Heavy quark $1/m_Q$ contributions in semileptonic $B$ decays to orbitally excited $D$ mesons

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Exclusive semileptonic decays of $B$ mesons to orbitally excited $D$ mesons are considered beyond the infinitely heavy quark limit in the framework of the relativistic quark model based on the quasipotential approach. This model agrees with the structure of heavy quark mass corrections predicted by the heavy quark effective theory and allows the determination of corresponding leading and subleading Isgur-Wise functions. It is found that both relativistic and $1/m_Q$ contributions significantly influence the decay rates. Thus, relativistic transformations of the meson wave functions (Wigner rotation of the light quark spin) already contribute at leading order of heavy quark expansion and result in a suppression of $B\to D^*_0 e\nu$ and $B\to D^*_7 e\nu$ decay rates. On the other hand, the vanishing of the decay matrix elements at zero recoil of a final $D^{**}$ meson in the infinitely heavy quark mass limit makes the $1/m_Q$ corrections very important, and their account results in a substantial enhancement of $B\to D_1 e\nu$ and $B\to D^*_0 e\nu$ decay rates.

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I. INTRODUCTION

The investigation of semileptonic decays of $B$ mesons to excited $D$ meson states is an important problem for heavy flavor physics. In particular, these decays can provide an additional source of information for the determination of the Cabibbo-Kobayashi-Maskawa matrix element $V_{cb}$ as well as on the relativistic quark dynamics inside heavy-light mesons. The experimental data on these decays are becoming available now [1–3], and the $B$ factories will provide more accurate and comprehensive data. It is necessary to note that experiment shows that only approximately 60% of the inclusive semileptonic $B$ decay rate is due to the decays to ground-state pseudoscalar $D$ and vector $D^*$ mesons. Thus the rest of these decays should go to excited $D$ meson and continuum states.

The presence of the heavy quark in the initial and final meson states in these decays considerably simplifies their theoretical description. A good starting point for this analysis is the infinitely heavy quark limit, $m_Q \to \infty$ [4]. In this limit the heavy quark symmetry arises, which strongly reduces the number of independent weak form factors [5]. The heavy quark mass and spin then decouple and all meson properties are determined by light-quark degrees of freedom alone. As a result the heavy quark degeneracy of energy levels emerges. The spin $s_q$ of the light quark couples with its orbital momentum $l (j = l \pm s_q)$, resulting for $P$-wave mesons in two degenerate $j = 3/2$ states ($J^P = 1^+, 2^+$), and two degenerate $j = 1/2$ states ($0^+, 1^+$). The heavy quark symmetry also predicts that the form factors for $B\to D^{**} e\nu$ decays, where $D^{**}$ is a generic $P$-wave $D$ meson state, can be expressed in terms of two independent Isgur-Wise functions [5]. However, in the infinitely heavy quark limit the decay matrix elements between a $B$ meson and an orbitally excited $D$ meson vanish at zero recoil because of the heavy quark spin-flavor symmetry [5]. The kinematically allowed range for these decays is not broad. As a result the role of relativistic and finite heavy quark mass contributions not vanishing at zero recoil is considerably more important here than in the decays to ground-state $D$ mesons. Thus, the magnitude of such corrections might be comparable with the leading-order result.

Recently the first-order $1/m_Q$ corrections to the exclusive semileptonic $B$ decays into excited charmed mesons were investigated within the heavy quark effective theory (HQET) [6]. The structure of the $1/m_Q$ corrections to decay matrix elements, which follows from QCD and heavy quark symmetry, was determined. It was found that at the first order of heavy quark expansion the $B\to D_1, B\to D^*_0$, and $B\to D^*_7$ matrix elements do not vanish at zero recoil and can be expressed at this kinematical point in terms of the leading Isgur-Wise functions. Away from the zero recoil point new subleading Isgur-Wise functions arise, which cannot be de-

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1Here $J = j \pm 1/2$ is the total angular momentum, and the superscript $P$ denotes the meson parity.

2For concrete $P$-wave meson states the standard notations $D_1$, $D^*_2$, $D^*_3$, and $D^*_5$ are used.
termed from symmetry considerations alone. Thus for their
determination, additional model-dependent assumptions are
necessary. In Ref. [6] an estimation of these functions, based
on the nonrelativistic quark model as well as on some addi-
tional assumptions, was made. It is just here that we can
apply the relativistic quark model to get a more consistent
calculation of relevant Isgur-Wise functions and decay rates.

Our relativistic quark model is based on the quasipotential
approach in quantum field theory with a specific choice of
the quark-antiquark interaction potential. It provides a
consistent scheme for the calculation of all relativistic correc-
tions at a given \( \nu^2/c^2 \) order and allows for the heavy quark
\( 1/m_Q \) expansion. In preceding papers we applied this model
to the calculation of the mass spectra of orbitally and radially
excited states of heavy-light mesons [7], as well as to a de-
scription of weak decays of \( B \) mesons to ground-state heavy
and light mesons [8,9]. The heavy quark expansion for the
ground-state heavy-to-heavy semileptonic transitions [10]
has been found to be in agreement with model-independent
predictions of the HQET. We considered the exclusive semi-
leptonic decays of \( B \) mesons to orbitally excited \( D \) mesons in
the infinitely heavy quark limit in [11] and found the impor-
tant relativistic contribution to the leading Isgur-Wise func-
tions arising from the relativistic transformation of the meson
wave function.

