Interpreting the wave function and its evolution in terms of new motion of particle

Gao shan
Institute of Quantum mechanics
11-10, NO.10 Building, NO.21, YueTan XiJie DongLi, XiCheng District
Beijing 100045, P.R.China
E-mail: gaoshan.iqm@263.net

Aiming at providing an objective motion picture for the microscopic object described by the wave function, new analysis about motion is presented by use of the point set theory in mathematics, through which we show that a new kind of motion named quantum discontinuous motion is the general motion mode of the particle, while classical continuous motion is just one kind of extremely peculiar motion, and the wave function in quantum mechanics proves to be the very mathematical complex describing the particle undergoing the quantum discontinuous motion. Furthermore, Schrödinger equation of the wave function is shown to be the simplest nonrelativistic evolution equation for the particle undergoing the new motion, and the consistent axiom system of quantum mechanics is also deduced out. At last, we demonstrate that present quantum measurement theories just confirm the existence of the new motion of the microscopic particle described by the wave function, and the weird displays of the wave function in microscopic world are also physically explained in terms of the new motion.

I. INTRODUCTION

Owing to the springing up of the new concepts about the wave function such as quantum cosmology and protective measurement [6] [11] [12], the demand for the last objective interpretation of the wave function is skyrocketing in recent years. On the other hand, the analyses about motion have never ceased since the old Greece times, but from Zeno Paradox to Einstein’s relativity [1], only classical continuous motion is discussed. In fact, there exist some other motion modes in nature, and the mysterious quantum motion in microscopic world may be one of these snubbed motion modes. In this paper we present one kind of new motion, which is called quantum discontinuous motion, and after a deep and reasonable analysis we find that the last reality underlying the wave function is just the new motion undergone by the microscopic particle.

From a historical point of view, interpreting the wave function in terms of new motion of particle inherits the essence of thought accumulated since the foundation of quantum mechanics, the first essential concept is the ontology concept widely adopted in interpreting quantum mechanics, it aims at finding the last reality behind the mysterious wave function, this concept can be traced back to Einstein, Schrödinger, and the followers Bohm [5], Bell [10] et al, especially in recent years, the protective measurement [11] [12] presented by Aharonov et al has not only consolidated this concept from the inside of quantum mechanics, but also provided the experiments to confirm it, thus this advance cries for an objective elucidation about the reality underlying the wave function more strongly than ever, and it may be the time to disclose the whole quantum mystery now.

The second is the particle concept hold by Born’s generally-accepted probability interpretation [3] and nearly most following interpretations of quantum mechanics, according to this particle concept the last reality described by the wave function in quantum mechanics is essentially a particle, which is in only one position in space at any instant. The most predominant character of this concept is that it can not only describe the particle picture in nonrelativistic quantum mechanics, but also be extended to describe the field picture in relativistic quantum field theory when unifying new motion with special relativity; on the other hand, it provides a precondition to further study the boring collapse process as one kind of objective process.

The third is the renunciation of classical continuous motion, for example, it is widely demonstrated and accepted by ontologists that the microscopic particle can not pass through only one slit without influenced by another slit in two-slit experiment, and sometimes they even say that the microscopic particle “passes through both slits” or “in two positions at the same time” [11] [12], although this kind of description is too vague to form a strict definition or elucidation about the objective quantum motion undergone by the microscopic particle, which may be extremely
different from classical continuous motion, needless to say, the renunciation of classical continuous motion sheds light on the road to interpret the wave function in terms of new motion other than classical continuous motion, and the vague description "in two positions at the same time" indeed grasps something real and moves a peg along this road.

Certainly, there exist other deeper logical reasons leading to interpreting the wave function in terms of new motion of particle, some of them are as follows:

1. The linear superposition principle of wave function in quantum mechanics strongly implies that the real object described by the wave function, if any, is not a field, but a particle, since as to the field its different branches will generally interact, and the linear superposition principle of the wave function describing the field will be broken; while as to the particle, it is in only one position in space at any instant, and in nonrelativistic quantum mechanics the transfer of interaction is instantaneous, thus for the wave function describing the quantum discontinuous motion of particle, its different branches will not interact.

2. Since the founding of science people have been studying the classical continuous motion of particle, and taking it for granted that it is the only type of motion in Nature, but from the mathematical analyses it is shown that there exists one kind of complementary motion mode of classical continuous motion, which can be called quantum discontinuous motion, they form the complete set of motion modes of particle, and the former seems more natural than the latter. In physics, as we will demonstrate, the macroscopic world is governed by the classical continuous motion, while the microscopic world is governed by the quantum discontinuous motion.

3. In order to unify particle and field, we have been searching for the lost connection between them, now the new motion—quantum discontinuous motion is just the lost golden bridge. On the one hand, the object undergoing the new motion is a particle, this is the particle-like aspect of the new motion, owing to this particle-like property, the new motion of particle provides a basic basis for the objective collapse process of the wave function during quantum measurement, since the definite measurement result such as one spot on the photoplate, is produced only in one local region, not in many different local regions in space; On the other hand, the particle undergoing the new motion moves throughout the whole space with a certain position measure density $\rho(x,t)$ during infinitesimal time interval, this is the field-like aspect of the new motion, owing to this field-like property, the new motion of particle can describe the objective evolution process of the wave function during normal evolution, which determines the interference pattern of the particle, and will provide the objective origin of probability in quantum mechanics.

4. When the new motion marries with special relativity, we will find that it can be extended to describe the quantum field picture, and quantum mechanics will naturally be taken over by quantum field theory, concretely speaking, when in relativistic quantum field the transfer velocity of interaction is finite, thus for the wave function describing the quantum discontinuous motion of particle, its different branches will interact through the transfer particle of the interaction, for example, in quantum electrodynamics the different branches of the electron wave function will interact through the photon, which is characterized by the interaction term $\psi\gamma^\mu A_\mu\psi$.

5. The new motion of particle will provide a broader framework for objectively studying the microscopic process, in which the notorious collapse problem may be naturally solved, for example, there may exist many kinds of concrete motion modes among the new motion, and the new motion may display differently in the nonrelativistic and relativistic domains, especially when involving relativistic gravity the new motion of particle may naturally provide the origin of randomness in the collapse process, and further result in the objective collapse process.

