Phases and Structures of Interstellar Gas

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ABSTRACT

The thermal and chemical phases of the cool component of interstellar gas are discussed. Variations with galactocentric radius and from galaxy to galaxy are mostly the result of changes in the ambient interstellar pressure and radiation field. Interstellar structure that is hierarchical or fractal in the cloudy parts and has large and connected empty regions between these clouds is probably the result of turbulence. Such structure opens up the disk to the transmission of OB star light into the halo, and it provides for a diffuse ionized component that tapers away gradually from each dense HII region. Fractal cloud structure may also produce the cloud and clump mass functions, and perhaps even the star cluster mass function.

1. Phases of Interstellar Gas

Interstellar gas in the solar neighborhood has a variety of thermal and chemical phases, including cold atomic gas inside or mixed with cold molecular clouds (e.g., Sato & Fukui 1978), cool and warm atomic gas on the envelopes of molecular clouds (Chromey et al. 1989; Andersson & Wannier 1993; Wannier et al. 1993) and in the diffuse, low density medium (Kulkarni & Heiles 1987), cold molecules in dark and dense clouds (Combes 1991) and in translucent clouds (Magnani, Blitz & Mundy 1985), warm molecules in dense clouds near embedded luminous stars (e.g., Cesaroni et al. 1994), dense photo-ionized gas in the neighborhoods of O and B-type stars, low-density ionized gas between these stars (Walterbos & Braun 1994; Reynolds 1995; Ferguson et al. 1996), and hot, shock-heated gas near supernovae and windy stars (Ostriker & McKee 1988).

There is also considerable variety in the types of molecules that are present inside dark clouds, ranging from primarily H$_2$ in translucent clouds (Spitzer & Jenkins 1975; Magnani et al. 1998), with a possible trace of PAH or long-chain molecules (Leger & Puget 1984; Tulej et al. 1998), to a mixture of complex molecules in dense clouds (e.g., Langer et al. 1997; see reviews by van Dishoeck and Thaddeus, this conference).

The thermal and chemical states of the cold and cool gas have such a great variety because the clouds shield themselves to different levels from photodissociative radiation and radiant heating. This makes the molecular abundance and temperature depend on density, column density, metallicity, age, and local radiation field, all of which vary from cloud to cloud and with radius in the Galaxy. The nature of cloud structure does not change this much, because it is largely the result of turbulence, self-gravity, and local explosions, which seem to act the same way everywhere.

Here we discuss the dominant processes that affect the molecular and thermal states of the gas, and we review the some of the aspects of cloud structure that are likely to arise from turbulence.
1.1. Cloud Self-Shielding and Molecule Formation

Interstellar clouds shield themselves from photo-dissociative radiation when the molecular formation rate (in cm$^{-3}$s$^{-1}$) integrated over the cloud radius exceeds the molecular dissociation flux (in cm$^{-2}$s$^{-1}$) from incident starlight (Jura 1975; Federman, Glassgold, & Kwan 1979; van Dishoeck & Black 1988; Sternberg 1989; Hollenbach & Tielens 1997). This means that clouds determine their own molecular abundances for any particular radiation field, depending on their density, column density, and metallicity. Dense clouds with only moderate column densities can have the same H$_2$/H ratio as lower density clouds with larger column densities. As a result, regions with larger pressures tend to be more molecular, i.e., higher pressures lead to higher densities and greater self-shielding at all cloud masses.

One implication of this result is that most galaxies have a radial gradient in the molecular fraction outside the nuclear or barred region because the pressure is lower at larger radii (Elmegreen 1993; Honma, Sofue, & Arimoto 1995). The total (turbulent+magnetic+thermal) interstellar medium pressure varies approximately as $P \approx \left(\frac{\pi}{2}\right) G\sigma_g \sigma_T$ for gas column density $\sigma_g$ and total gas+star column density in the gas layer, $\sigma_T$. Typically, $\sigma_g \propto \sigma_T$, so $P \propto \sigma_T^2$. This means that as $\sigma_g$ decreases exponentially with radius in a typical spiral galaxy, the total pressure decreases with radius too, and it has half the scale length. This corresponds to a rather rapid decrease of interstellar pressure with galactocentric radius.

Radial pressure gradients lead to the appearance of “molecular rings” in galaxies that have inner disk cutoffs from a bar or bulge. Molecular rings are not true rings, like galactic resonance rings, but only molecular emission concentrations that result from smooth exponential disks with inner cutoffs. The rapid decrease of molecular fraction with increasing radius beyond the “ring” results primarily from large-scale pressure changes.