The paper is organized as follows. In Sec. II we briefly
present the necessary HQET results on the \( B \to D^{**} \)
transition-matrix elements obtained in Ref. [6]. In Sec. III we
describe our relativistic quark model, putting special empha-
sis on the calculation of decay matrix elements and on the
relativistic transformation of a meson wave function from the
rest reference frame to the moving one. The heavy quark
expansion for decay matrix elements is carried out up to the
first order \( 1/m_Q \) corrections and compared to model indepen-
dent HQET predictions in Secs. IV–VI. There we present
our results for leading and subleading Isgur-Wise functions
and compare predictions for decay rates with and without the
\( 1/m_Q \) corrections being taken into account. Taking account
of \( 1/m_Q \) corrections leads to a substantial enhancement of
\( B \to D_1^{*} e \nu \) and \( B \to D_0^{*} e \nu \) decay rates and gives better agree-
ment between theoretical predictions and available experi-
mental data. We also present the electron spectra for the
considered decays and test the fulfillment of the Bjorken sum
rule. Section VII contains our conclusions.

II. HQET RESULTS FOR DECAY MATRIX ELEMENTS

A. \( B \to D_1 e \nu \) and \( B \to D_0^{*} e \nu \) decays

The matrix elements of the vector and axial-vector cur-
rrents between \( B \) mesons and \( D_1 \) or \( D_2^{*} \) mesons can be pa-
rametrized in the following way:

\[
\frac{\langle D_1(V', \epsilon) | \gamma^{\mu} b | B(\nu) \rangle}{\sqrt{m_D m_B}} = f_{V_1} \epsilon^{* \mu} + (f_{V_2} \nu^{\mu} + f_{V_3} \nu^{\nu})
\times (\epsilon^{*} \cdot \nu),
\]

\[
\frac{\langle D_2^{*}(V', \epsilon) | \gamma^{\mu} b | B(\nu) \rangle}{\sqrt{m_{D_2^{*}} m_B}} = f_{V_1} \epsilon^{* \mu} + (f_{V_2} \nu^{\mu} + f_{V_3} \nu^{\nu})
\times (\epsilon^{*} \cdot \nu),
\]

where \( \nu(V') \) is the four-velocity of the \( B(D^{**}) \) meson,
\( e^{\mu}(e^{*}) \) is a polarization vector (tensor) of the final vector
tensor) charged lepton, and the form factors \( f_i \) and \( k_i \) are
dimensionless functions of \( w = \nu \cdot V' \). The double differential
decay rates expressed in terms of the form factors read as follows [5,6]:

\[
d^2\Gamma_{D_1} = 3\Gamma_0 r_1^3 \sin^2 \theta_{(w-r_1)} f_{V_1} + (w^2 - 1)
\times (f_{V_2} r_f V_1^2) + (1 - 2r_1w + r_1^2)
\times [(1 + \cos^2 \theta) f_{V_1} + (w^2 - 1) f_{V_2}]
\times [4 \cos \theta_{(w-r_1)} f_{V_1}],
\]

\[
d^2\Gamma_{D_2^*} = \frac{3}{2} \Gamma_0 r_2^3 (w^2 - 1)^{3/2}
\times \left[ 4 \frac{3}{\sin^2 \theta_{(w-r_2)} k_{A_1}}
\times (1 - 2r_2w + r_2^2)
\times [(1 + \cos^2 \theta)(k_{A_1}^2 + (w^2 - 1) k_{A_2}^2)
\times [4 \cos \theta_{(w-r_2)} k_{A_1} k_{A_2}],
\right]
\]

where \( \Gamma_0 = G_f^2 |V_{cb}|^2 m_B^7/(192 \pi^2) \), \( r_1 = m_D/m_B \),
\( r_2 = m_{D_2^*}/m_B \), and \( \theta \) is the angle between the charged lepton
and the charmed meson in the rest frame of the virtual \( W \)

boson.

The main predictions of HQET for the structure of the
\( B \to D_1 e \nu \) form factors are as follows [6]:

\[
\sqrt{6} f_{V_1} = -(w + 1) \tau - e_b [(w - 1) \tau - (2w + 1) \tau_1
\times (- \tau_2)] + (w + 1) \eta_b - e_c [4(w \tau - \tau_3) \tau - (w - 1)
\times (w - 1) + \eta_b - e_c [4(w + 1)(w \tau - \tau_3) \tau - (w - 1)
\times (3 \tau_1 - 3 \tau_2 - \eta_b + 2 \eta_1 + 3 \eta_3)],
\]

\[
\sqrt{6} f_{V_2} = (1 - w^2) \tau - e_b (w^2 - 1) [(w + 1) \tau - (2w + 1) \tau_1
\times (w - 1)] - e_c [4(w + 1)(w \tau - \tau_3) \tau - (w - 1)
\times (3 \tau_1 - 3 \tau_2 - \eta_b + 2 \eta_1 + 3 \eta_3)],
\]
where $\varepsilon_Q = 1/(2m_Q)$ and $\hat{\Lambda}(\hat{\Lambda}') = M(M') - m_Q$ is the difference between the heavy ground-state (orbitally excited) meson and heavy quark masses in the limit $m_Q \to \infty$. The form factor $\tau$ is the leading-order Isgur-Wise function ($\tau = \sqrt{3}$ times the function $\tau_{2/3}$ of Refs. [5,11]). The subleading Isgur-Wise functions $\tau_1$ and $\tau_2$ originate from the $1/m_Q$ corrections to the $b \to c$ flavor-changing current, while $\eta_{ke}$ and $\eta_k$ form factors result from kinetic energy and chromomagnetic corrections to the HQET Lagrangian.

