Weaker magnetic fields in chiral quark models

I. Introduction

The magnetic field in the vacuum is a source of the electric charge density. The effect of the electric charge density on the magnetic field is

\[ \mathbf{E} = \nabla \times \mathbf{B} \]

where \( \mathbf{E} \) is the electric field and \( \mathbf{B} \) is the magnetic field. The electric field is generated by the electric charge density, which is the divergence of the electric current:

\[ \mathbf{J} = \nabla \cdot \mathbf{E} \]

where \( \mathbf{J} \) is the electric current density. The magnetic field is generated by the electric current and the magnetic current:

\[ \mathbf{J} = \nabla \times \mathbf{B} \]

where \( \mathbf{J} \) is the electric current density and \( \mathbf{B} \) is the magnetic field. The magnetic current is a source of the electric field, which is the curl of the magnetic field:

\[ \mathbf{E} = \nabla \times \mathbf{B} \]

The magnetic field is generated by the magnetic current and the electric current:

\[ \mathbf{J} = \nabla \cdot \mathbf{E} \]

where \( \mathbf{J} \) is the electric current density and \( \mathbf{E} \) is the electric field. The electric field is generated by the electric current density and the magnetic current:

\[ \mathbf{J} = \nabla \times \mathbf{B} \]

where \( \mathbf{J} \) is the electric current density and \( \mathbf{B} \) is the magnetic field. The magnetic field is generated by the electric current density and the magnetic current:

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\[ \mathbf{J} = \nabla \cdot \mathbf{E} \]

where \( \mathbf{J} \) is the electric current density and \( \mathbf{E} \) is the electric field. The electric field is generated by the electric current density and the magnetic current:

\[ \mathbf{J} = \nabla \times \mathbf{B} \]
Here $G_A \equiv g^{QM}/f_1^{QM}$, with $g_1^{QM} \equiv \langle B | \lambda_3 \otimes 1 | B \rangle$ and $f_1^{QM} \equiv \langle B | \lambda_3 \otimes \sigma^2 | B \rangle$. The $\lambda_{3\nu}$ is the SU(3) matrix that ef-fectuates the flavor transition and the $\sigma^2$ operator measures the spin polarizations of the quarks in the baryons.

### 2.1 The weak axial-vector form factors

The weak axial-vector form factors $G_A = g_1^{QM}/f_1^{QM}$ can be obtained from the SU(6) QM expressed in terms of the parameters $F$ and $D$ [10]. In the $\chi$QM, the $G_A$'s are expressed in the quark spin polarizations of the proton, i.e., $\Delta u$, $\Delta d$, and $\Delta s$. These spin polarizations differ considerably from the ones in the SU(6) QM due to the depolarization of the quark spins by the Goldstone bosons (GBs). The spin polarizations in the $\chi$QM are calculated with one GB emission. They are [6]

\[
\begin{align*}
\Delta u &= \frac{2}{3} - \frac{\sqrt{3}}{3} a, \\
\Delta d &= -\frac{1}{3} - \frac{2}{3} a, \\
\Delta s &= -a,
\end{align*}
\]

where $a$ is the parameter which measures the probability of emission of a GB from a quark. Using the relations $F = \frac{1}{3} (\Delta u - \Delta s)$ and $D = \frac{1}{3} (\Delta u + 2 \Delta d + \Delta s)$ [11], we have

\[
\begin{align*}
G_A^{\Sigma^+} &= \Delta u - \Delta d, \\
G_A^{\Sigma^0} &= \frac{1}{3} (\Delta u - \Delta s), \\
G_A^{\Sigma^-} &= \frac{1}{3} (\Delta u - 2 \Delta d + \Delta s), \\
G_A^{\Xi^+} &= \Delta u - \Delta s
\end{align*}
\]

for the $\Delta S = 0$ decays and

\[
\begin{align*}
G_A^{\Sigma^+} &= \Delta d - \Delta s, \\
G_A^{\Sigma^0} &= \Delta u - \Delta d, \\
G_A^{\Xi^+} &= \frac{1}{3} (\Delta u + \Delta d - 2 \Delta s), \\
G_A^{\Xi^-} &= \frac{1}{3} (2 \Delta u - \Delta d - 2 \Delta s), \\
G_A^{\Xi^- \Xi^+} &= \Delta u - \Delta d
\end{align*}
\]

for the $\Delta S = 1$ decays.