High latitude clouds, shells, spiral-arm dust lanes, and other pressurized fronts should also have large molecular fractions compared to other clouds with the same column density. A good example of the influence of pressure on molecule formation is shown by the nearby L1457 cloud, which has H$_2$ primarily in the southern part, where it was recently compressed (Moriarty-Schieven, Andersson & Wannier 1997).

The thermal state in clouds is generally related to the molecular state, at least in the solar neighborhood, because once a cloud begins to shield itself in the H$_2$ lines, it also becomes optically thick from dust. Then it removes its principle quiescent heat source, which is the photoejection of electrons off grains (de Jong 1977; Draine 1978; Bakes & Tielens 1994). Molecular clouds need not always be cold, though. Different photons are involved with photodissociation, which is limited to spectral lines, and photoejection, which is a continuum process. Besides, density and column density can vary separately. Consequently, there are warm molecular clouds in the diffuse phase, i.e., high density, low column-density clouds (Spitzer & Cochran 1973). These clouds tend to dominate the diffuse cloud population in the inner (high pressure) regions of bright galaxies (Polk et al. 1988; Honma, Sofue, & Arimoto 1995). There are also diffuse or dense molecular clouds in the high pressure regions of dim galaxies that are so cold they do not emit much CO radiation (Allen et al. 1995). Such temperature variations for molecular gas inversely affect the conversion factor from integrated CO linewidth to molecular hydrogen. High temperature molecular clouds have proportionally less molecular matter per unit CO emission than low temperature molecular clouds.

Figure 1 shows a schematic diagram of the cold and cool interstellar cloud phases in the parameter space that is defined by density and column density. The rising line that separates diffuse from self-gravitating
gas comes from the relation
\[
\frac{P}{G \sigma^2} = 60 \left( \frac{n}{60 \, \text{cm}^{-3}} \right) \left( \frac{N}{5 \times 10^{20} \, \text{cm}^{-2}} \right)^{-2} \left( \frac{c}{2 \, \text{km s}^{-1}} \right)^2 = 1
\]  
for velocity dispersion \( c \). This is the threshold for strong self-gravity in a cloud. The falling line that separates molecular from atomic gas comes from the equation
\[
\left( \frac{Z}{Z_0} \phi/\phi_0 \right) \left( \frac{n}{60 \, \text{cm}^{-3}} \right) \left( \frac{N}{5 \times 10^{20} \, \text{cm}^{-2}} \right)^{2/3} = 1
\]
for metallicity \( Z \) and radiation field \( \phi \), normalized to solar neighborhood values. This relation is based on the balance between the \( \text{H}_2 \) formation rate (on dust, hence the \( Z \) dependence) along a column of gas, \( \propto Z n N \), and the photodissociation rate per unit area, which is proportional to the absorbed photon flux, \( \phi(N) \propto \phi N^{1/3} \). The column density dependence in \( \phi(N) \) accounts for extra absorption in the line wings as \( N \) increases (Federman, Glassgold, & Kwan 1979).

Equation 2 was considered in more detail by Sternberg (1989), who included a temperature dependence for the molecule formation rate and dust extinction. Sternberg also approximated the column density dependence of \( \phi(N) \) as \( \propto N^{1/2} \) for pure line-wing absorption. According to Sternberg, dust dominates molecules for the absorption of light in the shielding layer when \( \alpha G \gg 1 \), which translates to
\[
\alpha G \sim \frac{260 \, \text{cm}^{-3} \phi/\phi_0}{n \left( [T/10 \, \text{K}] [Z/Z_0] \right)^{1/2}} \gg 1.
\]
(This assumes Sternberg’s molecule formation rate, \( R \), and dust cross section, \( \sigma \), both scale with metallicity, \( Z \).) If we scale the Federman, Glassgold & Kwan (1979) result with \( \phi \propto N^{1/2} \) instead of \( N^{1/3} \) and add a \( T^{1/2} \) dependence to the molecule formation rate (from the thermal speed at which \( \text{H} \) atoms impact grains), then the equation of detailed balance implied by equation 2, \( Z n NT^{1/2} \propto \phi(N) \), gives a molecular column density for self-shielding of \( N \propto (\phi/nZ)^2 / T \), and a corresponding dust opacity \( \tau \propto Z N \propto (\phi^2/n^2 Z T) \).

The square root of this opacity is essentially the quantity \( \alpha G \) in Sternberg (1989), to within a factor of 2. The column density at this opacity limit is approximately \( 5 \times 10^{20} \, \text{cm}^{-2} \) for standard interstellar extinction, which means that points to the right of \( N(\text{HI}) / 5 \times 10^{20} \, \text{cm}^{-2} \) at high density \( n \) in figure 1 are primarily shielded by dust, and points to the left are self-shielded by molecules. In both cases, the clouds are molecular, as indicated.