The analogous formulas for $B \to D_s^* e \nu$ have the form [6]

$$ k_v = -\tau - \varepsilon_b [(\hat{\Lambda} + \hat{\Lambda}) - (2w + 1) \tau_1 - \tau_2] - \varepsilon_c [(2w - 1) \tau_1 + \tau_2 + \eta_b]. $$

where $\varepsilon_b = 1/(2m_b)$ and $\hat{\Lambda}(\hat{\Lambda}') = M(M') - m_b$. The analogous formulas for $D_s^* e \nu$ decays are as follows:

$$ \langle D_s^*(\nu')|c \bar{\gamma}^\mu b|B(\nu)\rangle = 0, $$

$$ \langle D_s^*(\nu')|c \bar{\gamma}^\mu \gamma_5 b|B(\nu)\rangle = g_+(\nu^\mu + \nu'^\mu) + g_-(\nu^\mu - \nu'^\mu), $$

$$ \langle D_s^*(\nu', \nu)|c \bar{\gamma}^\mu b|B(\nu)\rangle = g_V_1 \varepsilon_+^\mu + (g_1 \varepsilon_+^\mu + g_1 \varepsilon_-^\mu) \times (\varepsilon_+^\mu \cdot \nu). $$

$$ \langle D_s^*(\nu', \nu)|c \bar{\gamma}^\mu \gamma_5 b|B(\nu)\rangle = i g_A \varepsilon_+^\mu \gamma_5 \varepsilon_+ \varepsilon_- \varepsilon_- \varepsilon_-, $$

where the form factors $g_i$ are functions of $w$. In terms of these form factors the double differential decay rates for $B \to D_s^* e \nu$ and $B \to D_s^* e \nu$ decays can be expressed in the following way [6]:

$$ \frac{d^2 \Gamma_{D_s^*}^{B}}{d w d \cos \theta} = \frac{3 \Gamma_{D_s^*}^{B} (w^2 - 1)^{3/2} \sin^2 \theta[(1 + r_0^*) g_+ - (1 - r_0^*) g_-]^2, $$

$$ \frac{d^2 \Gamma_{D_s^*}^{B}}{d w d \cos \theta} = \frac{3 \Gamma_{D_s^*}^{B} (w^2 - 1)^{3/2} \sin^2 \theta[(1 + r_0^*) g_+ - (1 - r_0^*) g_-]^2, $$

where $\Gamma_0 = G_F^2 |V_{cb}|^2 m_b^5 f(B(192 \pi^3))$, $r_0 = m_{D_s^*}/m_B$ and $r_1^* = m_{D_s^*}/m_B$.

The HQET predictions for the form factors of the decay $B \to D_s^* e \nu$ are given by [6]

$$ g_+ = \varepsilon_c [2(w - 1) \zeta_3 - 3 \zeta_3 w \hat{\Lambda}^* - \hat{\Lambda}] $$

$$ - \varepsilon_b \left[ \frac{w \hat{\Lambda}^* - \hat{\Lambda}}{w + 1} \right], $$

$$ g_- = \zeta + \varepsilon_c [\chi_{ke} + 6 \chi_1 - 2(w + 1) \chi_2] + \varepsilon_b \chi_b. $$

The analogous formulas for the decay $B \to D_s^* e \nu$ are as follows:

$$ g_A = \zeta + \varepsilon_c \left[ \frac{w \hat{\Lambda}^* - \hat{\Lambda}}{w + 1} \right] $$

$$ - \varepsilon_b \left[ \frac{w \hat{\Lambda}^* - \hat{\Lambda}}{w + 1} \right], $$

$$ g_{V_1} = (w - 1) \zeta + \varepsilon_c [(w \hat{\Lambda}^* - \hat{\Lambda}) \zeta + (w - 1) (\chi_{ke} - \hat{\Lambda})], $$

$$ g_{V_2} = 2 \varepsilon_c (\xi_1 - \chi_2). $$

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where $\alpha_s$ is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge, and $k = p - q$. $\gamma_\mu$ and $\nu(p)$ are the Dirac matrices and spinors

$$ u^\nu(p) = \sqrt{\frac{e(p) + m}{2e(p)}} \left( \frac{1}{\epsilon p} \right) \chi^\nu $$

with $\epsilon(p) = \sqrt{p^2 + m^2}$. The effective long-range vector vertex is given by

$$ \Gamma_\mu(k) = \gamma_\mu + \frac{i\kappa}{2m} \sigma_\mu \gamma^5, $$

where $\kappa$ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$ V_{\text{conf}}^V(r) = (1 - \varepsilon)(Ar + B), \quad V_{\text{conf}}^S(r) = \varepsilon(Ar + B), $$

reproducing

$$ V_{\text{conf}}(r) = V_{\text{conf}}^V(r) + V_{\text{conf}}^S(r) = Ar + B, $$

where $\varepsilon$ is the mixing coefficient.

