CCFM prediction for $F_2$ and forward jets at HERA

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Predictions of the CCFM evolution equation for $F_2$ and forward jets at HERA energies are obtained from a modified version of the Monte Carlo program SMALLX. The treatment of the non-Sudakov form factor $\Delta_{ns}$ is discussed as well as the effect of the so called "consistency constraint". For the first time a good description of $F_2$ and the forward jet data is obtained from the CCFM equation.

1. Introduction

The parton evolution at small values of $x$ is believed to be best described by the CCFM evolution equation [1], which for $x \to 0$ is equivalent to the BFKL evolution equation [2] and for large $x$ reproduce the standard DGLAP equations. The CCFM evolution equation takes coherence effects of the radiated gluons into account via angular ordering. On the basis of this evolution equation, the Monte Carlo program SMALLX [3] has been developed already in 1992. In 1997 the Linked Dipole Chain [4] was developed as a reformulation of the original CCFM equation. Predictions of the CCFM equation for hadronic final state properties were studied in [5] paying special attention to non-leading effects. All approaches [4,5] found a good description of $F_2$ but failed completely to describe the forward jet data of HERA experiments, which are believed to be a signature for new small $x$ parton dynamics.

In the following I discuss the treatment of the non-Sudakov form factor $\Delta_{ns}$ as well as the effects of the so-called “consistency constraint”, which was found to be necessary to include non-leading contributions to the BFKL equation [6]. I show that a good description of $F_2$ and the forward jet data can be achieved.

2. Implementation of CCFM in SMALLX

The implementation of the CCFM [1] parton evolution in the forward evolution Monte Carlo program SMALLX is described in detail in [3]. Here I only concentrate on the basic ideas and discuss the treatment of the non-Sudakov form factor.

The initial state gluon cascade is generated in a forward evolution approach from a starting distribution of the $k_t$ unintegrated gluon distribution according to:

$$xG_0(x, k_t^2) = N \cdot (1 - x)^4 \cdot \exp\left(-k_t^2/k_0^2\right) \tag{1}$$

where $N$ is a normalization constant and $k_0^2 = 1$ GeV$^2$. Gluons then can branch into a virtual ($t$-channel) gluon $k_{i+1}$ and a final gluon $q_{i+1}$ according to the CCFM splitting function [1]:

$$dP_i = \hat{P}_g^i(z_i, q_{t_i}^2, k_{t_i}^2) \cdot \Delta_s dz_i \frac{d^2 q_{t_i}^2}{q_{t_i}^2} \cdot \Theta(q_{t_i}^2 - z_i q_{t_{i-1}}^2) \cdot \Theta(1 - z_i - \epsilon_i) \tag{2}$$

with $q_{t_i}^2 = q_{t_{i-1}}^2/(1 - z_i)$ being the rescaled transverse momentum, $z_i = x_i/x_{i-1}$, $\epsilon_i = Q_0/q_{t_i}$ being a collinear cutoff to avoid the $1/(1-z)$ singularity and $\Delta_s$ being the Sudakov form factor:

$$\Delta_s(q_{t_i}^2, z_i q_{t_{i-1}}^2) = \exp\left(-\int\frac{d^2 q_{t_i}^2}{q_{t_i}^2} \int dz \frac{\alpha_s}{1 - z}\right) \tag{3}$$

which at an inclusive level cancels against the $1/(1-z)$ collinear singularity and is used to gen-
erate $q_{ti}$. The gluon splitting function $\tilde{P}^i_g$ is given by:

$$\tilde{P}^i_g = \frac{\alpha_s(q_i^2)}{1 - z_i} + \frac{\alpha_s(k_t^2)}{z_i} \Delta_{ns}(z_i, q_{ti}, k_t^2)$$

(4)

with the non-Sudakov form factor $\Delta_{ns}$ being defined as:

$$\log \Delta_{ns} = -\alpha_s(k_t^2) \int \frac{dz'}{z'} \int \frac{dq^2}{q^2}$$

$$\cdot \Theta(k_{ti} - q) \Theta(q - z'q_{ti})$$

(5)

which gives for the region $k_t^2 > z_i q_{ti}^2$:

$$\log \Delta_{ns} = -\alpha_s(k_t^2) \log \left( \frac{1}{z_i} \right) \log \left( \frac{k_t^2}{z_i q_{ti}^2} \right)$$