Low density clouds are generally atomic except at very high column density, at which point they may turn molecular if the conditions are right. However, real clouds with very low densities are not likely to turn molecular at high column density if their spatial extent is so large that they include many field stars inside them. In that case, there is no proper boundary for self-shielding. Such clouds will still make a transition from diffuse to self-gravitating at moderate column densities, however. This explains why the largest HI clouds in spiral galaxies can be self-gravitating, and conversely, why the largest self-gravitating clouds are often atomic (Elmegreen & Elmegreen 1983; 1987; Rand 1995).

Figure 1 also suggests that high density clouds are generally molecular, except at very low column density. When the density is high because of a high pressure, molecular clouds can be diffuse. The internal density can also be high in a region with a low ambient pressure provided the cloud is self-gravitating; this makes the cloud molecular again.

In the Solar neighborhood, most clouds make a transition from atomic and diffuse at low column density to molecular and self-gravitating at high column density, with relatively little total cloud mass...
in the form of molecular diffuse gas or atomic self-gravitating gas. However, in the spiral arms of our Galaxy, atomic self-gravitating gas is prevalent in the form of giant HI cloud complexes (10⁷ M⊙), which have relatively small molecular cores (10⁵ − 10⁶ M⊙) that are also self-gravitating. Such clouds are often regularly spaced along the spiral arms. In other galaxies such as M51, these 10⁷ M⊙ spiral arm clouds are mostly molecular (Rand 1993). This is a sensible result for M51 because it has a relatively high total gas column density, and therefore a high pressure (P ∼ Gσ²).

Figure 2 makes these same points in a different way, showing the densities of diffuse and self-gravitating gas as functions of cloud mass. The Larson (1981) relation between density and mass is used for the self-gravitating clouds. It comes from the two equations $P ∼ GM^2/R^4$ and $a^2 ∼ GM/(5R)$.

At low mass, both diffuse and self-gravitating states are possible, as in the usual two-fold solution to the virial equation with external pressure:

$$3Ma^2 - \frac{3GM^2}{5R} = 4\pi R^3 P. \tag{4}$$

In this equation, for masses less than $M = 1.8a^4 (G^3 P)^{-1/2}$, there are stable states with low density and negligible self-gravity, bounded by external pressure, and there are stable states with high density and strong self-gravity, bounded by gravity, all at the same mass. Similarly, in the interstellar medium, there are low mass diffuse clouds as well as low mass self-gravitating clouds, such as Bok globules.

At high mass, equation 4 has only one stable solution and that is with self-gravity dominant. Similarly in the interstellar medium, most massive clouds are strongly self-gravitating. This mass limit changes when a magnetic field is present, but only slightly if the field has the equipartition energy density (Mouschovias & Spitzer 1976).

Also shown in figure 2 are short lines that indicate transitions from atomic to molecular phase as the mass increases ($M_M$), from a mixture of non-self-gravitating and self-gravitating clouds to pure self-gravitating clouds ($M_G$), and from molecular back to atomic gas ($M_A$). The origin of this latter transition, which occurs at around 10⁷ M⊙ for normal pressure, is that the density of a massive cloud is so low as a result of the high velocity dispersion and the requirement of pressure equilibrium, that the cloud can no longer shield itself from photodissociative radiation. The fact that the cloud is strongly self-gravitating does not matter. Examples of such clouds are again the giant HI complexes in spiral arms. At higher pressures, this transition from molecular to atomic self-gravitating clouds occurs at higher masses.

### 1.2. Thermal States of Interstellar Gas

Two thermal phases of atomic gas, cool and warm, can co-exist in the Solar neighborhood at the same pressure and radiation field because of inflections in the thermal cooling rate as a function of temperature (Field, Goldsmith & Habing 1969). At temperatures of around 100 K, ionized Carbon and neutral Oxygen cool the gas by collisions with neutral H, electrons, and, for OI, ionized H (Wolfire et al. 1995). These collisions are very strong coolants at temperatures near the energy levels of CII and OI, which are 94 K and 228 K, easily balancing the photoelectric heating rate that scales primarily with density. Thus the gas is thermally stable around 100 K. At lower temperatures, the CII and OI cooling transitions are barely excited, but the cooling rate does not drop much because the CI fine structure lines, which have lower excitation temperatures, become important coolants. Thus the gas remains stable below 100 K too. At higher temperatures and lower densities with the same pressure, the collision rates and cooling rates
Fig. 1.— Schematic diagram of interstellar cloud types on a density-column density plane. Clouds turn molecular at high column density or at high density because of self-shielding. Self-gravitating clouds require a sufficiently high column density for a given ambient pressure, so they tend to occur to the right in this diagram. Diffuse clouds require a low column density, so they are on the left. The transition point depends on pressure and radiation field and varies around the Galaxy. Figure from Elmegreen (1995).