The quasipotential for the heavy quarkonia, expanded in $\mathcal{O}(\alpha_s^2)$, can be found in Refs. [14,15] and for heavy-light mesons in [7]. All the parameters of our model, such as quark masses, parameters of the linear confining potential, mixing coefficient $\varepsilon$, and anomalous chromomagnetic quark moment $\kappa$, were fixed from the analysis of heavy quarkonia masses [14] and radiative decays [16]. The quark masses $m_q = 4.88$ GeV, $m_c = 1.55$ GeV, $m_s = 0.50$ GeV, $m_{u,d} = 0.33$ GeV, and the parameters of the linear potential $A = 0.18$ GeV and $B = -0.30$ GeV have the usual quark model values. In Ref. [10] we have considered the expansion of the matrix elements of weak heavy quark currents between pseudoscalar and vector meson states up to the second-order in inverse powers of the heavy quark masses. It has been found that the general structure of the leading, first, and second order $1/m_Q$ corrections in our relativistic model is in accord with the predictions of HQET. The heavy quark symmetry and QCD impose rigid constraints on the parameters of the long-range potential in our model. The analysis of the first-order corrections [10] allowed us to fix the value of the Pauli interaction constant $\kappa = -1$. The same value of $\kappa$ was found previously from the fine splitting of heavy quarkonia $3P_J$-states [14]. Note that the long-range chromomagnetic spin-independent interaction in our model is proportional to $(1 + \kappa)$ and thus vanishes for $\kappa = -1$ in agreement with the flux tube model [18]. The value of the mixing parameter of

\[\ldots\]
vector and scalar confining potentials $e = -1$ has been found from the analysis of the second-order corrections [10]. This value is very close to the one determined from radiative decays of heavy quarkonia [16].

In order to calculate the exclusive semileptonic decay rate of the $B$ meson, it is necessary to determine the corresponding matrix element of the weak current between meson states. In the quasipotential approach, the matrix element of the weak current $J^W = c \gamma^5 (1 - \gamma^5)b$ between a $B$ meson and an orbittally excited $D^{**}$ meson takes the form [19]

$$\langle D^{**}| J^W_{\mu}(0)|B \rangle = \int \frac{d^3 p d^3 q}{(2\pi)^6} \Psi_{D^{**}}(p) \Gamma_{\mu}(p,q) \Psi_B(q),$$

(17)

where $\Gamma_{\mu}(p,q)$ is the two-particle vertex function and $\Psi_{B,D^{**}}$ are the meson wave functions projected onto the positive energy states of quarks and boosted to the moving reference frame. The contributions to $\Gamma$ come from Figs. 1 and 2. In the heavy quark limit $m_{b,c} \rightarrow \infty$ only $\Gamma^{(1)}$ contributes, while $\Gamma^{(2)}$ contributes at 1/$m_Q$ order. They are

$$\Gamma^{(1)}_{\mu}(p,q) = \bar{u}_c(p_c) \gamma_{\mu} (1 - \gamma^5) u_b(q_b) (2\pi)^3 \delta(p - q),$$

(18)

and

$$\Gamma^{(2)}_{\mu}(p,q) = \bar{u}_c(p_c) u_b(q_b) \left( \frac{\Lambda_{b}^{(-)}(k)}{e_5(k) + e_5(p_c)} \right) \times \frac{\gamma_0^0}{\epsilon_{q}(q) + \epsilon_{q}(p)}$$

$$\times \frac{\Lambda_{c}^{(-)}(k')}{e_{c}(k') + e_{c}(q_b)} \gamma_{\mu}$$

$$\times (1 - \gamma_5^0) \bar{u}_b(q_b) u_q(q),$$

(19)

where the superscripts ‘‘(1)’’ and ‘‘(2)’’ correspond to Figs. 1 and 2, $Q = c$ or $b$; $k = p_c - \Delta$; $k' = q_b + \Delta = p_{D^{**}} - p_B$; $\epsilon(p) = (m^2 + p^2)^{1/2}$.

The contribution $\Gamma^{(2)}$ is the consequence of the projection onto the positive-energy states. Note that the form of the relativistic corrections resulting from the vertex function $\Gamma^{(2)}$ is explicitly dependent on the Lorentz structure of the $qq$ interaction.

\textbf{FIG. 1.} Lowest-order vertex function $\Gamma^{(1)}$ contributing to the current matrix element (18).

\textbf{FIG. 2.} Vertex function $\Gamma^{(2)}$ taking the quark interaction into account. Dashed lines correspond to the effective potential (12). Bold lines denote the negative-energy part of the quark propagator.

$$\Lambda^{(-)}(p) = \frac{\epsilon(p) - [m^2 + q^2]}{2\epsilon(p)}.$$

Here [19]

$$p_{c,q} = \frac{\epsilon_{c,q}(p)}{M_{D^{**}} \epsilon_{c,q}(p)} \sum_{i=1}^{3} n^{(i)}(p_{D^{**}}) p^i,$$

$$q_{b,q} = \frac{\epsilon_{b,q}(p)}{M_B \epsilon_{b,q}(p)} \sum_{i=1}^{3} n^{(i)}(p_B) q^i,$$

and $n^{(i)}$ are three four vectors given by

$$n^{(i)}(p) = \frac{p^i}{E - \sqrt{p^2 + M^2}}, \quad E = \sqrt{p^2 + M^2}.$$

The wave function of a $P$-wave $D^{**}$ meson at rest is given by

$$\Psi_{D^{**}}(p) = \Psi^{JM}_{D^{**}}(p) = \Upsilon^{JM}_j \psi_{D^{**}}(p),$$

(20)

where $J$ and $M$ are the total meson angular momentum and its projection, while $j$ is the light quark angular momentum; $\psi_{D^{**}}(p)$ is the radial part of the wave function, which has been determined by the numerical solution of Eq. (9) in [7]. The spin-angular momentum part $\Upsilon^{JM}_j$ has the following form:

$$\Upsilon^{JM}_j = \sum_{\sigma Q} \left\langle j M_{-\sigma} \sigma Q \left| \Upsilon^{JM}_j \right| j M_{-\sigma} \sigma Q \right\rangle,$$

