We consider the detection possibilities of isocurvature fluctuations in the future CMB satellite experiments MAP and Planck for different cosmological reference models. We present a simultaneous 10 parameter fit (8 for the case of open model) to determine the correlations between the cosmological parameters, including isocurvature cold dark matter contribution to the anisotropy. Assuming that polarization information can be fully exploited, we find that an isocurvature perturbation can be detected by the Planck Surveyor if the ratio of the initial isocurvature and adiabatic perturbation amplitude is larger than 0.07. In the absence of polarization data, the signal from isocurvature perturbations can be confused with tensor perturbations or early reionization effects, and the limit is larger by almost an order of magnitude.

PACS numbers: 98.70.Vc, 98.80.Cq

I. INTRODUCTION

A second generation of satellite experiments, MAP [1] and Planck [2], will soon provide highly detailed temperature maps of the cosmic microwave background (CMB). From the measured spectrum of temperature fluctuations one may infer the properties of the cosmic fluid, such as its density or the rate by which it comoves with the expansion of the Universe, and thus determine most cosmological parameters to an accuracy of a few percent [3–5]. In addition, the spectrum of temperature fluctuations allows us to study and constrain physics at very small distance scales, corresponding to energies much higher than can be achieved in any particle accelerator in the foreseeable future. This is so because the features on the temperature map depend on the nature of the primordial density fluctuation.

Cosmic inflation is currently the most popular explanation for the origin of the primordial perturbations, and it typically predicts a near scale invariant spectrum with Gaussian fluctuations. The perturbations are adiabatic with the number density proportional to entropy density so that \( \delta(n/s) = 0 \). This is so because the quantum fluctuations of the field responsible for inflation, called the inflaton, give directly rise to perturbations in the energy density of the inflaton field.

However, the inflaton may not be the only field which is subject to quantum fluctuations during inflation. In fact, any effectively massless scalar field will fluctuate by virtue of the nearly constant energy density of the inflation era with a root mean square amplitude \( H/(2\pi) \), where \( H \) is the Hubble parameter during inflation. Such fluctuations will contribute to the perturbations in the microwave background. They may be adiabatic, in which case it is difficult to entangle their effect from the inflation fluctuations. The other possibility is isocurvature fluctuations, which are fluctuations in the number rather than the energy density so that \( \delta(n/s) \neq 0 \). Examples of particle physics models giving rise to isocurvature fluctuations include axions [6,7], inflation with more than one inflaton field [8], and supersymmetric theories with flat directions in the potential [9].

Isocurvature perturbations [10,11] in a given particle species have \( \delta \rho = 0 \) with the overdensities balanced by perturbations in other particle species, such as radiation. At the last scattering surface (LSS) the compensation for the isocurvature perturbations can be maintained only for scales larger than the horizon, effectively generating extra power to photon perturbations at small angular multipoles \( l \). As a consequence, the spectrum of isocurvature perturbations differs a great deal from adiabatic perturbations, and a purely isocurvature cold dark matter (CDM) perturbation spectrum is in fact already ruled out [12] on the basis of COBE normalization and \( \sigma_8 \), the amplitude of the rms mass fluctuations in an 8h\(^{-1}\) Mpc\(^{-1}\) sphere. (There are suggestions that a decaying CDM model could sustain purely isocurvature perturbations [13]; however, such models are not motivated from particle physics point of view). A small isocurvature contribution might, however, be beneficial in the sense that it could help to improve the fit to the power spectrum in \( \Omega_0 = 1 \) CDM models with a cosmological constant [14,6].

The simplest possibility is that there is a single CDM which has both adiabatic and isocurvature perturbations. An example of such a situation is obtained in supersymmetric models with the so-called Affleck-Dine (AD) fields [15], which also provide a basis for baryogenesis. The AD fields are superpositions of squark and slepton fields corresponding to the flat directions. During inflation they will fluctuate along the flat directions forming a condensate. They are complex fields and, in the currently favoured D-term inflation models [16], is effectively massless during inflation. The condensate does not correspond to the state of lowest energy but fragments into non-topological solitons [17], which carry baryonic (and sometimes leptonic) number [18] and are called B-balls.
The fluctuations in the AD field are both adiabatic and isocurvature [9], and will be inherited by the B-balls, which are either stable or decay later into lightest supersymmetric particles, which form the CDM. In the latter case there will be a lower bound on the amplitude of the isocurvature perturbation.