Fig. 2.— Schematic diagram of interstellar cloud types on a density-mass parameter plane. Self-gravitating clouds satisfy the density-mass relation from Larson (1981) and occur for all masses, while the diffuse clouds, which are not self-gravitating, have about constant density and occur only at small mass. The transition mass \( M_M \) is where diffuse clouds turn from atomic into molecular at the local pressure and radiation field; \( M_G \) is where diffuse clouds turn self-gravitating for local conditions, and \( M_A \) is where molecular self-gravitating clouds turn atomic at higher mass because of their low density. Figure from Elmegreen (1993).
decrease, leading to an unstable region until \( T \geq 5000\text{K} \), at which point rapid cooling from Ly\( \alpha \) emission and electron recombination onto grains makes the gas stable again (Wolfire et al. 1995).

Figure 3 shows the thermal pressure, heating and cooling rates, ionization fraction, temperature, and ionization parameter \((G_0 T^{3/2}/n_e)\) for radiation field \(G_0\) scaled to the value in the Solar neighborhood, temperature \(T\), and electron density \(n_e\) for the “standard” model in Wolfire et al. (1995).

The local coexistence of two stable thermal states should not apply everywhere in a disk galaxy. In the inner parts, where the pressure is very high and the ambient radiation field is only moderately high, there may not be a warm atomic phase at all, and a high fraction of the atomic gas can be cool. Conversely, at large galactocentric radii, the pressure drops precipitously but the radiation field only a small amount, so there comes a point, at about 4 exponential scale lengths, where most of the HI becomes warm (Elmegreen & Parravano 1994; Braun 1997).

Local variations in the atomic state can arise because of variations in the radiation field and pressure. These variations often follow a spiral density wave. For example, most of the gas between the spiral arms is at low pressure, so it should be highly atomic, mostly diffuse, and warm. A population of low-mass, self-gravitating molecular clouds typically remains as well, from the previous arm. When this gas shocks to form a dust lane in the arm, the density and opacity to starlight go up in the atomic gas, the temperature drops, and the gas should turn molecular, even ultra-cold molecular. But if massive star formation occurs inside or near this shocked gas, the enhanced radiation can warm it up again and dissociate it, forming a concentration of cool or warm HI near the resulting OB association, as observed in M83 (Allen, Atherton & Tilanus 1985; Tilanus & Allen 1993), M51 (Tilanus & Allen 1989), M100 (Rand 1995), and M81 (Allen et al. 1997). Cold dense molecular material can coexist with this warm atomic gas if the molecules are in strongly self-gravitating clouds; the gravity preserves their high density and self-shielding even in the presence of strong radiation fields.

The pressure, radiation field, and metallicity vary both systematically and randomly from place to place and time to time in any one galaxy, and from galaxy to galaxy along the Hubble sequence. What applies to the Solar neighborhood will not generally apply elsewhere, and what typifies the Milky Way can be relatively rare in galaxies with different Hubble types. Nevertheless, most of the variations that have been observed in other galaxies can be explained as a consequence of the simple rules given above.

The hotter phases of interstellar gas require local excess energy sources such as hot stars and supernovae. These sources are relatively rare in a typical galactic disk (not in a nuclear starburst disk, though), but each commands such a wide domain of influence that the locally heated regions can sometimes overlap with each other, making the hot gas pervasive (Cox & Smith 1974; Salpeter 1976; McKee & Ostriker 1977). The domain of influence increases at lower density, so a single massive star at a large distance off the galactic plane can ionize a large volume in the halo. For example, an O5 star in a gas with a halo-type density of 0.03 cm\(^{-3}\) can ionize out to 3 kpc distance.

2. Structure of Interstellar Gas

2.1. Hierarchical Structure in Clouds

The origin of structure in interstellar gas is far less tangible than the origin of thermal and chemical states, because some of the structure is caused by supersonic turbulence, and no one understands yet how this works. Other structures, such as shells (Tenorio-Tagle & Bodenheimer 1988) and spiral shocks (Roberts
Fig. 3.— (top left) The thermal pressure is shown as a function of density for interstellar gas in thermal equilibrium. Only the rising parts of this curve are in stable equilibrium, and these produce the cool and warm neutral components locally. (lower left) The heating (dashed) and cooling rates for neutral gas are shown as functions of density. In the upper and lower right, the electron fraction, temperature, and ionization parameter are shown. All figures are from Wolfire et al. (1995).
1969), are somewhat easier to understand because they are regular and well defined. Turbulent structures, on the other hand, are irregular, boundary-free, and extremely intricate.

