STRONG NONLINEAR RESPONSE OF SUPERCONDUCTING TUNNEL JUNCTIONS DUE TO LOCALIZED TRAPS

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The responsivity of a single superconducting tunnel junction (STJ) for photon detection has been determined over a wide range of energy from optical to hard x-ray. An unexpected rapid decrease of responsivity is found in the ultraviolet regime. We show that the effect is due to the presence of localised traps – regions of locally reduced energy gap containing a number of confined states. The balance equations for non-equilibrium quasiparticles have been generalized to account for the effects of localized traps. Quantitative modelling yields the details of trap density and trapping probability.
I. INTRODUCTION.

One of the most exciting recent developments in astrophysical instrumentation in recent years has been the use of superconducting tunnel junctions (STJ) to detect single photons at optical wavelengths [1]. In contrast to charge coupled devices, STJs also possess intrinsic wavelength and time resolution, thus providing the basis for photon counting systems which are far superior to existing types. STJs have previously been used to detect ultraviolet and x-ray photons [2], however the STJ responsivity at these energies is invariably larger than for optical photons. This unexpected result cannot be explained on the basis of the standard Rothwarf-Taylor description of non-equilibrium quasiparticle (qp) dynamics [3]. According to this model responsivity should decrease monotonically with increasing photon energy due to the increasing self-recombination [3,4]. In this paper we present detailed experimental results on the responsivity of a single STJ device over wide range of energy, from the visible to the hard x-ray region (Fig. 1). The device used was a high quality epitaxially grown, Nb-Al-AlO$_x$-Al-Nb proximised structure which has enhanced responsivity due to multiple tunneling. We found that the responsivity remained constant throughout the optical and near ultraviolet range, then rose rapidly to a peak value of approximately 43 times the optical value, before falling nearly hyperbolically along the curve expected due to self-recombination. We shall show that the observed energy variation of these quantities is due to the presence of “pools” having reduced energy gap and giving rise to potential wells each containing a number of localized states in which the qps may be confined. These will be referred to as localized traps. Our results suggest that the localized traps play no less a significant role in superconductors than in semiconductors.

II. BALANCE EQUATIONS IN THE PRESENCE OF LOCALIZED TRAPPING STATES

To describe the time evolution of non-equilibrium qp’s and the phonon distributions generated by the absorption of a photon in the STJ, the phenomenological Rothwarf-Taylor (RT) equations may be written (for the situation in which due to fast phonon pair-breaking processes phonons remain in local equilibrium with qp’s) [3,4]

\[
\frac{dn_j}{dt} = -n_1(\Gamma_{t,1} + \Gamma_{QL,1} + \Gamma_{QPL,12} + \Gamma_{QPL,1}) \\
+ n_2(\Gamma_{t,2} + \Gamma_{QPL,21}) \\
\frac{dn_T}{dt} = -n_2(\Gamma_{t,2} + \Gamma_{QL,2} + \Gamma_{QPL,21} + \Gamma_{QPL,2}) \\
+ n_1(\Gamma_{t,1} + \Gamma_{QPL,12}).
\]

(1)

Here $n_j$ and $n_T$ are densities of non equilibrium and thermal qp’s in electrode $j$, $\Gamma_{t,j}$ is the tunnel rate from electrode $j$ and $\Gamma_{QPL,j}$, $\Gamma_{QPL,j}$ are loss rates due to recombination. $\Gamma_{QPL,j,i}$ corresponds to phonon escape from electrode $j$ into $i$ (phonon coupling) and $\Gamma_{QPL,j}$ accounts for phonon escape into surrounding materials (e.g. the substrate). It is given
by $\Gamma_{QL,j} = (n_j + 2n_T)R^*$ with the effective recombination coefficient $R^*$. Finally, $\Gamma_{QL,j}$ is the loss rate due to any process other than recombination, notably diffusion into leads and trapping by localized states. Due to the dependence of the recombination rate on qp density (self-recombination) Eqs 1 are non-linear. The measured charge $Q(E)$, where $E$ is the energy of the absorbed photon, is integral over the total excess tunneling current, and can be easily expressed in terms of solution of Eqs (1). We shall define a fundamental STJ responsivity $S(E)$ as $Q(E)$ per unit $E$ normalized to the limit of an infinitesimally small $E$. Thus

$$S(E) \equiv \frac{Q(E)}{E} \lim_{E \to 0} \left( \frac{E}{Q(E)} \right)$$

(2)

