INCLUSIVE STUDY OF HIGH-ENERGY MULTIPARTICLE PRODUCTION
AND TWO-BODY CORRELATIONS

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ABSTRACT

The present experimental information on multibody hadron production is examined in the framework of an inclusive and semi-inclusive approach. In the first part, the trends towards possible asymptotic limits is briefly discussed. The main part of this review is then devoted to the study of inclusive and semi-inclusive two-particle correlations.

1. SINGLE-PARTICLE INCLUSIVE DISTRIBUTIONS

In the last few years, the energy range over which the dynamics of strong interactions can be experimentally studied has increased by factors of 10 to 100 with the operation of the proton-proton intersecting storage rings (ISR) of CERN and of the 300 GeV proton synchrotron of FNAL. At these machines the first exploratory work was aimed at understanding to which extent the main features of hadronic interactions could be considered as asymptotic in this new energy domain. The first item of this study was the energy dependence of the total and elastic cross-sections, with the exciting and widely unexpected result of a general increase of hadronic total cross-sections¹⁾.

Extensive experimental studies have also been made on the structure of multi-body production. Owing to the large average multiplicity of the production, ranging from 9 to 18 for $\sqrt{s} = 20$ to 60 GeV, the exclusive study of each individual inelastic channel became more and more impractical, so that the experimenters have been forced to search for a more global approach. This has been identified in the concept of inclusive measurements, in which the production of a single particle in a given region of the phase space is studied independently of the structure of the individual event. The reader who approaches, for the first time, the field of high-energy multibody production should first read the Appendix to familiarize himself with this quantity and with the variables used throughout. At low energies, inclusive spectra consist mostly of the decay products of the resonant states in which the projectiles are excited. Indeed, at PS energies the available rapidity range is of the order of 4 units, while the interval covered by the decay of a resonance is of the order of 2 units. At ISR energies (corresponding to a maximum rapidity interval of ~ 8 units), particles associated with the projectiles' resonant excitation are instead separated and a new dynamical mechanism was expected for the production in the intermediate region. Two appealing theoretical speculations, describing the gross features of the production in the asymptotic limit, were compared with the data.

The limiting fragmentation hypothesis²⁾ postulates the existence of limiting momentum distributions of secondaries of the collision when studied in the rest frames of the interacting hadrons. The physical idea behind this statement is a picture of the interaction as independent fragmentations of the two impinging particles. The two groups of fragments are mixed together at low energy and tend to separate in the limit of very high energy, each one then becoming independent of the energy of the other projectile. This can be expressed as

$$E \frac{d\sigma}{d^3p}(s, y_{\text{proj}}, p_T) \xrightarrow{s \rightarrow \infty} f(y_{\text{proj}}, p_T)$$

where $E(d\sigma/d^3p)$ is the invariant inclusive cross-section, p_T and y_{proj} are, respectively, the transverse momentum and the rapidity of the selected particle in the projectile rest frame, and s is the square of the c.m. energy. Since at finite energies^{*)} no prescription can be given on how to identify the two groups of fragmentation products, the experimental test has been done by associating with each projectile the particles produced with small momenta in its rest frame, or, equivalently, in the corresponding forward cone in the c.m. system (the so-called fragmentation cone). It has been found empirically that the region of validity of this hypothesis covers about three rapidity units at ISR energies (see later). In this picture, however, no prediction can be made on the production of particles with small longitudinal momenta in the centre-of-mass system.

The scaling hypothesis states that at infinite energy the single-particle production cross-section depends on the ratio of the c.m. longitudinal component of the particle momentum (p_L^*) to its maximum value (p_0^*), and not separately on these two variables³⁾.

$$E \frac{d\sigma}{dp_L^* dp_T^2} \rightarrow f(p_T, x) ,$$

where

$$x = \frac{2p_L^*}{\sqrt{s}} \approx \frac{p_L^*}{p_0^*} .$$

Since (see Appendix) a finite value of x corresponds to a fixed distance from one edge of the available rapidity interval, Feynman scaling and limiting fragmentation hypotheses are equivalent in the fragmentation cones. The first one, however, tells us something also on the production in the central c.m. region, because the prediction that $f(x)$ tends to a constant at $x = 0$ implies that the inclusive rapidity distribution develops a constant plateau around $y^* = 0$. The value of this plateau was left undetermined and it was customary in 1971 to expect that the plateau level could not match with the distribution in the fragmentation region; it was even thought possible that the central region might be empty. An important result of the first ISR measurements was the observation that central production is abundant and that inclusive distributions show a nice continuity over the full rapidity plot.

An intuitive way to describe the scaling hypothesis is in terms of a thermodynamical analogy with the molecules of a gas. As in a real gas, where each

*) At infinite energy, secondaries of finite momentum in any frame moving with fixed velocity with respect to the projectile are fragments of it.

molecule can feel the presence of other ones only if they are in a limited region of space around it, in the longitudinal phase space a particle is affected by other particles only when these are produced at small rapidity distance from it. Therefore, at high enough energy, particles produced close to one border of the rapidity plot do not know how far the other border is and the production spectra become energy independent in these regions. The distance L within which a particle feels the presence of another one is called "correlation length". At even higher energies, the distance between the central region in the rapidity plot, around $y^* = 0$, and both edges becomes larger than L , so that the production in this region does not feel how distant these edges are and shows a flat and energy-independent cross-section.

In this picture, edge effects are put in correspondence with the fragmentation of the projectiles and the central yield is connected with the pressure in the gas. The continuity between the fragmentation region proper and the central plateau becomes then quite natural. Moreover, gas density fluctuations lead to short-range correlation effects, as those abundantly discussed later.

For more details on these theoretical hypotheses on multiparticle production, in particular on the expected rate of approach to these asymptotic limits, the reader is referred to the abundant literature published in the past few years⁴⁾.

In the present work, the experimental evidence on the validity of the asymptotic hypotheses will be reviewed. With this aim in mind, most of the data are taken from ISR experiments. There is some overlap with FNAL results and frequent references will be made to them. We shall be concerned only with data on particle production at small or moderate values of the transverse momentum. The unexpected features of the production at $p_T > 2 \text{ GeV}/c$ are a most interesting phenomenon in itself, and will be discussed in a separate topical review⁵⁾. It must be stressed, however, that these events represent a negligible minority of the over-all production which takes place with an average transverse momentum $\langle p_T \rangle$ of $\sim 350 \text{ MeV}/c$.

At the ISR, most inclusive distributions have been measured with single-arm magnetic spectrometers. In many cases the particle mass could be identified by means of threshold Čerenkov counters. As a rule, particles produced in beam-beam events were separated from beam-gas background by requiring the proper position of the source point, which could be reconstructed by tracing back the particle trajectory in the spectrometers. This mode of background rejection provides an unbiased inclusive trigger, while all other requirements, as for instance suitable coincidence with other particles emitted in the same event, would bias to some extent the distributions, by favouring particular classes of events. Spectrometers of this kind have been operated by

- i) the CERN-Holland-Lancaster-Manchester Collaboration (CHLM), for $35 < \theta < 180$ mrad⁶⁾ (see Fig. 1),
- ii) the Bologna-CERN Collaboration (formerly including Argonne) (BC) at intermediate angles, $80 < \theta < 300$ °⁷⁾ (Fig. 2),
- iii) the British-Scandinavian Collaboration (BS) between $\theta = 30^\circ$ and $\theta = 90^\circ$ °⁸⁾ (Fig. 3),
- iv) the Saclay-Strasbourg Collaboration (SS) at 90° °⁹⁾ (Fig. 4).

Angular distributions of charged secondaries have also been measured by the Pisa-Stony Brook (PSB) group, which employed a counter hodoscope system covering nearly the full solid angle (Fig. 5)¹⁰⁾. This apparatus was designed to record $\sim 97\%$ of all inelastic events, thus providing a clean inclusive trigger. Examples of angular distributions, plotted against the variable $\eta = -\ln \tan \theta/2$, which is a good approximation of the rapidity (see Appendix), are shown in Fig. 6.

At FNAL, single-particle inclusive distributions have been measured mostly in the 30 in. hydrogen bubble chamber. Taking advantage of the properties of this instrument, attention was concentrated on the study of the target fragmentation region and on the production of unstable particles (K^0, Λ) which cannot be easily studied at ISR. Some measurement of π -induced reactions have allowed very interesting comparisons with the most extensively studied proton-induced reactions. A detailed description of the results obtained at FNAL is given in a previous issue of this journal¹¹⁾.

Experiments with magnetic spectrometers, in which the momentum and angle of the produced particles are measured, were well suited for testing the validity of the scaling hypothesis. A collection of data obtained by the various groups at $p_T = 0.4$ GeV/c is plotted in Fig. 7 against the rapidity as measured in the projectile rest frame. In the fragmentation region ($y_{lab} < 2$) the ISR pion data do not show any appreciable energy dependence and are also very close to low-energy data obtained at the PS. This conclusion is valid within an average error of $\sim 10\%$, which must be attributed to the data mostly for systematic uncertainties on the spectrometer acceptances, on the measurement of the ISR luminosity^{*)}, and on the correction factors applied to the raw data. For K^- and $\bar{p}$, PS data are

*) At intersecting storage rings the relationship between the cross-section for a given process σ and its rate R is given by

$$R = L \cdot \sigma.$$

The luminosity L is a quantity describing the characteristics of the machine, the beam currents, and the dimensions of the interaction region. Typical values of the luminosity at the ISR are of the order of $10^{31} \text{ cm}^{-2} \text{ sec}^{-1}$, corresponding to $\sim 10^6$ proton-proton interactions per second.

Fig. 1 General layout of the CHLM experiment. The spectrometer is built on top of the downstream arm of beam 1. Septum magnets 1 and 2 and Cerenkov counters 1 and 2 are movable in order to vary the production angle (in the vertical plane). The momentum is generally measured twice for each track, once at the level of the septum magnets and then with bending magnets BM1, BM2, and BM3. Each track is reconstructed off-line to a precision $\Delta p/p \approx 0.5\%$, $\delta\theta \approx 0.5$ mrad. The solid angle of the spectrometer is $\Delta\Omega \approx 0.2$ msr and the momentum bite $\Delta p/p \approx 30\%$ full width at half maximum. The total systematic error, due mainly to the uncertainty on the ISR luminosity, and to the corrections for particle absorption and decay, is believed to be less than 10% .

Fig. 2 Scheme of the BC experiment. Particles are bent in the vertical plane, while the production angle is measured in the horizontal plane. The solid angle covered by the spectrometer is 0.02 msr , and the momentum acceptance is $\Delta p/p = 4.5\%$. Momenta between 1 and 10 GeV/c are measured with different magnet settings. Systematics errors are typically of the order of 10%, while statistical errors range between 3 and 15%.