The magnetic moments of the octet baryons and the weak axial-vector form factor $G_A^{\Sigma^+}$ can be used to fit the parameter $a$ and the quark magnetic moment $\mu_d$. Using $\mu_u = -2 \mu_d$ and $\mu_s = 2 \mu_d/3$ [12], we then obtain $a \approx 0.104$ and $\mu_d \approx -1.196 \mu_N$. This gives $\Delta u \approx 0.90$, $\Delta d \approx -0.36$, and $\Delta s \approx -0.10$.

In the $\chi$QM, the effective quark masses can be determined from the fitted value of $\mu_d$. The quark masses are then $m_{u}^{\text{eff}} = m_{d}^{\text{eff}} \approx 260$ MeV and $m_{s}^{\text{eff}} = 3m_{d}^{\text{eff}} / 2 \approx 390$ MeV. In the following, we will use $m_q \equiv m_q^{\text{eff}}$, where $q = u, d, s$.

The values of the $G_{A,B}^{R}$'s for the $\chi$QM are listed in Table 1, where for reference also the axial-vector form factors of the naive QM (NQM) are displayed.

### 3 The ratio $\rho_f$ and the “weak magnetism”

We will now concentrate on the “weak magnetism” form factor $\rho_f$, which is defined as

\[
\rho_f = \frac{f_1}{f_2}.
\]

Inserting Eqs. (4) and (5) in Eq. (19), we obtain

\[
\rho_f = \frac{\sqrt{3}}{3} G_A - 1.
\]

Since there are no linear terms in $E$ and $\epsilon$ in either $f_1$ or $f_2$, the formula for $\rho_f$ is valid up to terms of second order in $E$ and $\epsilon$. The quark masses in this and related formulas appear as effective masses, and the parametric dependence of the quark spin polarization $\Delta q$, where $q = u, d, s$, on the emission probability $a$ of GB incorporates effects of relativistic corrections and other possible dynamical effects on both the magnetic moments [18] and the $\rho_f$'s. When these effects are taken into account directly, in terms of a changed structure of the currents, the fits become worse [19].

The expression (20) for $\rho_f$ above is closely related to the corresponding formula for the magnetic moments $\mu_B$ of the octet baryons used in earlier studies. In the same approximation as here, we have

\[
\begin{align*}
\rho_1 &= Q_B, \\
\rho_2 &= \Sigma \sum_{q=u,d,s} \frac{e_q}{2m_q} \Delta q - Q_B \\
&= \frac{M_B}{M_N} \kappa_B,
\end{align*}
\]

where $e_q$ is the quark charge. $Q_B = 0, \pm 1$ is the charge of the baryon, and $\kappa_B$ is the anomalous magnetic moment of the baryon in nuclear magnetons. It is therefore in principle possible to convert expression (20) above to an expression in terms of the magnetic moments. This will eliminate the parametric model dependence. In the following, we will discuss how this can be done in a way that preserves the absence of terms linear in $E$ and $\epsilon$ in Eq. (20).