6. It is generally accepted that the main accepted objective interpretations of quantum mechanics are Everett’s relative state interpretation [8] and Bohm’s hidden variables interpretation [5], but besides other unsatisfactory characters they all interpret neither the wave function, nor Schrödinger equation of the wave function, as to the former, the wave function is directly taken as a physical entity with no a priori interpretation, while as to the latter, the wave function is taken as a real field in the configuration space, which is assumed to satisfy Schrödinger equation in quantum mechanics with no further interpretation, in fact, it does not interpret this objective field in real space either. Now, in this paper we will interpret both the wave function and its Schrödinger equation in terms of the new motion of particle.

The plan of this paper is as follows: In Sect. 2 we first give three general presuppositions, which relate physical reality with abstract mathematics, and are the basis of the following analysis about motion. In Sect. 3 we give a strict mathematical analysis about motion by use of point set theory, especially the discontinuous motion described by regular dense point set is analyzed in detail. In Sect. 4 a strict physical definition of the new motion—quantum discontinuous motion is given, the wave function in quantum mechanics is interpreted as a mathematical complex describing the particle undergoing the quantum discontinuous motion, and the consistent axiom system of quantum mechanics is deduced out, especially Schrödinger equation of the wave function is shown to be the simplest nonrelativistic evolution equations for the particle undergoing the quantum discontinuous motion. Furthermore, in Sect. 5 we demonstrate that the quantum measurement theories just confirm the existence of the quantum discontinuous motion of the microscopic particle described by the wave function. At last, in Sect. 6 the notorious characters of wave function is consistently interpreted, and two concrete examples are given to explain the weird displays of the wave function in microscopic world in terms of the quantum discontinuous motion of particle.
II. THREE GENERAL PRESUPPOSITIONS

First, we will give three general presuppositions about the relation between physical reality and abstract mathematics, which are basic conceptions and correspondence rules before we discuss the physical motion of particle, we suppose they are the commonness for all kinds of physical motion of particle.

1. Time and space in which the particle moves are both continuous point set.
2. The moving particle is represented by one point in time and space.
3. The motion of particle is represented by the point set in time and space.

For simplicity but lose no generality, in the following we will mainly analyze one-dimension motion, namely the point set in two-dimension time and space.

III. MATHEMATICAL ANALYSIS ABOUT NEW MOTION

A. Point set and its law

As we know, the point set theory has been deeply studied since the beginning of this century, nowadays we can grasp it more easily. According to this theory, we know that the general point set is dense point set, whose basic property is the measure of the point set, while the continuous point set is one kind of particular dense point set, its basic property is the length of the point set.

Naturally, as to the point set in two-dimension time and space, the general situation is the dense point set in this two-dimension space, while the continuous curve is one kind of extremely particular dense point set, surely it is a wonder that so many points bind together to form one continuous curve by order, in fact, the probability for its formation is zero.

Now, we will generally analyze the law of the point set, as we know, the law about the points in point set, which can be called point law, is the most familiar law, and it is widely taken as the only rational law, for example, as to the continuous curve in two-dimension time and space there may exist a certain expressible analytical formula for the points in this particular point set (people cherish this kind of point laws owing to their infrequent existence, but perhaps Nature detests and rejects them, since the probability of creating them is zero), but as to the dense point set in two-dimension time and space the point law does not exist, since the dense point set is discontinuous everywhere, even if the difference of time is very small, or infinitesimal, the difference of space can be very large, then infinitesimal error in time will result in finite error in space, thus even if it exists we can not formulate it in nature, and owing to finite error in time measurement, we can not confirm it either, let alone predict the evolution of the point set using it, in one word, there does not exist point law for dense point set in mathematics and physics.

Because of nonexistence of the point law for general dense point set, people cherish only the particular dense point set—continuous curve with point law, which corresponds to classical continuous motion, and detest the general dense point set without point law, let alone regard it as another kind of real motion mode. But when we consider the confirmation of law, we will find more truth about the law for point set, as we know, as to the point law of continuous curve, we must confirm it by means of the following process: $\Delta t \rightarrow dt \rightarrow 0$, among these processes the process $\Delta t \rightarrow dt$ is complete for confirming the differential law for point set, and the process $dt \rightarrow 0$ is only necessary for the confirmation of point law, but evidently this process can not be achieved in reality, in fact, only the process $\Delta t \rightarrow dt$ can possess real physical meaning through testing the law more and more accurately, thus there does not exist point law for both general dense point set and continuous curve, and the privilege of continuous curve and the corresponding classical continuous motion is also lost.

On the other hand, in physical there exist only the dynamical quantities defined during infinitesimal time interval, this fact can be seen from the familiar differential quantities such as $dt$ and $dx$, whereas the point quantities come only from mathematics, people always mix up these two kinds of quantities, this is a huge obstacle for the development of physics. Thus we can only discuss the quantities defined during infinitesimal time interval, as well as their differential laws if we study the point set corresponding to real physical motion.

B. Deep analysis about dense point set

Now, we will further study the differential description of point set in detail.

First, in order to find the differential description of the peculiar dense point set—continuous curve, we may measure the rise or fall size of the space $\Delta x$ corresponding to any finite time interval $\Delta t$ near each instant $t_j$, then at any instant $t_j$ we can get the approximate information about the continuous curve through the quantities $\Delta t$ and $\Delta x$ at
that instant, and when the time interval $\Delta t$ turns smaller, we will get more accurate information about the curve. Theoretically we can get the complete information through this infinite process, that is to say, in theory we can build up the basic description quantities for the peculiar dense point set—continuous curve, which are the differential quantities $dt$ and $dx$, then given the initial condition the relation between $dt$ and $dx$ at all instants will completely describe the continuous curve.