We now introduce the effect of localized trapping states by writing the Shockley-Read type balance equation [5] for the density of populated traps $n_t$

$$\frac{dn_t}{dt} = c_t (N_t - n_t) n - \gamma n_t$$

(3)

Here $N_t$ is the total density of traps, $c_t$ and $\gamma$ are the trapping coefficient and de-trapping rate respectively. By definition $c_t N_t \equiv \Gamma$ is the trapping rate by empty traps. Whilst in semiconductors the Shockley-Read equation describes trapping of an electron (hole) by a discrete level a localized trap in superconductor is envisaged to contain a large number of states. Thus Eq.(3) deals with properly averaged quantities.

De-trapping occurs either directly via the absorption of a phonon or through recombination of the trapped qp with a free qp of high enough energy such that the resultant emitted phonon may break a Cooper pair and produce two free qps. At temperatures normally used in experiments the latter process is dominant. De-trapping via recombination requires that the non-equilibrium qp's have, on average, an energy at least that of the trap depth above the superconducting edge $\Delta$. We write the de-trapping rate $\gamma(t) = R^+ n(t)$ with $R^+$ accounting for the events in which the recombination phonons are retained within the system, in contrast to $R^*$ where they are lost from the structure, and also for those in which the excess qp's are too close to the superconducting edge to produce de-trapping.

The overall trapping rate is constant as long as $N_t >> n_t$ and if $\gamma$ does not depend on qp density. These conditions are fulfilled for $E \to 0$. However, at high incident photon energy and specifically for the high quality STJ's now available, the number of excess qp's can approach or exceed the number of traps, therefore reducing the overall trapping rate and enhancing the de-trapping process. We have generalized the RT equations to include the kinetics of trapped qp's as follows.

First, we solve (3) for $n_t(t)$, assuming all traps are empty at time $t = 0$. Then we split the loss term $\Gamma_{QL,j} n_j$ of (1) into the sum of a diffusive contribution $\Gamma n_j$ and a trap contribution $-\frac{dn_t}{dt}$, and introduce dimensionless qp densities $y(t) = \frac{n_1(t)}{n_0}$ and $z(t) = \frac{n_2(t)}{n_0}$ with $n_0$ the initial qp density. We consider a symmetrical STJ, where all parameters are the same for both electrodes. Substituting $\frac{dn_t}{dt}$ into (1) with the symbols $\alpha = R^* n_0$, $\beta = \frac{n_T}{n_0} + \frac{P}{2\alpha}$ and $\zeta = (c_t + R^+) n_0$ leads to a new set of equations which replace (1):

3
\[
\frac{dy}{dt} = -\alpha(y^2 + 2\beta y) - \Gamma y \exp[-\zeta \int_0^t y(t')dt'] \\
-\Gamma(t)(y - z) - \Gamma_{QL12} + \Gamma_{QL21}z \\
\frac{dz}{dt} = -\alpha(z^2 + 2\beta z) - \Gamma z \exp[-\zeta \int_0^t z(t')dt'] \\
-\Gamma(t)(z - y) - \Gamma_{QL21} + \Gamma_{QL12}y.
\]

(4)

**III. ANALYTICAL SOLUTION FOR MULTIPLE TUNNELING SYSTEMS**

As will be discussed in Section IV, the above equations may be solved numerically for an arbitrary set of parameters. Fig.1 shows the modeled responsivity for a typical sample. However, many of the salient features may be derived analytically by considering the limiting behaviour of a multiple tunneling system. In this situation each qp on average tunnels several times before it is lost, i.e. \(\Gamma_t \gg \max(\alpha, \Gamma_{QL}, \Gamma, \Gamma)\). Under such circumstances the qp's become equally distributed between both electrodes before any loss occurs. Hence, the initial conditions can be taken as \(y(t = 0) = z(t = 0) = \frac{1}{2}\) and at any instance of time \(y(t) \approx z(t) \approx \frac{y(t) + z(t)}{2}\). Hence (4) can be reduced to a single equation for \(y(t)\) (or \(z(t)\)).

