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MASSES AND INTERNAL STRUCTURE
OF MESONS IN THE STRING QUARK MODEL

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Abstract


The relativistic quantum string quark model, proposed earlier, is applied to all mesons, from pion to $\Upsilon$, lying on the leading Regge trajectories (i.e., to the lowest radial excitations in terms of the potential quark models). The model describes the meson mass spectrum, and comparison with measured meson masses allows one to determine the parameters of the model: current quark masses, universal string tension, and phenomenological constants describing nonstring short-range interaction. The meson Regge trajectories are in general nonlinear; practically linear are only trajectories for light-quark mesons with non-zero lowest spins. The model predicts masses of many new higher-spin mesons. A new $K^*(1^-)$ meson is predicted with mass 1910 MeV. In some cases the masses of new low-spin mesons are predicted by extrapolation of the phenomenological short-range parameters in the quark masses. In this way the model predicts the mass of $\eta(1S)(0^{-})$ to be $9500 \pm 30$ MeV, and the mass of $B_c(0^-)$ to be $6400 \pm 30$ MeV (the potential model predictions are 100 MeV lower). The relativistic wave functions of the composite mesons allow one to calculate the energy and spin structure of mesons. The average quark-spin projections in polarized $\rho$-meson are twice as small as the nonrelativistic quark model predictions. The spin structure of $K^*$ reveals an 80% violation of the flavour $SU(3)$. These results may be relevant to understanding the “spin crises” for nucleons.

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Предложенная ранее релятивистская квантовая струйная кварковая модель применяется ко всем мезонам, от пиона до $\Upsilon$, лежащим на главных реджевских траекториях (т.е. к низшим радиальным возбуждениям в терминах потенциальных кварковых моделей). Модель описывает спектр масс мезонов, и сравнение с экспериментом позволяет определить параметры модели: токовые массы кварков, натяжение струны и феноменологические константы, описывающие неструйное взаимодействие на малых расстояниях. Мезонные реджевские траектории в общем случае оказываются нелинейными, практически линейны лишь траектории с нулевыми нижними спинами для мезонов, состоящих из легких кварков. Модель предсказывает массы многих новых мезонов с вышими спинами. По-видимому, существует новый $K^*(1^-)$ мезон с массой 1910 MeV. В некоторых случаях удается предсказать новые мезоны с малыми спинами, экстраполируя феноменологические параметры как функции масс кварков. Модель предсказывает массу $\eta(1S)(0^{-})$, равную 9500±30 MeV, и массу $B_c(0^-)$, равную 6400±30 MeV (предсказания потенциальных моделей на 100 MeV ниже). Релятивистские волновые функции составных мезонов позволяют вычислить энергетическую и спиновую структуру мезонов. Средние проекции спинов кварков в поляризованном $\rho$-мезоне вдвое меньше, чем в нерелятивистской кварковой модели. В спиновой структуре $K^*$ происходит 80% нарушение $SU(3)$ симметрии арматов. Эти результаты могут помочь понять так называемый “спиновый кризис” для нуклонов.

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1. Introduction

The naive quark model of hadron, attractive in its simplicity, is not so simple at a closer consideration. It is not relativistic since it contains a confinement potential proportional to a space distance. One can introduce a quasipotential dependent on the distance and momentum, which makes the wave equation Lorentz covariant, but then the phenomenological quasipotential is not simple. The model contains constituent quarks which are purely phenomenological notions. Their masses are not fundamental and can vary from mesons to hadrons and even from one meson to another. Their spins are not fundamental either, and the "spin crises" for nucleons suggests that they should be different from 1/2, or the naive quark model is too naive.

In Refs. [1,2] an alternative, a string quark model (SQM) has been proposed which contains neither potential nor constituent quarks. The physical origin of confinement and constituent quarks, the gluon field, is taken into account explicitly, in an approximation of quantum Nambu-Goto string. The string provides a confinement mechanism and, since the string is a physical object with its own energy-momentum and angular momentum, the quarks at the ends of the string are fundamental quarks with current masses and spin 1/2.