### 3.1 The weak magnetism and CVC

From SU(3) flavor symmetry the weak magnetism form factors can be related to the magnetic moments of the
nucleons. The result is [10]
\[ \rho_{f}^{p p} = (\mu(p) - \mu(n))/\mu_N - 1, \]  
(23)
\[ \rho_{f}^{S^{-} Z^{0}} = (\mu(p) + \frac{1}{2} \mu(n))/\mu_N - 1, \]  
(24)
\[ \rho_{f}^{p p} = -\sqrt{2}\mu(n)/\mu_N. \]  
(25)
\[ \rho_{f}^{S^{-} Z^{0}} = (\mu(p) + 2\mu(n))/\mu_N - 1, \]  
(26)
\[ \rho_{f}^{p p} = (\mu(p) + 2\mu(n))/\mu_N - 1, \]  
(27)
\[ \rho_{f}^{S^{-} Z^{0}} = (\mu(p) - \mu(n))/\mu_N - 1, \]  
(28)
\[ \rho_{f}^{S^{+} A^{0}} = \mu(p) + \mu(n)/\mu_N - 1, \]  
(29)
\[ \rho_{f}^{S^{+} A^{0}} = (\mu(p) - \mu(n))/\mu_N - 1. \]  
(30)
\[ \rho_{f}^{S^{+} A^{0}} = (\mu(p) - \mu(n))/\mu_N - 1. \]  
(31)
This is called the (extended) CVC hypothesis. In the NQM, using the formula (20) above, all these relations emerge by putting \( \Sigma = M_N/m \) and using \( \Delta u = 4/3, \Delta d = -1/3 \), and \( \Delta s = 0 \). Then all \( \rho_{f}^{s} \)’s can be expressed in terms of the quark magnetic moment \( \mu_q = -\mu_u / 2 = -1/(6m) \), which can be related to the proton and neutron magnetic moments.

It is, however, obvious that the symmetry breaking in the masses, neither of the quarks nor of the baryons are accounted for. In particular, \( \mu_s = \mu_d \) in this approximation.

On the next level of refinement, one could therefore try to use in Eq. (20) instead the real baryon masses together with \( m_s/m = 3/2 \), along with the SU(6) QM values for the spin polarizations \( G_A \). Since the magnetic moments are fairly well accounted for in the NM, this is probably a rather good improvement. In the \( \chi QM \), we also allow the spin polarizations to deviate from their SU(6) values, increasing the improvement still somewhat. The results are given in Table 2.

3.2 The weak magnetism in the chiral quark model.
The \( \Delta S = 0 \) cases

The formula (20) above is transformed into an expression in terms of the magnetic moments of the baryons, when \( G_A/\sigma \) is expressed in the magnetic moments through \( \Delta q/(2m_q) \).

Consider for example the \( n \to p + l^- + \bar{\nu}_l \) decay. We can then show, using \( \mu(p) = \Delta u \mu_u + \Delta d \mu_d + \Delta s \mu_s \) and the corresponding formula for \( \mu_n \), that
\[ \rho_{f}^{p p} = \frac{1}{2} \left( 1 + \frac{M_u}{M_f} \right) \left( \mu(p) - \mu(n) \right) \frac{1}{\mu_N} - 1 \]  
\[ \simeq \left( \mu(p) - \mu(n) \right) \frac{1}{\mu_N} - 1 = \kappa_{S^{-}} - \kappa_{S^{+}}. \]  
(32)
Here we have used the expression \( G_A^{p p} = \Delta u - \Delta d \) from Subsection 2.1 above and \( \mu_u = -2 \mu_d \). Equation (32) is exactly the CVC formula for the \( n \to p + l^- + \bar{\nu}_l \) decay.

For the other transitions among the octet baryons, the \( \chi QM \) predicts the symmetry breaking in these weak magnetic moments due to the symmetry breaking in the masses both of quarks and baryons. In the following, we study the symmetry breaking using isospin symmetry and begin with the \( \Delta S = 0 \) transitions.