Introducing the new function \(\varphi(t) = \exp[\zeta \int_0^t y(t')dt']\), measuring \(t\) in units of \(\Gamma_t^{-1}\), and integrating the resulting second order differential equation once with the initial condition \(\varphi'(0) = -\frac{\zeta}{\Gamma_t}\), we obtain the exact result

\[
\frac{\Gamma_t}{2\beta\zeta} \varphi' = -\left[1 + \frac{\Gamma}{2\beta(\zeta - \alpha)}\right] \varphi^{1+\frac{\zeta}{\Gamma_t}} + \varphi \\
-\frac{\Gamma}{2\beta(\zeta - \alpha)} \varphi^2.
\]

(5)

Since \(\frac{Q(E)}{E} = -\frac{2e\Gamma_{N0}}{\zeta E} \ln \varphi(\infty)\), where \(N_0\) is the initial number of qp's, a knowledge of the full time dependence of \(\varphi(t)\) is unnecessary. At \(t \to \infty\) we have \(\varphi' = -\zeta \varphi y \to 0\) because \(\varphi(\infty)\) remains finite while \(y \to 0\), so that (5) becomes

\[
(1 + \chi E - \frac{b}{c})X + \frac{b}{c}X^{c+1} = 1
\]

(6)

where \(\chi E = \frac{1}{4\beta}, b = \frac{\Gamma}{2\beta \alpha}, c = \frac{\zeta}{\alpha} - 1\) and \(X = [\varphi(\infty)]^{\frac{1}{c+1}}\). Note that \(b\) is primarily governed by the trapping rate \(\Gamma\), while \(c\) contains both trapping (\(e_t\)) and de-trapping (\(R^+\)) coefficients. The number of traps \(N_t\) influences only \(b\). \(\chi\) is mainly determined by the diffusive losses \(\Gamma\).

At \(E \to 0\) we obtain

\[
\lim_{E \to 0} \frac{Q(E)}{E} = \frac{e}{1.75\Delta \Gamma(\Gamma + \Gamma + 2R^*n_T)}.
\]

(7)

Using (7) and (2) we arrive at the general expression:
\[ S(E) = -(b + 1) \frac{\ln X}{\chi E}. \] (8)

As follows from Eq. (7) at low photon energy the STJ responsivity is essentially constant. Simple analysis shows that it rises significantly when the photon energy exceeds a certain threshold, then saturates and then gradually decreases at high energies as \( \frac{\ln(1+\chi E)}{E} \) due to the self-recombination process. The maximum value of responsivity occurs at an energy \( E_M \) obtained from setting the differential of \( S(E) \) to zero and using (5,6) and is given by

\[ S_M = \frac{b + 1}{bX_{M}^{c+1} + 1} X_M. \] (9)

Thus the shape of the responsivity curve versus energy is determined primarily by the two parameters \( b \) and \( c \). It will be of particular interest to determine the relative values of \( c_t \) and \( R^+ \), which appear combined in \( c \). Thus \( c_t \gg R^+ \) implies a limited number of traps, while the opposite case \( c_t \ll R^+ \) describes the situation of a much higher density of traps, but with effective de-trapping.