(21)

where $\left\langle jm_1,jm_2|JM\right\rangle$ are Clebsch-Gordan coefficients, $Y^m$ are spherical harmonics, and $\chi(\sigma)$ (where $\sigma = \pm 1/2$) are spin-wave functions:

$$\chi(1/2) = \left| \begin{array}{c} 1 \\ 0 \end{array} \right|, \quad \chi(-1/2) = \left| \begin{array}{c} 0 \\ 1 \end{array} \right|.$$

It is important to note that the wave functions entering the weak current matrix element (17) are not in the rest frame in general. For example, in the $B$ meson rest frame, the $D^{**}$ meson is moving with the recoil momentum $\Delta$. The wave function of the moving $D^{**}$ meson $\Psi_{D^{**}}(p)$ is connected

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with the $D^{*\star}$ wave function in the rest frame $\Psi_{D^{*\star}0} = \Psi_{D(i)}$ by the transformation $[19]$

$$\Psi_{D^{*\star}0}(p) = D_{c}^{1/2}(R_{L_{\Delta}}^{W})D_{q}^{1/2}(R_{L_{\Delta}}^{W})\Psi_{D^{*\star}0}(p),$$

(22)

where $R^{W}$ is the Wigner rotation, $L_{\Delta}$ is the Lorentz boost from the meson rest frame to a moving one, and the rotation matrix $D^{1/2}(R)$ in spinor representation is given by

$$D_{c}^{1/2}(R_{L_{\Delta}}^{W}) = S^{-1}(p_{c,q})S(\Delta)S(p),$$

(23)

where

$$S(p) = \sqrt{\frac{e(p) + m}{2m}} \left( 1 + \frac{\alpha \sigma p}{e(p) + m} \right)$$

is the usual Lorentz transformation matrix of the four spinor. For electroweak $B$-meson decays to $S$-wave final mesons such a transformation contributes at the first order of the $1/m_{Q}$ expansion, while for the decays to excited final mesons it gives a contribution already to the leading term due to the orthogonality of the initial and final meson wave functions.

IV. LEADING AND SUBLEADING ISGUR-WISE FUNCTIONS

Now we can perform the heavy quark expansion for the matrix elements of $B$ decays to orbitally excited $D$ mesons in the framework of our model and determine leading and subleading Isgur-Wise functions. We substitute the vertex functions $\Gamma^{(1)}$ and $\Gamma^{(2)}$ given by Eqs. (18) and (19) in the decay matrix element (17) and take into account the wave function properties (20)–(22). The resulting structure of this matrix element is rather complicated, because it is necessary to integrate both over $d^3p$ and $d^3q$. The $\delta$ function in expression (18) permits us to perform one of these integrations and thus this contribution can be easily calculated. The calculation of the vertex function $\Gamma^{(2)}$ contribution is more difficult. Here, instead of a $\delta$ function, we have a complicated structure, containing the $Q\bar{Q}$ interaction potential in the meson. However, we can expand this contribution in inverse powers of heavy $(b,c)$ quark masses and then use the quasipotential equation in order to perform one of the integrations in the current matrix element. We carry out the heavy quark expansion up to first order in $1/m_{Q}$ . It is easy to see that the vertex function $\Gamma^{(2)}$ contributes already at the subleading order of the $1/m_{Q}$ expansion. Then we compare the arising decay matrix elements with the form factor decompositions (1) and (5) and determine the corresponding form factors. We find that, for the chosen values of our model parameters (the mixing coefficient of vector and scalar confining potential $\varepsilon = -1$ and the Pauli constant $\kappa = -1$), the resulting structure at leading and subleading order in $1/m_{Q}$ coincides with the model-independent predictions of HQET$^{6}$ given by Eqs. (3), (4), (7), and (8). We get the following expressions for leading and subleading Isgur-Wise functions:

(i) $B \to D_{1} e\nu$ and $B \to D_{2}^{*} e\nu$ decays

$$\tau(w) = \sqrt{\frac{2}{3}} \frac{1}{(w+1)^{3/2}}$$

$$\times \int \frac{d^3p}{(2\pi)^3} \bar{\psi}_{D_{1}(3/2)}(p) \frac{2\varepsilon_{q}}{M_{D_{1}(3/2)}(w+1)} \Delta$$

$$\times \left[ -2\varepsilon_{q} \frac{\partial}{\partial p} + \frac{p}{\varepsilon_{q} + m_{q}} \right] \psi_{B}(p),$$

(24)

$$\tau_{1}(w) = \frac{\bar{\Lambda}^\prime + \bar{\Lambda}}{w+1} \tau(w),$$

(25)

$$\tau_{2}(w) = -\frac{w}{w+1}(\bar{\Lambda}^\prime + \bar{\Lambda}) \tau(w).$$

(26)