For the \( \Sigma^{-} \to \Sigma^{0} \) transition, we have
\[ \rho_{f}^{S^{-} Z^{0}} = \frac{2M_{\Sigma}}{2m_{\Sigma}} \left( \Delta u - \Delta s \right) - 1. \]  
(33)
Using the expressions for \( \mu(\Sigma^{-}) \) and \( \mu(\Sigma^{+}) \), we find in this case
\[ \Delta u - \Delta s = \frac{1}{3\mu_d} \left( \mu(\Sigma^{-}) - \mu(\Sigma^{+}) \right), \]  
(34)
which leads to
\[ \rho_{f}^{S^{-} Z^{0}} = \frac{M_{\Sigma}}{2M_{\Sigma}} \left( \mu(\Sigma^{+}) - \mu(\Sigma^{-}) \right) - 1 \]  
(35)
\[ \rho_{f}^{S^{-} Z^{0}} = \frac{M_{\Sigma}}{2M_{\Sigma}} \left( \mu(\Sigma^{+}) - \mu(\Sigma^{-}) \right) - 1 \]  
(36)

In a similar way, we obtain for the \( \Xi^{-} \to \Xi^{0} \) transition
\[ \rho_{f}^{S^{-} Z^{0}} = \frac{M_{\Xi}}{2M_{\Xi}} \left( \mu(\Xi^{0}) - \mu(\Xi^{-}) \right) - 1 \]  
(37)
\[ \rho_{f}^{S^{-} Z^{0}} = \frac{M_{\Xi}}{2M_{\Xi}} \left( \mu(\Xi^{0}) - \mu(\Xi^{-}) \right) - 1 \]  
(38)

Direct computation in the \( \chi QM \) using the formula (20) with the parameters \( \Delta q \), where \( q = u, d, s \), gives \( \rho_{f}^{S^{-} Z^{0}} \approx -2.27 \), whereas the formula (36) above gives \( \rho_{f}^{S^{-} Z^{0}} \approx -1.84 \) when the experimental magnetic moments are inserted. The discrepancy is due to the relatively poor agreement between the \( \chi QM \) prediction of the magnetic moments of the \( \Xi^{0} \) and \( \Xi^{-} \) and their experimental values.

For \( f_1 = 0 \), we calculate instead \( f_2 = \Sigma g_{\Sigma}^{QM}/\sigma \). This replaces \( \rho_{f}^{S} \) for \( \Sigma^{+} \to \Lambda \). The result can be expressed in terms of the \( \Sigma \Lambda \) magnetic moment transition matrix element:
\[ \mu(\Sigma \Lambda) = -\frac{1}{2\sqrt{3}} (\Delta u - 2\Delta d + \Delta s) (\mu_u + \mu_d). \]  
(39)
We then obtain
\[ \rho_{f}^{S^{-} A^{0}} = -\sqrt{2}(M_A + M_{\Sigma}) |\mu(\Sigma \Lambda)|. \]  
(40)
In all the cases above, there is an inherent ambiguity in the choice of magnetic moments, since in the \( SU(6) \) QM the form factors \( G_A \) can be expressed in only two polarization differences, say \( \Delta u = \Delta d \) and \( \Delta u = \Delta s \). In our approximation, the different choices are related by the sum-rule
\[ \mu(p) - \mu(n) + \mu(\Sigma^{-}) - \mu(\Sigma^{+}) + \mu(\Xi^{0}) - \mu(\Xi^{-}) = 0, \]  
(41)
which follows under quite general assumptions on the spin polarizations and the magnetic moments of the quarks, and in particular from the \( SU(6) \) QM. This sum-rule is valid to within about 0.5 \( \mu_N \) on the left-hand side.

In these cases, \( \sigma \approx 2m \) and the quark magnetic moments \( \mu_q = \pm 2\mu_d \) are related to \( 1/\sigma \) without any symmetry breaking. When we pass to anomalous magnetic moments in Eqs. (32), (35), and (36) it is no longer possible to use Eq. (39), since extra linear terms in \( E \) will then appear.
3.3 The $\Delta S = 1$ cases

The cases with $\Delta S = 1$ are less straightforward, and there is no "natural" way to express the spin polarizations in terms of the magnetic moments, since many different possibilities give the same formal result. To begin with, there is a complication that $\sigma = m + m_s$ in these cases. The crucial factor in the transformation of Eq. (20) into an expression in terms of magnetic moments is, up to normalization, given by an expression of the form

$$A \simeq \frac{1}{\sigma(p_d + p_s)}$$

where $x$ is a real parameter. This expression must not contain terms linear in the small quantity $\epsilon = \delta/\sigma$. It is easy to see that the condition for this is given by $x = 1$.