Then, we will deeply analyze the differential description of the general dense point set. As to this kind of point set, we still need to study the concrete situation of the point set corresponding to finite time interval near every instant. Now, when time is during the interval $\Delta t$ near instant $t_j$, the points in space are no longer limited in the local space interval $\Delta x$, they distribute throughout the whole space instead, so we should study this new nonlocal point set, which is also dense point set, for simplicity but lose no generality, we consider finite space such as $x\in[0,1]$, first, we may divide the whole space in small equal interval $\Delta x$, then calculate the measure of the local dense point set in the space interval $\Delta x$ near each $x_i$, which can be written as $M_{\Delta x,\Delta t}(x_i,t_j)$, since the measure sum of all local dense point sets in time interval $\Delta t$ just equals to the length of the continuous time interval $\Delta t$, we have:

$$
\sum_i M_{\Delta x,\Delta t}(x_i,t_j) = \Delta t
$$

(1)

On the other hand, since the measure of the local dense point set in the space interval $\Delta x$ and time interval $\Delta t$ will also turn to be zero when the intervals $\Delta x$ and $\Delta t$ turn to be zero, it is not an useful quantity, and we have to create a new quantity on the basis of this measure. Through further analysis, we find that a new quantity $\rho_{\Delta x,\Delta t}(x_i,t_j) = M_{\Delta x,\Delta t}(x_i,t_j)/(\Delta x \cdot \Delta t)$, which can be called average measure density, will be an useful one, it generally does not turn to be zero when $\Delta x$ and $\Delta t$ turn to be zero, especially if the limit $\lim_{\Delta x \to 0, \Delta t \to 0} \rho_{\Delta x,\Delta t}(x_i,t_j)$ exists, it will no longer relate to the observation sizes $\Delta x$ and $\Delta t$, so it can accurately describe the whole dense point set, as well as all local dense point sets near every instant, now we let:

$$
\rho(x,t) = \lim_{\Delta x \to 0, \Delta t \to 0} \rho_{\Delta x,\Delta t}(x,t)
$$

(2)

then we can get:

$$
\int_{\Omega} \rho(x,t)dx = 1
$$

(3)

this is just the normalization formula, where $\rho(x,t)$ is called position measure density, $\Omega$ denotes the whole integral space, we call this kind of dense point set regular dense point set.

Now, we will analyze the new quantity $\rho(x,t)$ in detail, first, the position measure density $\rho(x,t)$ is not a point quantity, it is defined during infinitesimal interval, this fact is very important, since it means that if the measure density $\rho(x,t)$ exists, then it will be continuous relative to both $t$ and $x$, that is to say, contrary to the position function $x(t)$, there does not exist the discontinuous situation for the measure density function $\rho(x,t)$. Furthermore, this fact also results in that the continuous function $\rho(x,t)$ is the last useful quantity for describing the regular dense point set; Secondly, the essential meaning of the position measure density $\rho(x,t)$ lies in that it represents the dense degree of the points in the point set in two-dimension space and time, and the points are denser where the position measure density $\rho(x,t)$ is larger.

C. The evolution of regular dense point set

Now, we will further discuss the evolution law for regular dense point set.

Just like the continuous position function $x(t)$, although the continuous position measure density function $\rho(x,t)$ completely describes the regular dense point set, it is one kind of static description about the point set, and it can not be used for prediction itself, so in order to predict the evolution of the regular dense point set we must create some kind of quantity describing its change, enlightened by the theory of fluid mechanics we can define the fluid density for the position measure density $\rho(x,t)$ as in the following:

$$
\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial j(x,t)}{\partial x} = 0
$$

(4)

we call this new quantity $j(x,t)$ position measure fluid density, it is evident that this quantity just describes the change of the measure density of the regular dense point set, thus the general evolution equations of the regular dense point set can be written as in the following:
\[
\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial j(x,t)}{\partial x} = 0 \quad (5)
\]

\[
\frac{\partial j(x,t)}{\partial t} + f(\rho, j, \frac{\partial \rho}{\partial x}, \frac{\partial j}{\partial x}, \ldots) = 0 \quad (6)
\]

where \( f(\rho, j, \frac{\partial \rho}{\partial x}, \frac{\partial j}{\partial x}, \ldots) \) is a certain function containing \( \rho(x,t), j(x,t) \) and their partial derivatives relative to \( x \).

**IV. PHYSICAL ANALYSIS ABOUT NEW MOTION**

In this part, we will return to the physical world and analyze the physical motion.

**A. The definition of new motion**

Through the presuppositions presented in the beginning of this paper, we will give a strict physical definition of new motion in three-dimension space, the definition for other abstract spaces or many-particle situation can be easily extended.

1. The new motion of particle in space is described by regular dense point set in four-dimension space and time.
2. The new motion state of a particle in space is described by the position measure density \( \rho(x,y,z,t) \) and the position measure fluid density \( j(x,y,z,t) \) of the corresponding regular dense point set, in the simplest situation they form an integrated abstract wave function \( \psi(x,y,z,t) = \rho^{1/2} \cdot e^{iS(x,y,z,t)/\hbar} \), where \( S(x,y,z,t) = m \int_{-\infty}^{\infty} j(x',y,z,t)/\rho(x',y,z,t)dx' \), \( m \) is the mass of the particle and \( \hbar \) is a constant quantities.
3. The evolution of new motion corresponds to the evolution of regular dense point set, and one of its evolution equations containing the wave function \( \psi(x,y,z,t) \) is Schrödinger equation in quantum mechanics.

We call the new motion quantum discontinuous motion, compared with classical continuous motion, the commonness of these two kinds of motion is that the particle exists only in one position in space at one instant, their difference lies in the behavior of the particle during infinitesimal time interval \([t,t+dt]\), for classical continuous motion, the particle is limited in a certain local space interval \([v,v+dv]\), while for quantum discontinuous motion, the particle moves throughout the whole space with a certain position measure density \( \rho(x,y,z,t) \).