(ii) $B \to D_{0}^{*} e\nu$ and $B \to D_{1}^{*} e\nu$ decays

$$\xi(w) = \sqrt{\frac{2}{3}} \frac{1}{(w+1)^{1/2}}$$

$$\times \int \frac{d^3p}{(2\pi)^3} \bar{\psi}_{D_{1}(1/2)}(p) \frac{2\varepsilon_{q}}{M_{D_{1}(1/2)}(w+1)} \Delta$$

$$\times \left[ -2\varepsilon_{q} \frac{\partial}{\partial p} - \frac{2p}{\varepsilon_{q} + m_{q}} \right] \psi_{B}(p),$$

(27)

$$\xi_{1}(w) = \frac{\bar{\Lambda}^* + \bar{\Lambda}}{w+1} \xi(w).$$

(28)

The contributions of all other subleading form factors, $\eta_{i}(w)$ and $\chi_{i}(w)$, to decay matrix elements are suppressed by an additional power of the ratio $(w-1)/(w+1)$, which is equal to zero at $w=1$ and less than $1/6$ at $w_{\max} = (1+r^2)/(2r)$ ($r=r_{1}, r_{2}, r_{0}^{*}$, or $r_{1}^{*}$, respectively). Since the main contribution to the decay rate comes from the values of form factors close to $w=1$, these form factors turn out to be unimportant. This result is in agreement with the HQET-motivated considerations [6] that the functions parametrizing the time-ordered products of the chromomagnetic term in the HQET Lagrangian with the leading-order currents should be small.

$^{6}$In particular we note that at the zero recoil point $w=1$, the scalar potential gives no contribution to the decay matrix elements and the correct reproduction of HQET relations is provided by the contribution of the vector potential and crucially depends on the specific choice of $\varepsilon = -1$. 

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The arrow over $\partial / \partial p$ in Eqs. (24) and (27) indicates that the derivative acts on the wave function of the $D^{**}$ meson. All the wave functions and meson masses have been obtained in [7] by the numerical solution of the quasipotential equation. We use the following values for HQET parameters: $\bar{\Lambda} = 0.51$ GeV, $\bar{\Lambda}' = 0.80$ GeV, and $\bar{\Lambda}^* = 0.89$ GeV [7].

The last terms in the square brackets of the expressions for the leading-order Isgur-Wise functions $\tau(w)$ Eq. (24) and $\zeta(w)$ Eq. (27) result from the wave-function transformation (22) associated with the relativistic rotation of the light quark spin (Wigner rotation) in passing to the moving reference frame. These terms are numerically important and lead to the suppression of the $\zeta$ form factor compared to $\tau$. Note that if we had applied a simplified nonrelativistic quark model [5,20] these important contributions would be missing. Neglecting further the small difference between the wave functions $\psi_{D(1/2)}$ and $\psi_{D(3/2)}$, the following relation between $\tau$ and $\zeta$ would have been obtained [6]:

$$\zeta(w) = \frac{w+1}{\sqrt{3}} \tau(w). \quad (29)$$

However, we see that this relation is violated if the relativistic transformation properties of the wave function are taken into account. At the point $w = 1$, where the initial $B$ meson and final $D^{**}$ are at rest, we find instead the relation

$$\frac{\tau(1)}{\sqrt{3} - \frac{1}{2}} \approx \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \frac{\psi_{D^{**}}(p)}{p_{||} + m_q} \psi_b(p). \quad (30)$$

obtained by assuming $\psi_{D(3/2)} \approx \psi_{D(1/2)} \approx \psi_{D^{**}}$. The relation (30) coincides with the one found in Ref. [21], where the Wigner rotation was also taken into account.

V. NUMERICAL RESULTS AND PREDICTIONS

In Table I we present our numerical results for the leading-order Isgur-Wise functions $\tau(1)$ and $\zeta(1)$ at zero recoil of the final $D^{**}$ meson, as well as their slopes $\rho_{3/2} = -(1/\tau)(\partial / \partial w) \tau |_{w=1}$ and $\rho_{1/2} = -(1/\zeta)(\partial / \partial w) \zeta |_{w=1}$, in comparison with other model predictions [6,21–26]. We see that most of the above approaches predict close values for the function $\tau(1)$ and its slope $\rho_{3/2}$, while the results for $\zeta(1)$ significantly differ from one another. This difference is a consequence of a specific treatment of the relativistic quark dynamics. Nonrelativistic approaches predict $\zeta(1) \approx (2/\sqrt{3}) \tau(1)$ [see Eq. (29)], while the relativistic treatment leads to $(2/\sqrt{3}) \tau(1) > \zeta(1)$ [see Eq. (30)]. The more relativo-

![FIG. 3. Isgur-Wise functions $\tau(w)$ (upper curve), $\tau_1(w)$ (middle curve), and $\tau_2(w)$ (lower curve) for the $B \rightarrow D_{1,2}e^+\nu$ decay.](Image 1)

![FIG. 4. Isgur-Wise functions $\zeta(w)$ (upper curve) and $\zeta_1(w)$ (lower curve) for the $B \rightarrow D_{0,1}^* e^+\nu$ decay.](Image 2)

![FIG. 5. Isgur-Wise functions $\tau(w)$ (upper curve), $\tau_1(w)$ (middle curve), and $\tau_2(w)$ (lower curve) for the $B \rightarrow D_{1,2}e^+\nu$ decay.](Image 3)
zero recoil of the final trix elements due to the heavy quark spin-flavor symmetry at leading-order contribution. This is the consequence of the vents \( \frac{1}{m_Q} \)

