String theoretic axion coupling and the evolution of cosmic structures

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Abstract

We examine the effects of the axion coupling to $R \tilde{R}$ on the evolution of cosmic structures. It is shown that the evolutions of the scalar- and vector-type perturbations are not affected by this axion coupling. However the axion coupling causes an asymmetric evolution of the two polarization states of the tensor-type perturbation, which may lead to a sizable polarization asymmetry in the cosmological gravitational wave if inflation involves a period in which the axion coupling is important. The polarization asymmetry produced during inflation are conserved over the subsequent evolution as long as the scales remain in the large-scale limit, and thus this may lead to an observable trace in the cosmic microwave background radiation.
String or $M$-theory has been received much attention as the best candidate for the unified theory of all fundamental forces [1]. In view of that its typical energy scale is too high to be probed by laboratory experiments\(^1\), cosmology can be one of the best testing grounds for string or $M$-theory. In this regard, it is important to search for a realistic inflation model within the framework of string or $M$-theory. The next step would be to find out whether this inflation model allows a successful structure formation. As is well known, due to inflation microscopic quantum fluctuations can be magnified to macroscopic classical structures which can evolve into the large-scale structures we observe today such as the galaxy distribution and the temperature fluctuations in the cosmic microwave background radiation (CMBR). According to this paradigm, the amplitude and spectrum of the large angular scale fluctuations in the CMBR can work as a window to the early universe, and particularly to the inflation era. Such studies in the context of low-energy effective action of string theories have been done before [3], some of which include the effects of higher order corrections in the expansion in either the inverse string tension or the string coupling [4,5].

One of few model-independent predictions of string or $M$-theory is the existence of axions which couple to $F\tilde{F} = \eta^{abcd} F_{ab} F_{cd}$ and/or to $R\tilde{R} \equiv \eta^{abcd} R_{ab}^{\quad ef} R_{cdef}$ [6,7] where $\eta^{abcd}$ is a (totally antisymmetric) Levi-Civita tensor density. It is rather easy to see that for spatially homogeneous and isotropic background the axion coupling to $F\tilde{F}$ affects neither the evolution of background nor the evolution of perturbations in linear approximation [8]. The axion coupling to $R\tilde{R}$ does not affect the evolution of background also, however it can affect the evolution of perturbations in the early universe [9].

Compared to the Ricci scalar curvature term, $R\tilde{R}$ is higher order in the dimensionful gravitational coupling $\kappa^2 = 8\pi G_N$. Although suppressed by $\kappa^2$, it may lead to sizable effects in some string inflation models (pre-big-bang) which encounter a curvature singularity when

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\(^1\)Recently, it has been noted that some string theories allow the Kaluza-Klein scale or the string scale to be far below the Planck scale, even as low as TeV [2].
the cosmological evolution is governed by the lowest order effective action [3]. In such inflation models, it is expected [5] that higher dimensional terms of curvature tensor regulate the curvature singularity to the period of large but finite curvature $\sim 1/\kappa^2$ which would smoothly evolve into the standard radiation dominated flat universe. Clearly in the high curvature period, higher dimensional terms of curvature tensor such as the axion coupling to $\tilde{R}\tilde{R}$ and/or the moduli coupling to the Gauss-Bonnet combination [4] can be important as much as the Ricci scalar term [10–13].

Recently, authors of [9] considered the effect of axion coupled $\tilde{R}\tilde{R}$ term on gravitational waves. They derived the gravitational wave equation in Minkowsky background and showed that $\tilde{R}\tilde{R}$ coupling term leads to asymmetric equations for the left and right handed polarization states of gravitational waves. In this paper, we wish to generalize the study of [9] by discussing the effects of the axion coupling on the evolution of cosmological structures of all types in the context of evolving cosmological background. It turns out that the evolutions of scalar- and vector-type structures are not affected by the axion coupling. However, as noted in [9] it causes an asymmetric evolution of the two polarization states of tensor-type perturbation. This may lead to a sizable polarization asymmetry in tensor-type perturbation if inflation involves a period during which the axion coupling is important, such as the high curvature period in pre-big-bang model. Moreover, in the large-scale limit, the amplitudes of both polarization states are seperately conserved as can be seen in Eq. (18). Thus once the polarization asymmetry in tensor-type perturbation was generated during inflation, it will be preserved over the subsequent evolution stage as long as the scale remains in the large-scale limit. This then may lead to an observable trace in the large angular scale polarization asymmetry in the CMBR [9].

