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COMPATIBILITY OF THE DUAL POMERON WITH UNITARITY AND
THE ABSENCE OF GHOSTS IN THE DUAL RESONANCE MODEL

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ABSTRACT

It is shown that the number of states coupling in the Veneziano model when the space time dimension, d , is 26 ($d = 10$ for the Neveu-Schwarz model) is reduced in such a way that the Pomeron is probably a pole. A simple consequence of this counting is the absence of ghosts for $d \leq 26$ ($d \leq 10$ for Neveu-Schwarz).

The Veneziano n point function ¹⁾, as originally proposed, suffered from a number of fundamental defects, even as the Born term of a dual perturbation series. Striking amongst these was the presence of ghosts. Virasoro ²⁾ pointed out that an infinity of gauge conditions appear when the intercept $\alpha(0)=1$, and in the two years since his paper considerable evidence that these conditions indeed remove the ghosts has been amassed ^{3),4)}. Further, other dual resonance models incorporating similar algebraic structures for ghost elimination have been proposed ^{5),6)} and versions of these avoid the presence of tachyons ⁷⁾.

A major obstacle to interpreting these models as Born terms of a complete theory is the Pomeron like singularity generated in dual loop diagrams, which being a cut in the momenta, violates unitarity. Sometime ago, Lovelace ⁸⁾ pointed out that this Pomeron would become a pole if the dimension of space time, d , were 26, and there were two infinite sets of gauge conditions for the physical states to satisfy. At first sight this second set of conditions is missing and the present work originated in a closer examination of this situation. We found that, remarkably, precisely when $d=26$ so many of the physical states (satisfying the Virasoro gauge conditions) become null states (i.e., zero norm states orthogonal to all physical states) that, effectively, an extra infinite set of gauge conditions appears. The number of states which couple is then just that corresponding to 24 dimensional oscillators. Thus, provided there are no unexpected modifications to the loop integrand and the Pomeron factorizes consistently, compatibility with unitarity will be regained.

It is a simple corollary of these results that there are no ghosts in the Veneziano model if and only if $d \leq 26$. Since Del Giudice, Di Vecchia and Fubini (DDF) ⁹⁾ have shown how to construct an orthonormal set of positive norm physical states whose number is that which would be given by $(d-2)$ dimensional oscillators, these are all that couple if $d=26$. The absence of ghosts for $d < 26$ follows a fortiori because then the d dimensional space may be regarded as a subspace of the 26 dimensional one and the 26 dimensional gauge operators equal the d dimensional ones plus operators which are zero on the subspace.

We first prove our results taking advantage of the explicit DDF construction. Secondly, we give a more abstract version of the proof which extends immediately to the Neveu-Schwarz model. Here the critical dimension is reduced to 10. This might induce one to hope that it will be four in some more realistic model.

The Virasoro conditions (for the original model with $\alpha(0)=1$) can be written

$$L_n |\psi\rangle = 0 \quad (1)$$

where

$$[\alpha_m^\mu, \alpha_n^\nu] = m g^{\mu\nu} \delta_{m,-n}, \quad \mu, \nu = 1, 2, \dots, d \quad *)$$

$$L_n = \frac{1}{2} : \sum_{r=-\infty}^{\infty} \alpha_{-r} \alpha_{n+r} :$$

$\alpha_m^\mu = \alpha_{-m}^{\mu\dagger}$, $L_n = L_{-n}^\dagger$, $\alpha_0 = p_0$, the momentum operator. The states which couple also satisfy the mass shell condition

$$(L_0 - 1) |\psi\rangle = 0 \quad (2)$$

and the L_n satisfy the Virasoro algebra

$$[L_m, L_n] = (m-n) L_{m+n} + \frac{d}{12} m(m^2-1) \delta_{m,-n} \quad (3)$$

For a given value of $R = L_0 - p_0^2$, N , say, one would expect ⁴⁾ that the number of solutions of (1) and (2) is the number of states created by $(d-1)$ dimensions of operators $A_n^{i\dagger}$ (with that value of R), $T^{d-1}(N)$, rather than the full d dimensions. As progress towards this the DDF construction ⁹⁾ gives $T^{d-2}(N)$ orthonormal positive norm solutions by introducing operators A_n^i , $i=1, \dots, d-2$, $n=1, 2, \dots$, such that ¹⁰⁾