The application of SQM in Ref. [1] was confined to a particular type of leading meson Regge trajectories. Here we consider all the four types of them, obtain relativistic wave functions of composite mesons, and calculate the internal (energy and spin ) structure of mesons.

As in Ref. [1], we consider only the simplest string configuration — the rotating straight line, which is responsible for the leading Regge trajectories of mesons. The daughter trajectories (i.e., the higher radial excitations in terms of potential models) correspond to vibrations of the string.

The model is quantized in accord with Poincaré invariance and, due to account of quark spins, contains no tachyons.

The model predicts that the Regge trajectories for light-quark mesons with lowest spin 1 (ρ-type and b1-type) are practically linear. The corresponding trajectories for heavy-
light-quark mesons are not linear, but, to a good approximation, can be represented by straight lines for spins less than 6 by replacing the argument $m^2$ by $(m - m_h)^2$, where $m_h$ is the heavy-quark mass. The slopes of these straight lines are bigger than for the light-quark mesons, and increase with $m_h$, the limit value being twice as big as for the light-quark mesons. The trajectories for heavy quarkonia are essentially nonlinear.

The Regge trajectories with lowest spin 0 ($\pi$-type and $a_0$-type) are always nonlinear in the low-spin region.

The model describes masses of all the mesons, from pion to $\Upsilon$, lying on the leading Regge trajectories. The main parameters of the model, the universal string tension and the current quark masses, have been determined in [1] by comparison with experimental meson masses lying on the $\rho$-type trajectories. So, for each other trajectory (without mixing), we have only one unknown short-range parameter. Experiment suggests that these parameters for the $\pi$-type and the $b_1$-type trajectories are equal. The short-range parameters do not strongly depend on the quark masses, and in some cases can be obtained from known parameters by a safe extrapolation in the quark masses.

As a result, the model predicts masses and other quantum numbers of many high-spin mesons and some low-spin mesons. For instance, the model predicts a new $K^*(1^-)$ meson with mass 1910 MeV (without extrapolation) and the masses of $\eta(1S)(0^{-+})$ and $B_d(0^-)$ to be 9500 ± 30 MeV (bigger than the $\Upsilon$-mass) and 6400 ± 30 MeV, respectively. The corresponding predictions of a potential quark model (PQM) [5] are 100 MeV lower. This number can characterize difference between many SQM and PQM predictions, so that further systematic experimental study of meson spectrum with accuracy capable to distinguish these predictions seems to be important for the understanding of confinement.

The SQM relativistic wave functions of composite mesons allow one to calculate the meson internal structure. The separate string and quark contributions into meson masses are obtained. The average spin projections of $u$- and $d$-quark in polarized $\rho^+$, divided by the same projection of the total meson spin, are found to be 0.22 and 0.23, respectively, i.e., twice as small as the nonrelativistic quark model prediction 0.5.

The corresponding numbers for $u$- and $d$-quark within $K^{*+}$ are 0.22 and 0.42, respectively. This means that the flavour $SU_3$ is violated up to 80% for the spin structure in the relativistic model.

The results on the meson spin structure suggest a new approach to understanding the nucleon spin structure, obtained from polarized deep inelastic lepton-nucleon scattering and extrapolated to low $Q^2$ (the so-called spin crises).

At the same time, the above numbers for the $\rho$ spin structure are different from 0.17, the number corresponding to vanishing quark masses, so that one can hope that future polarization experiments will allow one to estimate the light-quark current masses from experiment.

The outline of the paper is the following. In Sec. 2, the string physics is described in the classical approximation. It follows, of course, from the string equations of Sec. 3, but can be described in familiar terms of the pointlike-particle mechanics, if only few basic properties of the string are taken from the equations. This description shows that the string, presumably realized by the gluon field inside mesons, is quite a new object
from mechanical viewpoint. Sec. 2 also clarifies the origin and properties of the string functions, relevant to the quantum case.