Our results for decay rates both in the infinitely heavy quark mass limit and taking into account the first-order \( 1/m_Q \) corrections as well as their ratio

\[
R = \frac{\text{Br}(B \to D^{**} e\nu) \text{with } 1/m_Q}{\text{Br}(B \to D^{**} e\nu) \text{in } m_Q \to \infty}
\]

are presented in Tables II and III. We see that the inclusion of \( 1/m_Q \) corrections considerably influences the results and for some decays their contribution is as important as the leading-order contribution. This is the consequence of the vanishing of the leading-order contribution to the decay matrix elements due to the heavy quark spin-flavor symmetry at zero recoil of the final \( D^{**} \) meson [6], while nothing prevents \( 1/m_Q \) corrections to contribute to the decay matrix element at this kinematical point. In fact, from Eqs. (1) and (5), we see that decay matrix elements at zero recoil are determined by the form factors \( f_{\nu_1}(1) \), \( g_+(1) \), and \( g_{\nu_1}(1) \), which receive nonvanishing contributions from first-order heavy quark mass corrections. From Eqs. (3), (7), and (8) one can find [6]

\[
\sqrt{6} f_{\nu_1}(1) = -8 e_c (\bar{\Lambda}' - \bar{\Lambda}) \tau(1),
\]

\[
g_+(1) = -\frac{3}{2} (e_c + e_b) (\bar{\Lambda}^* - \bar{\Lambda}) \xi(1),
\]

\[
g_{\nu_1}(1) = (e_c - 3e_b) (\bar{\Lambda}^* - \bar{\Lambda}) \xi(1).
\]

Since the kinematically allowed range for these decays is not broad (\( 1 \leq w \leq w_{\text{max}} = 1.32 \)), the contribution to the decay rate of the rather small \( 1/m_Q \) corrections is substantially increased [6]. This is confirmed by numerical calculations. From Tables II and III we see that the decay rate \( B \to D^{**}_e e\nu \), for which all contributions vanish at zero recoil, is only slightly increased by subleading \( 1/m_Q \) corrections. On the other hand, \( B \to D_1 e\nu \) and \( B \to D^*_0 e\nu \) decay rates receive large \( 1/m_Q \) contributions. The situation is different for the \( B \to D_1^* e\nu \) decay. Here the \( 1/m_Q \) contribution at zero recoil is not equal to zero, but it is suppressed by a very small factor \( (e_c - 3e_b) \) [see Eq. (33)], which is only \( 0.015 \text{ GeV}^{-1} \) for our model parameters. As a result the \( B \to D^*_1 e\nu \) decay rate receives \( 1/m_Q \) contributions comparable to those for the \( B \to D^*_0 e\nu \) rate. The above discussion shows that the sharp increase of \( B \to D_1 e\nu \) and \( B \to D^*_0 e\nu \) decay rates by first-order \( 1/m_Q \) corrections does not signal the break-

**TABLE II.** Decay rates \( \Gamma \) (in units of \( |V_{ud}|/0.04|^2 \times 10^{-15} \text{ GeV} \)) and branching ratios \( \text{BR} \) (in \%) for \( B \to D^{**} e\nu \) decays in the infinitely heavy quark mass limit and taking account of first-order \( 1/m_Q \) corrections. \( R \) is a ratio of branching ratios taking account of \( 1/m_Q \) corrections to branching ratios in the infinitely heavy quark mass limit.

<table>
<thead>
<tr>
<th>Decay</th>
<th>( m_Q \to \infty )</th>
<th>With ( 1/m_Q )</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B \to D_{1s} e\nu )</td>
<td>1.4</td>
<td>0.32</td>
<td>2.7</td>
</tr>
<tr>
<td>( B \to D_{2s} e\nu )</td>
<td>2.1</td>
<td>0.51</td>
<td>2.5</td>
</tr>
<tr>
<td>( B \to D_{1h} e\nu )</td>
<td>0.31</td>
<td>0.073</td>
<td>0.39</td>
</tr>
<tr>
<td>( B \to D_{2h} e\nu )</td>
<td>0.25</td>
<td>0.061</td>
<td>0.59</td>
</tr>
</tbody>
</table>

**FIG. 6.** Isgur-Wise functions \( \xi(w) \) (upper curve) and \( \xi'(w) \) (lower curve) for the \( B \to D^{**}_1 e\nu \) decay.
down of the heavy quark expansion, but is rather a result of the interplay of kinematical and dynamical effects. Thus we have good reasons to expect that higher-order $1/m_Q$ corrections will influence these decay rates at the level of 10–20%. In Table II we present the experimental data from CLEO and ALEPH, which are available only for the $B \rightarrow D_1 e \nu$ decay. For $B \rightarrow D_2^* e \nu$, these experimental groups present only upper limits, which require the use of some additional assumptions about the hadronic branching ratios of the $D_2^*$ meson. Our result for the branching ratio of the $B \rightarrow D_1 e \nu$ decay with the inclusion of $1/m_Q$ corrections is in good agreement with both measurements. On the other hand, our branching ratio for the $B \rightarrow D_2^* e \nu$ decay is only within the CLEO upper limit and disagrees with the ALEPH one. However, there are some reasons to expect that the ALEPH bound is too strong.