$$\begin{aligned} [L_n, A_m^i] &= 0 & [A_m^i, A_n^j] &= m \delta_{m,-n} \delta_{ij} \\ [R, A_m^i] &= -m A_m^i & A_m^i &= A_{-m}^{i\dagger} \end{aligned} \quad (4)$$

Our proof of the absence of ghosts is an immediate consequence of the following

Theorem When the dimension $d=26$ any solution of Eqs. (1) and (2) is of the form $|\psi\rangle = |f\rangle + |ns\rangle$ where $|f\rangle$ is in the space spanned by the DDF states and $|ns\rangle$ is a null spurious state. Thus only the DDF states couple.

*) $-g^{00} = g^{11} = g^{22} = g^{33} = \dots = 1.$

[Null means that the state is orthogonal to all other physical states, including itself and spurious means it is a linear combination of states of the form $L_{-n}|\chi\rangle$, $n > 0$.]

We approach the proof of the theorem by two lemmas. The states $|f\rangle$ of DDF are constructed in a specific kinematic configuration, such that if R has eigenvalue N , p_0 has eigenvalue $p+Nk$, where $p=(0,0,0,1)$ and $k=(\frac{1}{2},0,0,-\frac{1}{2})^*$. In addition to (1) and (2), the states $|f\rangle$ satisfy

$$K_n |f\rangle = 0 \quad n = 1, 2, \dots \quad (5)$$

where $K_n = k \cdot \alpha_n$, $K_n = K_{-n}^\dagger$, $[K_m, K_n] = 0$, $[L_m, K_n] = -nK_{m+n}$.

We extend the states $|f\rangle$ into a basis for the whole Hilbert space of states by the following

Lemma A If $|f, M, \kappa\rangle$ is an orthonormal basis for the DDF states with $R=M$ (κ being a degeneracy label) the states

$$L_{-1}^{\lambda_1} L_{-2}^{\lambda_2} \dots L_{-n}^{\lambda_n} K_{-1}^{\mu_1} \dots K_{-m}^{\mu_m} |f, M, \kappa\rangle$$

form a basis for the subspace of the whole Hilbert space with $R=M + \sum \lambda_r r + \sum \mu_s s$.

Proof The proof is a familiar type of argument [cf., Ref. 4] and is described in the Appendix.

The special nature of $d=26$ and the mass shell condition is brought out by the second Lemma.

Lemma B Let $\mathcal{S}^{\ell_0}$ be the space of spurious states with $L_0 = \ell_0$. Then L_1 defines a map $\mathcal{S}^1 \rightarrow \mathcal{S}^0$ and $\tilde{L}_2 = L_2 + \frac{3}{2}L_1^2$ defines a map $\mathcal{S}^1 \rightarrow \mathcal{S}^{-1}$ provided that $d=26$.

*) We can further extend the construction of DDF to give off-mass-shell physical states [i.e., relax (2)] by changing the momentum of the tachyon vacuum, from which $|f\rangle$ is created by A_{-n}^i operators from p to $p-\beta k$; then

$$L_0 |f\rangle = (1 + \frac{1}{2}\beta) |f\rangle.$$

Proof

Because the L_{-n} ($n \geq 1$) are generated by L_{-1}, L_{-2} , any $|s\rangle \in \mathcal{S}^1$ is of the form

$$|s\rangle = L_{-1}|\lambda, 0\rangle + \tilde{L}_{-2}|\lambda, -1\rangle$$

where $L_0|\lambda, n\rangle = n|\lambda, n\rangle$ and $\tilde{L}_{-2} = \tilde{L}_2^\dagger$. Then

$$L_1|s\rangle = L_{-1}L_1|s\rangle + \tilde{L}_{-2}L_1|\lambda, -1\rangle \in \mathcal{S}^0$$

and, similarly $\tilde{L}_2|s\rangle = |s'\rangle + (\frac{d}{2} - 13)|\lambda, -1\rangle$ where $|s'\rangle \in \mathcal{S}^{-1}$.