Adiabatic perturbations [19–21] are characterized by the gauge-invariant quantity $\zeta$, which equals $\delta \rho/\rho + p$ when the perturbation exits or enters the horizon. Outside the horizon, $\zeta$ does not change with time. Isocurvature perturbations [10,20,21] are characterized by the gauge-invariant entropy perturbation $S \equiv \delta (n_e/s_e) / \delta_e / (3/4) \delta_\gamma$, where $n_e$ is the number density of CDM, $s_\gamma$ is the entropy density associated with photons, and $\delta_e$ and $\delta_\gamma$ are the relative overdensities in the CDM and photon energy densities. During radiation domination $S \sim \delta_e \equiv \delta n_e / n_e$. Outside the horizon, $S$ does not change with time. “Initial” scale-free ($n = 1$) adiabatic and isocurvature perturbation spectra are given by

$$P_c(k) \equiv \langle |\delta_k|^2 \rangle = Ak^{-4} = Ak^{-3},$$

$$P_S(k) \equiv \langle |S_k|^2 \rangle = Bk^{-4} = Bk^{-3}. \quad (1)$$

Following these remarks, we assume in this paper that there is just a single species of CDM, and that it has both adiabatic and isocurvature fluctuations. As is natural when both fluctuations have their origin in inflation, we also assume that their power spectra are similar. The goal of the paper is then to study how well parameter error estimates, and degeneracies with respect to some cosmological parameters; different effects causing similar features in the power spectrum. Also the information from the large angular scales is severely limited by cosmic variance.

Important additional information can be obtained by the measurement of the polarization of the CMB [23–26]. Since polarization of the CMB is caused by scattering only, polarization carries information about the LSS only (including late scattering due to reionization), eliminating confusion with line-of-sight effects. Polarization is especially useful in separating out the effects of tensor fluctuations, isocurvature fluctuations, and reionization, all of which produce excess power at large scales.

Unfortunately the polarization is expected to be of rather small amplitude, at least an order of magnitude below the anisotropy signal, making it difficult to detect [24]. The contamination with the foreground may also be a worse problem than for the temperature anisotropy, since there are polarized foreground sources, e.g., galactic synchrotron emission. The MAP satellite is expected to provide a definite detection of CMB polarization. The higher sensitivity of the Planck Surveyor will then help in getting a reasonable power spectrum for the polarization.

The polarization pattern on the sky can be separated into two components, the E-mode and the B-mode [23]. Thus an experiment which also measures the polarization, will produce three maps: temperature, E-mode polarization, and B-mode polarization. The B-mode polarization of the CMB is uncorrelated with the temperature or the E-mode polarization. Thus the statistical information is carried by three angular power spectra

$$C_{TI} \equiv \langle \bar{a}_{lm}^T \bar{a}_{lm}^T \rangle,$$

$$C_{EI} \equiv \langle \bar{a}_{lm}^E \bar{a}_{lm}^E \rangle,$$

$$C_{BI} \equiv \langle \bar{a}_{lm}^B \bar{a}_{lm}^B \rangle, \quad (3)$$

and one correlation function

$$C_{CI} \equiv \langle \bar{a}_{lm}^T \bar{a}_{lm}^E \rangle, \quad (4)$$

giving the correlation between the temperature and the E-mode polarization [23–26]. The different spectra are demonstrated in Fig. 1.

These four spectra can be calculated for different values of the cosmological parameters $s_i$, and compared to data from CMB experiments. This way the true values of the cosmological parameters can be determined.