Fig. 3 Structure of the BS spectrometer used in the measurements of inclusive distributions. The use of two magnets allows a double determination of the particle momentum. The solid angle of the spectrometer is about 12 msr. The complete set-up can rotate from 90° to 36° around the intersection point of the beams. The momentum resolution is $\Delta p/p = \pm 4\%$. The main sources of systematic errors are: the measurement of ISR luminosity ($\pm 10\%$), the absorption of particles in the spectrometer itself, and the reproducibility of the data under various running conditions ($\pm 7\%$).

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Fig. 4 Layout of the SS spectrometer. The apparatus is fixed at 90° and has a solid angle acceptance of 85 msr. The momentum resolution is of $\pm 5\%$. An uncertainty of $\approx 5\%$ must be attributed to the absolute value of the measured cross-section because of the errors associated with the luminosity measurement⁹⁾.

Photo 1 ISR intersection region I2 during 1973. The BS apparatus is clearly visible at the centre of the photo. The spectrometer of the CHLM Collaboration is positioned above the ISR magnets on the extreme left-hand side.

lower than ISR ones and the asymptotic limit is approached slower for the heavier particles. An inspection of the p_T dependence of these trends shows that the approach to the limit is faster for low than for large values of p_T . No precise statement can be made, however, on the behaviour of the production for very low values of the transverse momentum ($p_T < 0.1$ GeV/c), where available data are still very poor. This is due to the experimental difficulty of measuring very low momentum particles outside the ISR vacuum pipes. Even if some anomalous effect takes place at very low p_T , its contribution to the inclusive yield is heavily reduced by the p_T factor in the phase-space integration.

A constant cross-section between $y_{lab} \approx 2$ and the value of y corresponding to $\theta_{cm} = 90^\circ$ (this value depends on the energy of the interaction) would show the existence of a flat and energy-independent central plateau, as required by the second statement of the Feynman scaling hypothesis. The results shown in Fig. 7 provide a semi-quantitative indication that indeed the distributions tend to develop a central plateau when the energy increases. This limit is approached from below. This effect is more evident for produced particles of smaller masses. An exception to this trend is provided, in Fig. 7, by protons which approach the asymptotic limit from above. The proton spectrum shows, however, other major differences with respect to the spectra of the other particles. First of all, it increases smoothly with decreasing y_{lab} . This peculiarity can be attributed to the leading-particle effect, namely to the fact that the primary particle carries, on the average, about one half of its original momentum after the interaction. A similar behaviour is shown by the data on neutron¹²⁾ and Λ ¹³⁾ production in pp reactions, plotted in Fig. 8, which are characterized by an abundant production at large values of x and by a broad bump around $x \approx 0.5$. When considering production of heavy particles, one should keep in mind that particles produced close in rapidity have momenta roughly in the ratio of their masses. For instance, in an N^* decay the proton momentum is on the average three times larger than the pion momentum. This partly explains the difference in shape between baryon and meson x -distributions. There is, however, a remarkable difference between the n and Λ spectra and the proton spectrum. The proton data show an additional increase and a sharp peak towards $y_{lab} = 1$, which is not present in the n and Λ data. This peak is to be associated with a well-defined interaction mechanism, namely the diffractive excitation of one of the interacting protons, while the observed one is merely quasi-elastically scattered.

A closer inspection of data in the central region shows that the value of the inclusive cross-section for pion production at $y_{cm} = 0$ increases by $(25 \pm 8)\%$

a) φ -hodoscope

b) L - box

c) TB counters

d) θ -hodoscope

Fig. 5 Schematic layout of the PSB experiment. (Top figure). The structure of the hodoscopes is described by (a) to (d). Each hodoscope $H_1 \dots H_4$ is split into azimuthal octants. Two hodoscopes, H_2 and H_4 , have an additional θ -plane consisting of circular strips. The L-box surrounds the intersecting point over almost the full azimuthal angle. A particle is identified by a coincidence between any two hodoscopes H_1H_2 , H_3H_4 , or L_sL . The trigger requires at least one particle in each hemisphere¹⁾.

Photo 2 ISR intersection region I8 before (a) and after (b) the installation of the PSB experiment.

Fig. 6 Angular distributions of charged particles as measured at the ISR by PSB at $\sqrt{s} = 23$ and 63 GeV for various observed multiplicities. Data have been corrected for spurious counts due to secondary interactions and for particle losses by means of a Monte Carlo calculation¹⁰⁾. A systematic error of about 10%, not shown in the data, is attached to these corrections. Statistical errors are negligible. The dip at $\eta = 0$ can be accounted for by the distortion introduced by the use of η in place of y .

Fig. 7 Inclusive distributions of particles of fixed transverse momentum versus rapidity in the projectile rest frame. Data have been collected by the following groups: BC, CHLM, BS and SS. This compilation can be found in Ref. 7. The integral of each spectrum over y and p_T gives the partial average multiplicity. This procedure was adopted by BC⁷⁾ to evaluate the energy dependence of the average charged multiplicity $\langle n \rangle$ between $\sqrt{s} = 20$ and $\sqrt{s} = 60$, as shown later in Fig. 17.

Fig. 8 Neutron and Λ spectra up to the highest ISR and FNAL energies^{12,13}.

in the ISR energy range¹⁴⁾ (see Fig. 9). This increase becomes larger for larger values of p_T and for the production of heavier particles. This scaling violation must be compared with the 12% increase of the total proton-proton cross-section which has been found in the same energy range. Since the scaling hypothesis was proposed under the assumption of an asymptotically constant cross-section, we can suspect that the scaling quantity is not the inclusive cross-section but the particle density, defined as

$$\rho(y) = \frac{1}{\sigma} \frac{d\sigma}{dy}$$

where σ = inelastic cross-section.

For the pion density, the scaling violation is thus reduced to $(12 \pm 8)\%$ over the full energy range covered by the ISR. It must be stressed that this stability, when increasing s by a factor of 10, even if approximate to the 10% level, represents one of the most remarkable effects discovered at the ISR.

When searching for scaling properties, one should not only look for the most appropriate function but also choose the right variables. For instance, the range of validity of scaling at subasymptotic energies can be greatly extended if the variable $\tilde{x}$ is used in place of x , as proposed by Yen¹⁵⁾, where

$$\tilde{x} = \frac{E^*}{E_0^*} = \frac{2E^*}{\sqrt{s}}$$

and E_0^* and E^* are the maximum energy and the particle energy in the c.m. system, respectively. This variable differs from x only at finite energies, when the mass or the transverse momentum of the produced particle are not negligible with respect to p_L . In a very symmetric form, this variable can be expressed as

$$\tilde{x} = \left(x^2 + x_T^2 + \frac{4m^2}{s} \right)^{1/2}$$

with $x_T = 2p_T/\sqrt{s}$. A beautiful example of the early scaling shown by the data when plotted versus this variable is given in Fig. 10.

It is interesting to compare the inclusive spectra of particles produced in proton-proton collisions with the analogous data obtained at FNAL in π^-p collisions. Figure 11 shows that the inclusive π^- spectrum is not symmetric around $x = 0$ at $p_{inc} = 100 \text{ GeV}/c$ ¹⁶⁾. For negative values of x (proton fragmentation region) the π^- distribution is very similar to that obtained in pp reactions. For $x > 0$ (π^- fragmentation region) the cross-section decreases gently between $x = 0$ and $x = 0.5$, remains flat up to $x \approx 0.8$, and then shows a pronounced peak. Again we can attribute these features rather to a general leading-particle effect (large

Fig. 9 BS data on the invariant inclusive cross-section at $\theta_{cm} = 90^\circ$. ●: π^+ data, x: π^- data.

cross-section for $x > 0.5$), than to a large cross-section for diffractive dissociation of the target proton (peak at $x \approx 1$). The asymmetry around $x = 0$ shows in a very clear way that, at this energy, no "asymptotic" central plateau has been yet reached because, even at $\theta_{\text{cm}} = 90^\circ$, the production remembers on which side was the proton and on which the pion.

The experimental test of the limiting fragmentation hypothesis (LFH) has been performed at the ISR with a better accuracy, to the level of $\pm 2\%$, by taking advantage of the flexibility in operation of the storage rings. LFH requires that the distribution in momentum and angle of secondaries associated with the fragmentation of one projectile does not depend on s . This can be checked in the projectile rest frame or in any reference frame obtained from the rest frame by a finite boost. The basic idea of the measurement was to study the angular distribution of the particles produced at relatively small angles with respect to one beam (1), the energy of which was kept constant, while the energy of the other beam (2) was varied. The first check was done by the PSB group by fixing the energy of beam 1 at 15.4 GeV and by varying the energy of beam 2 between 15.4 and 26.6 GeV¹⁷⁾. The comparison of the raw distributions obtained at 15.4 + 15.4, at 26.6 + 26.6, and at 15.4 + 26.6 GeV, is shown in Fig. 12a. For a constant energy of one beam, the angular distribution at small angles around it does not feel the energy variation of the other beam. The ratio of the two angular distributions, shown in Fig. 12b, is consistent with one in nearly the full hemisphere, within statistical errors of $\pm 2\%$. Systematic errors, which heavily effect the measurements of absolute cross-sections, cancel in the determination of this ratio.

More recently, this test has been repeated by the Aachen-California-CERN Collaboration (ACC) for a more specific channel

$$pp \rightarrow (p\pi^+\pi^-) + x$$

with complete magnetic analysis and mass reconstruction¹⁸⁾. The proton and the two pions were detected in a spectrometer consisting of two septum magnets, one above and one below the beam (see Fig. 13). Also, in this case, the energy of the beam running along the spectrometer was kept fixed (at 26.6 GeV), while the energy of beam 2 was varied between 11.8 and 26.6 GeV. The mass spectra and the p_T^2 distribution of the produced resonance were found to be identical within the errors (see Fig. 14) and

$$\frac{\sigma(\sqrt{s} = 35)}{\sigma(\sqrt{s} = 53)} = 1.03 \pm 0.05 .$$

As in the previous measurement, a small error could be attributed because a number of systematic errors cancelled in the cross-section ratio.

Fig. 10 Inclusive differential cross-section for π^- production as a function of $\tilde{x} = 2E^*/\sqrt{s}$. Data range from $p_{inc} = 12 \text{ GeV}/c$ to $p_{inc} = 1500 \text{ GeV}/c$ ¹⁵⁾.