For example, a single molecular cloud, such as the Orion cloud, probably contains a total number of dense tiny clumps and cloud pieces (each with masses of $M < 10^{-2} \, M_\odot$ in the $10^5 \, M_\odot$ Orion cloud) that exceeds the total number of all other Orion-type clouds in all of the galaxies out to and including the Virgo cluster (considering $\sim 10^3 - 10^4$ Orion-type clouds per spiral galaxy the size of the Milky Way). Obviously there are far too many pieces of structure in a typical molecular cloud to catalog or map at the present time, and far too many to simulate on a computer. Yet these tiny pieces are very important: they probably contain most of the mass in the cloud, as demonstrated by the generally high excitation densities for many molecular transitions (e.g., Falgarone, Phillips, & Walker 1991; Lada, Evans & Falgarone 1997), and therefore control most of the chemistry, and ultimately, star formation. How should we proceed to understand interstellar matter in the face of such complexity (Scalo 1990)?

One of the most fundamental properties of turbulent clouds seems to be the arrangement of gas into hierarchical structures that are fractal or multi-fractal, with increasing density and local volume filling factor on smaller scales. Scale-free structures like this make a certain amount of sense for turbulent gas because the velocities that continuously form them have a scale-free nature as well, reminiscent of the Kolmogorov velocity law in incompressible laboratory turbulence. Scale-free velocities should make scale-free structures, as long as thermal pressure and gravity are not important structuring agents as well. When gravity is important, the gas becomes centrally condensed and builds up a pressure gradient. Most star-forming clouds are centrally condensed already.

We would like to know whether non-linear gas motions (i.e. “turbulence”) with a magnetic field can produce structures and velocities similar to what is observed in space. Figure 4 shows one time step in a computer simulation of a supersonically turbulent, magnetic gas. The density is plotted as a grayscale, with two cycles of gray representing the span of density in the central “cloudy” region of a two-dimensional MHD simulation. The initial conditions are a uniform density (value 1), and pressure (value 1, giving a sound speed of $5/3$), with a uniform magnetic field oriented vertically in the figure. The initial Alfven speed for this field is 10 in these units, which is supersonic. There is no self-gravity. The full grid measures 800 cells in the vertical direction, and 640 cells horizontally (only the central 480 cells in the vertical direction are shown here). After this initial setup, the magnetic field lines at the top and bottom of the grid are accelerated back and forth, horizontally, with random accelerations for random durations at random times, taking big swaths of field lines all at once. The distribution of swath size is the same as we measured for the distribution of interstellar cloud sizes, namely a power law with $n(R)dR \propto R^{-3.3}dR$ (Elmegreen & Falgarone 1996). What this result simulates is the non-linear excitation of field lines by moving neighboring clouds.

The gas in the simulation has two stable thermal states, one at a temperature corresponding to a sound speed of 1, and another at a temperature corresponding to a sound speed of $10^{0.5}$ (i.e., 10 times hotter). The magnetic field-line excitations at the top and bottom of the grid heat up the gas there to the high temperature state, and also drive the motion of gas toward the grid center. This forms a dense “cloud” at the vertical center of the grid, and this cloud takes the cool thermal state. The simulation was run for $10^5$ timesteps, with each time step equal to one-tenth of the time it takes an initial Alfven wave to travel over a distance of one cell.

The figure shows the dense central part of the grid, measuring 640 cells horizontally and 480 cells vertically. Substructure inside the dense part is apparently hierarchical, with some “clumps” containing 4
Fig. 4.— One time step in a computer simulation of compressible magnetic turbulence, with grey scale from black to white, and then black to white again, representing increasing density. The structure is hierarchical with at least 4 levels of clumps within clumps visible here.
levels of substructure. The larger pieces move with a mildly supersonic speed, and last for about one sound crossing time; during this time, they are “bound” by ram pressure from their constant motion. That is, the $\mathbf{v} \cdot \nabla \mathbf{v}$ term in the equation of motion acts like a binding pressure. The thermal pressure is actually lower between the clumps than inside the clumps, so they are not bound by thermal pressure; i.e., there is no warm interclump medium.

Simulations like these suggest that hierarchical structure is a natural consequence of non-linear magnetic wave interactions. This gives an indication that perhaps turbulence is causing some of the structures seen in interstellar clouds. We have a long way to go before the picture is complete.