Assuming the actual values of the parameters are $s_i$, the information obtained on these parameters by observation is described by the Fisher information matrix [3,28,23,4,5,27]

$$F_{ij} = \sum_l \sum_{X,Y} \frac{\partial C_{XI}}{\partial s_l} C_{ov}^{-1}(C_{XI}, C_{YJ}) \frac{\partial C_{YJ}}{\partial s_j}, \quad (5)$$

where $C_{XI} = C_{TI}, C_{EI}, C_{BI}$, or $C_{CI}$.
The angular power spectra for SCDM (see Table I) adiabatic (a) and isocurvature (b) perturbations. These are 

\[
\text{Cov}(C_{BI}C_{BI}) = N_l \left( C_{BI} + \left[ \sum_c \frac{w_c^P B_{l,c}^2}{\sigma_{T,c}^2} \right]^{-1} \right)^2,
\]

\[
\text{Cov}(C_{C1}C_{C1}) = N_l \left[ C_{C1}^2 + \left( C_{T1} + \left[ \sum_c \frac{w_c^P B_{l,c}^2}{\sigma_{T,c}^2} \right]^{-1} \right) \times \left( C_{EL} + \left[ \sum_c \frac{w_c^P B_{l,c}^2}{\sigma_{T,c}^2} \right]^{-1} \right) \right],
\]

\[
\text{Cov}(C_{C1}C_{C1}) = N_l C_{C1}^2,
\]

\[
\text{Cov}(C_{T1}C_{C1}) = N_l C_{C1} \left( C_{T1} + \left[ \sum_c \frac{w_c^P B_{l,c}^2}{\sigma_{T,c}^2} \right]^{-1} \right),
\]

\[
\text{Cov}(C_{EL}C_{EL}) = N_l C_{C1} \left( C_{EL} + \left[ \sum_c \frac{w_c^P B_{l,c}^2}{\sigma_{T,c}^2} \right]^{-1} \right),
\]

depend on the sensitivity and angular resolution of the instrument with

\[
w_c^P = \frac{1}{\sigma_{T,c}^2 \theta_{\text{fwhm},c}^2},
\]

\[
w_c^P = \frac{1}{\sigma_{T,c}^2 \theta_{\text{fwhm},c}^2},
\]

and

\[
B_{l,c}^2 = e^{-l(l+1)\theta_{\text{fwhm},c}^2/8 \ln 2}
\]

where \( \sigma_c \) is the pixel noise (sensitivity) and \( \theta_{\text{fwhm},c} \) is the beamwidth (full width at half maximum) of instrument channel \( c \).

The prefactor is given by

\[
N_l = \frac{2}{(2l+1)f_{\text{sky}}},
\]

where \( f_{\text{sky}} \) takes into account the fact that the whole sky cannot be mapped because of foreground contamination. We adopt \( f_{\text{sky}} = 0.65 \), i.e., assume that foreground can be completely removed from 65% of the sky and the rest of the sky is unusable.

The Planck Surveyor carries two instruments, HFI and LFI. For MAP and LFI we use the three highest frequency channels, for HFI the three lowest frequency channels for our estimates (see Table I).

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Frequency</th>
<th>( \theta_{\text{fwhm}} )</th>
<th>( \sigma_T )</th>
<th>( \sigma_P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAP</td>
<td>40 GHz</td>
<td>0.47°</td>
<td>35 ( \mu )K</td>
<td>( \sqrt{2} \times 35 \mu )K</td>
</tr>
<tr>
<td></td>
<td>60 GHz</td>
<td>0.35°</td>
<td>35 ( \mu )K</td>
<td>( \sqrt{2} \times 35 \mu )K</td>
</tr>
<tr>
<td></td>
<td>90 GHz</td>
<td>0.21°</td>
<td>35 ( \mu )K</td>
<td>( \sqrt{2} \times 35 \mu )K</td>
</tr>
<tr>
<td>LFI</td>
<td>44 GHz</td>
<td>23°</td>
<td>2.4 \times 10^{-6}</td>
<td>( \sqrt{2} \times 2.4 \times 10^{-6} )</td>
</tr>
<tr>
<td></td>
<td>70 GHz</td>
<td>14°</td>
<td>3.6 \times 10^{-6}</td>
<td>( \sqrt{2} \times 3.6 \times 10^{-6} )</td>
</tr>
<tr>
<td></td>
<td>100 GHz</td>
<td>10°</td>
<td>4.3 \times 10^{-6}</td>
<td>( \sqrt{2} \times 4.3 \times 10^{-6} )</td>
</tr>
<tr>
<td></td>
<td>100 GHz</td>
<td>10°</td>
<td>1.7 \times 10^{-6}</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>143 GHz</td>
<td>8.0°</td>
<td>2.0 \times 10^{-6}</td>
<td>3.7 \times 10^{-6}</td>
</tr>
<tr>
<td></td>
<td>217 GHz</td>
<td>5.5'</td>
<td>4.3 \times 10^{-6}</td>
<td>8.9 \times 10^{-6}</td>
</tr>
</tbody>
</table>