Let us study this for the case of the $\Sigma^- \to n$ transition. We have

$$\rho_j^{\Sigma^- n} = \frac{M_{\Sigma} + M_N}{m + m_s} \mu(\Sigma^-) - \mu(n) - \frac{1}{2} \mu(\Sigma^+)$$

This can be rewritten as

$$\rho_j^{\Sigma^- n} = (M_{\Sigma} + M_N) A_1 \left( \mu(n) - \mu(\Sigma^-) \right) - 1,$$

where $A_1 = -1/(m + m_s)\mu_\sigma$. It is easy to check that this expression is linear in $\epsilon$, since $x = 1/2$. In fact, using $m_s = 3m/2$, we get $A_1 = 9/10$, so the deviation from 1 is 10%.

An alternative way of obtaining the spin polarization for the $\Sigma^- \to n$ transition is to use

$$\mu_{\Sigma^-} = \mu(n) + \frac{1}{2} \mu(p) - \mu(\Sigma^-) - \frac{1}{2} \mu(\Sigma^+)$$

$$= -\frac{3}{2} (\Delta d - \Delta s)(\mu_d + \mu_s).$$

This gives

$$\rho_j^{\Sigma^- n} = (M_{\Sigma} + M_N) A_2 \mu_{\Sigma^-} - 1,$$

where

$$A_2 = -\frac{2}{3(m + m_s)(\mu_d + \mu_s)} \simeq 3 + \mathcal{O}(\epsilon^2).$$

In fact, for $m_s = 3m/2$, we obtain $A_2 = 24/25$, which is only 4% from 1. In the following, this term will therefore be put equal to 1. We can then write

$$\rho_j^{\Sigma^- n} = \frac{M_{\Sigma} + M_N}{2 M_N} \kappa_{\Sigma^-},$$

where $\kappa_{\Sigma^-} \equiv \kappa_\pi + \frac{1}{2} \kappa_{\kappa} - \kappa_{\Sigma^+} - \frac{1}{2} \kappa_{\Sigma^-}$. Next consider

$$\rho_j^{\Sigma^- n} = \frac{M_{\Sigma} + M_N}{m + m_s} (\Delta d - \Delta s) - 1$$

$$= \left( \mu(p) - \mu(n) \right) \frac{1}{\mu_N} - 1 = \kappa_p - \kappa_n,$$

where we have used $M_{\Sigma} + M_N \simeq 3(m + m_s)$. This expression happens to coincide exactly with CVC (see Eq. (28)).

However, to neglect the hyperfine interaction in the mass formulas for the baryons means to discard terms linear in the hyperfine interaction constant, which is generally of the order 50 MeV. This is almost of the same order as the symmetry breaking mass difference $\delta$ between the quark masses. We should therefore not be satisfied with this approximation.

Again, it is possible to use another combination to express the axial-vector coupling constant. This is given by

$$\mu_{\Sigma^+} \equiv \mu(\Sigma^+) + \frac{1}{2} \mu(\Sigma^-) - \mu(\Xi^0) - \frac{1}{2} \mu(\Xi^-)$$

$$= -\frac{3}{2} (\Delta u - \Delta d)(\mu_d + \mu_s).$$

Using this gives

$$\rho_j^{\Xi^- \pi^0} = (M_{\Xi} + M_{\Sigma}) \mu_{\Xi^+} - 1.$$  

(48)

For later use this can be rewritten as

$$\rho_j^{\Xi^- \pi^0} = \frac{M_{\Xi} + M_{\Sigma}}{2 M_N} (\kappa_{\Sigma^+} + \frac{1}{2} \kappa_{\Sigma^-} - \kappa_{\Xi^+} - \frac{1}{2} \kappa_{\Xi^-}).$$

(49)

Since all $G_A$’s can be expressed in terms of the spin polarization differences $\Delta d - \Delta s$ and $\Delta u - \Delta d$ it is possible to express all other $\Delta S = 1$ transitions in terms of $\mu_{\Sigma^+}$ and $\mu_{\Xi^+}$. However, if we want to convert the result from magnetic moments to anomalous magnetic moments, we must also avoid terms that are linear in the mass ratios $E = \Delta/\Sigma$.