In Table IV we present our predictions for the ratios of decay rates $B \rightarrow D_2^* e \nu$, $B \rightarrow D_0^* e \nu$, $B \rightarrow D_1^* e \nu$, and of the sum of all $B \rightarrow D^{**} e \nu$ decay rates to the rate $B \rightarrow D_1 e \nu$ both in the limit $m_Q \rightarrow \infty$, and taking into account the $1/m_Q$ corrections. It is reasonable to consider such ratios in order to normalize to a measured rate. In Ref. it is argued that a ratio $\text{Br}(B \rightarrow D_2^* e \nu)/\text{Br}(B \rightarrow D_1 e \nu) = 1.55 \pm 0.15$ is a mere consequence of the heavy quark symmetry. In the heavy quark limit we confirm this result. However, the inclusion of $1/m_Q$ corrections strongly influences this prediction and considerably reduces this ratio to a value close to 1. Such a reduction seems to be favored by available experimental
data. In the last row of Table IV we give the sum of all $B \rightarrow D^{**} e \nu$ branching ratios. We see that our model predicts that 1.45% of $B$ meson decays go to the first orbitally excited $D$ meson states. This result means that approximately 2.5% of $B$ decays should go to higher excitations.

In Fig. 7 we plot the electron spectra $(1/\Gamma_0)\langle d\Gamma/dy \rangle$ for $B \rightarrow D^{**} e \nu$ decays. Here $y = 2E_\gamma/m_B$ is the rescaled lepton energy. These differential decay rates can be easily obtained from double differential decay rates $(2), (6)$, using the relation $y = 1 - w^2/r^2 - 1/2 \cos \theta$ and then integrating in $w$ over $[(1 - y)^2 + r^2]/(2r(1 - y)) < w < (1 + r^2)/(2r)$. We present our results both in the heavy quark limit $m_Q \rightarrow \infty$ (dashed curves) and with the inclusion of first-order $1/m_Q$ corrections (solid curves).

VI. BJORKEN SUM RULE

Finally, we test the fulfillment of the Bjorken sum rule [27] in our model. This sum rule states

$$\rho^2 = \frac{1}{4} + \sum_m |\xi^{(m)}(1)|^2/4 + 2 \sum_m |\tau^{(m)}(1)|^2/3 + \ldots,$$

where $\rho^2$ is the slope of the $B \rightarrow D^{(*)} e \nu$ Isgur-Wise function, $\xi^{(m)}$ and $\tau^{(m)}$ are the form factors describing the orbitally excited states discussed here and their radial excitations, and ellipses denote contributions from nonresonant channels. We see that the contribution of the lowest lying $P$-wave states implies the bound

$$\rho^2 > 1/4 + |\xi(1)|^2/4 + 2 |\tau(1)|^2/3 = 0.81,$$

which is in agreement with the slope $\rho^2 = 1.02$ in our model [10] and with experimental values [28].

VII. CONCLUSIONS

In this paper we have applied the relativistic quark model to the consideration of semileptonic $B$ decays to orbitally excited charmed mesons, in the leading and subleading order of the heavy quark expansion. We have found an interesting interplay of the relativistic and finite heavy quark mass contributions. In particular, it has been found that the Lorentz transformation properties of meson wave functions play an important role in the theoretical description of these decays. Thus, the Wigner rotation of the light quark spin gives a significant contribution already at the leading order of the heavy quark expansion. This contribution considerably reduces the leading-order Isgur-Wise function $\xi$ with respect to $\tau$. As a result, in this limit, the decay rates $B \rightarrow D^{*}_0 e \nu$ and $B \rightarrow D^{*}_{1e} e \nu$ are approximately an order of magnitude smaller than the decay rates $B \rightarrow D_{1e} e \nu$ and $B \rightarrow D^{*}_{2e} e \nu$. On the other hand, inclusion of the first-order $1/m_Q$ corrections also substantially influences the decay rates. This large effect of subleading heavy quark corrections is a consequence of the vanishing of the leading-order contributions to the decay matrix elements due to heavy quark spin-flavor symmetry at the point of zero recoil of the final charmed meson. However, the subleading-order contributions to $B \rightarrow D_{1e} e \nu$, $B \rightarrow D^{*}_0 e \nu$, and $B \rightarrow D^{*}_{2e} e \nu$ do not vanish at this kinematical point. Since the kinematical range for these decays is rather small, the role of these corrections is considerably increased. Their account results in an approximately twofold enhancement of the $B \rightarrow D_{1e} e \nu$ and $B \rightarrow D^{*}_0 e \nu$ decay rates, while the $B \rightarrow D^{*}_{2e} e \nu$ and $B \rightarrow D^{*}_{2e} e \nu$ rates are increased only slightly. The small influence of $1/m_Q$ corrections on the $B \rightarrow D^{*}_{2e} e \nu$ decay rate is the consequence of the additional interplay of $1/m_c$ and $1/m_b$ corrections at the zero recoil point [see Eq. (33)]. We thus see that these subleading heavy quark corrections turn out to be very important and considerably change the infinitely heavy quark limit results. For example, the ratio of branching ratios $Br(B \rightarrow D^{*}_{2e} e \nu)/Br(B \rightarrow D_{1e} e \nu)$ changes from the value of about 1.6 in the heavy quark limit, $m_Q \rightarrow \infty$, to the value of about 1 after subleading corrections are included.

In conclusion, we have presented here the first self-consistent dynamical calculation of subleading heavy quark corrections in the framework of the relativistic quark model, which are found to be in agreement with the HQET predictions.

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