### 2.2. Density wave structure in clouds

Optical images of galaxies like ours show such dominance by spiral arms that we should always question whether any local interstellar structures might have been caused by density waves. Obviously, the string of giant molecular clouds along the Sagittarius-Carina arm, including the M17 and M16 clouds in the first quadrant, the eta Carina cloud in the third quadrant, and many others (e.g., Cohen et al. 1985), should look like pieces in a giant spiral arm dust lane if viewed from outside the Galaxy. There are probably streaming motions along this dustlane too.

Spiral arm streaming motions (Roberts 1969) are well documented for other galaxies (e.g., Kuno & Nakai 1997) and for some regions in our Galaxy, particularly along the tangent points of the inner spiral arms (Burton 1992). These streaming motions affect our estimates for the kinematic distances to molecular clouds. For example, the longitude and velocity of the M17 cloud, $l = 15^\circ$ and $v = 20$ km s$^{-1}$, give it a kinematic distance of 2.5 kpc in the standard Galactic rotation model. But Hanson, Howarth & Conti (1997) found a spectroscopic distance in the range of 1.1 to 1.7 kpc. At the preferred distance of 1.3 kpc, its radial velocity should be only 10 km s$^{-1}$. This implies that the M17 molecular cloud has a streaming motion in the radial direction of $\sim 10$ km s$^{-1}$, moving away from the Sun. This is consistent with the expected sense of spiral wave streaming, i.e., directed radially inward along the arm inside corotation, or it corresponds to a purely azimuthal streaming of $\sim 30$ km s$^{-1}$. Thus, the true streaming speed is somewhere between 10 and 30 km s$^{-1}$. In any case, the observed radial speed gives it a kinematic distance that is wrong by nearly a factor of 2! Such an error affects the mass estimate for M17 and other clouds as well as our perception of Galactic spiral structure.

### 3. Further Implications of Hierarchical/Fractal Structure

#### 3.1. Intercloud Medium

If turbulence makes some clouds and cloud clumps, then it also makes some of the low density regions between these clouds and clumps. These are the spaces cleared out by the turbulent motions when the gas is moved into denser regions. For a fractal cloud population, the intercloud medium has certain regular properties that also come from fractal geometry.

One of these properties is the size distribution of holes. In a hierarchical model with clumps inside other clumps, each level in the hierarchy has a constant proportion of the number of empty regions of a particular size to the number of clumps of that size. This means that the hole size distribution is the same as the clump size distribution, which is $n(S)d\log S \propto S^{-D}d\log S$ for size $S$ and fractal dimension $D$.
The dimension for this type of fractal is defined by the relation
\[ D = \log N / \log L \]
for number of subclumps in a clump, \( N \), and for size ratio of clump to subclump \( L \). It may also be defined from the scaling of mass with size \( M \propto S^D \). If \( D \approx 2.3 \) from the cloud size distribution and mass-size relation (Elmegreen & Falgarone 1996), then the hole size distribution should be \( S^{-2.3} d \log S \) too.

The fractal Brownian motion model proposed by Stutzki et al. (1998) also has a size distribution for clouds equal to the size distribution for holes. This model begins with a white noise spectrum, and then convolves this with a power law that has decreasing power at high frequencies. The result is a noise model with the large scale features emphasized. These features, like the original noise, are equally likely to have positive or negative excursions around the mean density. If we identify the positive excursions with clouds and cloud clumps and the negative excursions with holes between the clouds, then the size distribution for holes will be the same as the size distribution for clumps.

These predicted size distributions are steeper than the shell size distribution found by Oey & Clarke (1997), who got a power law with a slope of \(-1.3\) (instead of \(-2.3\)) on a log-log plot. Oey & Clarke’s objects are real shells, however, not just random regions with locally low densities. There are no studies yet of the distribution of hole sizes, but consideration of this might prove fruitful.

The volume filling factor of low density gas is another important consideration for comparisons with observations. For the hierarchical fractal model discussed above, the low density volume filling factor equals

\[ f_{\text{holes}} = 1 - C^{(D/3) - 1} \approx 80\% \]  

for density contrast \( C \) between the highest and lowest levels (Elmegreen 1997). To evaluate this, we used \( C = 10^3 \) considering densities from 0.1 cm\(^{-3}\) to 100 cm\(^{-3}\) in the diffuse medium, or from 100 cm\(^{-3}\) to 10\(^5\) cm\(^{-3}\) in molecular clouds. The large value of this filling factor indicates that a turbulent interstellar medium should be mostly empty, with most of the mass in the form of clouds that occupy only a small fraction of the volume.

