon. Cross-sections per nucleon are calculated for $\alpha = 0.87$. The J/ψ background relative to silicon includes 9.4 ± 2.4 events from combinatorial background and 5.5 ± 0.4 events where the interaction is in the target.

	W	Cu	Si
Luminosity (nb/nucleus) $^{-1}$	1.35 ± 0.11	7.78 ± 0.39	12.80 ± 0.87
J/ψ candidates	346	763	72
J/ψ background	25.8 ± 2.5	44.4 ± 3.8	14.9 ± 2.4
J/ψ signal	$320 \pm 19 \pm 3$	$719 \pm 28 \pm 4$	$57.1 \pm 8.5 \pm 2.4$
$\epsilon_{J/\psi}$ (%)	$19.87 \pm 0.17 \pm 0.81$	$19.06 \pm 0.11 \pm 0.74$	$2.04 \pm 0.05 \pm 0.08$
$\sigma(J/\psi)$ ($\mu\text{b/nucleus}$)	$19.8 \pm 1.2 \pm 1.9$	$8.07 \pm 0.31 \pm 0.58$	$3.63 \pm 0.54 \pm 0.36$
$\sigma_0(J/\psi)$ (nb/nucleon)	$212 \pm 12 \pm 20$	$218 \pm 8 \pm 16$	$200 \pm 30 \pm 20$
ψ' candidates	13	30	—
ψ' background	6.70 ± 0.70	16.7 ± 3.3	—
ψ' signal	$6.3 \pm 3.6 \pm 0.7$	$13.3 \pm 5.5 \pm 3.3$	—
$\epsilon_{\psi'}$ (%)	$21.14 \pm 0.39 \pm 0.79$	$19.41 \pm 0.20 \pm 0.75$	—
$\sigma(\psi')$ ($\mu\text{b/nucleus}$)	$2.6 \pm 1.5 \pm 0.4$	$1.05 \pm 0.43 \pm 0.28$	—
$\sigma_0(\psi')$ (nb/nucleon)	$28 \pm 16 \pm 4$	$28 \pm 12 \pm 8$	—

Table 2: Statistical and systematic uncertainties on the J/ψ and ψ' cross-sections, for interactions on tungsten, copper and silicon.

Source of uncertainty:	J/ψ			ψ'	
	$\Delta\sigma_W$ (%)	$\Delta\sigma_{Cu}$ (%)	$\Delta\sigma_{Si}$ (%)	$\Delta\sigma_W$ (%)	$\Delta\sigma_{Cu}$ (%)
Statistics	5.8	3.8	14.9	57.2	41.2
Luminosity	7.8	5.0	6.8	7.8	5.0
Branching fraction for decay to $\mu^+\mu^-$	3.2	3.2	3.2	7.1	7.1
Background	0.8	0.5	4.3	11.1	24.8
Acceptance:					
x_F dependence	2.4	2.3	2.3	1.8	2.2
detector effects	2.5	2.3	2.4	2.5	2.3
polarization	2.2	2.2	2.2	2.2	2.2
simulation statistics	0.9	0.6	2.6	1.9	1.0
Total systematics	9.4	7.1	9.8	15.9	26.6

Table 3: Results of fitting the spectra of x_F and p_T^2 for J/ψ , for $D\bar{D}$ pairs and for charmed mesons ($D^0, \bar{D}^0, D^+, D^-$). Average values of x_F and p_T^2 are also reported.

		J/ψ	$D\bar{D}$ pairs	$D^0, \bar{D}^0, D^+, D^-$
$\langle x_F \rangle$		0.22	0.23	0.17
$\langle p_T^2 \rangle$ (GeV/c) ²		1.96	1.94	1.36
$d\sigma_0/dx_F$ fit function				
$\propto (1 - x_F)^n$	$\chi^2/d.o.f.$ n	2.1 2.74 ± 0.27	1.6 1.80 ± 0.22	8.3 4.69 ± 0.57
$\propto (1 - x_1)^{n_1}(1 - x_2)^{n_2}/(x_1 + x_2)$	$\chi^2/d.o.f.$ n_1 n_2	0.5 2.50 ± 0.21 7.8 ± 4.2	1.7 1.80 ± 0.20 11.9 ± 1.3	4.2 5.0 ± 2.9 8.5 ± 5.8
$\propto x_1^a(1 - x_1)^k(1 - x_2)^{k+2}/(x_1 + x_2)$	$\chi^2/d.o.f.$ a k	0.5 0.27 ± 0.28 2.67 ± 0.63	1.4 0.23 ± 0.29 1.43 ± 0.51	5.5 0.30 ± 0.12 5.73 ± 0.52
$d\sigma_0/dp_T^2$ fit function				
$\propto e^{-bp_T^2}$ ($p_T^2 < 6$)	$\chi^2/d.o.f.$ b (GeV/c) ⁻²	1.6 0.64 ± 0.03	2.2 0.51 ± 0.04	3.4 0.86 ± 0.02
$\propto (1 + p_T^2/\beta^2)^{-c}$	$\chi^2/d.o.f.$ β (GeV/c) c	0.6 2.09 ± 0.20 4.12 ± 0.50	2.9 1.80 ± 0.10 3.47 ± 0.22	2.6 2.27 ± 0.07 6.24 ± 0.29