Consider therefore next

$$\rho_j^{\Xi^- A} = \frac{1}{3} \frac{M_{\Xi} + M_A}{m + m_s} (\Delta u + \Delta d - 2 \Delta s) - 1.$$  

(50)

To avoid terms that are linear in $E$, we must in this case use

$$\mu_{\Xi A} \equiv \mu(p) + 3 \mu(A) - \mu(\Xi^0) - \frac{1}{2} \mu(\Sigma^+) - \frac{1}{2} \mu(\Xi^-)$$

$$= -\frac{3}{2} (\Delta u + \Delta d - 2 \Delta s)(\mu_d + \mu_s).$$

(51)

We then obtain, in the same approximation,

$$\rho_j^{\Xi^- A} = (M_{\Xi} + M_A) \frac{1}{3} \mu_{\Xi A} - 1 = \frac{M_{\Xi} + M_A}{2 M_N} \frac{1}{3} \kappa_{\Xi A} + A_3,$$

(52)

where $\kappa_{\Xi A} \equiv \kappa_\pi + 3 \kappa_A - \kappa_{\Xi^0} - 3 \kappa_{\Xi^+} - 3 \kappa_{\Xi^-}$ and $A_3 = \frac{1}{(3M_{\Sigma} + M_{\Xi})} (1/(2M_{\Xi}) - 1/(4M_{\Sigma}) + 3/(4M_{\Xi})) - 1$. It is easy to verify that $A_3 = \mathcal{O}(E^2)$ and may therefore be neglected.

Finally, we can express $\rho_j^{\Xi^-}$ in terms of magnetic moments in the same way starting from

$$\rho_j^{\Xi^-} = \frac{1}{3} \frac{M_{\Xi} + M_N}{3 m + m_s} (2 \Delta u - \Delta d - \Delta s) - 1.$$  

(53)

The best way of expressing the form factor uses

$$\mu_{\Xi} \equiv \mu(p) + \frac{3}{2} \mu(\Xi^0) - \frac{1}{2} \mu(\Xi^-)$$

$$= -\frac{3}{2} (2 \Delta u - \Delta d - \Delta s)(\mu_d + \mu_s).$$  

(54)
Omitting second order mass differences, this gives
\[
\rho_f^{\mu} = (M_A + M_N) \frac{1}{3} \kappa_{\mu}^{\mu} - 1 = \frac{M_A + M_N}{2M_N} \frac{1}{3} \kappa_{\mu}^{\mu} ,
\]
where \( \kappa_{\mu}^{\mu} \equiv \kappa + \frac{5}{3} \kappa_p + \frac{1}{3} \kappa_S - \frac{1}{3} \kappa_A - \kappa Z^- .

### 3.4 The weak magnetism in the chiral quark soliton model

The weak magnetic form factors have also been calculated in the chQSM. The result, after normalization in our convention, is given in \( [7] \) as