Fig. 11 Spectrum of π^- produced in $\pi^- p$ reactions at 100 GeV/c ¹⁶).

Fig. 12a Raw inclusive angular distributions for symmetric and asymmetric beam energies. O: 26.6 + 26.6 GeV (B); $\diamond$: 15.4 + 15.4 GeV (A); $\bullet$: 15.4 + 26.6 GeV (C);

12b Δ : ratio C/A; $\blacktriangle$: ratio C/B.

These two experiments, even if similar, are not completely equivalent and show nicely the difference between an inclusive and an exclusive approach. The PSB experiment checks that the inclusive cross-section at a specific value of x is energy independent, without excluding the possibility that the relative contributions of different channels depend on the energy. The ACC measurement shows, on the contrary, that the cross-section for a specific process (in particular a diffractive one) is energy independent. This does not forbid other channels, and therefore the inclusive cross-section, from depending on s .

As a conclusion to this section, we can say that the basic hypothesis of scaling and limiting fragmentation are approximately valid in the energy range covered by the ISR, and that they provide a useful framework in which more detailed dynamical studies can be pursued.

2. SEMI-INCLUSIVE MEASUREMENTS

The study of single-particle inclusive spectra can provide only a general outlook on the dynamics of hadron production. In order to obtain more detailed information, always in the spirit of an inclusive approach, we should proceed to measure two-, three-, etc.-particle inclusive cross-sections. At the very end, a complete set of inclusive cross-sections of all orders would provide the same amount of information as the study of all exclusive channels; it would also, however, require the same experimental effort and would, therefore, be outside the practical possibilities. Since complete exclusive and inclusive studies are both unachievable, the present trend is towards a more flexible approach: an exclusive study of the simplest final states (elastic scattering, quasi-two-body reactions, low-multiplicity channels) and a semi-inclusive analysis of higher multiplicity events. For instance, this second class of measurements would have to answer the following question: how single-particle inclusive spectra are modified if the events are selected according to a particular request?

Typical examples are the measurements of

- i) y and p_T spectra for events characterized by the presence of a particle at a given value of y and/or p_T (two-body correlations),
- ii) single-particle rapidity distributions inside samples of events of fixed charged multiplicity,
- iii) charge rapidity distributions for events characterized by a particle of a given charge at a given value of y ,
- iv) rapidity distributions of identified particles (for instance protons or antiprotons) when a second identified mass is detected in a fixed region of the phase space.

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Fig. 13 General layout of the ACC spectrometer. It consists of two magnets, one above and one below the beam, of 32 planes of multiwire proportional chambers, and of two four-element Cerenkov counters. It covers, altogether, a cone of ± 100 mrad aperture¹⁸⁾.

Fig. 14 Invariant mass reconstruction and p_T^2 distribution for the three body system at 26 + 26 GeV [open circles in (b) and histograms in (2), (c) and (d)] and at 11 + 26 [solid circles]¹⁸⁾.

These and similar questions, taken separately or coupled together, form the basic instruments of the semi-inclusive studies of multibody production, which have produced in the last few years a large amount of interesting information. For instance, this approach has been able to put beautifully into evidence the existence of a large diffractive component. This can be described as the excitation of one of the interacting hadrons into a higher mass state, while the other hadron remains unaffected.

This component ($\sim 20\%$ of the total inelastic production) was first discovered at the ISR as a spike in the inclusive spectrum for values of x very close to $x = 1$ ¹⁹⁾ [see Fig. 15²⁰⁾]. At the same time, semi-inclusive studies of the clustering properties of particles inside individual events of fixed multiplicity allowed the selection of a class of events characterized by a large rapidity gap between a single particle (the unaffected proton) at small angles and the other ones²¹⁾. This qualitative evidence for the diffractive component is illustrated by Fig. 16. These two pieces of experimental information give independent direct evidence for a diffractive production mechanism. The combined study of the proton momentum peak and of the associated cluster of the remaining particles will provide, in the near future, more information on this mechanism. Several experiments on this subject are at present in progress at the ISR and at FNAL, or are planned for the SPS. A detailed discussion of this process will be the subject of a separate review.

3. TWO-BODY RAPIDITY CORRELATIONS

The study of two-body correlations deals with the modifications induced in the single-particle spectra by the requirement that a particle is detected in a given region of the phase space. The correlation function depends on 5 variables, which can be chosen as the rapidity and the transverse momentum of the two selected particles, and the relative azimuthal angle ϕ .

Since in high-energy hadron interactions the dynamics is such that the large majority of the produced particles have small transverse momentum, one would expect possible dynamical correlations to show up in longitudinal phase space. For this reason this section will deal only with rapidity correlations integrated over p_{T1} , p_{T2} and ϕ . In Section 6 it will be shown that azimuthal correlations, even if small, are also present and can provide very powerful insight into the picture of the production mechanism which will develop from the analysis of rapidity correlations. From the definition of inclusive distributions, we obtain the rapidity densities

$$\rho^{(1)}(y) = \frac{1}{\sigma} \frac{d\sigma}{dp_T^2 dy} dp_T^2 \quad (1)$$

Fig. 15 Invariant cross-section for inclusive proton production plotted against x at $s = 6.7^{20}$ and 30 GeV. ISR data are from the BS, BC and CHLM Collaborations. The similarity of the two spectra and the knowledge that the forward peak in the low-energy data is due to the excitation of $I = \frac{1}{2}$ resonances has led to the association of the high-energy peak with a diffractive production mechanism.

31.4+31.4 GeV/c

Fig. 16 Distribution of events versus size and position of the diffractive cluster. For each event of a given multiplicity the two extreme tracks in the η -plot are disregarded. The remaining ones are used to calculate the average $\bar{\eta} = 1/(n-2) \sum \eta_i$ and the dispersion, $\delta(\bar{\eta}) = (\sum (\eta_i - \bar{\eta})^2)/(n-3)$. Each event is then represented by a point in a two-dimensional plot, versus δ and $\bar{\eta}$. For diffractive events $\bar{\eta}$ is large and δ is small, while for non-diffractive events $\bar{\eta} \approx 0$ and δ is large. Data (more than 10^6 events) are replaced by an interpolated analytical fit. We see, qualitatively, that the two diffractive peaks are found to dominate at low multiplicity, while they become negligible for $n > 10$.

$$\rho^{(2)}(y_1, y_2) = \frac{1}{\sigma} \int \frac{d\sigma}{dp_{T_1}^2 dp_{L_1} dp_{T_2}^2 dp_{L_2}} dp_{T_1}^2 dp_{T_2}^2, \quad (2)$$

where σ is the total inelastic cross-section.

In terms of these quantities we can define the two-body correlation function²²⁾

$$C(y_1, y_2) = \rho^{(2)}(y_1, y_2) - \rho^{(1)}(y_1)\rho^{(1)}(y_2) \quad (3)$$

as the difference between the two-body inclusive density and the product of the two independent single-particle densities. By integrating over y_1 and y_2 we obtain

$$\int \rho^{(1)}(y) dy = \langle n \rangle \quad (4)$$

$$\int \rho^{(2)}(y_1, y_2) dy_1 dy_2 = \langle n(n-1) \rangle \quad (5)$$

$$\int C(y_1, y_2) dy_1 dy_2 = \langle n(n-1) \rangle - \langle n \rangle^2 = f_2, \quad (6)$$

where f_2 is the second moment of the multiplicity distribution.

Quite independent of a direct measurement, one can make the following statements on $C(y_1, y_2)$:

- a) The production of a particle at a given rapidity must affect the production of the remaining particles through energy and momentum conservation. Since the selection of a particle of momentum $\vec{p}$ forces the remaining particles to arrange their momenta in order to balance it, these correlations are expected to be locally negative and positive for particles emitted at rapidities of opposite sign. This is called a long-range effect, to the extent that it affects all the produced particles. This has to be contrasted with purely short-range effects which would involve particles which are near by in phase-space; for example, those associated with the decay of a particular resonance.
- b) The integral of the $C(y_1, y_2)$ function is connected through Eq. (6) to the width of the multiplicity distribution. Since f_2 is found to be positive for laboratory momenta larger than 60 GeV/c and increases sharply with increasing energy [see Fig. 17²³⁾], some dynamical mechanism, dominant at high energies, must be responsible for abundant positive correlations. Indeed, a striking phenomenon discovered experimentally at the ISR is the importance of positive short-range correlations.

Before presenting the experimental results, we shall describe the main features of the experiments that have produced the bulk of the data. As in the description of the data on single-particle inclusive distributions, we shall limit ourselves to low and moderate values of p_T . The reader is referred to a forthcoming issue of this Journal for the study of correlations with particles of large transverse momentum⁵⁾.

At FNAL the main source of information has been the 30 in. hydrogen bubble chamber. Data have been collected

- a) in an energy range between 100 and 400 GeV/c,
- b) with direct measurement of the rapidity of each individual track,
- c) without identification of the particle nature, but with recognition of the sign of the charge,
- d) with poor statistics,
- e) with good pattern recognition of each single event.

At the ISR two-body correlations have been studied with the following detectors:

i) CERN-Hamburg-Vienna Collaboration (CHV)

A set of lead-glass Čerenkov counters detected photons (from π^0 decay) produced at various angles (see Fig. 18), while two rings of counters detected the charged particles produced at $y = 0$ and $y = -2.5$ ²⁴⁾.

The trigger of the experiment and the selection of beam-beam events was obtained by means of two sets of counters downstream of each beam.

ii) Pisa-Stony Brook Collaboration (PSB)

A large system of scintillation counter hodoscopes covered nearly the full solid angle around the interaction region, (see Fig. 5), splitting it into ~ 250 separate cells of different production angle and azimuth. The trigger was defined by the request of at least one particle in each hemisphere, and was able to include $\sim 97\%$ of all inelastic events.

iii) Aachen-CERN-Munich Collaboration (ACM)

Two big streamer chambers, above and below the interaction region, allowed the detection of charged particles produced between $y = -2.5$ and $y = 2.5$ (Fig. 19). The trigger was given by two systems of scintillation counters positioned downstream of the interaction point and detecting $\sim 90\%$ of the inelastic production²⁵⁾.

a Momentum dependence of $\langle n_{ch} \rangle$ as measured at Serpukhov and FNAL (●²²), and estimated at ISR (O⁷).

b Momentum dependence of the second moment of the multiplicity distribution for negative particles produced in pp collisions²³.

Fig. 17

82809

Fig. 18 Schematic layout of the CHV experiment²⁴).

In the three above experiments only production angles were directly measured. Therefore the variable used to present the data was η instead of the rapidity.