(6)

The constraint $k_t^2 > z_i q_{ti}^2$ is often referred to as the “consistency constraint” [6]. However the upper limit of the $z'$ integral is constraint by the $\Theta$ functions in eq.(5) by $^1$: $z_i \leq z' \leq \min(1, k_{ti}/q_{ti})$, which results in the following form of the non-Sudakov form factor [7]:

$$\log \Delta_{ns} = -\alpha_s(k_t^2) \log \left( \frac{z_0}{z_i} \right) \log \left( \frac{k_t^2}{z_0 z_i q_{ti}^2} \right)$$

(7)

where

$$z_0 = \begin{cases} 
1 & \text{if } k_{ti}/q_{ti} > 1 \\
 k_{ti}/q_{ti} & \text{if } z < k_{ti}/q_{ti} \leq 1 \\
z & \text{if } k_{ti}/q_{ti} \leq z 
\end{cases}$$

(8)

giving no supression in the region $k_{ti}/q_{ti} \leq z$ we have $\Delta_{ns} = 1$. The Monte Carlo program SMALLX [3] has been modified to include the non-Sudakov form factor according to eq.(7) and the scale in $\alpha_s$ was changed to $k_t^2$ in the cascade and in the matrix element. To avoid problems at small $k_t^2$, $\alpha_s(k_t^2)$ is restricted to $\alpha_s(k_t^2) \leq 0.6$.

The “consistency constraint” was introduced to account for next-to-leading effects in the BFKL equation, and which was found [6] to simulate about 70% of the full next-to-leading corrections to the BFKL equation. Since in LO BFKL the true kinematics of the branchings are neglected, they can be interpreted as next-to-leading effects, and this constraint is often also called “kinematic constraint”. In the CCFM equation energy and momentum conservation is already included at LO, and it is not clear, whether the arguments coming from BFKL also apply to CCFM. In the following the effects of the $1/(1 - z)$ terms and the “consistency constraint” are studied in more detail.

3. Predictions for $F_2$ and forward jets at HERA

With the modifications on the treatment of the non-Sudakov form factors described above, the predictions of the CCFM evolution equation, as implemented in the program SMALLX [3], for the structure function $F_2(x, Q^2)$ are shown in Fig. 1 without applying any additional “consistency constraint”. Here the following parameters were used: $Q_0 = 1.1$ GeV, $\Lambda_{QCD} = 0.2$ GeV, $N = 0.4$ and the masses for light (charm) quarks were set to $m_q = 0.25(1.5)$ GeV. The scale of $\alpha_s$ in the off-shell matrix element was set to $k_t^2$. With this parameter settings very good description

![Figure 1. The structure function $F_2(x, Q^2)$ compared to H1 data [8]. The solid (dashed) line is the prediction of the Monte Carlo without (with) applying the “consistency constraint” (c.c.) and the dotted line includes only the $1/(1 - z)$ term.](image-url)
of $F_2$ over the range $0.5 \cdot 10^{-5} < x < 0.05$ and $3.5 < Q^2 < 90$ GeV$^2$ is obtained. In Fig. 2 the prediction for the forward jets using the same parameter setting is shown. The data are nicely described. The effect of the $1/(1-z)$ term in the splitting function is shown in Figs. 1 and 2 separately with the dashed line. It is obvious that these terms are important for a reasonable description of $F_2$ and the forward jet data.

Including the “consistency constraint” the $x$ dependence of the cross section changes, but a similarly good description of $F_2$ is obtained by changing $Q_0$ to $Q_0 = 0.85$ GeV. However the forward jet cross section becomes smaller, as shown with the dotted line in Figs. 1 and 2. It is interesting to note, that this prediction is very similar to the one obtained from the BFKL equation as shown in [6] for $k_t^2$ as the scale in $\alpha_s$.

4. Acknowledgments

I am very grateful to B. Webber for providing me with the code of SMALLX. I am grateful to B. Andersson, G. Gustafson, H. Kharrazia, J. Kwiecinski, L. Lönnblad, A. Martin, S. Munier, R. Peschanski and G. Salam for many very helpful discussions about CCFM.

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