\[
\rho_f^{\Sigma^+} = \frac{M_p + M_N}{2M_N} \left( \kappa_{\Sigma^+} - \kappa_{\Sigma^-} \right) ,
\]
\( \rho_f^{\Sigma^0} = \frac{M_p + M_N}{2M_N} \left( \kappa_{\Sigma^0} - \frac{1}{2} \kappa_{\Sigma^+} - \frac{1}{2} \kappa_{\Sigma^-} \right) ,
\]
\( \rho_f^{\Xi^+} = \frac{M_p + M_N}{2M_N} \left( \kappa_{\Xi^+} - \kappa_{\Xi^-} \right) ,
\]
\( \rho_f^{\Xi^0} = \frac{M_p + M_N}{2M_N} \left( \kappa_{\Xi^0} - \frac{1}{2} \kappa_{\Xi^+} - \frac{1}{2} \kappa_{\Xi^-} \right) ,
\]
\( \rho_f^{\Xi^-} = \frac{M_p + M_N}{2M_N} \left( \kappa_{\Xi^-} + \frac{1}{2} \kappa_{\Xi^+} - \frac{1}{2} \kappa_{\Xi^0} - \frac{1}{2} \kappa_{\Xi^-} \right) ,
\]
\( \rho_f^{\Sigma^-} = \frac{M_p + M_N}{2M_N} \left( \kappa_{\Sigma^-} + \frac{1}{2} \kappa_{\Sigma^0} - \frac{1}{2} \kappa_{\Sigma^-} \right) ,
\]
\( \rho_f^A = \frac{M_A + M_N}{2M_N} \left( \kappa_A - \kappa_{\Sigma^-} \right) .
\]

We have here neglected the possible change in the transition from the anomalous magnetic moments to the full magnetic moments that might be related to the change in normalizaiton. By this, we mean that in the above expressions, \( \kappa_p = \mu_B / \mu_N - Q_B M_N / M_B \), and the normalization, of course, is of relevance when the symmetry is broken.

However, our understanding is that, apart from a factor of \( M_B / (M_B + M_P) \), these differences in our normalization are of the order \( O(m_f^2) \) or \( O(m_f^2 / m_N) \), i.e. in terms that are anyhow neglected in the above formulas \( [7] \), and the corresponding terms of second order or higher in the mass ratios that are neglected in our calculations.

From Section 3 it should be clear then, that the method of expressing \( \rho_f \), that avoids introducing linear terms in the symmetry breaking masses, in general produces expressions that coincide with those above. This shows that the chQSM and the chQSM give the same results when linear terms in the symmetry breaking are eliminated.

### 4 Discussion

The particular choice of combinations of magnetic moments, that enables one to express the \( \rho_f \)'s in term of anomalous magnetic moments, are enforced from the cancellation of linear terms in both the quark and baryonic mass differences. The analysis presented here shows that when this is done the \( \chi \) QM and the \( \chi \) QSM give the same results. The earlier noticed numerical differences are related to the difficulty to reproduce the octet baryon magnetic moments in the \( \chi \) QM without symmetry breaking in the spin polarizations. This can be seen e.g. in the case of \( \Xi^0 \) and \( \Xi^- \).

It is of course possible to stop and be satisfied at the level where the \( \rho_f \)'s are expressed in terms of magnetic moments. Then, since all \( G_A \) can be expressed in terms of only two spin polarization differences, there are several equivalent relations for the \( \rho_f \)'s related to sum rules for the \( G_A \)'s. On top of that, there is in this case also the possibility to use the sum-rules for the magnetic moments to find alternative ways to express the \( \rho_f \)'s.

For the \( \rho_f \)'s the existing experimental data is given in Table 2.

Let us consider this table.

The CVC values listed are in a way half experimental results, since the use the measured values of the anomalous magnetic moments for the nucleons as input data to calculate these values.

Since the magnetic moments of the quarks are fitted to the magnetic moments of the proton and neutron, the SU(6) QM results should coincide with CVC in the absence of symmetry breaking and are not listed.

All values obtained for the \( \rho_f \)'s in the \( \chi \) QM lie within the experimental errors, where experimental data exist. (The experimental results have large errors, though.)

In one case, that of neutron decay, we can see that \( \rho_f(\chi \text{QM}) \approx \rho_f(\text{CVC}) \). For the other decays, the \( \rho_f \)'s of the \( \chi \) QM incorporate effects of vector current non-conservation due to the mass differences between the isomultiplets as well as depolarization of the spin due to GB emission.