On the other hand, in order to analyze the physical evolution of the new motion, one point needs to be emphasized, according to the above definition of new motion, the particle is in only one position at any instant, but this is not a direct physical statement, but a simple metaphysical statement about the time-division existence of the reality undergoing the new motion, which is logically deduced from the experiment fact that the wave function satisfies the linear superposition principle, or there does not exist self-interaction for the wave function in quantum mechanics, strictly speaking, the particle may be not in only one position at one instant( we can not confirm it in physics either ), the only rational physical requirement is that the instants set, at any instant of which the particle is in only one position, forms a dense point set, whose measure equals to the length of the time interval. In fact, the following physical states of the new motion are all defined during infinitesimal time interval in the meaning of measure, not at one instant, for example, the momentum eigenstate \( \psi_p(x,t) = e^{ipx-iEt} \), especially even the position eigenstate \( \psi(x,t) = \delta(x-x_0) \) are still defined during infinitesimal time interval.

**B. The evolution of new motion**

In the following, we will give the main clues for finding the possible evolution equations of the new motion, at the same time, the axiom system of quantum mechanics will be deduced out by use of a logical analysis, and Schrödinger equation will prove to be just the simplest nonrelativistic evolution equations of the new motion. For simplicity but lose no generality, we mainly analyze one-dimension motion, but the results can be easily extended to three-dimension situation.

1. Two kinds of description bases

As we know, contrary to classical continuous motion, the state of quantum discontinuous motion is generally nonlocal, and the particle undergoing this new motion moves throughout the whole space with a certain position measure density \( \rho(x,t) \) during infinitesimal time interval, thus there exist two kinds of description bases for quantum discontinuous motion in essence, one should be the position of particle, which is local description basis; the other will
be the corresponding nonlocal description basis, in the following we will demonstrate that it is just the momentum of particle.

In fact, in order to find the nonlocal description basis, we only need to analyze the simplest situation of the free evolution of the new motion, where $\rho(x,t)$ is constant during the evolution, for this situation we can easily find that $j(x,t)$ is also constant, and we have:

$$\frac{\partial \rho(x,t)}{\partial t} = 0$$  \hspace{1cm} (7)

$$\frac{\partial j(x,t)}{\partial x} = 0$$  \hspace{1cm} (8)

$$\frac{\partial j(x,t)}{\partial t} = 0$$  \hspace{1cm} (9)

$$\frac{\partial \rho(x,t)}{\partial x} = 0$$  \hspace{1cm} (10)

but the mathematical and physical meaning of these four equations need to be analyzed, firstly, these equations can also hold in classical fluid mechanics, but as to describing the evolution of the new motion of a particle, their meaning will be very different, according to these four equations, the quantities position measure density $\rho(x,t)$ and position measure fluid density $j(x,t)$ will be constant irrelevant to both $x$ and $t$, generally we can let $\rho(x,t)=1$, then we have $j(x,t)=\rho(x,t) \cdot v_0=v_0=\frac{p_0}{m}$, where $m$ is the mass of the particle, these two results mean that for the free particle undergoing the new motion with one constant momentum, its position will spread throughout the whole space with the same position measure density, thus we demonstrate that the momentum of particle is just the nonlocal description basis, and the momentum state of new motion is completely nonlocal.

2. One-to-one relation

Now we have shown there are two kinds of description bases for quantum discontinuous motion, but it is evident that there exists only one definite motion state at any instant, so the state description using these two kinds of description bases should be equivalent, this means that there exists a one-to-one relation between these two descriptions, and this relation is irrelevant to the concrete motion state, in the following we will mainly discuss how to find this one-to-one relation, and our analysis will also show that this relation essentially determines the distinct evolution for quantum discontinuous motion, as well as the axiom framework of Hilbert space for quantum mechanics.

Like the measure density $\rho(x,t)$ and measure fluid density $j(x,t)$ for local position $x$ of the particle, we can also define the measure density $f(p,t)$ and measure fluid density $J(p,t)$ for the nonlocal momentum $p$ of the particle, and according to the above analysis there should exists a one-to-one relation between the local position description $(\rho, j)$ and nonlocal momentum description $(f, J)$. First, it is evident that there exists no direct one-to-one relation between the measure density functions $\rho(x,t)$ and $f(p,t)$, since even for the above simplest situation, we have $\rho(x,t)=1$ and $f(p,t) = \delta^2(p-p_0)$ (this result can be directly obtained when considering the general normalization relation $\int_0^\infty \rho(x,t)dx = \int_0^{\infty} f(p,t)dp$, and there is no one-to-one relation between them.

Then in order to obtain the one-to-one relation, we have to create new properties on the basis of the above position description $(\rho, j)$ and momentum description $(f, J)$, this needs a little more mathematical trick, here we only give the main clues and the detailed mathematical demonstrations are omitted, first, we disregard the time variable $t$ and let $t=0$, as to the above free evolution state with one momentum, we have $(\rho,j) = (1,\frac{p_0}{m})$ and $(f,J) = (\delta^2(p-p_0),0)$, thus we need to create a new position state function $\psi(x,0)$ using 1 and $p_0/m$, a new momentum state function $\varphi(p,0)$ using $\delta^2(p-p_0)$ and 0, and find the one-to-one relation between these two state functions, this means there exists an one-to-one transformation between the state functions $\psi(x,0)$ and $\varphi(p,0)$, we generally write it as follows:

$$\psi(x,0) = \int_{-\infty}^{+\infty} \varphi(p,0)T(p,x)dp$$  \hspace{1cm} (11)