So, $\tilde{L}_2|s\rangle \in \mathcal{S}^{-1}$ if and only if $d=26$.

We can now proceed to the conclusion of the proof.

Proof of the Theorem

Let $|\psi\rangle$ satisfy (1) and (2), and $\mathcal{H}^{\ell_0}$ be the space of states with basis $K_{-1}^{\mu_1} \dots K_{-m}^{\mu_m} |f, M, \kappa\rangle$ satisfying $L_0 = \ell_0$. Then, by Lemma A, we can write

$$|\psi\rangle = |\varphi\rangle + |s\rangle \quad |\varphi\rangle \in \mathcal{H}^1, \quad |s\rangle \in \mathcal{S}^1$$

Further, by Lemma B, and the algebra of the L 's and K 's, L_1 maps $\mathcal{S}^1$ into $\mathcal{S}^0$ and $\mathcal{H}^1$ into $\mathcal{H}^0$. As these are independent spaces $L_1|\psi\rangle = 0$ implies $L_1|\varphi\rangle = L_1|s\rangle = 0$. Similarly $\tilde{L}_2|\varphi\rangle = \tilde{L}_2|s\rangle = 0$, and, as $L_1, \tilde{L}_2$ generate all the L_n , $n \geq 1$,

$$L_n|\varphi\rangle = L_n|s\rangle = 0 \quad n=1, 2, \dots \quad (6)$$

So $|s\rangle$ is both physical and spurious and consequently a null physical state; it remains to show that $|\varphi\rangle$ is a DDF state. To do this, express

$$|\varphi\rangle = \sum C_{\{\mu\}}^{\kappa M} K_{-1}^{\mu_1} K_{-2}^{\mu_2} \dots K_{-m}^{\mu_m} |f, M, \kappa\rangle$$

For given κ, M , we can order the $C_{\{\mu\}}^{\kappa M}$ by considering first those involving $K_{-m}^{\mu_m}$ for the largest m , and amongst these first those with the largest power μ_m , and then amongst these, order similarly by $K_{-m+1}^{\mu_{m-1}}$ if necessary, and so on. Then if $C_{\{\lambda\}}^{\kappa M}$ denotes the first term in this sense $L_1^{\lambda_1}, L_2^{\lambda_2}, \dots, L_m^{\lambda_m}|\varphi\rangle$ involves only the first terms, with non-zero coefficients. As those are independent for different κ, M , we can prove each successive $C_{\{\mu\}}^{\kappa M} = 0$ unless every $\mu_i = 0$. In other words, $|\varphi\rangle$ is a DDF state and the theorem is proved.

[The decomposition used in proving this theorem is illustrated by the state $|\psi\rangle = \{\tilde{L}_{-2} + (\frac{d}{2} - 13)(K_{-2} + 2K_{-1}^2)\}|0\rangle$ at level $R=2$, which is physical for each value of d , having norm $\langle\psi|\psi\rangle = (13 - \frac{d}{2})$.]

Modifications and extensions to the Neveu-Schwarz model

While it was a strong motivation in this work, the Lemmas and Theorem can be formulated without explicit reference to the DDF construction. Apart from the advantage of making the argument more self-contained this reformulation immediately extends to the Neveu-Schwarz model where a detailed construction of $(d-2)$ dimensions of physical states is yet to be found. This rephrasing is effected by using as defining equations for the DDF states (1) and (5) and replacing Lemma A by the following two results.