A strong non-linearity of the STJ response results from either scenario. In the former case the sharp rise of the responsivity is the result of trap saturation, while in the latter it is the effective de-population of the traps which makes such traps inefficient as a qp loss mechanism. Both mechanisms result in an increase of the qp lifetime such that the remaining free qp's carry on tunneling and contributing to the STJ response. With a further increase in qp density the responsivity saturates since the self-recombination becomes dominant and eventually causes its decrease as \( \frac{\ln E}{E} \). The second mechanism described above can be considered as self-heating since the traps are effectively de-populated, or heated up. In systems with multiple tunneling, after each tunneling event the qp emerges with an excess energy \( eV_b \), where \( V_b \) is bias voltage, so that the qp distribution is raised in energy above \( \Delta \). If the bias voltage is large enough, the number of tunnels per each qp is large such that the mean qp energy is high. In this situation the de-population factor \( R^+ \) may be close to the absolute recombination factor \( R \), thus favoring self-heating.

**IV. COMPARISON WITH EXPERIMENT**

Using the above equations, we have modelled the responsivity of a Nb-Al-AlO\(_x\)-Al-Nb STJ, in the range 1 eV to 6000 eV as shown in Fig.1. This type of STJ is fully described in [2]. The device is diamond-shaped, 20\( \times \)20 \( \mu \)m\(^2 \) deposited epitaxially on polished sapphire. The Nb and Al layers are 100 nm and 120 nm thick respectively, both for top and base film. The residual resistance ratio (RRR) of the epitaxial base Nb film is \( \sim 70 \) and that of polycrystalline top film \( \sim 5 \). The AlO\(_x\) barrier has an estimated thickness of \( \leq 1 \)nm and a resistance of \( \sim 2.2 \cdot 10^{-6} \Omega \cdot \text{cm}^2 \). The bandgap of the device is 0.44 meV. In the range 40 eV-2000 eV, the device was irradiated using monochromatic synchrotron radiation. It was installed in a He\(^3\) cryostat at a temperature of \( \sim 320\text{mK} \) with the chip mounted so as to fully illuminate the top electrode. It was biased at 0.18mV. A magnetic field of \( \sim 8\text{mT} \) was applied parallel to the junction in order to suppress the Josephson current and the Fiske steps.
Measurements in the optical range (1 eV to 5 eV) and at 6 keV were performed at 300mK in a separate experiment [2], leading to a signal of \( \sim 2750 \) electrons per 1 eV in the optical and \( \sim 14400 \) electrons per 1 eV at 6 keV. These results are consistent with the synchrotron data. The whole set is shown in Figure 1 (crosses +), with a maximum responsivity \( S_M \sim 43.4 \) at \( \sim 525 \) eV.

In order to fit the data, a numerical code was developed to solve Eqs.(3) and (4) with extra terms accounting for the qp diffusion from the absorption site. Parameters \( b \) and \( c \) are first estimated from the experimental results, using the analytical solutions described in the previous section. Eqs (3) and (4) are then solved numerically using these values and the input parameters tuned to achieve a more accurate fit to the observations. Parameter \( \chi \) may be determined independently, by noting that at maximum responsivity the signal decay is mostly controlled by residual losses. At 525 eV the measured time is 47 \( \mu \)s, giving \( \Gamma \lesssim 2 \cdot 10^4s^{-1} \), hence \( \chi \).