where $T(p,x)$ is called transformation function and generally continuous and finite for finite $p$ and $x$, since the function $\varphi(p,0)$ will contain some form of the basic element $\delta^2(p-p_0)$, normally we may expand it as $\varphi(p,0) = \sum_{i=1}^{\infty} a_i \delta^i(p-p_0)$, while the function $\psi(x,0)$ will contain the momentum $p_0$, and be generally continuous and finite for finite $x$, then it is evident that the function $\varphi(p,0)$ can only contain the term $\delta(p-p_0)$, because the other terms will result in infiniteness.
On the other hand, since the result $\varphi(p,0) = \delta(p-p_0)$ implies that there exists the relations $f(p,0) = \varphi(p,0)^* \varphi(p,0)$ and $\rho(x,0) = \psi(x,0)^* \psi(x,0)$, we may let $\psi(x,0) = e^{iG(p,x)}$ and have $T(x,p) = e^{iG(p,x)}$, then considering the symmetry between the properties position and momentum (this symmetry essentially stems from the equivalence between these two kinds of descriptions, the direct implication is for $\rho(x,0) = \delta^2(x-x_0)$ we also have $f(p,0) = 1$) we have the extension $G(p,x) = \sum_{i=0}^{\infty} b_i (px)^i$, but the symmetry between the properties position and momentum further results in the symmetry between the transformation $T(x,p)$ and its reverse transformation $T^{-1}(p,x)$, where $T^{-1}(p,x)$ satisfies the relation $\varphi(p,0) = \int_{-\infty}^{+\infty} \psi(x,0) T^{-1}(p,x) dp$, thus we can only have the term $px$ in the function $G(p,x)$, for this situation the symmetry relation between these two transformations is $T^{-1}(p,x) = T^*(p,x) = e^{-ipx}$, and we let $b_1 = 1/h$, where $h$ is a constant quantity, for simplicity we let $h = 1$ in the following discussions. Then mainly owing to the essential symmetry involved in the new motion we work out the basic one-to-one relation, it is $\psi(x,0) = \int_{-\infty}^{+\infty} \varphi(p,0) e^{ipx} dp$, where $\psi(x,0) = e^{-ip0x}$ and $\varphi(p,0) = \delta(p-p_0)$.

In fact, there may exist other complex forms for the state functions $\psi(x,0)$ and $\varphi(p,0)$, for example, they are not the above simple number functions but multidimensional vector functions such as $\psi(x,0) = (\psi_1(x,0), \psi_2(x,0), ..., \psi_n(x,0))$ and $\varphi(p,0) = (\varphi_1(p,0), \varphi_2(p,0), ..., \varphi_n(p,0))$, but the above one-to-one relation still exists for every component function, and these vector functions still satisfy the above modulo square relations, namely $\rho(x,0) = \sum_{i=1}^{n} \psi_i(x,0)^* \psi_i(x,0)$ and $f(p,0) = \sum_{i=1}^{n} \varphi_i(p,0)^* \varphi_i(p,0)$, these complex forms will correspond to more complex theories, say, involving more inner properties of the particle such as charge and spin etc.

Now, since the one-to-one relation between the position state description and momentum state description is irrelevant to the concrete motion state of the new motion, the above one-to-one relation for the free motion state with one momentum should apply for any motion state of the new motion, and the states which satisfy the one-to-one relation will be the possible motion states of the new motion. Furthermore, it is evident that this one-to-one relation will directly result in the famous uncertainty relation $\Delta x \cdot \Delta p \geq h/2$; and as we will demonstrate, it will essentially result in the consistent axiomatic system of quantum mechanics.

3. Axiom I of quantum mechanics

Now, the direct linear superposition of the above momentum state $\psi_p(x,0) = e^{-ipx}$, which can be called momentum eigenstate, will evidently satisfy the one-to-one relation, and should be the possible motion state of the new motion, at the same time, Fourier analysis further shows that any normal motion state can be expanded as the linear superposition of the momentum eigenstates, then all normal motion states of the new motion will form an abstract complex linear space, which has been called Hilbert space, thus we have shown that any motion state of the system undergoing the new motion will correspond to the state vector in Hilbert space, namely the Axiom I of quantum mechanics [7] [9], for example, the free new motion state with two momenta corresponds to the state vector in Hilbert space, which is a certain kind of linear superposition of two free new motion states with one momentum in this space.

4. Axiom II of quantum mechanics

From a mathematical point of view, the above one-to-one relation will also physically determine the linear operator structure in Hilbert space, which may give a more abstract but deeper description of the new motion, thus the Axiom II of quantum mechanics [7] [9] is further included, namely, every observable of the system corresponds to the self-adjoint operator in the Hilbert state space, the self-adjoint requirement guarantees the real value of the corresponding physical observable, which may be the position observable $x$, the momentum observable $p$ and the energy observable $E$ etc., and in general the relations of these observables can be extended to the corresponding operators.

Furthermore, through defining the linear operators $\hat{x}$ and $\hat{p}$, the above one-to-one relation describing the new motion will result in the famous noncommuting relation $[\hat{x}, \hat{p}] = i\hbar$, which is taken as the basis for quantizing anything.

5. Axiom III of quantum mechanics

Now, we will demonstrate that the Axiom III of quantum mechanics [7] [9], as well as the irreducibility of probability in quantum mechanics, results from the objective nature of the quantum discontinuous motion, first, the proper measurement in physics should reflect the property of the measured system as truly as possible, and it is also rational to presuppose the existence of such proper measurement for the new motion, as well as for other realities; secondly, if we assume that each measurement about the system undergoing the new motion will bring about only one definite result, then the objective measure density $\rho$ of the new motion will naturally result in the probability distribution of the measurement results, which is $P = |\psi|^2$ for discrete observable and $P = \int \psi^* \psi^2 dE$ for continuous observable, where $|E_1, E_2|$ is the result interval, this is just the Axiom III of quantum mechanics. Furthermore, there does not exist one kind of point description for the quantum discontinuous motion in essence, so the probability for a single definite measurement result is irreducible, it is essentially determined by the discontinuity and value-dispersion nature of the quantum discontinuous motion.

6. Axiom IV of quantum mechanics

Now, we will work out the dynamical evolution law of the new motion, namely the Axiom IV of quantum mechanics.

First, in order to find how the time variable $t$ is included in the functions $\psi(x,t)$ and $\varphi(p,t)$, we may consider the
linear superposition of two momentum eigenstates, namely \( \psi(x, t) = \frac{1}{\sqrt{2}}[e^{ip_1x-ic_1t} + e^{ip_2x-ic_2t}] \), then the position measure density is \( \rho(x, t) = [1 + \cos(\Delta c(t) - \Delta px)]/2 \), where \( \Delta c(t) = c_2(t) - c_1(t) \) and \( \Delta p = p_2 - p_1 \), now we let \( \Delta p \to 0 \), then we have \( \rho(x, t) \to 1 \) and \( \Delta c(t) \to 0 \), especially using the conservation relation we can get \( dc(t)/dt = dp/p/m \), namely \( dc(t) = d(p^2/m) \cdot t \) or \( dc(t) = dE \cdot t \), where \( E = p^2/m \), is the energy of the particle in the nonrelativistic domain, thus as to any momentum eigenstate we have the time-included formula \( \psi(x, t) = e^{ipx-itE} \).