In a few other experiments, single-particle magnetic spectrometers were used in conjunction with large coverage counter detectors. In this way the rapidity and the mass of particle 1 were directly measured, while the correlation function was studied as a function of η . This was the case of the following detectors:

iv) CHLM Collaboration

A barrel of scintillation counters allowed the study of the production in the central region associated with a particle detected in the forward spectrometer²⁶⁾ (see Fig. 20);

v) PSB-CERN-Roma Collaboration

The PSB hodoscope system was triggered by a negative particle produced at 0° with $0.4 < x < 1$ and detected in a magnetic spectrometer (see Fig. 21)²⁷⁾.

In these experiments the measurement of the absolute value of the correlation function requires the correction of the data for spurious particle production in the pipe and the material surrounding the beams and for the particle loss due to the finite resolution of detectors. Depending on the experiment, these corrections can be rather complex. Therefore, it was often customary to introduce a "divided" correlation function

$$R(y_1, y_2) = \frac{C(y_1, y_2)}{\rho^{(1)}(y_1)\rho^{(1)}(y_2)} = \frac{\rho^{(2)}(y_1, y_2)}{\rho^{(1)}(y_1)\rho^{(1)}(y_2)} - 1, \quad (7)$$

which is less sensitive to experimental errors than $C(y_1, y_2)$. In fact, to the extent that the double efficiency function (containing all corrections mentioned above) $\varepsilon(y_1, y_2)$ is equal to the product of the two single uncorrelated efficiencies

$$\varepsilon(y_1, y_2) = \varepsilon(y_1)\varepsilon(y_2),$$

these functions cancel out in the definition of $R(y_1, y_2)$.

Figure 22 gives an example of the data from which the values of $R(y_1, y_2)$ are calculated. The double differential cross-section is divided by the product $\rho^{(1)}(y_1)\rho^{(1)}(y_2)$ and the ratio is compared to 1.

A very instructive, even if somehow qualitative, description of the behaviour of $R(y_1, y_2)$ is given by two-dimensional plots in which continuous lines connect points of constant R . These plots were first produced by the PSB Collaboration, studying the charged-charged correlation function, $R_{cc}(y_1, y_2)$. Since pions

Photo 3 Typical streamer chamber event as detected by the ACM Collaboration.
As shown in the diagram below, photons are converted in lead absorbers.

Fig. 19 Experimental set-up of the ACM Collaboration²⁵⁾. The distance between the chambers is 8 cm. Each chamber has a volume of $50 \times 125 \times 270 \text{ cm}^3$. The length of the tracks ranges between 13 and 30 cm. Typical errors are $\Delta\phi \approx 5^\circ$, $\Delta\eta \approx 0.1$. Converted γ -rays and secondaries of nuclear interactions introduce a contamination of $\sim 3\%$ in the central rapidity region.

Fig. 20 CHLM central detectors for correlation measurements in conjunction with the small-angle spectrometer (SAS) on the left-hand side. The azimuthal interval covered by these counters with respect to SAS is $70^\circ < \phi < 110^\circ$.

represent $\sim 80\%$ of the over-all production, this quantity can be considered a good approximation of the pion-pion correlation function $R_{\pi\pi}$.

Figures 23 a to d show this function at 100 and 400 GeV/c incident momentum [$\sqrt{s} = 13.7$ and 27 GeV, respectively, FNAL data^{28, 29)}], and at $\sqrt{s} = 23$ and 63 GeV [ISR data³⁰⁾]. The most relevant feature of these data is the large mountain, positioned at the centre of the plots, which expands along the diagonal $y_1 = y_2$ when the energy increases. At the highest energies the value of $R(y_1, y_2)$ is almost constant for $y_1 = y_2$ and drops sharply as soon as one moves along the diagonal $y_1 = -y_2$ (see Fig. 24). This effect gives direct evidence for the existence of short-range positive correlations: the request that a particle be present at a given rapidity favours the production of other particles in the same region of rapidity. The numerical value of the correlation function at the top, 0.6, was first considered as an indication that short-range correlations were particularly strong. If we interpret the pion yield as coming only from ρ production and decay, a much smaller value (~ 0.2) was expected. This is due to the fact that the probability of detecting (associating) simultaneously the two pions coming from the same ρ is heavily reduced by the large value of the particle density in the central region ($\rho \approx 2$) integrated over the rapidity range over which the two pions coming from the same ρ are spread. We shall, however, see in the following that only a part of the observed effect is due to short-range correlations.

If we cut these two-dimensional plots along lines of fixed y_1 , we observe that the correlation function has a peculiar triangular shape. Figure 25 shows a set of data obtained at various energies for $y_1 = 0$. The top value of the function, $R(0,0)$, is constant over the full energy range, while the width increases regularly with increasing energy.

When we select different, fixed values of y_1 (see Fig. 26), we see that the peak of $R(y_1, y_2)$ follows the position of the chosen particle. At low energies the top value drops quickly when y_1 is changed, while at top ISR energies it remains constant over an interval of $\sim \pm 2$ rapidity units.

A second interesting feature shown by the two-dimensional plots is the positive value of $R(y_1, y_2)$ for large $y_1(y_2)$ and $-y_2(-y_1)$ values (see also Fig. 24). This correlation connects a particle of large rapidity (fast particle at small angle) to a burst of particles produced on the opposite side of the rapidity plot. Events showing this kind of correlation are to be associated with the diffractive component of the inelastic cross-section.

The scaling properties of the R function are very clearly shown by Fig. 27, where the values of $R(\eta_1, \eta_2)_{\eta_1=\eta_2}$ are plotted, at two different energies, against the variable $q = \sqrt{s}/2M \tan \theta$ (θ is the c.m. production angle, M is the proton mass).

Fig. 21 Scheme of the CERN-Roma spectrometer used in the correlation experiment with the PSB detector. The first ISR magnet sweeps negative particles produced at $\theta \approx 0^\circ$ away from the beam into a set of MPWC (F_1, F_2, B_1, B_2), a large bending magnet (4 Tm maximum field integral) and 4 threshold Čerenkov counters. The solid angle covered by the spectrometer is $\sim 4 \mu sr$; its momentum acceptance is $\pm 12\%$. Momentum and angle resolutions are $\pm 1\%$ and ± 0.3 mrad, respectively.

Fig. 22 Inclusive two-particle differential cross-section (●) and product $\rho^{(1)}(y_1)\rho^{(1)}(y_2)$ (Δ) at 31 + 31 GeV. Data are from the PSB Collaboration.

Fig. 23 Two-dimensional plot of $R(y_1, y_2)$: (a) at 102 GeV/c; (b) at 400 GeV [FNAL data^{28, 29}], and (c) at 11 + 11 and (d) 31 + 31 GeV [ISR data³⁰]. The qualitative aspect of the plot does not change significantly over the full ISR energy range.

Fig. 24 Two-body rapidity correlations $R(\eta_1, \eta_2)$; $\circ$: along the diagonal $\eta_1 = \eta_2$; $\bullet$: along the diagonal $\eta_1 = -\eta_2$. Data are from PSB.

Fig. 25 Correlation function $R(\eta_1, \eta_2)$ for $\eta_1 = 0$ at various energies.
x: $\sqrt{s} = 14$ GeV ²⁸⁾, O: $\sqrt{s} = 23$ GeV, ●: $\sqrt{s} = 63$ GeV ³⁰⁾.

Since (see Appendix)

$$q \cong \exp \left(\frac{Y}{2} - y^* \right)$$

(where Y is the over-all rapidity interval), the logarithm of q can be used, in experiments where only angles are measured, to approximate the rapidity of the produced particle in the beam rest frame. The energy independence of data at small q shows that, although the correlation contours expand with increasing energy (see Figs. 23 and 25), the amount of this phenomenon is as required by scaling. Indeed scaling (or limiting fragmentation, applied to correlations, requires that inside a fragmentation cone [namely at small $(Y/2) - y^*$ or small $\ln q$, as for data in Fig. 27] the function $R(\eta_1, \eta_2)_{\eta_1=\eta_2}$ depends only on $(Y/2) - y^*$ and not on s .

A similar plot is shown for $R(\eta_1, \eta_2)_{\eta_1=-\eta_2}$ in Fig. 28 (for $q < 10$, small-angle production). The peak at the lowest q -values corresponds to the enhancement seen in the corners of the two-dimensional plots, at the extremes of the diagonal $\eta_1 = -\eta_2$. Again this function is found to be energy-independent. This result can be interpreted by saying that long-range correlations inside diffractive events depend only on the rapidity distance from the parent particle and not on s .

At FNAL the ability of the bubble chamber to identify the sign of the charge of each particle has allowed the separate study of the functions $R^{+-}(y_1, y_2)$, $R^{++}(y_1, y_2)$, and $R^{--}(y_1, y_2)$. The general features of these functions are very similar to those shown by $R_{cc}(y_1, y_2)$, the main difference consisting in a higher value of $R^{+-}(0,0)$ and a smaller value of $R^{--}(0,0)$ and $R^{++}(0,0)$. Again these values are fairly energy independent and do not depend on the nature of the incoming projectile, as shown in Fig. 29³¹⁾. The difference between R^{+-} and R^{--} (R^{++}) will be very relevant in the following, when correlation data will be interpreted in terms of cluster production, by indicating that neutral clusters are favoured with respect to double charged ones.

Examples of $R^{+-}(y_1, y_2)$, $R^{--}(y_1, y_2)$ and $R^{++}(y_1, y_2)$ for $p_{inc} = 400$ GeV/c are given in the two-dimensional plots of Fig. 30²³⁾. A comparison of these plots with those obtained for R_{cc} shows that $R^{+-}(y_1, y_2)$ has the characteristic shape of an elongated ridge along the diagonal $y_1 = y_2$, as $R_{cc}(y_1, y_2)$. On the contrary, $R^{--}(y_1, y_2)$ and $R^{++}(y_1, y_2)$ are much more symmetric around the centre, $y_1 = y_2 = 0$, and are more similar to the data obtained at 100 GeV/c than at 400 GeV/c. This could be taken as an indication that the "asymptotic" limit is reached for like particles at higher energies than for charged or unlike particles. In this sense, measurements of R^{++} and R^{--} performed at the ISR energies could add an important piece of information in order to clarify our present understanding of the asymptotic trends of correlations.

Fig. 26 Correlation function $R(\eta_1, \eta_2)$ at values of $\eta_1 \neq 0$. Data are from PSB³⁰; O: at $\sqrt{s} = 23$; ●: at 62 GeV.