In Sec. 3, the classical and quantum SQM is formulated and the meson wave functions are obtained. To go from the classical SQM to the real one, we take into account quark spins, canonical quantization and nonstring, short-range, quark-antiquark interaction. All these effects are of the same order and all are necessary for consistency of the model. The quark spins are introduced at the classical level with the help of anticommuting variables obeying constraints [3]. We add a special term to the Lagrangian to ensure conservation of the spin constraints, which renders the total SQM Lagrangian supersymmetric. The canonical quantization implies finding out all the constraints between canonical variables, and using a first form method [4] to obtain the Poisson brackets of physical variables. As a result, the meson wave function satisfies two Dirac equations and a spectral condition, into which we introduce a nonstring, short-range contribution. In general, the spectral condition may contain contributions dependent on the meson spin. Since we believe that the long-range contribution is given by the string term, then the additional short-range contribution cannot increase with the meson spin. Experiment suggests that the short-range contribution does not depend on the spin or, for heavy quarkonia, has an additional, decreasing with the spin, term [1]. In this way we have phenomenological short-range parameters which depend on the type of the trajectory (i.e., on the space and charge-conjugation parity of the wave function) and on the quark masses. They obey the chiral symmetry (then the model obeys this symmetry) and, at present, are to be obtained from experiment.

In Sec. 4 and Appendix C, the spectral conditions for different meson wave functions are compared with the experimental meson spectrum, the model parameters are obtained and predictions of masses and other quantum numbers of new mesons are made. The results of SQM are compared with that of a potential quark model [5].

Knowing the parameters of the model, we calculate in Sec. 5 the internal structure of mesons: the average values of string and quark energies, and projections of quark spins and orbital momentum for polarized mesons, as well as average total quark spin and orbital momentum squared, and spin-orbit correlation.

Sec. 6 contains conclusions. Some mathematical and phenomenological details are considered in Appendices A, B and C.

2. Classical string physics

The behaviour of a straight-line Nambu-Goto string, with or without point spinless quarks at the string ends, follows from the Lagrangian of the next Section. This behaviour can be described in terms of the point-particle relativistic mechanics if we take from the Lagrangian three properties of the string. Let the string be in its rest frame, where the string is at rest as a whole, i.e., its 4-momentum is \((m, 0)\). The specific string properties are the following:

I. The internal self-interaction string parameter \(a\), called string tension, can be used as a "rest mass density" of the string.
II. The ends of an open string move with the velocity of light perpendicular to the string direction. The open string rotates in a plane around its center with an angular velocity

$$\omega = \frac{2}{d},$$

(1)

where \(d\) is the string length, Fig. 1a.

![Diagram of string](image)

Fig. 1. A straight-line Nambu-Goto string in its rest frame, open(a) and with current quarks at the ends (b). \(O\) is the rotation center, \(m_1\) and \(m_2\) are the current quark masses and \(d = l_1 + l_2\) is the string length.

III. Point quarks at the ends of a rotating string do not move along the string. The string with quarks rotates in a plane. Its angular velocity and position of the rotation center are determined by equality of the centrifugal force and the string-tension force

$$\frac{m_i \omega^2 l_i}{\sqrt{1 - \omega^2 l_i^2}} = a \sqrt{1 - \omega^2 l_i^2},$$

(2)

Fig. 1b. For zero-mass quarks, Eq. (2) is equivalent to Eq. (1). For heavy quarks \(\omega l_i \ll 1\), and Eq. (2) reduces to

$$m_i \omega^2 l_i = a.$$

(3)

We see that the main peculiarity of the string is that it always rotates, and cannot be stopped. If the quarks at the string ends are heavy and move slowly, so that their velocities \(v_i \rightarrow 0\), then, from Eq. (3), \(l_i = v_i^2 m_i/a \rightarrow 0\), and the string disappears. All the points of the string cannot be at rest, and the notion "rest mass density" is not applicable literally to the string. Property I above is a definition, following from the string Lagrangian. The string dynamics cannot be reduced to the point-particle dynamics, although all other properties of the string can be obtained with its help.