All calculated values for the \( \chi \) QM have the same sign as the CVC values and they are also close in magnitude. The numerical results cannot be expected to be much better than within 10%. Already isospin is violated to a few percent.

For comparison, we have in two cases calculated the \( \rho_f \)'s obtained by neglecting the hyperfine interaction, since it is rather small. The results are

\[
\rho_f^{\Xi^+} = (\mu(p) - \mu(n))/\mu_N - 1 \approx 3.71
\]

and

\[
\rho_f^{\Xi^-} = (\mu(\Xi^+) + \mu(\Xi^-))/3 \mu_N - 1 = 0.01 \pm 0.02,
\]

which both are very close to the values using Eqs. (49) and (52), respectively.

### 5 Summary and conclusions

We have studied the baryonic weak magnetism form factors in detail in the spirit of the \( \chi \) QM and compared the
results with the $\chi$QSM. The comparison shows that the results are in good agreement, and that the differences are of the order of reliability of the results in all cases. This might indicate that the main part of the symmetry breaking is accounted for in these formulas. The numerical results are presented in Tables 1 and 2.

The present investigation has used the SU(3) symmetric coupling in the $\chi$QSM and the static approximation for the quarks. A natural improvement would be to incorporate lowest order non-static effects and further SU(3) symmetry breaking effects [15, 16], to obtain better agreement with experimental data. In particular, we expect that this would lead to a closer agreement with the $\rho_2$ ratios obtained from direct application of Eq. (20), since symmetry breaking can be expected for the octet baryon magnetic moments [12]. SU(3) symmetry breaking also leads to better agreement for $g_2^B$ [12, 15, 16].

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References

1. R.P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958)

Table 1. Weak axial-vector form factors, $G_A^{\pi\pi}$. The values in the NQM column are the SU(6) values for the weak axial-vector form factors and the values in the $\chi$QSM column are obtained from the quark spin polarizations. $g_1^{QM,\pi\pi}$ are given instead of $G_A^{\pi\pi}$, since $f_1^{QM,\pi\pi} = 0$. The experimental values for $G_A^{\pi\pi}$ have been obtained from Ref. [17], except for the $g_1^{QM,\pi\pi} = 0$, which are CERN WA2 [18,19] results from branching ratio measurements.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Experimental value</th>
<th>NQM</th>
<th>$\chi$QSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_A^{\pi\pi}$</td>
<td>1.2670 ± 0.0035</td>
<td>1.26</td>
<td>1.26</td>
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<tr>
<td>$G_A^{\pi\pi}$</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_1^{QM,\pi\pi}$</td>
<td>0.580 ± 0.016</td>
<td>1.06</td>
<td>0.62</td>
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<tr>
<td>$\rho_1^{QM,\pi\pi}$</td>
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<tr>
<td>$G_A^{\pi\pi}$</td>
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<td>$G_A^{\pi\pi}$</td>
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</table>

Table 2. The ratios $\rho_{2,3,4}^{\pi\pi} = f_{2,3,4}^{\pi\pi} / f_{2,3,4}^{\pi0}$. The experimental values have been obtained from Ref. [18] (see also Ref. [19]). $f_{2,3,4}^{\pi\pi}$ are given instead of $\rho_{2,3,4}^{\pi\pi}$, since $f_{2,3,4}^{\pi\pi} = 0$.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Experimental value</th>
<th>CVC</th>
<th>$\chi$QSM</th>
</tr>
</thead>
<tbody>
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<td>$\rho_1^{\pi\pi}$</td>
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<td>$\rho_1^{\pi\pi}$</td>
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<tr>
<td>$\rho_1^{\pi\pi}$</td>
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<tr>
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<tr>
<td>$\rho_1^{\pi\pi}$</td>
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<td>-1.82</td>
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<tr>
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<tr>
<td>$\rho_1^{\pi\pi}$</td>
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