Fig. 27 Correlation function $R(\eta_1, \eta_2)$, measured at $\eta_1 = \eta_2$, as a function of q ($|q| = \sqrt{s}/2M |\tan \theta|$) ($\circ$: $\sqrt{s} = 23 \text{ GeV}$; $\bullet$: $\sqrt{s} = 62 \text{ GeV}$).

Fig. 28 Correlation function $R(\eta_1, \eta_2)$, measured for $\eta_1 = -\eta_2$ versus $|q| = \sqrt{s}/2M |\tan \theta|$. O: $\sqrt{s} = 23$ GeV; ●: $\sqrt{s} = 62$ GeV ³⁰.

Correlations between photons and charged particles have also been measured at ISR²⁴⁾ and at FNAL³²⁾. The results, shown in Fig. 31, are very similar to the data on $R_{cc}(y_1, y_2)$. This is not a surprising result since practically all photons are produced via π^0 and η decay and these parent particles are expected to follow the same dynamical behaviour as charged hadrons.

As a summary, we can conclude that data on R_{cc} , R^{+-} , and R^{--} provide clear indications for:

- i) strong positive short-range correlations in the central region,
- ii) long-range correlations due to the diffractive component,
- iii) scaling properties in the fragmentation cones similar to those of single-particle spectra,
- iv) larger values of R^{+-} as compared to R^{--} and R^{++} , indicating the dominance of the production of neutral pairs of particles over the production of charged pairs.

4. INTERPRETATION OF INCLUSIVE CORRELATION DATA

The study of particle production in the central region at very high energy represents the best tool to investigate the properties of hadronic matter, independently of the characteristics of the projectile particles. Since short-range correlations are dominant in this kinematical region, it is clear that they can be used as a powerful key to understanding the dynamics of the production. For this reason many models have been proposed in the last few years to explain this phenomenon.

The two classes of models which have been successfully employed at reproducing and predicting the observed behaviour of correlations are

- i) two- or multi-component models³³⁻³⁷⁾,
- ii) cluster models³⁸⁻⁴⁰⁾, which improve on the first ones.

We shall outline in the following the main features of each class of models, because both of them are needed to explain the data presented in the previous section and, more specifically, the results on correlations at fixed multiplicity which will be presented in Section 5. For a detailed discussion of the various models the reader is referred to the review of Bialas⁴¹⁾ and to the references quoted therein.

In the framework of the original two-component model, inelastic events are separated into two categories: diffractive^(D) and non-diffractive^(π). These classes of events have different rapidity and multiplicity distributions: the

Fig. 29 Energy dependence of $R^+(0,0)$ and $R^-(0,0)$ at Serpukhov and FNAL energies. (x): pp interactions; (O): π^-p ; (●): π^+p . This compilation is due to Ko³¹.

Fig. 30 Correlation function $R(y_1 y_2)$ for various charge combinations at 400 GeV/c ²³).

Fig. 31 γ -charged particle correlations at different energies and for two different values of y_{ch} ($\circ$: $y_{ch} = 0$; $\bullet$: $y_{ch} = -2.5$)²⁴).

multiplicity distribution is expected to be mostly energy independent and centred at low values of n for D-events and to move towards larger multiplicities with increasing energy for π -events. The rapidity distribution of D-events is characterized by a single particle on one side and by a bunch of particles on the other side. The same distribution for π -events is symmetric around zero c.m. rapidity and scales asymptotically in the central region. Each event is assumed to have a definite structure (D or π), so that there is no interference between the two components.

The dispersion of the over-all multiplicity distribution, in terms of the dispersions of the separate multiplicity distributions D_D^2 and D_π^2 , is given by

$$D^2 = \alpha_\pi D_\pi^2 + \alpha_D D_D^2 + \frac{\alpha_D}{\alpha_\pi} \left(\langle n \rangle - \langle n_D \rangle \right)^2, \quad (8)$$

where

$$\alpha_\pi = \frac{\sigma_\pi}{\sigma}, \quad \alpha_D = \frac{\sigma_D}{\sigma},$$

while $\langle n \rangle$ and $\langle n_D \rangle$ are the average values of the multiplicity for all and for diffractive events, respectively.

If $\langle n \rangle$ increases faster than $\langle n_D \rangle$ with increasing energy, the third term of expression (8) explains quite naturally the broadening of the over-all multiplicity distribution and the increasing positive value of f_2 at high energies.

The inclusive correlation function can be expressed in terms of the correlations inside each group of events

$$C(y_1, y_2) = \alpha_\pi C_\pi(y_1, y_2) + \alpha_D C_D(y_1, y_2) + \frac{\alpha_D}{\alpha_\pi} \left[\rho^{(1)}_\pi(y_1) - \rho^{(1)}_D(y_1) \right] \left[\rho^{(1)}_\pi(y_2) - \rho^{(1)}_D(y_2) \right]$$

[Notice that Eq. (9), integrated over y_1 and y_2 , gives again Eq. (8)].

Even if C_π and C_D were negligible, the third ("crossed") term of Eq. (9) would provide a positive correlation function, because $\rho^{(1)}_D(y) \ll \rho^{(1)}_\pi(y)$ in the central region. It is interesting to notice that, in this case, correlations would result only from the difference of single-particle spectra of diffractive and non-diffractive events and would not correspond to any dynamical effect in individual events. In this case $C(y_1, y_2)$ and $R(y_1, y_2)$ should exhibit the scaling properties of single-particle spectra and should develop a central plateau at asymptotic energies. Since all basic features of the two-component model are experimentally observed, one should expect the crossed term to play an important

role in the explanation of correlations. It is clear, however, that this term alone cannot explain all experimentally observed features of $C(y_1, y_2)$, in particular the sharp peak at $y_1 = y_2$ which can only be attributed to the $C_\pi(y_1, y_2)$ term. Several attempts have been made to separate this term from the crossed term, but the present experimental information on α_D and $\rho_D^{(1)}(y)$ is too poor to allow a model-independent separation^{39, 40}.

A simple interpretation of the short-range component is given in the framework of cluster models, according to which particles are produced in a two-step process:

- i) production of clusters (or fireballs),
- ii) decay of the clusters into final-state particles.

The collective motion of the cluster fragments gives a natural explanation of the short-range, positive correlations.

While different models make different assumptions on the cluster production mechanism [multiperipheral, thermodynamical, uncorrelated models⁴¹] they generally agree on a set of simplifying assumptions which can be made without altering the essential features of the models:

- i) absence of correlations among clusters,
- ii) isotropic decay of clusters in their rest frames, and
- iii) energy independence of the decay parameters.

The average transverse momentum of secondaries can be simply related to the average momentum of the cluster decay products in the cluster rest frame. Knowledge of this parameter and the assumed isotropy of the cluster decay allow one to calculate the rapidity distribution of particles associated with the decay of the same cluster. One finds that this distribution is well approximated by a Gaussian form⁴²)

$$D(y, y_c) = \frac{1}{\sqrt{2\pi} \delta} e^{-(y-y_c)^2/2\delta^2}, \quad (10)$$

where y_c is the rapidity of the cluster and $\delta = 0.6-0.9$.

In this approach, the double differential cross-section splits into two terms according to whether both particles come from the decay of the same cluster or from the decay of different clusters. Short-range correlations are contained in the first term, to the extent that increasing the energy allows for production of more clusters, but that the properties of these clusters are well defined. The short-range correlations, associated with cluster formation and decay, are energy independent. This is clearly shown by Fig. 25. Under the hypothesis of

cluster-independent emission, the second term almost cancels with the uncorrelated product $\rho^{(1)}(y_1)\rho^{(1)}(y_2)$ in the correlation formula.

We get finally

$$C(y_1, y_2) = A e^{-(y_1 - y_2)^2 / 4\delta^2} + B \rho^{(1)}(y_1) \rho^{(1)}(y_2) . \quad (11)$$

where A and B are expressed in terms of the parameters characterizing the cluster production and decay (average and width of both multiplicity distributions). These parameters are too many to be determined by means of a single measurement without further model-dependent assumptions on these distributions⁴³.

5. TWO-BODY CORRELATIONS AT FIXED MULTIPLICITY

The most successful way to separate long- and short-range effects has been the study of correlations inside events of fixed multiplicity.

Let us define the topological density of particles as

$$\rho_n^{(1)}(y) = \frac{1}{\sigma_n} \frac{d\sigma_n}{dy} \quad (12)$$

and

$$\rho_n^{(2)}(y_1, y_2) = \frac{1}{\sigma_n} \frac{d^2\sigma_n}{dy_1 dy_2} . \quad (13)$$

We can again define the correlation function as

$$C_n(y_1, y_2) = \rho_n^{(2)}(y_1, y_2) - \rho_n^{(1)}(y_1) \rho_n^{(1)}(y_2) . \quad (14)$$

In this case, at fixed values of y_1

$$\int C_n(y_1, y_2) dy_2 = -\rho_n(y_1) \quad (15)$$

and

$$\iint C_n(y_1, y_2) dy_1 dy_2 = -n . \quad (16)$$

The correlation function can also be defined in a slightly different way, such that, for uncorrelated production it would be identically zero:

$$C'_n(y_1, y_2) = \frac{\rho_n^{(2)}(y_1, y_2)}{n(n-1)} - \frac{\rho_n^{(1)}(y_1)}{n} \frac{\rho_n^{(1)}(y_2)}{n} \quad (17)$$

Notice that, if in events of multiplicity n , particles are produced uncorrelated from each other,

$$\frac{d^2\sigma_n}{dy_1 dy_2} = \frac{d\sigma_n}{dy_1} \frac{n-1}{n} \rho_n^{(1)}(y_2) .$$

The experimental problems involved in the measurement of semi-inclusive two-body correlations are rather severe.

These correlations, even assuming integration over p_T , must be studied as a function of 4 variables, y_1 , y_2 , n , and s . The interest of the measurements depends clearly on the completeness of the analysis, over a wide range of the variables, outside the region of maximum rate $y_1 \approx y_2$, $n \approx \langle n \rangle$. Therefore, high statistics and simultaneous coverage of the full solid angle are required from the experiments. In addition, each event must be reconstructed well enough to identify the multiplicity and to measure the momenta, and not only the production angles, of all particles. These two requirements are conflicting in practice, and have not yet been met completely by any present experiment.

Data available at present have been collected at FNAL in the 30 in. bubble chamber and at the ISR by the ACM and PSB groups. Typical FNAL experiments deal with some thousands of completely reconstructed events⁴⁴⁻⁴⁶). The data obtained by the ACM²⁵) group are based on a similar number of events, at two energies. In both experiments the visual technique allowed a clear identification of the tracks, a recognition of spurious secondary interactions, and a precise measurement of the event multiplicity.