TABLE I. The beamwidths and sensitivities of the satellite experiments [1,2].
The 1-σ error on parameter \( s_i \) is
\[
\Delta s_i = \sqrt{(F^{-1})_{ii}},
\]
when all parameters are estimated from the same data. (If all the other parameters were somehow known a priori, the error would be \( \Delta s_i = (F_{ii})^{-\frac{1}{2}} \). This is a lower limit for the error; but when the error is small it is usually a good approximation for the error [28,5].

### III. THE PARAMETERS AND MODELS

We consider a simultaneous fit of 10 parameters, \( C_2, \Omega_0 \equiv \Omega_\Lambda + \Omega_m, \Omega_\Lambda - \Omega_m, \Omega_b h^2, \Omega_{\text{HDM}}, H_0, \tau, n_\gamma, r, \text{ and } \alpha \). Here \( \tau \) is the optical depth to LSS and \( r \equiv C_2^T/C_2^S \) is the tensor/scalar ratio. We assume three matter components, \( \Omega_m = \Omega_{\text{CDM}} + \Omega_{\text{HDM}} + \Omega_b \). Note that we have defined \( \Omega_b \) to include the vacuum energy; it is the parameter which determines the curvature of space. We assume \( T_0 = 2.728 \, \text{K}, Y_{\text{He}} = 0.24, \) and \( N_p = 3 \). For hot dark matter we assume all 3 neutrinos have equal masses.

The isocurvature contribution to anisotropy is given by the parameter \( \alpha \), which gives the fraction of \( C_2^S \) due to isocurvature perturbations. Thus
\[
\alpha \equiv C_2^\text{iso}/(C_2^\text{iso} + C_2^\text{ad}).
\]

Our \( \alpha \) is the same as \( 1 - \alpha \) of Stompor et al [14]. The isocurvature fluctuations are assumed to be primordial density fluctuations in CDM, initially balanced by radiation+baryons+neutrinos to make the total energy density homogeneous. The isocurvature spectral index is assumed to be the same as for adiabatic fluctuations; in principle, they could be different, but if both fluctuations originate from inflation, and in particular from massless field fluctuations during inflation, then their power spectra are guaranteed to be equal.

The large-scale anisotropy due to isocurvature perturbations is [21]
\[
\delta T/T = -\frac{1}{3} S + \frac{1}{3} \Phi,
\]
where
\[
\Phi = -\frac{1}{5} S
\]
is the Newtonian gravitational potential [29,20,22], whereas for adiabatic perturbations we have just
\[
\frac{\delta T}{T} = \frac{1}{3} \Phi,
\]
where the potential is related to the gauge-invariant \( \zeta \) by
\[
\Phi = -\frac{3}{2} \zeta \tag{15}
\]

Thus the “observational” isocurvature parameter \( \alpha \) is related to the amplitudes of the initial isocurvature and adiabatic spectra by
\[
\frac{\alpha}{1 - \alpha} = \frac{(-\delta T/T)^\text{iso}}{(-\delta T/T)^\text{ad}} = \left(\frac{4S^2}{5\zeta^2}\right) = \frac{16}{25} \left(\frac{B}{A}\right). \tag{16}
\]

In a given particle physics model \( A \) and \( B \) are known in terms of the model parameters, and Eq. (17) tells how these parameters are related to the observed isocurvature parameter \( \alpha \).