Other parameters are the thermal population \( n_T \) and the recombination coefficients \( R_1 \), \( R^+ \) and \( R^- \). Using the densities of states for qp’s and Cooper pairs from the proximity effect model [7], and applying the general expression for the recombination rate given in [6] to the thermal population, one has \( n_T \sim 0.0827q^2p^2/\mu m^3 \), and \( R \sim 600\mu m^3/s \) for qp’s close to the energy gap \( \Delta \); for qp’s of energy \( \sim 1.25\Delta \), \( R \sim 1200\mu m^3/s \). The proximity effect model of [7] was extended to the case of arbitrary layer thicknesses, to account for relatively thick Al layers. The effective recombination coefficient is given by \( R^+ = R \left( 1 + \frac{\tau_{PL}}{\tau_{PB}} \right)^{-1} \), with \( \tau_{PL} \) the phonon escape time out of the electrode, and \( \tau_{PB} \) the Cooper pair breaking time for the phonons, calculated to be \( \sim 100 \) ps by averaging the characteristic times in Nb and Al over the density of states. The credibility of the proximity effect model on which our calculations for \( \Gamma, R \) and \( \tau_{PB} \) are based is supported by independent measurements of I-V curves, gap energy and critical current versus temperature. The phonon loss time for Nb-based junctions similar to the device reported here has been empirically determined in [4] as \( \sim 12ns \), giving \( R^+ \sim 46 \mu m^3/s \) at the energy gap. Finally, \( R^+ \) is given by \( \eta R \left( 1 + \frac{\tau_{PB}}{\tau_{PL}} \right)^{-1} \), where \( \eta \leq 1 \) accounts for the availability of only a fixed fraction of the excess qp’s for detrapping. If \( \eta = 1 \), one has \( R^+ \sim 510\mu m^3/s \). Allowing \( 600\mu m^3/s < R < 1200\mu m^3/s \) and \( 10^3s^{-1} < \Gamma < 2 \cdot 10^4s^{-1} \) leads to \( 0.018eV^{-1} < \chi < 0.072eV^{-1} \).

Parameter \( b \) can be estimated using Equation (9) or from experimental measurements of \( S_M \). At optical energies only trapping loss \( \Gamma \) is of importance. Using 2\( \mu \)s measured signal decay time at optical range yields \( \Gamma \sim 5 \cdot 10^5s^{-1} \). Finally, applying Equation (6) to the maximum responsivity point, using the range specified for \( \chi \) and the estimate of \( b \), one has \( 1.1 < c < 5.9 \). The resultant best fit is given by the solid line in Fig.1. The best fit parameters, listed in Table 1, all lie within the estimated ranges. The fit matches the data with a standard deviation of 1.4\%, with slight discrepancy around the maximum. A value of 2\( cm^2/s \) for \( D \) indicates that diffusion in the structure is rather slow. Faster diffusion would produce flatter responsivity below 300 eV, and sharper rise towards the maximum. The tunnel time of 87 ns accounts for a response of 2750 electrons per eV at 1 eV, whilst the proximity effect model predicts a tunnel time of 90 ns at the bandgap with a slow rise over the broad range of qp energies reaching 150ns by 2.5\( \Delta \). This does not exclude the possibility that qp’s have finite distribution above \( \Delta \). Indeed the best fit value for \( R \) corresponds to the qp energy of 1.25\( \Delta \), while \( \eta \) is not negligibly small. Finally, we note that \( c_0 \) and \( R^+ \) are very similar, implying that this specific device is in the intermediate range between the
self-heating and trap saturation mechanisms.

Finally, since the theoretical model introduces trap behaviour phenomenologically it describes the effect of local traps of any kind, for example the locally suppressed gap regions which occur due to known effects of Nb oxidation [8,9]. However, the traps might also be more fundamental in origin, for example due to crystallographic imperfections at grain boundaries, STJ edges, Nb/Al interfaces. In the latter case the lateral fluctuations of the main mesoscopic parameters determining the specific characteristics of a proximity effect may easily provide extra localized traps.

V. CONCLUSIONS

In summary we have derived the generalization of the Rothwarf-Taylor balance equations to incorporate the effects of localized trap kinetics. It has been shown that the population and de-population of traps can lead to a strong non-linearity of the STJ response. The theory is shown to agree well with experimental observations in Nb/Al based STJ. The observation of the spectral maximum in the STJ responsivity profile provides a direct method of determining the trap density.


TABLE I. Parameters used for the fit of Fig. 1. All quantities are defined in the text.

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<th>$N_t$</th>
<th>$\Gamma^{-1}$</th>
<th>$\tau_{PB}$</th>
<th>$R$</th>
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<td>($\mu m^3$/s)</td>
<td>(st/$\mu m^3$)</td>
<td>($\mu s$)</td>
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**FIG. 1** The responsivity vs photon energy: data (+) together with the theoretical fit (solid line).