Figure 1: View (x - z projection) of the experimental setup (upper picture), including target (T), silicon-microstrip detectors (SMD), multi-wire proportional chambers (MW), butterfly-shaped hodoscopes (BH), drift chambers (D), electromagnetic calorimeter (EC), muon filter comprising beam dump (BD) and resistive-plate chambers (RPC). Enlargements of the interaction region (lower pictures), show Beam Hodoscope (BmH), In-target detector (IT), Decay Detector (DkD) and Vertex Detector (VxD).

Figure 2: Invariant-mass distributions for $\mu^+\mu^-$ pairs (points with error bars) after application of the selection criteria described in the text, for interactions on different materials. The distributions for $\mu^\pm\mu^\pm$ pairs (shaded histograms) give an indication of the combinatorial background. Enlargements of the mass region around the ψ' mass are shown as inserts, with a bin size of 50 MeV/c². The superimposed curves represent the results of fits to the data with a sum of four Gaussians (two for the J/ψ signal and two for the ψ' signal) and two exponentials.

Figure 3: The x_F distribution (upper plot) and p_T^2 distribution (lower plot) for J/ ψ events (after background subtraction but without acceptance correction), all data samples combined; and for background events in the signal region of the $\mu^+\mu^-$ invariant-mass spectrum. The background distributions are obtained from entries in the side bands, appropriately normalized. The histogram represents the simulation tuned to the data. Errors are statistical only.

Figure 4: Acceptance as a function of x_F (upper plot) and p_T^2 (lower plot) for simulated J/ψ events, with production in silicon, copper and tungsten. Efficiencies for silicon are multiplied by 10, to allow the shapes of the curves to be compared with those for copper and tungsten. The statistical uncertainty on the acceptance varies from about 2%, for $x_F = 0.5$ and for $p_T^2 = 0.5 \text{ GeV}/c^2$, to about 20% for $x_F = 0.9$ and for $p_T^2 = 13.5 \text{ GeV}/c^2$.

Figure 5: Relative J/ψ acceptance (continuous line), as a function of the μ^+ emission angle, θ^* , calculated in the J/ψ rest frame with respect to the J/ψ direction in the laboratory frame; and (shaded circles) the distribution of the μ^+ emission angles obtained after background subtraction and acceptance correction, for all data samples combined. The dashed curve superimposed on the measured angular distribution represents the result of a fit of the function $dN/d \cos \theta^* \propto (1 + \lambda \cdot \cos^2 \theta^*)$, yielding $\lambda = -0.01 \pm 0.19$ as the polarization magnitude.

Figure 6: Measurements of J/ψ cross-section (shaded circles) and ψ' cross-section (unshaded circles) in different materials. Uncertainties include statistical and systematic contributions. The continuous line represents the result of a fit to the J/ψ data of $\sigma = \sigma_0 A^\alpha$, where σ_0 and α are fit parameters. The dashed line is the interpolation of ψ' data points.

Figure 7: Measurements of α as a function of x_F (left) and p_T^2 (right) for J/ψ (shaded circles) and for charmed mesons (unshaded circles), all data from WA92 (as discussed in section 4.4). Uncertainties include statistical and systematic contributions.

Figure 8: Cross-section per nucleon for J/ψ production in π^- interactions as measured in different experiments, for various targets and energies. Where necessary, published data have been adjusted so as to represent the case $\alpha = 0.87$, $B(J/\psi \rightarrow \mu^+ \mu^-) = 6.01 \pm 0.19\%$ and $x_F > 0$. Statistical and systematic uncertainties have been added quadratically. The continuous line represents a fit to the data of a second-order polynomial. Predictions from the Colour-Evaporation Model (dotted) and from the Factorization Approach (dashed) are shown for two choices of the factorization scale: $\mu = m_c$ (lower) and $\mu = 0.8 m_c$ (upper), where m_c is fixed at $1.41 \text{ GeV}/c^2$ for the Colour-Evaporation Model and at $1.5 \text{ GeV}/c^2$ for the Factorization Approach.

Figure 9: Invariant differential cross-sections with respect to x_F (upper plot) and p_T^2 (lower plot), for J/ψ , for DD pairs and for D mesons. The uncertainties shown are statistical only. The curves represent the results of fits to the J/ψ data of $d\sigma_0/dx_F \propto (1-x_1)^{n_1}(1-x_2)^{n_2}/(x_1+x_2)$ (upper plot) and $d\sigma_0/dp_T \propto (1+p_T^2/\beta^2)^{-c}$ (lower plot).