The PSB experiment has produced a few million events at each energy that allow, in principle, a detailed study of the correlation functions over very wide intervals of the relevant variables¹¹). However, each track is identified in this experiment only by the fact that a counter is fired in the hodoscopes surrounding the interaction region, so that it is impossible to reject in a particular event possible spurious tracks produced in a secondary interaction. In addition, more particles falling in the same counter are counted as a single one. An efficiency matrix $\epsilon(y,n,s)$ has been determined for each counter by means of a Monte Carlo calculation. It has been used to correct in an average way the measured distributions.

A comparison of a number of features typical of these three experiments is shown in Table 1.

Table 1

Mean multiplicities for rapidity intervals
taken from three experiments

Exp	Variable	Δy	$\langle n \rangle$	Events	Corrections
FNAL	y	full	8.5	2.5×10^3	small
ACM	η	± 2.5	5.5	2.5×10^3	$\sim 10\%$
PSB	η	full	9.2-8.2	5×10^6	10-40%

A detailed comparison of the results obtained in the three experiments is made difficult by the use of different variables or different ranges of the same variable.

The main features of C_n are however clearly identified in all sets of data.

The semi-inclusive correlations measured for $y_1 = 0$ are shown in Figs. 32, 33, 34 and 35. A narrow spike around $y_1 = 0$ stands up over a broad negative pedestal. It should be observed that C_n is forced to be mostly negative by its over-all normalization

$$\int C_n(0, y_2) dy_2 = -\rho_n(0) .$$

The central peak is much narrower than the peak shown by the inclusive correlations and is found to be remarkably independent in shape of the multiplicity and of the energy.

The first interesting result of this analysis is obtained in the study of the relationship between inclusive and semi-inclusive correlation data. Since samples of events of different multiplicity may be treated formally as different components of the production, the inclusive correlation function can be expressed as

$$C(y_1, y_2) = \sum \alpha_n C_n(y_1, y_2) + \sum \alpha_n \times \left(\rho^{(1)}(y_1) - \rho_n^{(1)}(y_1) \right) \left(\rho^{(1)}(y_2) - \rho_n^{(1)}(y_2) \right) \quad (18)$$

with $\alpha_n = \sigma_n / \sigma$.

In complete analogy [see formula (9)] with the previously discussed case of the two-component model, the inclusive correlation function is therefore expressed

in terms of

- a) the average of the semi-inclusive C_n functions over the multiplicity distribution,
- b) a crossed term containing the difference between inclusive and semi-inclusive particle densities.

The relevant difference between terms (a) and (b) consists in the fact that terms (b) can be completely predicted on the basis of inclusive and semi-inclusive single-particle distributions $[\rho^{(1)}(y)$ and $\rho_n^{(1)}(y)]$, while term (a) needs the direct measurement of two-particle densities in individual events. If short-range correlations contain a truly dynamical effect, this must show up in term (a); on the other hand, the presence of a diffractive component is mostly felt in terms (b) and at low multiplicities, so that it can be eliminated by selecting $n > \langle n \rangle$.

In the PSB experiment the simultaneous measurement of $\rho^{(1)}(\eta)$, $\rho_n^{(1)}(\eta)$, $\rho^{(2)}(\eta_1, \eta_2)$ and $\rho_n^{(2)}(\eta_1, \eta_2)$ allowed checking the relative importance of the two terms. This is done in Fig. 36, which shows that the inclusive correlation function at $\sqrt{s} = 23$ and $\sqrt{s} = 63$ GeV are heavily dominated by terms (b), while terms (a) add only a narrow central spike at the centre of the broad peak. This central peak provides, however, the clearest evidence for the existence of short-range correlations and allows a quantitative study of their properties.

A simple empirical fit to the data is obtained by means of the expression ^{*)}

$$C_n(y_1, y_2) = A_n e^{-(y_1 - y_2)^2 / 4\delta_n^2} - B_n \rho_n(y_1) \rho_n(y_2), \quad (19)$$

in which the parameter δ_n gives the width of the correlation effect and A_n its strength. The good quality of this fit is shown in Figs. 37 and 38, where the Gaussian term has been isolated in ACM and PSB experiments.

The parameters A_n and B_n have a transparent meaning if interpreted in the framework of the cluster model^{43, 47, 48}). In this approach, the semi-inclusive correlation function is expressed as

$$C_n(y_1, y_2) \Big|_{y_1=0} = \left[\frac{\langle K(K-1) \rangle}{\langle K \rangle} \right]_n \rho_n^{(1)}(0) e^{-(y_1 - y_2)^2 / 4\delta_n^2} - \left(1 - \left[\frac{\langle K(K-1) \rangle}{\langle K \rangle} \right]_n \right) \rho_n^{(1)}(0) \rho_n^{(1)}(y_2), \quad (20)$$

where K is the multiplicity of the cluster decay.

^{*)} The Gaussian term is written in this form to make easier the comparison with the predictions of the cluster model.

Fig. 32 Semi-inclusive correlations (C_n) obtained by PSB¹¹) at $\sqrt{s} = 23$ GeV. The three sets of data are labelled with the measured multiplicity. The corresponding average multiplicities at the production are: (a) $n = 9.11$, (b) $n = 14.0$, (c) $n = 21.7$.

Fig. 33 Semi-inclusive correlations as measured by PSB at $\sqrt{s} = 63 \text{ GeV}^{11)$. As in the previous figure: (a) $n = 8.29$, (b) $n = 19.2$, (c) $n = 30.3$.

Fig. 34 Weighted average of the semi-inclusive correlation functions (calculated according to definition (17) as obtained by ACM²⁵⁾ for $6 < n < 15$. The average is defined as $\langle (n-1)C'_n(\eta_1, \eta_2) \rangle = 1/\sigma \sum_{n=6}^{15} (n-1)\sigma_n C'_n(\eta_1, \eta_2)$. Open squares ($\square$) give the same quantity as determined in the PSB experiment and show the good agreement between results obtained in two experiments using completely different techniques.

Fig. 35 AFIMM data at $n = 8$ and $200 \text{ GeV}/c$ ⁴⁶). Correlations are selected according to the charge of the detected particles. The short-range effect is present in all sets of data.

Both factors A_n and B_n contain the quantity $[\langle K(K-1) \rangle / \langle K \rangle]_n$, which depends on the average multiplicity and on the width of the multiplicity distribution of cluster decay. If, for instance, this distribution was a δ -function, with a fixed number of secondaries K_0 , this factor would be independent of n

$$\frac{K(K-1)}{K} = K_0 - 1.$$

If the distribution was Poisson-like, the observed multiplicity distribution of final-state particles would be the convolution of the multiplicity distributions of cluster production and decay. The selection of a definite value of n would therefore favour a particular interval of multiplicities in the decay and the factor $[\langle K(K-1) \rangle / \langle K \rangle]_n$ would depend on n .

An approximate relationship would be in this case⁴³:

$$\left[\frac{\langle K(K-1) \rangle}{\langle K \rangle} \right] \approx \frac{\langle K \rangle}{\langle n \rangle} (u - 1),$$

implying a linear growth of this factor with n around $n = \langle n \rangle$. The study of the dependence of this factor on n allows one therefore to learn something of the multiplicity distribution of cluster decay.

PSB data provide a value of δ_n which is independent of n for $n > 8$ and is the same at both energies $\delta = 0.62 \pm 0.05$. A completely consistent result ($\delta = 0.65$) has been obtained in the analysis of ACM data. A more detailed analysis (performed by ACM) of this parameter shows that the quoted value is the average of δ over the azimuthal angle ϕ ranging from $\delta = 0.5$ for $\phi < \pi/2$ and $\delta = 0.87$ for $\phi > \pi/2$ (see Section 6). The absolute value of δ and its energy and multiplicity independence are in fair agreement with the predictions of the cluster model.

The ACM and PSB Collaborations agree quite well also in the determination of $\langle K(K-1) \rangle / \langle K \rangle$. The ACM Collaboration finds an average value

$$\frac{\langle K(K-1) \rangle}{\langle K \rangle} = 1.2 \pm 0.2.$$

The large statistics of the data available allowed the PSB Collaboration to perform a detailed study of the n -dependence of this quantity. The results (see Fig. 39) show that this ratio is sensibly energy independent if plotted against $n/\langle n \rangle$ and increases with $n/\langle n \rangle$. This increase, less than linear, tells us that the multiplicity distribution is broader than a δ -function and narrower than a

exaggerated line (a) $\delta = 0.62$ at $\sqrt{s} = 2.76$ GeV (b) $\delta = 0.65$ at $\sqrt{s} = 2.76$ GeV (c) $\delta = 0.62$ at $\sqrt{s} = 2.76$ GeV (d) $\delta = 0.65$ at $\sqrt{s} = 2.76$ GeV (e) $\delta = 0.62$ at $\sqrt{s} = 2.76$ GeV (f) $\delta = 0.65$ at $\sqrt{s} = 2.76$ GeV (g) $\delta = 0.62$ at $\sqrt{s} = 2.76$ GeV (h) $\delta = 0.65$ at $\sqrt{s} = 2.76$ GeV (i) $\delta = 0.62$ at $\sqrt{s} = 2.76$ GeV (j) $\delta = 0.65$ at $\sqrt{s} = 2.76$ GeV (k) $\delta = 0.62$ at $\sqrt{s} = 2.76$ GeV (l) $\delta = 0.65$ at $\sqrt{s} = 2.76$ GeV (m) $\delta = 0.62$ at $\sqrt{s} = 2.76$ GeV (n) $\delta = 0.65$ at $\sqrt{s} = 2.76$ GeV (o) $\delta = 0.62$ at $\sqrt{s} = 2.76$ GeV (p) $\delta = 0.65$ at $\sqrt{s} = 2.76$ GeV (q) $\delta = 0.62$ at $\sqrt{s} = 2.76$ GeV (r) $\delta = 0.65$ at $\sqrt{s} = 2.76$ GeV (s) $\delta = 0.62$ at $\sqrt{s} = 2.76$ GeV (t) $\delta = 0.65$ at $\sqrt{s} = 2.76$ GeV (u) $\delta = 0.62$ at $\sqrt{s} = 2.76$ GeV (v) $\delta = 0.65$ at $\sqrt{s} = 2.76$ GeV (w) $\delta = 0.62$ at $\sqrt{s} = 2.76$ GeV (x) $\delta = 0.65$ at $\sqrt{s} = 2.76$ GeV (y) $\delta = 0.62$ at $\sqrt{s} = 2.76$ GeV (z) $\delta = 0.65$ at $\sqrt{s} = 2.76$ GeV

Fig. 36 Inclusive correlation function at $\sqrt{s} = 23$ GeV (a) and 63 GeV (b) and separate contributions to $C(\eta_1, \eta_2)$ of $\sum_n C_n(\eta_1, \eta_2)$ ($\blacksquare$: at $\sqrt{s} = 23$; $\triangle$: at $\sqrt{s} = 63$) and of the crossed term (x: at $\sqrt{s} = 23$; $\bullet$: at $\sqrt{s} = 63$ GeV).