To make illustrative estimates, we shall use the experimental value of \(a\)

$$a = 0.176 \text{ GeV}^2 \approx 1 \text{ GeV/fm}.$$

(4)

It is a huge "mass density" on the macroscopic scale.

*Open string.* From I and II, the energy of an open string in Fig. 1a, equal to its mass, is

$$E_0 = m = \int_{-d/2}^{d/2} \frac{ad}{\sqrt{1 - \omega^2 x^2}} = \frac{1}{2} \pi a d,$$

(5)

or the length of the string is proportional to its mass

$$d = \frac{2}{\pi a} m.$$

(6)
The heavier a light-quark meson, the bigger it is. The lightest meson, the pion, would have \( d \approx 0.1 \text{ fm} \).

From II, the angular velocity of the string is inversely proportional to its mass

\[
\omega = \frac{\pi a}{m}. \tag{7}
\]

For the pion it would be \( \omega \approx 20 \text{ fm}^{-1} \approx 10^{24} \text{ Hz} \). We shall see that the pion is not "the smallest top", but it is — "the fastest one".

In the same way we can calculate the angular momentum of the string with respect to its rotation center

\[
L = \int_{-d/2}^{d/2} x \frac{\omega x}{\sqrt{1 - \omega^2 x^2}} \, dx = \frac{\pi a}{2 \omega^2}, \tag{8}
\]

or

\[
L = \frac{1}{2\pi a} m^2. \tag{9}
\]

Both sides of this equation are observable. This is a well-known linearly (with respect to \( m^2 \)) rising Regge trajectory.

For the pion Eq. (9) yields \( L \approx 0.02 \), a comfortably small number.

**Heavy-quark mesons.** Let us introduce a meson mass excess

\[
m_E = m - m_1 - m_2, \tag{10}
\]

where \( m \) is the meson mass and \( m_1 \) and \( m_2 \) are the current quark masses, and let us consider

\[
m_E/m_i \ll 1. \tag{11}
\]

Then the motion is nonrelativistic, and the energies of the string and the quarks in Fig. 1b are

\[
E_0 = ad, \tag{12}
\]

\[
E_i = m_i + \frac{1}{2} a l_i. \tag{13}
\]

The last equation follows from Eq. (3). Summing all these equations, we get

\[
d = \frac{2}{3a} m_E. \tag{14}
\]

The contribution of the string energy to the meson mass is small, but the contribution to \( m_E \) is not small,

\[
E_0/m_E = 2/3 = 67\%, \tag{15}
\]

and do not depend on the quark masses.

Eq. (14) resembles Eq. (6), where \( m \) is replaced by \( m_E \) and the slope is slightly bigger, to the extent that 3 is smaller than \( \pi \).

We see that the string length in this case can be very small if \( m_E \) is small. Indeed, for the strange-quark current mass 0.22 GeV (Sec. 4), the diameter of the \( \eta \)-meson is
smaller than that of the pion by 20%. The smallest particle is $\Upsilon$, the $b$-quark mass being 4.71 GeV (Sec. 4). The $\Upsilon$ diameter is 0.02 fm, 1/5 the pion's diameter.

On the contrary, the behaviour of the angular velocity of the heavy-quark mesons is quite different from the open-string case. From Eqs. (3) and (14) it is easy to get

$$\omega = \frac{a}{\sqrt{\frac{2}{3} \mu m_E}},$$

where $\mu$ is the reduced quark mass

$$\mu = m_2 m_2/(m_1 + m_2).$$

The angular velocity of $\Upsilon$ is 1/5 the pion's velocity.