Now, as to the free motion state with one momentum, namely the momentum eigenstate \( \psi(x, t) = e^{ipx-itE} \), using the nonrelativistic relation \( E = p^2/m \) and including the constant quantity \( h \) we can easily find its nonrelativistic evolution law, which is

\[
\frac{i\hbar}{\partial t} \frac{\partial \psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x, t)}{\partial x^2} \tag{12}
\]

then owing to the linearity of this equation, this evolution equation also applies to the linear superposition of the momentum eigenstates, namely all possible free motion states, or we can say, it is the free evolution law for the new motion; Secondly, we will consider the evolution law for the new motion under outside potential, when the potential \( U(x, t) \) is a constant \( U \), the evolution equation will be

\[
\frac{i\hbar}{\partial t} \frac{\partial \psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x, t)}{\partial x^2} + U \cdot \psi(x, t) \tag{13}
\]

then when the potential \( U(x, t) \) is related to \( x \) and \( t \), the above equation will also essentially determine the evolution equation, it is

\[
\frac{i\hbar}{\partial t} \frac{\partial \psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x, t)}{\partial x^2} + U(x, t) \cdot \psi(x, t) \tag{14}
\]

for three-dimension situation the equation will be

\[
\frac{i\hbar}{\partial t} \frac{\partial \psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(x, t) + U(x, t) \cdot \psi(x, t) \tag{15}
\]

this is just the Schrödinger equation in quantum mechanics, thus we have deduced the Axiom IV of quantum mechanics [7] [9].

On the other hand, according to the Axiom II of quantum mechanics, every observable of the system corresponds to a self-adjoint operator in the Hilbert state space, and in general the relations of these observables can be extended to the corresponding operators, thus the above evolution equation of new motion can be reduced to the familiar nonrelativistic energy equality \( E = p^2/m + U \), and the equation can be rewritten as \( E \psi(x, t) = \frac{\hat{p}^2}{2m} \psi(x, t) + U(\hat{x}, t) \psi(x, t) \), where \( \hat{E} = i\hbar \partial/\partial t \), and \( \hat{p} = -i\hbar \nabla \).

At last, the above analysis also shows that the state function \( \psi(x, t) \) provides a complete description of the quantum discontinuous motion, since the new motion is completely described by the measure density \( \rho(x, t) \) and measure fluid density \( j(x, t) \), and according to the above evolution equation the state function \( \psi(x, t) \) can be formulated by these two functions, namely \( \psi(x, t) = e^{iS(x,t)}/h \), where \( S(x, t) = \int_{-\infty}^{x} j(x', t) \rho(x', t)dx' \) (Note: when in three-dimension space, the formula for \( S(x,y,z,t) \) will be \( S(x,y,z,t) = \int_{-\infty}^{x} \int_{-\infty}^{y} j(x', y', z, t) \rho(x', y', z, t)dx'dy' = \int_{-\infty}^{x} \int_{-\infty}^{y} \int_{-\infty}^{z} j(x, y, z', t) \rho(x, y, z', t)dz' \), since in general there exists the relation \( \nabla \times \{ j(x, y, z, t) / \rho(x, y, z, t) \} = 0 \) when \( \rho(x, y, z, t) \neq 0 \), and these two functions can also be expressed by the state function, namely \( \rho(x, t) = |\psi(x, t)|^2 \) and \( j(x, t) = [\psi^* \partial \psi / \partial t - \psi \partial \psi^* / \partial t]/2i \), thus there exists a one-to-one relation between \( \rho(x, t) \) and \( j(x, t) \), and the state function \( \psi(x, t) \) also provides a complete description of the quantum discontinuous motion.

On the other hand, we can see that the absolute phase of the wave function \( \psi(x, t) \), which may depends on time in nonrelativistic domain, is useless for describing the new motion, since according to the above analysis it disappears in the measure density \( \rho(x, t) \) and measure fluid density \( j(x, t) \), which completely describe the quantum discontinuous motion, thus from the point of view of the new motion it is natural that the absolute phase of the wave function possesses no physical meaning.

7. Axiom V of quantum mechanics

As we know, quantum mechanics is self-consistent when defined by the above four axioms, since the elucidation about the corresponding relation between the physical reality and mathematical language is just a conditional statement, namely if the measurement about the quantum system brings about only one definite result, then the probability distribution of the measurement results will satisfy the formula, say for the discrete observable, \( P = |\psi|^2 \); but if only
the above four axioms are included, quantum mechanics will be evidently incomplete, since it does not describe the real measurement result, thus its founders resorted to the projection postulate or Axiom V \cite{7} \cite{9} in order to account for the measurement process, whereas this postulate is still a direct description about the measurement result, it says nothing about how the measurement can and does bring about one definite result, so it needs to be further explained in physics, now the new motion of particle just provide such a broad framework for objectively studying the microscopic world that it may solve the collapse problem, for example, there may exist many kinds of concrete motion modes among the new motion, and the new motion may display differently in the nonrelativistic and relativistic domains, especially when involving gravity the new motion of particle may naturally result in the objective collapse process, but owing to the weird difficulty of this problem, we will tackle it in another paper.