Fig. 37 Short-range term according to ACM data²⁵⁾, at the extreme ISR energies. Data are collected at fixed values of $|\eta_1 - \eta_2|$ and are summed over n ($6 < n < 15$ at 23 GeV and $6 < n < 20$ at $\sqrt{s} = 63$ GeV).

Fig. 38 Short-range term, as obtained by PSB ¹¹⁾ after subtraction of the negative pedestal at two values of the multiplicity.

Poisson-like distribution. At $n \approx \langle n \rangle$ the PSB Collaboration obtains, at the two extreme ISR energies,

$$\left[\frac{\langle K(K-1) \rangle}{\langle K \rangle} \right]_{\langle n \rangle} = 1.0 \pm 0.1 .$$

The small value of this figure, implying an average number of charged particles $\langle K \rangle \leq 2$, suggests that well-known resonances, such as vector or scalar bosons, play an important role in the cluster structure of central multibody production⁴⁹⁾.

These data provide a first piece of information on a possible cluster structure of multibody hadron production. A direct way to obtain more details on the multiplicity distribution of cluster decay would be the measurement of three-body correlations. Attempts have been made in this direction by the ACM Collaboration²⁵⁾, but the results are still not significant because of the lack of statistics.

A more promising tool for the study of clustering effects seems to be the simultaneous measurement of y and ϕ (azimuthal angle) correlations at fixed multiplicity and for definite combinations of charges and masses, as shown in the next sections.

Before passing to the description of these data, it is interesting to mention that a different approach to this problem, complementary to the analysis of two-body rapidity correlations, is the study of the distribution of rapidity gaps (r) between subsequent particles inside individual events.

Let us assume that we are working at asymptotic energies, such that

$$\rho^{(1)}(y) = \rho = \text{constant}$$

and the rapidity interval covered by the experiment tends to infinity.

If correlations are absent, it is easily seen that the gap distribution is given by

$$P(r) = \rho e^{-\rho r} .$$

If we now switch on the production of clusters, with the corresponding short-range correlations of width δ , the exponential fall-off of $P(r)$ remains valid for $r > \delta$, where the particles involved come from different clusters. Thus, at large r , $\rho = \rho_c$ gives the density of cluster production. For $r \leq \delta$, the two particles defining the gap have a big chance of being produced in the same cluster and $P(r)$ is enhanced. We expect, therefore, an exponential dependence at large r , and a similar but faster decrease at small r . This qualitative behaviour is indeed

Fig. 39 Dependence of the factor $\left[\frac{\langle K(K-1) \rangle}{\langle K \rangle} \right]_{n=23 \text{ GeV}}$ as a function of $n/\langle n \rangle$ at the two extreme ISR energies¹¹; ●: $\sqrt{s} = 23 \text{ GeV}$; ×: $\sqrt{s} = 31 \text{ GeV}$.

shown by the data, as remarked by Quigg et al.⁵⁰⁾ in the analysis of FNAL data (see Fig. 40). A preliminary analysis based on the assumptions quoted above has produced a value of cluster multiplicity in reasonable agreement with that obtained in the study of semi-inclusive correlations. Unfortunately, the basic assumption $\rho = \text{constant}$ is far from being realistic at FNAL and also at ISR energies, and a quantitative conclusion must wait for a careful analysis which would have to take into account the distortion of the exponential fall-off due to the limited rapidity interval available and to the shape (different from a constant) of the $\rho^{(1)}(y)$ distribution. It has also been stressed⁵¹⁾ that energy and momentum conservation reduce strongly the sensitivity of this method at FNAL energies.

6. AZIMUTHAL CORRELATIONS

The selection of a particle with a definite value of the transverse momentum forces the other particles produced in the event to arrange their momenta in order to balance the first one. The limited phase space available in the transverse direction, due to the sharp cut-off which is characteristic of the p_T distribution, allowed the prediction that energy and momentum conservation would be the dominant effect in azimuthal correlations. These correlations were therefore expected to be negative for $\Delta\phi \approx 0$ and positive for $\Delta\phi \approx \pi$.

The first experimental studies were directed to check the validity of this statement and to understand whether the p_T compensation was localized in a limited region of y around the chosen particle or was spread out along the full rapidity interval. The first measurements of azimuthal correlations^{52, 53)} were then interpreted in terms of the quantities

$$F_{Y, \text{ch}}(y, \phi) = a + b(y) \cos \phi, \quad (21)$$

or, simply,

$$A = \left[\int_0^{\pi/2} \frac{d\sigma}{d\phi} d\phi - \int_{\pi/2}^{\pi} \frac{d\sigma}{d\phi} d\phi \right] \left(\int_0^{\pi} \frac{d\sigma}{d\phi} d\phi \right)^{-1}. \quad (22)$$

Both coefficients b and A were found to become more and more negative with increasing $|p_T|$ (see Fig. 41). These data show that the asymmetry is essentially independent of y , suggesting that the transverse momentum is balanced on equal grounds by all produced particles.

A more recent and accurate measurement of these correlations has been performed by the ACM Collaboration²⁵⁾, by studying the function

Fig. 40 Distribution of gap width at 300 GeV/c as obtained in Ref. 50; ○: for charged particles; ●: for negative particles. The smaller deviation of data for negative particles from an exponential agrees with the smaller value of the correlations in (--) with respect to (charged-charged).

$$C'(\Delta\phi) = \langle (n-1)C'_n(\phi_1, \phi_2) \rangle,$$

where the average is performed for $n > 6$ and $C'_n(\phi_1, \phi_2)$ is defined in analogy with formula (17)

$$C'_n(\phi_1, \phi_2) = \frac{\rho_n^{(2)}(\phi_1, \phi_2)}{n(n-1)} - \frac{\rho_n^{(1)}(\phi_1)}{n} \frac{\rho_n^{(1)}(\phi_2)}{n}.$$

These data are plotted in Fig. 42 and show that the correlation is negative for $\Delta\phi \approx 0$ and positive for $\Delta\phi \approx \pi$, as expected.

More significant information can be obtained by a simultaneous study of rapidity and azimuthal correlations, possibly at fixed multiplicity and for different selections of the charge combinations. A first source of interest lies in the additional tests that these measurements can provide on cluster-model predictions. Furthermore, this study has allowed putting into evidence a new and more hidden mechanism responsible for correlations between like particles, which we have not discussed so far. The basic argument generalizes to multiple boson production the idea of the Brown-Twiss correlation experiment in astronomy⁵⁴⁾ and takes into account the effect of Bose-Einstein statistics on the production of like particles⁵⁵⁾. Let us suppose that two identical particles are produced with four-momenta p_1 and p_2 by two sources very close in space and in time, such that in a particular experiment one cannot distinguish which particle comes from each source. The amplitude for double production contains then two terms and the cross-section contains the corresponding interference term, which will depend on the difference $|p_1 - p_2|$. This interference term must be present in the correlations between like pions and absent in the unlike combinations.

The study of this effect in multiparticle production as a function of the difference $|p_1 - p_2|$ should allow in principle the determination of the dimensions of the hadronic interaction volume and of the collision time. This new source of correlations is not exclusive of the cluster mechanism, but it works in addition to it, both when the particles come from the decay of a single cluster (the dimensions of the source are then those of the cluster itself) or from two separate clusters⁵⁶⁾. Since this effect drops quickly down with increasing $|p_1 - p_2|$ (it is expected to disappear for $|p_1 - p_2| > 0.2 \text{ GeV}/c$) it could not be discovered in the analysis of rapidity correlations, where data were integrated over p_T and ϕ . But this effect was expected to show up in the azimuthal correlations inside a given narrow bin of y in the central region, because in this case, for average values of $|p_T|$ and for $\Delta\phi \approx 0$ the difference in total momentum between the two particles is small.

Fig. 41a) Coefficient b of the azimuthal distribution [formula (21)] plotted as function of y for different ranges of p_T ⁵²).

41b) Parameter A [formula (22)] versus p_T ⁵³) integrated over y .

The joint study of rapidity and azimuthal angle correlations has been pursued by the ACM Collaboration²⁵⁾ and by the AFIMM Collaboration⁴⁵⁾ and their results are shown in a pictorial form in Figs. 43 and 44. In Fig. 43 is displayed the weighted average

$$\langle J_n C'_n(\Delta y, \Delta\phi) \rangle = \frac{\sum J_n C'_n(\Delta y, \Delta\phi) \sigma_n}{\sum \sigma_n},$$

where

$$C'_n(\Delta y, \Delta\phi) = \frac{\rho_n^{(2)}(y_1\phi_1, y_2\phi_2)}{n(n-1)} - \frac{\rho_n^{(1)}(y_1\phi_1)}{n} \frac{\rho_n^{(1)}(y_2\phi_2)}{n}$$

and J_n is the number of involved particles minus one (for instance, for $C_{\pi-\pi-}$, $J_n = n_{\pi-} - 1$). The sum runs from $\langle n \rangle \approx 6$ to $2\langle n \rangle \approx 12$. Even if the linear n -dependence of C'_n , which is assumed to justify this average, is not strictly valid over the full multiplicity range (it would require the quantity plotted in Fig. 39 to be independent of n), this assumption is not too bad and allows the collection of reasonable statistics of events even in this multidimensional plot and to present the most significant qualitative behaviours.

The same procedure and the same selection of multiplicities is adopted in the presentation of ACM data in Fig. 44. A comparison of these data with those shown in Fig. 43c shows that:

- i) the short-range positive correlation is maximum at $\phi = 0$ and $\phi = \pi$, and smaller at intermediate angles,
- ii) the correlation peak is narrower for $\phi = 0$ than for $\phi = \pi$.

Quantitatively, for ACM data,

$$\delta(\phi < \pi/2) = 0.54 \pm 0.04$$

$$\delta(\phi > \pi/2) = 0.89 \pm 0.05$$

at $\sqrt{s} = 23$ GeV and

$$\delta(\phi < \pi/2) = 0.49 \pm 0.02$$

$$\delta(\phi > \pi/2) = 0.86 \pm 0.04$$

at $\sqrt{s} = 53$ GeV.