Our starting point is an effective action including the axion coupling to $\tilde{R}\tilde{R}$, which may correspond to the low energy effective action of string or $M$-theory [1]:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R - \frac{1}{2} \omega(\phi) \phi^c \phi_c - V(\phi) + \frac{1}{8} \nu(\phi) \tilde{R}\tilde{R} + L_m \right],$$

where $\kappa^2 = 8\pi G_N$ is the reduced Newton constant, $R$ is the scalar curvature, $V(\phi)$, $\omega(\phi)$
and $\nu(\phi)$ are some generic functions of the axion field $\phi$, $R \tilde{R} \equiv \eta^{abcd} R_{ab} \epsilon^e f R_{ef}$, and $L_m$ is the Lagrangian including the fields other than the axion field $\phi$. Here we are interested in how the evolution of axion background affects the evolution of cosmic structures through the coupling to $R \tilde{R}$, and thus other axion couplings, e.g. the axion coupling to moduli, are ignored for the sake of simplicity. Generically string or $M$-theory predicts many axions [6,7], and then $\phi$ may be identified as the axion combination which couples to $R \tilde{R}$. The gravitational field equation is given by

$$R_{ab} - \frac{1}{2} R g_{ab} = \kappa^2 \left[ \omega(\phi) (\phi_a \phi_b - \frac{1}{2} \phi \phi_{abc} g_{ab}) - V(\phi) g_{ab} + \tilde{T}_{ab} + T^{(m)}_{ab} \right],$$

where $X_a$ and $X_{\alpha}$ denote the normal and covariant derivatives of $X$ respectively, $X_{(ab)} \equiv \frac{1}{2} (X_{ab} + X_{ba})$, and $T^{(m)}_{ab}$ is the additional energy-momentum tensor arising from $L_m$ which would include the contribution from cosmic fluid.

As the spacetime metric, we consider a spatially flat, homogeneous, and isotropic background including the most general form of space-time dependent perturbations:

$$ds^2 = -a^2 (1 + 2\alpha) d\eta^2 - a^2 (\beta_{\alpha} + B_{\alpha}) d\eta dx^\alpha$$

$$+ a^2 \left[ \delta_{\alpha\beta} (1 + 2\varphi) + 2\gamma_{\alpha\beta} + 2C_{\alpha\beta} + 2C_{\alpha\beta} \right] dx^\alpha dx^\beta,$$

where $a(t)$ is the cosmic scale factor with $dt \equiv a d\eta$. Here $\alpha(x, t)$, $\beta(x, t)$, $\varphi(x, t)$ and $\gamma(x, t)$ characterize the scalar-type perturbations, $B_{\alpha}(x, t)$ and $C_{\alpha}(x, t)$ are the transverse vector-type perturbation, and finally $C_{\alpha\beta}(x, t)$ stands for the tracefree, transverse tensor-type perturbation. The spatial indices are based on the metric $\delta_{\alpha\beta}$. We also decompose the energy-momentum tensor and the axion field as

$$T^a_b(x, t) = \tilde{T}^a_b(t) + \delta T^a_b(x, t), \quad \phi(x, t) = \bar{\phi}(t) + \delta \phi(x, t).$$

Here we use the effective action written in the Einstein frame. The axion-dependence of the Kähler metric $\omega$ and the potential $V$ can arise from nonperturbative effects in string theory.
In the following, the overbar in background configuration will be omitted unless necessary. Since the three types of perturbations decouple from each other due to the symmetry of the background and also the assumed linearity of the structures, we can handle them individually.

It is well known that $\nu(\phi)R\tilde{R}$ does not affect the equation for background [5]. After some calculation we can show that for the scalar-type perturbation we have $\tilde{T}_{ab} = 0$, and as a result the axion coupling does not affect the evolution of the scalar-type perturbation as well. This can be explained by that we cannot form a scalar ($\tilde{T}_{00}$) or a vector ($\tilde{T}_{0\alpha}$) or a symmetric tensor ($\tilde{T}_{\alpha\beta}$) which contains $\eta^{abcd}$ with the derivatives of scalar and $\delta_{\alpha\beta}$ only. Thus the evolution of scalar-type perturbation is the same as that in the case without the axion coupling which has been studied in [13].