The string angular momentum is negligible in this case and the total angular momentum is a sum of the quark angular momenta

$$L = \sum m_i \omega l_i = \frac{a^2}{\omega^2 \mu},$$

or

$$L = \frac{1}{a} (\frac{2}{3} m_E)^{3/2} \mu^{1/2}.$$

This is also an observable Regge trajectory, nonlinear in this case, but determined by the same parameter $a$.

Asymmetric mesons. Let one quark, with mass $m_1$, be heavy, and the other one be very light, i.e.,

$$m_E/m_1 \ll 1, \quad m_2/m_E \ll 1,$$

where

$$m_E = m - m_1.$$

The string and quark energies in Fig. 1b are

$$E_0 = a l_1 + \frac{1}{2} \pi a l_2,$$

$$E_1 = m_1 + \frac{1}{2} a l_1,$$

where, to a first approximation, $l_1$ can be neglected, and we obtain

$$d = \frac{2}{\pi a} m_E,$$

$$E_0/m_E = 1,$$

$$\omega = \frac{\pi a}{2 m_E},$$

$$L = L_0 = \frac{\pi a}{4 \omega^2}.$$
or

\[ L = \frac{1}{\pi a} m_E^2. \]  \hspace{1cm} (28)

The diameter (24) has the same slope as that for the open string, Eq. (6), the string gives the main contribution to the meson mass excess \( m_E \) and the Regge trajectory (28), as a function of \( m_E^2 \), has the slope twice as big as that for the open string, Eq. (9), although the corrections to the first approximation, which can be easily worked out, are not negligible in practice.

**General mesons.** For arbitrary quark masses

\[ m = E_0 + \sum E_i = a \int_{-l_i}^{l_i} \frac{dx}{\sqrt{1 - \omega^2 x^2}} + \sum \frac{m_i}{\sqrt{1 - \omega^2 l_i^2}}, \]  \hspace{1cm} (29)

\[ L = L_0 + \sum L_i = a \omega \int_{-l_i}^{l_i} \frac{x^2 dx}{\sqrt{1 - \omega^2 x^2}} + \sum \frac{m_i l_i^2 \omega}{\sqrt{1 - \omega^2 l_i^2}}, \]  \hspace{1cm} (30)

where \( l_i \) is given by Eq. (2).

Introducing

\[ l = 1/\omega, \]  \hspace{1cm} (31)

\[ l_i = \sqrt{l_i^2 + m_i^2/(4a^2)} - m_i/(2a), \]  \hspace{1cm} (32)

\[ G(l) = a \int_{-l_i}^{l_i} \sqrt{P^2 - x^2} \, dx + \sum m_i \sqrt{l_i^2 - l_i^2} \]  \hspace{1cm} (33)

we can rewrite Eqs. (29) and (30) in the form

\[ m = G_i(l) \equiv y \sum (\arctan t_i + t_i^{-1}), \]  \hspace{1cm} (34)

\[ L = K(l) \equiv \frac{1}{2a} (ym - \sum m_i^2 t_i), \]  \hspace{1cm} (35)

where index \( l \) means derivative with respect to \( l \), \( y = a l_i \), \( t_i = (a l_i/m_i)^{1/2} \) and

\[ K(l) = lG_i(l) - G(l). \]  \hspace{1cm} (36)

Eqs. (34) and (35) define a Regge trajectory as an implicit function

\[ L = K(l(m)), \]  \hspace{1cm} (37)

where \( l(m) \) is the solution of Eq. (34), Fig. 2.
Fig. 2. Three types of classical Regge trajectories: $a$ is for light-quark mesons ($m_1 = m_2 = 0$), $b$ is for asymmetric mesons ($m_2 = 0$) and $c$ is for heavy quarkonia ($m_1 = m_2$). Masses are in the $\sqrt{2\pi a}$ units.