The additional splitting of these data, according to the charge combinations, presented by the AFIMM Collaboration (Fig. 43) shows that this behaviour is the average of very different phenomena.

Fig. 43 Joint rapidity and azimuthal angle correlations for different charge combinations in the pp and π^-p channels (AFIMM data) ⁴⁶.

For instance the (--) channel is characterized by a strong peak at $\phi = 0$, while the (-+) channel is dominated by a broader peak centred at $\phi = \pi$. Also (--) and (++) correlations show relevant differences in the importance of the positive signal. Finally, the width of the short-range correlation peak varies significantly according to the value of ϕ and of the selected charges.

These data are very recent and it is too early to derive firm conclusions from the over-all pattern. Two interesting indications have, however, been emphasized by the authors:

- i) The different behaviour of (--) and (-+) correlations is reasonably well accounted for by the identical particle effect induced by Bose-Einstein statistics.
- ii) In the framework of the cluster concept, the general structure of combined ϕ - y correlations is compatible with an abundant production of low mass resonances.

A more controversial point concerns the direct detection of ρ^0 production in the central region. While an important ρ^0 production has been found in the $\pi^- p \rightarrow \rho^0 x$ channel at 200 GeV ⁵⁷⁾, this evidence has not been confirmed by the study of inclusive reactions at 100 and 300 GeV/c in the proton-proton channel and at 100 GeV/c in the $\pi^- p$ channel performed by the AFIMM Collaboration⁴⁵⁾.

The richness of the landscape which characterizes these multidimensional plots is finally confirmed by the strongly asymmetric structure of the correlation function measured in $\pi^- p$ inclusive reactions (Fig. 43).

It is expected that a continuation of the correlation studies with improved statistics and resolutions will allow a detailed understanding of the various processes contributing to multibody production. This will be even better when the masses of the produced particles are also identified. A first example of the usefulness of this information is given by the study of correlations with a fast particle of identified mass, obtained at the ISR, and described in the next section. These measurements approach the limit beyond which the separation between semi-inclusive and exclusive channels becomes artificial.

7. LEADING CLUSTERS

We have described up to now the properties of correlations between particles produced in the central region of the rapidity plot, and we have found that the concept of cluster is very useful for the interpretation of experimental data on multibody production. If we now focus our attention on the fragmentation regions we must investigate the connections between a possible cluster production and the

$\sqrt{s} = 53 \text{ GeV}$

$\sqrt{s} = 23 \text{ GeV}$

Fig. 44 Joint rapidity and azimuthal correlations obtained by ACM²⁵. The vertical scale is in units of 10^{-2} .

well-known leading-particle effect. Quite consistently with the picture of the interaction discussed in the previous sections, one is lead to search for a possible excitation of the incoming projectile into a "leading cluster" and to compare the characteristics of this object with those of ordinary clusters. Such clusters may differ from the central ones; nevertheless they should lead to important positive short-range correlations, which are most likely to involve a few pions together with the observed proton. Indeed, a first indication for the existence of leading clusters comes from CHLM²⁶⁾ data on correlations between a particle selected in the forward spectrometer and charged particles detected in a set of counters covering the central region (see Fig. 20). Figure 45 shows the results obtained by selecting π^+ , p, and $\bar{p}$. In these plots, different curves correspond to different values of the rapidity of the selected particle, ranging between $y = 2$ and $y = 4$. The first conclusion we can draw from these data is that a strong, short correlation peak is present in all curves at $y - \eta_c \approx 0$. Since proton data do not show any qualitative difference with respect to π^+ or $\bar{p}$ data, we conclude that leading protons behave like ordinary produced particles from the point of view of correlations. The statement that we are dealing with leading protons and not protons produced in $p\bar{p}$ pairs is justified by the rate of protons being 20 times larger than the rate of antiprotons. If this effect is interpreted again in terms of cluster production²⁶⁾, 50% of the clusters produced in this rapidity range contain a proton and are therefore leading clusters.

The cluster structure of the leading particle excitation is shown in an indirect way also by recent results of the PSBCR Collaboration²⁷⁾. Figure 46 shows the data collected by triggering the PSB 4π detector with negative particles produced at 0° and $x = 0.4$. The important effect is a narrow correlation peak centred around the rapidity of the selected particle, superimposed over a smoothly decreasing distribution, whose features can be explained in terms of energy and momentum conservation. This peak is again similar in shape for π^- , K^- , and $\bar{p}$.

An analysis of the n -dependence of this peak in the π^- measurement shows that its presence dominates the low multiplicity channels, in particular the states with $n = 3-4$. In this case, a qualitative fit to the data can be obtained in terms of the reaction

where C is a leading cluster with average mass around 2 GeV.

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Fig. 45 Correlation function $H(y, p_T, \eta_c) = [d^3\sigma(y, p_T, \eta_c)/dy dp_T d\eta_c] / [d^2\sigma(y, p_T)/dy dp_T] \cdot -1$.
 The relationship between H and R is given by $R(y, p_T, \eta_c) = \sigma H(y, p_T, \eta_c) / \rho(\eta_c) - 1$.
 a) π^+ production, $\sqrt{s} = 44.7$ GeV, $2.3 < y_{\pi^+} < 3.9$, $0.3 < p_T < 0.5$ GeV/c.
 b) p production, $\sqrt{s} = 44.7$ GeV, $1.7 < y < 3.1$, $0.3 < p_T < 0.5$ GeV/c.
 c) $\bar{p}$ production $\sqrt{s} = 52.8$ GeV, $2.1 < y < 2.6$, $0.12 < p_T < 0.2$ GeV/c.

Fig. 46 Correlations with a particle emitted at 0° and $x = 0.4$ ²⁷⁾.

APPENDIX

DEFINITION OF KINEMATIC VARIABLES
CURRENTLY USED IN INCLUSIVE STUDIES

In an inelastic interaction

$$a + b \rightarrow c + X ,$$

where a and b are the incoming projectiles, c is the particle under study and X contains all the rest of the production, the momentum distribution of particle c is commonly described by the inclusive invariant cross-section

$$E_c \frac{d^3\sigma}{dp^3} = F_{ab}^c(p_L, p_T, s) \quad (A.1)$$

where p_T and p_L are, respectively, the transverse and longitudinal components of the momentum of particle c , E_c is its energy and s is the square of the total energy in the c.m. system.

Since an event with n_c particles of type c enters n_c times into the definition of F_{ab}^c , the integral of this function is given by

$$\int F_{ab}^c(p_L, p_T, s) \frac{d^3p}{E_c} = \langle n_c \rangle \sigma , \quad (A.2)$$

where σ is the total inelastic cross-section.

The longitudinal variable which is commonly used in the description of inclusive spectra is the rapidity y , defined as

$$y = \sinh^{-1} \frac{p_L}{\sqrt{p_T^2 + m^2}} = \frac{1}{2} \ln \frac{E + p_L}{E - p_L} . \quad (A.3)$$

Since this variable is additive under longitudinal Lorentz transforms, its use allows an easy comparison of data collected in different rest frames.

The relationship between y and the Feynman variable $x = 2p_L^*/\sqrt{s}$ is given by

$$x = \frac{2\sqrt{p_T^2 + m^2}}{\sqrt{s}} \sinh y^* \quad (A.4)$$

(an asterisk indicates variables measured in the c.m. system). In terms of the

width of the rapidity interval $y = \ln s/(p_T^2 + m^2)$, such that $-Y/2 \leq y^* \leq Y/2$, formula (A.4), can be rewritten as

$$n = \exp \left[- \left(\frac{1}{2} Y - y^* \right) \right] - \exp \left[- \left(\frac{1}{2} Y + y^* \right) \right]. \quad (A.5)$$

The exponents $\left(\frac{1}{2} Y - y^* \right)$ and $\left(\frac{1}{2} Y + y^* \right)$ represent the distances of the produced particle from the edges of the over-all rapidity interval. In the two fragmentation regions, $\left[\text{fixed } \left(\frac{1}{2} Y - y^* \right) \text{ or } \left(\frac{1}{2} Y + y^* \right) \right]$, one of the two terms dominates asymptotically over the other one and, consequently, also x is fixed. Because of the additivity property of the rapidity, this means that a given value of the c.m. variable x corresponds to a fixed value of the rapidity in the reference system associated with one projectile, independently of s . Through the definition (A.3), this statement is equivalent to saying that a fixed value of x implies a fixed value of p_L in the corresponding projectile rest frame.

The rapidity is often approximated by the quantity

$$\eta = - \ln \operatorname{tg} \theta/2 ,$$

which can be determined in experiments in which only angles, and not momenta, are measured.

One gets $y \approx \eta$ if in Eq. (A.3) one makes the approximation $m^2/p^2 \ll 1$. This approximation is bad for the production at very small angles ($p_T \leq m \ll p_L$). This is reflected by the fact $-\infty < \eta < \infty$, while at any finite energy y ranges over a finite interval. Some approximation is involved also for centrally produced particles (p_L very small), since they are no more relativistic in view of the p_T cut-off. Indeed the ratio between $d\sigma/dy$ and $d\sigma/d\eta$ is proportional to $E/p \approx 1/\langle v \rangle$. The error introduced by this approximation is of the order of 15%. In practice, if one integrates the inclusive cross-section over p_T at fixed θ (as is done in the experiments on angular distributions), one finds that the average value of y differs from $\eta = - \ln \operatorname{tg} \theta/2$ by less than 10%, over nearly the full rapidity range.

Using this approximate expression for the rapidity, we can define a variable which is useful to illustrate the scaling properties of inclusive distributions, even in experiments in which only angles are measured. This variable is expressed as

$$q = \frac{\sqrt{s}}{2M} \operatorname{tg} \theta ,$$

where M is the proton mass. From the definition of x , we obtain

$$x = \frac{2p_T}{\sqrt{s} \operatorname{tg} \theta} = \frac{p_T}{Mq} , \quad (A.6)$$

so that

$$F(x, p_T, s) = F\left(\frac{p_T}{Mq}, p_T, s\right).$$

Any universal function of x (scaling function) must therefore be independent of s also when plotted against q .

We notice, finally, that, by means of Eqs. (A3) and (A6) and of the definition of Y , we can write

$$\rho \approx \frac{\sqrt{s}}{2M \sinh y^*} \approx e^{Y/2 - y^*}$$

for $p_T^2 \ll m^2$ and large values of y^* . This relationship shows that, in the fragmentation region, the variable $\ln q$ measures approximately the rapidity of the produced particle in the projectile rest frame.

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