8. The value of $\hbar$ in quantum mechanics

Up to now, one problem is still left, it is how to determine the value of $\hbar$ in quantum mechanics or our world, according to the above analysis we only know that the constant $\hbar$ possesses a finite nonzero value, certainly, just like the other physical constants such as $c$ and $G$, its value can be determined by the experience, but its existence need to be explained, and the above analysis about the new motion can provide the answer, namely the existence of $\hbar$ essentially results from the irreducibility of the nonlocal momentum definition, or nonexistence of the velocity or local momentum of the particle undergoing the new motion, this kind of irreducibility denotes that momentum is no longer related to space-time or velocity as for classical continuous motion, it is also an essential property of the new motion just like position, especially it provides an equivalent nonlocal description of the new motion, while position provides a local description of the new motion, this equivalence further results in the one-to-one relation between these two kinds of descriptions, then it is just this one-to-one relation which requires the existence of a certain constant $\hbar$ to cancel out the unit of the physical quantities $px$ and $Et$ in the relation, at the same time, the existence of $\hbar$ also indicates some kind of balance between the properties( concretely speaking, their value distribution dispersions ) limited by the one-to-one relation ( there is no such limitation for classical continuous motion and $\hbar = 0$ ), or we can say, the existence of $\hbar$ essentially indicates some kind of balance between the nonlocality and locality of the new motion in space-time.

Certainly, the new motion provides a broader motion framework for the particle in microscopic world, in which we can understand the weird displays of the microscopic objects objectively and consistently, which can not be grasped consistently in the old framework of classical continuous motion, but it can not give the concrete value of $\hbar$ in our world by itself, as special relativity can not determine the value of light velocity $c$, or general relativity can not determine the value of gravity constant $G$, surely, there may exist some deeper reasons for the particular value of $\hbar$ in our universe, but the new motion can not determine this value alone, the solution may have to resort to other subtle realities in this world, for example, gravity $(G)$, space-time$(c)$, or even the existence of mankind.

V. THE CONFIRMATION OF THE NEW MOTION

In the following, we will give two main methods to confirm the new motion underlying the wave function in microscopic world.

A. Protective measurement

The first method is called protective measurement \cite{11} \cite{12}, it aims at measure the new motion state of a single particle through repeatedly measuring it without destroying its state, in real experiment a small ensemble of similar particles may be required. By use of this kind of measurement, the new motion state or wave function of a particle does not change appreciably when the measurement is being made on it, its clever way is to let the system undergo a suitable interaction so that it is in a non-degenerate eigenstate of the whole Hamiltonian, then the measurement is made adiabatically so that the new motion state described by the wave function neither changes appreciably nor becomes entangled with the measurement device, this suitable interaction is called the protection.

In the following, we will demonstrate how to use the protective measurement to confirm the new motion of a single particle in microscopic world, which is described by the wave function and Schrödinger equation, for simplicity but lose no generality, we only consider a particle in a discrete nondegenerate energy eigenstate $\psi(x)$, the interaction Hamiltonian for measuring the value of an observable $A_n$ in this state is: $H = g(t)PA_n$, which couples the system to a measuring device, with coordinate and momentum denoted respectively by $Q$ and $P$, where $A_n$ is the normalized projection operator on small regions $V_n$ having volume $v_n$, namely:

\begin{equation}
V_n = \frac{1}{\hbar V} |\psi(x)|^2 dx
\end{equation}


\[ A_n = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \not\in V_n. \end{cases} \]  

the time-dependent coupling \( g(t) \) is normalized to \( \int_0^T g(t)dt = 1 \), we let \! g(t) \! = \! 1/T \! \text{ for most of the time } T \! \text{ and assume that } g(t) \! \text{ goes to zero gradually before and after the period } T \! \text{ to obtain an adiabatic process when } T \to \infty, \! \text{ the initial state of the pointer is taken to be a Gaussian centered around zero, and the canonical conjugate } P \! \text{ is bounded and also a motion constant not only of the interaction Hamiltonian, but of the whole Hamiltonian.} \!

Now using this kind of protective measurement, the measurement of \( A_n \) yields the result:

\[ \langle A_n \rangle = \frac{1}{v_n} \int_{V_n} |\psi|^2 dv = |\psi_n|^2 \]

the result \( |\psi_n|^2 \) is just the average of the measure density \( \rho(x) = |\psi(x)|^2 \) over the small region \( V_n \), so when \( v_n \to 0 \) and after performing measurements in sufficiently many regions \( V_n \) we can find the measure density \( \rho(x) \) of the new motion state of the measured particle.

Then we will measure the measure current density \( j(x) \) of the new motion state, namely we need measure the value of an observable \( B_n \) in this state, where \( B_n = \frac{1}{2i}(A_n \nabla + \nabla A_n) \), the measurement result will be \( \langle B_n \rangle \), ant it is just the average value of the measure fluid density \( j(x) = \frac{1}{2i}(\psi^* \nabla \psi - \psi \nabla \psi^*) \) in the region \( V_n \), so when \( v_n \to 0 \) and after performing measurements in sufficiently many regions \( V_n \) we can also find the measure fluid density \( j(x) \) of the new motion state of the measured particle.

Thus we have demonstrated that the new motion of a single particle, which is described by the measure density \( \rho(x) \) and measure fluid density \( j(x) \), or the abstract wave function \( \psi(x) \), can be confirmed through the above protective measurement.

B. Standard impulse measurement

Certainly, the standard impulse measurement in quantum mechanics [7] can also confirm the new motion of a single particle described by the wave function in microscopic world, its wrinkle lies in that we first prepare an ensemble of a large number of particles in the same state of new motion, then using this kind of measurement measure every particle in the ensemble only one time, and we need not repeatedly measure the same particle, thus even if the state of the new motion or wave function of a single particle will be destroyed after each measurement so that the following measurement will no longer reveal the real information about the original state of new motion, but according to quantum mechanics, all the individual measurement results about the ensemble can reveal the state of the ensemble, and also the state of the new motion of a single particle in the ensemble, since every particle in the ensemble is in the same state of new motion.