If the string moves as a whole with a velocity $v$, its rotation slows down: the angular velocity acquires a factor $\sqrt{1 - v^2}$. Its length, in general, is not conserved. The length oscillates between its minimal (rest-frame) value $d$, when the string is perpendicular to the velocity, and its maximal value $d/\sqrt{1 - v^2}$, when the string is parallel to the projection of the velocity on the rotation plane $v_{pl}$.

The classical description might be not only illustrative for $L \gg 1$. To make the model realistic, we must quantize it and take into account quark spins and nonstring short-range interactions. This will be done in the next Section.

3. Quantum string physics

We shall use Lorentz- and gauge-covariant variables. The straight-line string is a straight line in the 4-dimensional space-time

$$X(\tau, \sigma) = \tau(\tau) + f(\tau, \sigma)q(\tau),$$

where $\tau$ and $\sigma$ are time-evolution and space-position parameters, respectively, $r$ is a 4-vector of a point on the straight line, $q$ is an affine 4-vector of its direction, $f$ is a Lorentz scalar labelling points on the string, and $f_i = f(\tau, \sigma_i)$, $i = 1, 2$ correspond to the string ends. The covariant description introduces superfluous, from a physical viewpoint, variables, therefore, the string action must be invariant with respect to three $\tau$-dependent gauge transformations: shift of $\tau$ along $q$, multiplication of $q$ by a function of $\tau$, and reparametrization of $\tau$. The Lagrangian must be invariant with respect to the first two transformations and have a property $\mathcal{L}(c\hat{z}) = c\mathcal{L}(\hat{z})$, where $\hat{z}$ is every $\tau$-derivative and $c$ is a function of $\tau$. There is only one string variable which is Poincaré- and gauge-invariant

$$l = \sqrt{\tau^2}/b,$$
(not to consider higher \( \tau \)-derivatives), where \( \dot{r}_i^\mu \) is the string velocity, perpendicular to the rotation plane,

\[
\dot{r}_i^\mu = (g^{\mu\nu} + n^\mu n^\nu - \dot{n}^\mu \dot{n}^\nu / n^2) \dot{r}_\nu,
\]

\[
n = q / \sqrt{-q^2},
\]

and \( b \) is an angular velocity of the string with respect to the auxiliary time \( \tau \)

\[
b = \sqrt{-\ddot{n}^2}.
\]

\( b \) is gauge-dependent, but the condition

\[
b \neq 0,
\]

which we assume, is gauge-independent since \( \tau \) is monotonous. Then there is a physically distinguished point on the string, the instantaneous rotation center, and we can label the points on the string, in a gauge-invariant way, with respect to this center

\[
x = \sqrt{-q^2} f - \dot{r} \dot{n} / b^2.
\]

The classical Lagrangian of a meson in SQM consists of three terms

\[
\mathcal{L} = \mathcal{L}_{str} + \sum \mathcal{L}_i + \mathcal{L}_{ss},
\]

the first one being the Nambu-Goto Lagrangian for a straight-line string, Eqs. (38)-(44),

\[
\mathcal{L}_{str} = -ab \int_{z_i}^{z_f} \sqrt{1 - \dot{x}^2} \, dx,
\]

where \( a \) is a string-tension parameter.

The second term in Eq. (45) is sum of the Lagrangians for point massive spinning quarks [BM] having velocities of the string ends

\[
\mathcal{L}_i = -m_i \sqrt{X_i^2} - \frac{i}{2} \xi_i^M \dot{\xi}_i^M - i \left( \frac{\dot{X}_i \xi_i}{\sqrt{X_i^2}} - \xi_i \right) b \lambda_i,
\]

where \( m_i \) is the quark current mass and \( \xi_i^M \) and \( \lambda_i \) are quark-spin variables \((M = \mu, 5, \text{ and } g^{MN} = \text{diag}\{1, -1, -1, -1, -1\})\), which anticommute with each other (and commute with other variables, including spin variables of the other quark).