This result challenges the usual interpretation of the open frothy structure of the interstellar medium as an indication of stellar winds and supernovae (Brand & Zealey 1975; Hunter & Gallagher 1990). More likely, the frothy structure is from a combination of these direct stellar processes plus pervasive turbulent motions. For the turbulent motions, the energy can be put into the gas in non-explosive forms (e.g., shear, magnetic wave interactions, etc.), although some of it may ultimately come from explosions around stars too. A recent fractal interpretation of the frothy structure in HI maps of the SMC was made by Stanimirovic et al. (1998).

Equation 5 gives the total filling factor of low density gas from all levels in the hierarchy. There is another filling factor for a hierarchical cloud distribution and that is the filling factor of regions with a certain average gas density, \( < \rho > \),

\[ f ( < \rho > ) d \log < \rho > \equiv S^3 n (R[< \rho >]) d \log S \propto \frac{1}{< \rho >} d \log < \rho > . \]  

This result is easily derived using the intermediate step shown, along with the size distribution function \( n(R)d \log R \propto R^{-D} d \log R \) and the density-size relation \( < \rho > \propto S^{D-3} \), which comes from the mass-size relation that defines the fractal dimension \( D \) in this model, \( M \propto S^D \). Equation 6 indicates that the summed volume of regions with a certain average density occupies a smaller and smaller fraction of the total volume as that density increases. Equation 5 above suggests further that as this average density increases (and \( C \to 1 \)), the gas inside each dense piece becomes more and more uniform. Thus we recover the standard
model of interstellar clouds composed of numerous, tiny, near-uniform clumps, arranged in a fractal pattern with the average gas density varying inversely with size (for \( D \) near 2).

We also get an explanation for the common perception that clumps in emission line surveys generally occupy 1–10% of the volume (e.g., Blitz 1993). Evidently, this is a selection effect caused by the limitations of molecular emission maps which are typically sensitive to only a factor of \( 10 \) – \( 100 \) range in density for any one survey.

Berkhuijsen (1998) has obtained a remarkable result recently in plotting the volume filling factor of ionized interstellar gas as a function of its local average density. The result, shown in figure 5, is a \( 1/\rho \) distribution, as expected for a turbulent fluid with any fractal dimension. This does not mean that the ionized fluid itself is turbulent in this way; in most regions, the ionization structure is probably just a reflection of the neutral structure of the gas before it was ionized.

### 3.2. Cloud and Intercloud Mean Free Paths

Hierarchical or fractal structure in the interstellar medium implies that clouds are highly clumped, i.e., that any one cloud is likely to have another cloud nearby, and, conversely, any empty region is not particularly likely to have a cloud nearby. Such clumping makes the mean free path for intersecting a cloud on the line of sight different for regions near clouds than for regions between clouds. Thus the intercloud mean free path can be larger than the average mean free, which has always been obtained in the past by assuming a uniform density. This result has important implications for the distance ionizing photons can travel once they leave a molecular cloud/OB association complex.

The mean free path is generally defined as \( \lambda = (nS^2)^{-1} \) for number density of clouds \( n \) and cross sectional area \( S^2 \). When \( n \) is not uniform, but clumped instead, this expression has a different value on lines of sight near cloud complexes than it does on lines of sight between cloud complexes. The hierarchical cloud model discussed above (Elmegreen 1997) can be used to show that the number of sub-clouds on a line of sight through a fractal cloud is

\[
N_{\text{sub-clouds}} = C^{(D−2)/3} \sim 2.5. \tag{7}
\]

If there are 8 interstellar absorption lines kpc\(^{-1}\) on average (Blauuw 1952), then these correspond to \( 8C^{−(D−2)/3} \sim 3 \) fractal cloud complexes per kiloparsec. Indeed, most of the local gas is not randomly distributed with a constant space density of clouds everywhere. It is highly clumped into cloud complexes associated with the OB associations in Orion, Sco-Cen, and Perseus, and with the Taurus clouds. These are the types of clouds that occur with the rate of 3 kpc\(^{-1}\).

The average mean free path on lines of sight between the fractal cloud complexes should be \( C^{(D−2)/3}/8 \sim (3/\text{kpc})^{-1} \sim 330 \) pc. This may be defined as the pure intercloud mean free path. It exceeds the Galactic scale height, which means that once photons get into the disk intercloud medium, they are likely to reach the halo. For this reason, the ionization of the halo, giving the “Reynolds” layer (Reynolds 1995), may be much easier than previously thought (compare this result to the uniform cloud model by Miller & Cox 1993).
MILKY WAY

\[ f_V(n_e) \text{ of HII} \]

\[ \log f_V \]

\[ \log n_e (\text{cm}^{-3}) \]