We have nontrivial contributions from the axion coupling $\nu(\phi)R\tilde{R}$ only in the equations for vector- and tensor-type perturbations. (For explicit calculations, Appendices of [14,15] will be useful.) For the vector-type perturbation, Eq. (2) leads to

$$\frac{k^2}{2a^3}(a\Psi_\alpha - \kappa^2 \dot{\nu} \epsilon_\alpha \gamma^\delta \Psi_{\gamma\delta}) = \kappa^2 \delta T^{(m)0}_\alpha, \quad (5)$$

$$\frac{1}{a} \frac{\partial}{\partial t} \left[ a(a\Psi_{(\alpha} - \kappa^2 \dot{\nu} \epsilon_{\gamma\delta} \epsilon_{(\alpha} \gamma^\delta)_{,\beta}) \right] = \kappa^2 \delta T^{(m)}_{\alpha\beta}. \quad (6)$$

where we have introduced

$$\epsilon^{\alpha\beta\gamma} \equiv a^4 \eta^{0\alpha\beta\gamma}, \quad \Psi_\alpha \equiv B_\alpha + a\dot{C}_\alpha \quad (7)$$

with the overdot denoting the derivative w.r.t. $t$. When expressed in terms of the following notation based on the vector-type harmonic function $Y^{(v)}_\alpha$ with $Y^{(v)}_{\alpha\beta} \equiv -k^{-1}Y_{(\alpha,\beta)}^{(v)}$ [15],

$$\delta T^{(m)0}_\alpha \equiv (\mu + p)\nu_\omega Y^{(v)}_\alpha, \quad \delta T^{(m)}_{\alpha\beta} \equiv \pi^{(v)} Y^{(v)}_{\alpha\beta},$$

Eqs. (5) and (6) lead to

$$\frac{1}{a^4} \frac{\partial}{\partial t} \left[ a^4 (\mu + p) \times \nu_\omega \right] = -\frac{k}{2a} \pi^{(v)}, \quad (9)$$

where $\nu_\omega$ is related to the rotational velocity of vector-type fluid perturbations and $\pi^{(v)}$ is the anisotropic stress of the fluid which can work as the sink or source of the rotation [15]. If we
ignore the anisotropic stress, the angular-momentum of the fluid whose energy momentum tensor is given by $T^{(m)}_{ab}$ is conserved as

$$a^4 (\mu + p) \times v_\omega(x, t) \sim L(x). \quad (10)$$

Therefore, the presence of the axion coupling does not affect the rotational type perturbation of the fluid component which is described again by the angular momentum conservation, although the evolution of the associated metric components $\Psi_\alpha$ is affected by the axion coupling as is apparent in Eq. (5).

In fact, it is the tensor-type perturbation that may receive an observable impact from the axion coupling. Eq. (2) gives the following equation for the evolution of tensor-type perturbation:

$$D_{\alpha\beta} - \frac{2\kappa^2}{a} \epsilon_{(\alpha}^{\gamma\delta}[\dot{(\nu - H\dot{\nu})}\dot{C}_{\beta)\gamma} + \dot{\nu} D_{\beta)\gamma}]_\delta = \kappa^2 \delta T_{\alpha\beta}^{(m)}, \quad (11)$$

where

$$D_{\alpha\beta} \equiv \ddot{C}_{\alpha\beta} + 3H\dot{C}_{\alpha\beta} - \frac{1}{a^2} \Delta C_{\alpha\beta}. \quad (12)$$

(Here $\Delta$ denotes the spatial Laplacian.) Similar equation of gravitational wave in the presence of axion coupling was derived in [9], but only for the Minkowski background. Let us expand the tensor perturbation [16] as

$$C_{\alpha\beta}(x, t) \equiv \sqrt{\text{Vol}} \int \frac{d^3 k}{(2\pi)^3} \sum_\ell e^{(\ell)}_{\alpha\beta}(k) h_{\ell k}(t) e^{ik\cdot x}, \quad (13)$$

where $e^{(\ell)}_{\alpha\beta}$ is the circular polarization tensor ($\ell = L, R$) with the property $ik_{\gamma} \epsilon_{\alpha}^{\gamma\delta} e^{(\ell)}_{\beta\delta} = k\lambda_{\ell} e^{(\ell)}_{\alpha\beta}$ ($\lambda_L = -1$ and $\lambda_R = +1$) where $k \equiv |k|$. Then ignoring the anisotropic stress of the additional fluid, Eq. (11) becomes

$$\frac{1}{a^3(1 + \lambda_{\ell} \kappa^2 \dot{\nu} k/a)} \frac{\partial}{\partial t} \left[ a^3(1 + \lambda_{\ell} \kappa^2 \dot{\nu} k/a) \dot{h}_{\ell k} \right] + \frac{k^2}{a^2} h_{\ell k} = 0. \quad (14)$$

If we set $\psi_{\ell k} \equiv z_{\ell} h_{\ell k}$ with $z_\ell \equiv a\sqrt{1 + \lambda_{\ell} \kappa^2 \dot{\nu} k/a}$, we can easily show

$$\psi''_{\ell k} + \left( k^2 - \frac{z''_{\ell}}{z_\ell} \right) \psi_{\ell k} = 0, \quad (15)$$
where the prime denotes the derivative w.r.t. the conformal time $\eta$.