C. Understanding the wave function in terms of new motion

As we know, many ontological interpretations of quantum mechanics still consider the wave function as one kind of objective field, as Bell said, "No one can understand this theory until he is willing to think of \( \psi \) as a real objective field rather than a 'probability amplitude'" [10], but the difficulties involved in this kind of ontological interpretation had been pointed out since the beginning of quantum mechanics, for example, the existence of complex wave function, the multidimensionality of the wave function and the representation problem etc can not be solved in the framework of objective field, these difficulties greatly prevented physicists from accepting the objective view about the microscopic object described by the wave function. Now, according to the new quantum discontinuous motion, we can easily overcome these difficulties in the framework of objective particle:

(1). As to the problem of complex wave function, since the wave function is not one kind of objective field at all, it is just one kind of indirect abstract mathematical symbol, which is used to describe the objective quantum discontinuous motion of the particles in microscopic world, concretely speaking, it is an abstract complex of the measure density \( \rho(x, t) \) and measure fluid density \( j(x, t) \), which directly describe the state of the new motion in physics, and its appearance essentially results from the symmetry involved in the new motion and the resulting linear evolution principle, thus whether the wave function is complex or not is not a problem for the new motion described by the wave function.
The multidimensionality of the wave function is very natural from the point of view of new motion of particle, as we know, the measure density \( \rho(x, t) \), or wave function \( \psi(x, t) \) for a single particle depends on three space variables, and as to two particles generally we can not define their respective measure densities \( \rho_1(x, t) \) and \( \rho_2(x, t) \), or wave functions \( \psi_1(x, t) \) and \( \psi_2(x, t) \), whereas we should define their joint measure density \( \rho(x_1, x_2, t) \) according to point set theory, which means the joint measure density of particle 1 in position \( x_1 \) and particle 2 in position \( x_2 \), this further results in that the one-to-one relation is two-manifold Fourier transformation involving the wave function \( \psi(x_1, x_2, t) \), which depends on six space variables, namely

\[
\psi(x_1, x_2, t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \varphi(p_1, p_2, t) \rho_1(p_1) \rho_2(p_2) dp_1 dp_2 \quad (16)
\]

and Schrödinger equation for this two-particle situation is

\[
i\hbar \frac{\partial \psi(x_1, x_2, t)}{\partial t} = -\frac{\hbar^2}{2m} \left[ \nabla_1^2 + \nabla_2^2 \right] \psi(x_1, x_2, t) + U(x_1, x_2, t) \cdot \psi(x_1, x_2, t) \quad (17)
\]

Certainly, when these two particles are independent, the joint measure density \( \rho(x_1, x_2, t) \) can be reduced to \( \rho_1(x_1, t)\rho_2(x_2, t) \), and the joint wave function \( \psi(x_1, x_2, t) \) can also be reduced to \( \psi_1(x_1, t)\psi_2(x_2, t) \).

Certainly, as Bohr had taught us, if one does not wander about quantum mechanics he surely has not understood this theory yet. Indeed, the quantum discontinuous motion in microscopic world is extremely different from the familiar classical continuous motion in macroscopic world, so it is natural that the experience from classical mechanics contradicts the picture of new motion, especially the particle undergoing the new motion can move far away in a very small time interval, or even infinitesimal time interval, which appears to conflict with the local spreading of energy, but in fact, this just provides the objective origin of quantum nonlocality, which has been confirmed in experiments; on the other hand, when confirming this kind of weird display of new motion we have to consider the more weird quantum measurement, which needs to be further studied in another paper, but here we should keep in mind something, namely when we go into the new field of quantum discontinuous motion, we should re-understand the meaning of everything in principle, including energy and the accepted law that it moves locally or even the concept of particle, if we still use them, in one word, our understanding about reality can only be determined by the reality itself, not our belief.

**VI. EXPLAINING THE WEIRD DISPLAY OF THE WAVE FUNCTION IN TERMS OF NEW MOTION**

At last, we will give two familiar examples, which have evidently manifested the existence of new motion in microscopic world, to explain the weird displays of the wave function in terms of the new motion of particle. The first is the base state of Hydrogen atom, its position distribution density, which can be found through the above measurements, is written in the following:

\[
\rho(x) = |\psi(x)|^2 = \frac{4}{a_0} \cdot \exp \left( -\frac{2r}{a_0} \right) \quad (18)
\]

According to the new motion of the particle, at any instant the electron will be in only one position in space, but during infinitesimal time interval \([t, t+dt]\), the electron will move throughout the whole space where the above function does not equal to zero, and its position measure density will be the same as the above position distribution density function obtained from the wave function, thus during infinitesimal time interval \([t, t+dt]\), according to Gauss theorem the charge distribution of the whole system will be equivalent to the zero charge distribution for the outside observer, and there exists no change of the whole charge distribution either, so it can be easily understood that no energy is radiated during finite time interval, as well as during infinitesimal time interval, this is just the objective origin of the mysterious stability in atom world.

The second example is the double-slit experiment, people have been trying to understand the formation process of the double-slit interference pattern objectively, but few people can give an ontological description for it up to now, the essential reason, as we think, is that people all ignored the difference between instant and infinitesimal time interval. By means of the new quantum discontinuous motion of the particle, the mystery of this process can be disclosed, the real process should be that the particle undergoing the new motion passes through both slits in the double-slit experiment, this means that the particle is still in only one of the two slits at any instant, but during the time interval \(\Delta t\), which can approach to zero, the particle moves throughout both slits and passes through them, and the position measure density of the particle always satisfies the function \( \rho(x, t) = |\psi(x, t)|^2 \), which is finite and the same in both slits. Since the particle undergoing the new motion can pass through both slits in this objective way, we can more easily understand the forming of double-slit interference pattern, which is not a simple superposition of two one-slit interference patterns.
VII. CONCLUSIONS

On the whole, a new point of view about the motion of the particle is presented to interpret the weird display of the wave function describing the microscopic object, in mathematics, the point set theory casts a new light on the study of physical motion, especially through deeply analyzing the regular dense point set, we present a new kind of motion, which is called quantum discontinuous motion contrary to classical continuous motion. Then the physical meaning of this new motion is carefully examined, at the same time, we give a strict physical definition about the quantum discontinuous motion, and show that the notorious wave function is just a mathematical complex describing the new motion of the microscopic particle, and Schrödinger equation in quantum mechanics is just the simplest nonrelativistic evolution equations for the new motion, especially the consistent axiom system of quantum mechanics is also deduced out. At last, the protective measurement and standard impulse measurement are used to confirm the existence of the new motion of the microscopic particle described by the wave function, and two famous examples are also given to explain the weird display of the wave function in terms of new motion.

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