We have chosen to consider three different reference cosmological models (see Table II), which are the same as discussed by Wang et al. [30]: the standard CDM model with \( \Omega_\Lambda = 1 \), a CDM model with a cosmological constant \( \Omega_\Lambda = 0.7 \) and \( \Omega_\Lambda = 1 \), and an open CDM model with \( \Omega_\Lambda = 0.4 \).

To find the derivatives \( \partial C_{Xl}/\partial s_i \), we used CMBFAST [31] to run variations around these three reference models, with steps \( \Delta s_i \) or \( \Delta S_i/s_i \) between 0.01 and 0.05.

We COBE-normalized [32] \( C_2 \) for the three reference models. Note that as \( C_2 \) is one of the parameters to be determined from the data, we do not COBE-renormalize it for the variations around these reference models.

Wang et al. [30] considered 7-parameter fits. Our additional parameters are \( \Omega_{\text{HDM}}, r, \) and \( \alpha \). For OCDM we considered 8-parameter fits only, keeping \( r \equiv 0 \) and \( \Omega_{\text{HDM}} \equiv 0 \), since CMBFAST does not allow nonzero values for these parameters in open models.

### IV. RESULTS

We are interested in how accurately the isocurvature contribution can be determined and what is the smallest contribution that can still be detected. In the following tables (III, IV, and V) we give the 1-σ error on the isocurvature parameter \( \alpha \) for the three experiments, MAP, Planck LFI, and Planck HFI, and an ideal experiment with zero noise and infinitely fine resolution. Full sky coverage would reduce these errors by a factor of \( \sqrt{0.65} \).

We see that the addition of the two parameters, \( \Omega_{\text{HDM}} \) and \( r \), increased the error in \( \alpha \) determination significantly for the case without polarization data, but had only a small effect for the case with polarization. This is because the temperature spectra from tensor and isocurvature fluctuations resemble each other, resulting in a degeneracy between the parameters \( r \) and \( \alpha \). (Adding \( \Omega_{\text{HDM}} \) as a free parameter changes \( \Delta \alpha \) at most by 10%).
Because only tensor fluctuations have B-mode polarization, polarization measurements break this degeneracy.

Note that the tensor and isocurvature parameters, $r$ and $\alpha$ are non-negative by definition. It is quite likely that their true values are smaller than the errors in their determination, so that we will only obtain an upper limit. For the case without polarization information, there will be a degeneracy between these two parameters, such that their sum can be obtained with a higher accuracy than the parameters themselves (see Fig. 2). Since the upper limit to the sum is also an upper limit to the individual parameters, we can then obtain a tighter upper limit to $\alpha$ than the $\Delta \alpha$ given in Table III. This upper limit is essentially the $\Delta \alpha$ given in Table IV, where we have made the restriction $r \equiv 0$.

For MAP, there are also other significant degeneracies among the 10 parameters considered here. For Planck without polarization, the only other important degeneracy is with the optical depth $\tau$ due to reionization. Polarization breaks this degeneracy, since early reionization produces a polarization signal at large angular scales which cannot be mimicked by other parameter combinations [33]. For Planck with polarization there are no significant degeneracies between $\alpha$ and the other parameters (except, of course, a degeneracy between $C_2$ and $\alpha$), and so the errors would not become much smaller even if all the other parameters were assumed known (see Table V).