In many prototype inflation models, $z''_{\ell}/z_{\ell} = n_{\ell}/\eta^2$ with $n_{\ell} = \text{constant}$ provides a good approximation \cite{16}, and then the solution of Eq. (15) is given by

$$h_{\ell k}(t) = \frac{\sqrt{|\eta|}}{a\sqrt{1 + \lambda_{\ell}\kappa^2\dot{\nu}k/a}} \left[ \hat{c}_{1\ell}(k) H_{\nu_{\ell}}^{(1)}(k|\eta|) + \hat{c}_{2\ell}(k) H_{\nu_{\ell}}^{(2)}(k|\eta|) \right], \quad (16)$$

where $H_{\nu_{\ell}}^{(1)}$ and $H_{\nu_{\ell}}^{(2)}$ are Hankel functions of the first and second kinds with $\nu_{\ell} \equiv \sqrt{n_{\ell} + 1}/4$. Even for generic form of $z_{\ell}$, we can still derive the asymptotic solution of Eq. (15). In the small-scale limit where $k^2$ term dominates, we have a general solution given by

$$h_{\ell k}(t) = \frac{1}{a\sqrt{1 + \kappa^2\dot{\nu}k/a}} \left[ c_{1\ell}(k)e^{ik\eta} + c_{2\ell}(k)e^{-ik\eta} \right], \quad (17)$$

where $c_{1\ell}$ and $c_{2\ell}$ are integration constants of the left and right travelling waves. In the opposite limit of negligible $k^2$ term, which we call the large-scale limit, we have a general solution of the form

$$h_{\ell k}(t) = C_{\ell}(k) - D_{\ell}(k) \int_0^t \frac{dt}{a^3(1 + \lambda_{\ell}\kappa^2\dot{\nu}k/a)}, \quad (18)$$

where $C_{\ell}$ and $D_{\ell}$ are the coefficients of relatively growing and decaying modes, respectively. Ignoring the transient solution, the above solution manifestly shows that the amplitudes of both polarization states of the tensor-type perturbation are conserved in the large scale limit. Notice that the asymptotic solutions in Eqs. (17) and (18) are valid for generic forms of time varying $V(\phi)$, $\omega(\phi)$, and $\nu(\phi)$.

Remarkably, the conservation properties in Eqs. (10) and (18) are valid for generic forms of the axion potential $V(\phi)$, the axion Kähler metric $\omega(\phi)$, and also the axion coupling $\nu(\phi)$ to $R\tilde{R}$. The large-scale conservation property of tensor-type perturbation is particularly relevant in inflationary scenario. During the transition period from inflation to ordinary radiation era, the observationally relevant scales stay in the superhorizon size for which the large-scale condition may apply. As long as the scale remains in the large-scale limit, the conservation property of tensor type perturbation is valid independently of how the axion
field (and also other scalar fields which may have non-minimal coupling to gravity) is settled down to its vacuum expectation value during the transition period.

Since it distinguishes the states with different polarization as in Eq. (14), the axion coupling generically leads to polarization asymmetry in tensor-type perturbations. In particular, if inflation involves a period in which \( \dot{\nu} k/a \sim 1/\kappa^2 \), the resulting polarization asymmetry can be sizable. Of course, this would not take place if the cosmic evolution during the whole inflation period is determined by the dynamics at energy scales significantly below the Planck scale \( M_P \equiv 1/\sqrt{\kappa} \). However this can be a possibility in string cosmology scenario which encounters a curvature singularity during inflation at lowest order approximation. In such inflation models, higher dimensional terms are expected to regulate the curvature singularity to a value comparable to \( 1/\kappa^2 \), and then the axion coupling to \( R\tilde{R} \) can give a non-negligible effect although it is suppressed by \( \kappa^2 \). The evolution of tensor-type perturbation after inflation is mainly described by the large-scale solution (18) with conserved amplitudes. Thus once the polarization asymmetry were generated during inflation, it will be preserved over the subsequent evolution as long as the scale remains in the large-scale limit. This then may lead to an observable trace in the large angular scale polarization asymmetry in the CMBR [9]. For a more quantitative study of this problem, we will need specific inflation model for which our results can be easily applied.

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REFERENCES