---

Cordes et al. 1991, Habing + Israel 1979
- outer disk \( d \approx 10 - 100 \text{ pc} \)
- inner disk \( d \approx 1 - 100 \text{ pc} \)
- clumps \( d \approx 1 \text{ pc} \)
- Leahy 1987 \( d \approx 1 \text{ pc} \)
- Pynzar 1993
- P1 inter arm \( d \approx 300 - 600 \text{ pc} \)
- P2 arm \( d \approx 40 - 150 \text{ pc} \)
- P3 class. \( d \approx 1 - 50 \text{ pc} \)
- Heiles et al. 1996
- ELDWIM in arm \( d \approx 150 \text{ pc} \)
- Berkhuijsen
- \( R = 8.5 \text{ kpc} \), DIG + classical

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Fig. 5.— Volume filling factor of ionized gas in the Milky Way as a function of density, showing the inverse-density relation typical for hierarchical fractal structure. Figure from Berkhuijsen (1998).
3.3. The Diffuse Ionized Gas

HII regions expand away from giant molecular clouds into the intercloud medium. For fractal clouds, the neutral density varies inversely with distance $R$ as $\rho \propto R^{-D - 3} \sim R^{-0.7}$ for $D \sim 2.3$. When the ionization front reaches this neutral density, it heats and ionizes the gas, causing the smallest pieces to expand and mix with each other. This makes the local density more uniform on a timescale equal to the sound crossing time for the small pieces, but it does not affect the overall density gradient much because the sound crossing time is large on a large scale. Thus a young HII region should have a radial density gradient reflecting the $R^{-D - 3}$ gradient in the initial neutral cloud. As a result, there should be an extended component of diffuse ionized gas surrounding most HII regions. The fraction of the ionization outside the conventional Strömgren sphere can be large, perhaps 50% (Elmegreen 1997). Such extended ionization around HII regions presumably corresponds to the observations of diffuse ionized gas in our Galaxy (Reynolds 1995) and other galaxies (Walterbos & Braun 1994; Ferguson et al. 1996). This diffuse ionized gas is clearly associated with the outlying parts of individual HII regions in many cases (Walterbos & Braun 1994).

3.4. Cloud Mass Functions

The mass distribution of purely hierarchical structures should scale as $Mn(M) d\log M = \text{constant}$, or (Fleck 1996):

$$n(M) dM \propto M^{-2} dM.$$  \hspace{1cm} (8)

Actually, $n(M) \sim M^{-1.7 - 1.8}$ from clump surveys (Stutzki et al. 1998; Heithausen et al. 1998).

Elmegreen & Falgarone (1997) got $n \sim M^{-\alpha_M}$ with $\alpha_M \sim 1.7 - 1.8$ from a model of hierarchical cloud structure using the observed $n(S) dS \propto S^{-\alpha_S} dS$ for size $S$ with $\alpha_S = 1 + D \sim 3.3$, and the observed $M \propto S^\gamma$ with $\gamma \sim 3$, and then combining these relations with the standard conversion $n(M) dM = n(S) dS$, which gives $\alpha_M = 1 + D/\gamma$.

Stutzki et al. (1998) got $n(M) \sim M^{-1.8}$ with a cloud model based on fractal Brownian motion, using $\alpha_M = 3 - (8 - 2D)/\gamma$ and the same $D$ and $\gamma$. The physical difference between these two models is that hierarchical clouds have all their small clumps inside larger clumps, whereas fractal Brownian motion clouds have small structure with equal probability everywhere, although it is weaker outside the larger clumps. In theory, the Brownian motion model makes more sense for turbulence, which should have some density structure everywhere, but in practice, the low density regions outside of the main hierarchy in an interstellar cloud may be ionized or invisible in CO, in which case only the dense hierarchical core, which is common to both models, will be seen.

The $M^{-2}$ function also applies to the probability of choosing part of a hierarchical cloud from any level. If stars form at all levels in a hierarchy because all levels have dense gas in tiny clumps, then the resulting stars will be hierarchically clumped too. Such a distribution for stars is in fact commonly observed (Elmegreen & Efremov 1997, 1998). If, in addition, bound clusters form with some high and nearly constant efficiency, so $M_{\text{cluster}} \propto M_{\text{cloud piece}}$, then the mass distribution for star clusters will be about the same as the mass distribution for cloud structure (Elmegreen & Falgarone 1996).

Battinelli et al. (1994) found cluster mass functions from two samples that had power law slopes equal to $\alpha_{M,\text{clusters}} = -2.13 \pm 0.15$ and $-2.04 \pm 0.11$. Elmegreen & Efremov (1997) similarly found $\alpha_{M,\text{clusters}} \sim 2$ for three different age groups in the LMC. These cluster mass functions are very similar to the expectations from hierarchical clouds, given by equation 8.
REFERENCES


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