Lemma A' Suppose $|f\rangle$ satisfies

$$K_n |f\rangle = L_n |f\rangle = 0, \quad R |f\rangle = M |f\rangle \quad (7)$$

and that $\langle f|f\rangle \neq 0$. Then, for a given such $|f\rangle$, the states

$$|\{\lambda, \mu\}, f\rangle = L_{-1}^{\lambda_1} L_{-2}^{\lambda_2} \dots L_{-n}^{\lambda_n} K_{-1}^{\mu_1} \dots K_{-m}^{\mu_m} |f\rangle \quad (8)$$

$$\sum r \lambda_r + \sum s \mu_s = N - M > 0$$

form a linearly independent span of some subspace of the states with $R=N$. Further this subspace contains no solution of $L_n |\psi\rangle = K_n |\psi\rangle = 0$.

Proof See Appendix.

Lemma A'' The space $\mathcal{F}^M$ of states satisfying conditions (7) is positive definite and has dimension $T^{d-2}(M)$. If $\{|f, M, \nu\rangle\}$ is an orthonormal basis for $\mathcal{F}^M$ the states $|\{\lambda, \mu\}, f, M, \nu\rangle$, defined in (8), give a basis for the states with $R = \sum r \lambda_r + \sum s \mu_s + M = N$, and, as N varies, for the whole Hilbert space.

Proof The proof is by induction on N [cf., Ref. 4]. First construct the space $\mathcal{F}^1$ as the orthogonal complement of $L_{-1}|0\rangle, K_{-1}|0\rangle$ in the space with $R=1$. Now, for given N , consider the subspace $\mathcal{G}^N$ spanned by all states of the form $|\{\lambda, \mu\}, f, M, \nu\rangle, \sum r \lambda_r + \sum s \mu_s = N - M > 0$. Because the states $|\{\lambda, \mu\}, f, M', \nu\rangle, \sum r \lambda_r + \sum s \mu_s + M' = N' < N$ form a basis for the states with $R=N'$ by the inductive hypothesis, a state $|\psi\rangle$ with $R=N$ is orthogonal to $\mathcal{G}^N$ if and only if $L_n |\psi\rangle = K_n |\psi\rangle = 0, n > 0$, that is, it is in $\mathcal{F}^N$. Now by Lemma A', $\mathcal{G}^N$ contains no states in $\mathcal{F}^N$; so that it has no states which are orthogonal to it, i.e., no null states. Thus $\mathcal{G}^N$ together with its orthogonal complement $\mathcal{F}^N$ span the states with $R=N$. Also $\mathcal{F}^N$ has no null states because $\mathcal{G}^N$ has none; $\mathcal{F}^N$ has no negative norm states because of the conditions $K_n |f\rangle = 0$, and so it is positive definite. The dimension $T^{d-2}(N)$ follows by simple counting.

$\mathfrak{F}^M$ is just the subspace of DDF states and the Theorem now follows even more simply : Lemma A" now enables us to write a $|\psi\rangle$ satisfying (1) and (2) as $|\phi\rangle + |s\rangle$, as before, and Lemma B to infer $L_n|s\rangle = L_n|\phi\rangle = 0$. Now as $|\phi\rangle$ satisfies $K_n|\phi\rangle = 0$ it is in $\mathfrak{F}^M$ for some value of M and the proof is complete.

To extend the result to the Neveu-Schwarz model, where (1) is supplemented by $G_m|\psi\rangle = 0$, $m = \frac{1}{2}, \frac{3}{2}, \dots$, we define a subspace $\mathfrak{F}'^M$ by (7) and the additional conditions $G_m|\psi\rangle = H_m|\psi\rangle = 0$, $m = \frac{1}{2}, \frac{3}{2}, \dots$, where $H_m = k \cdot b_m$. Corresponding to Lemma A' we prove that, for fixed $|f\rangle \in \mathfrak{F}'^M$, $\langle f|f\rangle \neq 0$

$$G_{-\frac{1}{2}}^{\epsilon_1} G_{-\frac{3}{2}}^{\epsilon_2} \dots G_{-a+\frac{1}{2}}^{\epsilon_a} L_{-1}^{\lambda_1} \dots L_{-n}^{\lambda_n} H_{-\frac{1}{2}}^{\delta_1} \dots H_{-b+\frac{1}{2}}^{\delta_b} K_{-1}^{\mu_1} \dots K_{-n}^{\mu_n} |f\rangle$$