<table>
<thead>
<tr>
<th></th>
<th>MAP</th>
<th>LFI</th>
<th>HFI</th>
<th>ideal</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCDM</td>
<td>0.37</td>
<td>0.26</td>
<td>0.23</td>
<td>0.14</td>
</tr>
<tr>
<td>ΛCDM</td>
<td>0.38</td>
<td>0.23</td>
<td>0.20</td>
<td>0.17</td>
</tr>
<tr>
<td>OCDM</td>
<td>0.27</td>
<td>0.13</td>
<td>0.064</td>
<td>0.053</td>
</tr>
<tr>
<td></td>
<td>Without polarization</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SCDM</td>
<td>0.23</td>
<td>0.058</td>
<td>0.038</td>
<td>0.015</td>
</tr>
<tr>
<td>ΛCDM</td>
<td>0.21</td>
<td>0.062</td>
<td>0.044</td>
<td>0.018</td>
</tr>
<tr>
<td>OCDM</td>
<td>0.14</td>
<td>0.046</td>
<td>0.033</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>With polarization</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE III. $\Delta \alpha$ from 10-parameter fits.

<table>
<thead>
<tr>
<th></th>
<th>MAP</th>
<th>LFI</th>
<th>HFI</th>
<th>ideal</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCDM</td>
<td>0.21</td>
<td>0.056</td>
<td>0.037</td>
<td>0.015</td>
</tr>
<tr>
<td>ΛCDM</td>
<td>0.16</td>
<td>0.062</td>
<td>0.044</td>
<td>0.018</td>
</tr>
<tr>
<td>OCDM</td>
<td>0.14</td>
<td>0.046</td>
<td>0.033</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>Without polarization</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SCDM</td>
<td>0.053</td>
<td>0.049</td>
<td>0.047</td>
<td>0.046</td>
</tr>
<tr>
<td>ΛCDM</td>
<td>0.055</td>
<td>0.052</td>
<td>0.051</td>
<td>0.050</td>
</tr>
<tr>
<td>OCDM</td>
<td>0.041</td>
<td>0.038</td>
<td>0.036</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>With polarization</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE IV. $\Delta \alpha$ from 8-parameter fits ($\Omega_{\rm HDM} \equiv 0$, $r \equiv 0$).

<table>
<thead>
<tr>
<th></th>
<th>MAP</th>
<th>LFI</th>
<th>HFI</th>
<th>ideal</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCDM</td>
<td>0.053</td>
<td>0.039</td>
<td>0.030</td>
<td>0.013</td>
</tr>
<tr>
<td>ΛCDM</td>
<td>0.054</td>
<td>0.044</td>
<td>0.036</td>
<td>0.016</td>
</tr>
<tr>
<td>OCDM</td>
<td>0.041</td>
<td>0.030</td>
<td>0.026</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>Without polarization</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE V. $\Delta \alpha$ from 2-parameter fits ($C_2$ and $\alpha$). This table is shown to illustrate, by comparison to Table IV, the effect of degeneracies with other parameters. Because the best determinations of many of the other parameters have to come from the same CMB data, $\alpha$ cannot in reality be determined with this accuracy.
V. DISCUSSION

From Tables III and IV we find that the (1-σ) detection limit of isocurvature fluctuations is \( \alpha \sim 0.04 \) for the Planck Surveyor. This corresponds to a ratio \( B/A \sim 0.07 \) between the amplitudes of the initial isocurvature and adiabatic perturbation spectra. This is at the level predicted by certain B-ball models [9], which thus could be tested by Planck. It is likely that other particle physics models can also be constrained in a significant way on the basis (or the absence) of isocurvature fluctuation in CMB. However, to reach this sensitivity polarization measurements are crucial, as is evident from Tables III and IV.

Without polarization information, the contribution from isocurvature fluctuations to temperature anisotropies could be confused with tensor perturbations or early reionization effects. Polarization measurements break this degeneracy, since these other two effects both have a unique polarization signal.

Whether such sensitivity can be realized is still unclear; the foreground contamination for the polarization spectra may turn out to be a more severe problem than in the case of the temperature angular power spectrum. However, optimizing the polarization information should have a high priority, in particular for Planck; because of its higher sensitivity, it is in this respect that Planck is truly a superior instrument compared with MAP. CMB anisotropy by any experiment.

Finally, the cosmic variance sets an absolute lower theoretical limit, at about \( \alpha \sim 0.01 \), to how small isocurvature fluctuations can be detected from the CMB anisotropy by any experiment.

ACKNOWLEDGEMENTS

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