$$\epsilon_i, \delta_i = 0 \text{ or } 1, \quad \sum \epsilon_i (i - \frac{1}{2}) + \sum \delta_j (j - \frac{1}{2}) + \sum r \lambda_r$$

$$+ \sum s \mu_s = N - M > 0 \quad (9)$$

forms a linearly independent span of some subspace which contains no states in $\mathfrak{F}'^N$. Then replace Lemma A" by the result that $\mathfrak{F}'^N$ is positive definite, its dimension given by $(d-2)$ sets of α and b oscillators, and that the states (9) can be used to construct a basis for the whole space. Lemma B is replaced by a similar result for $G_{\frac{1}{2}}$ and $\tilde{G}_{\frac{3}{2}} = G_{\frac{3}{2}} + 2L_1 G_{\frac{1}{2}}$ for $d=10$ and a result strictly analogous to the main theorem follows at this critical dimension.

Comments

A simple structure has emerged for the physical states in the Veneziano model at the critical dimension, $d=26$. Then just the $T^{d-2}(N)$ states constructed by DDF couple and it seems that the Pomeron becomes a pole, unfortunately with intercept two. Exactly the same thing happens in the Neveu-Schwarz model at $d=10$, the Pomeron even having the same intercept ¹¹⁾.

The infinite momentum limit is particularly attractive when d reaches the critical value, because the $(d-2)$ dimensions of states which survive in this limit are precisely those of DDF ($A_n^i \rightarrow \alpha_n^j$, $j = i+1$ or i as $i \leq 3$). The model really does look like a non-relativistic oscillator in the $(d-2)$ transverse dimensions. We may hope that this structure may result in an increased understanding of the spectra of dual resonance models and in particular a construction of multiloop amplitudes which clearly maintains duality and the absence of ghosts.

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A P P E N D I X

To prove Lemma A we first note that, because of the number of states, if we prove independence, the states form the required basis. Next if $A_{\kappa M}^\dagger$ is the combination of the A_{-n}^i needed to create $|f, M, \kappa\rangle$ from the vacuum, by acting with various $A_{\kappa M}$ we separate the various $|f, M, \kappa\rangle$ states in any linear sum. As $A_{\kappa M}|f, M, \kappa\rangle$ is proportional to the vacuum, the problem is reduced to providing the independence of

$$L_{-1}^{\lambda_1} L_{-2}^{\lambda_2} \dots L_{-n}^{\lambda_n} K_{-1}^{\mu_1} \dots K_{-m}^{\mu_m} |0\rangle$$

where the vacuum state has the appropriate momentum $p' = p - \beta R$. This can be achieved as in Ref. 4) by considering first those states with the highest excitations of the lowest modes $L_{-1}^{\lambda_1} K_{-1}^{\mu_1} |0\rangle$ and noticing that they contain a part $(p' \alpha_{-1})^{\lambda_1} (k \alpha_{-1})^{\mu_1}$ which cannot be cancelled, and so on.

The independence part of Lemma A' can be proved by the same method. Now suppose $|\psi\rangle$ is in the subspace of Lemma A' and $L_n |\psi\rangle = K_n |\psi\rangle = 0$. For each of the basis states (8) consider the numbers $\lambda = \sum r \lambda_r$ and $\mu = \sum s \mu_s$. Under any K_n these basis states are mapped into states for which μ is not decreased. In fact, under K_1 only those terms arising from $[K_1, L_{-1}] = -\frac{1}{2}$ keep this minimal μ value. So using the independence just proved, we can infer the absence first of the terms in $|\psi\rangle$ with μ least and $\lambda_1 \neq 0$, and so eventually infer that $|\psi\rangle$ has only components with $\lambda_i = 0$. Then, as in the proof of the theorem we can infer each $\mu_i = 0$, which is a contradiction.

R E F E R E N C E S

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