The Lagrangian (47) contains spin-independent part

\[
\mathcal{L}_{s0} = -m_i \sqrt{X_i^2},
\]

spin-velocity term showing that the spin variables \( \xi_i^M \) are canonically self-conjugate, and spin-constraint term, proportional to a Lagrange multiplier \( \lambda_i \). The spin constraints must be conserved. To ensure this conservation, we shall find out the third term in Eq. (45)
(which restores a supersymmetry of the total Lagrangian). Toward this end, let us first consider the spin-independent part of the Lagrangian

$$\mathcal{L}_0 = \mathcal{L}_{\text{str}} + \sum \mathcal{L}_{\text{i}}.$$  \hfill (49)

The quark velocity $\dot{X}_i$ must be perpendicular to the string direction. This is a property of the minimal surface formed by straight lines, which follows from the Euler-Lagrange equations for the full string under the assumption Eq. (38). The proof of this property is given in Appendix A.

Introducing orthonormal vectors

$$v^0 = \dot{r}/(b l), \quad v^1 = \dot{n}/b,$$  \hfill (50)

we can write

$$\dot{X}_i = b(lv^0 + x_i v^1).$$  \hfill (51)

The extremum condition for $\mathcal{L}_0$ with respect to $x_i$ yields

$$x_i = (-1)^{l_i},$$  \hfill (52)

(Eq. (32)), and the Lagrangian takes the form

$$\mathcal{L}_0 = -b G(l),$$  \hfill (53)

where $G(l)$ is given by Eq. (33).

Let us rewrite this Lagrangian in the phase-space. The momenta conjugate to the coordinates $r$ and $q$

$$p = -\partial \mathcal{L}/\partial \dot{r}, \quad \pi = -\partial \mathcal{L}/\partial \dot{q};$$  \hfill (54)

are equal to

$$p = G(l)v^0,$$  \hfill (55)

$$\pi = (-q^2)^{-1/2}(\dot{r}v^1/b)p + K(l)v^1,$$  \hfill (56)

where index $l$ stands for derivative with respect to $l$ and $K(l)$ is given by Eq. (36). The momentum $p$ is conserved due to translation invariance. It is the total meson momentum, and the meson mass is

$$m = \sqrt{p^2}.$$  \hfill (57)

We shall use the notations

$$n^0 = p/m, \quad \pi^\mu_p = (g^{\mu\nu} - n^{0\mu}n^{0\nu})\pi_\nu.$$  \hfill (58)

From Eqs. (55) and (56)

$$\pi_p = (-q^2)^{-1/2}Kv^1.$$  \hfill (59)

We see that the phase-space variables obey three constraints

$$pq = 0, \quad \pi q = 0,$$  \hfill (60)
\[ m = G_i(l), \]  
\[ \sqrt{q^2 \pi^2} = K(l). \]  
(61) (62)

The third constraint is given by two Eqs. (61) and (62): we must solve one of them (e.g., the first one) to find out \( l \) as a function of \( m \), and put this solution into the second equation. The l.-h.s. of Eq. (62) with constraints Eqs. (60) is (orbital) angular momentum of our system

\[ \sqrt{q^2 \pi^2} = \sqrt{-L^2}, \]  
\[ L_{\mu} = \epsilon_{\mu \nu \rho \sigma} p^\nu L^{\rho \sigma}/2m, \quad L^{\mu \nu} = r^{[\mu \nu]} + q^{[\mu \nu]}. \]  
(63) (64)

The canonical Hamiltonian of a \( \tau \)-reparametrization-invariant system is zero, and the Hamiltonian of our system is a linear combination of the constraint functions [6]. We can rewrite the Lagrangian (49), (53) in the form

\[ L_0 = -(1/2)(pr - rp) - (1/2)(\pi \dot{q} - q \dot{\pi}) - c \left( \sqrt{-L^2} - K(l(m)) \right) - c_1 pq - c_2 \pi q. \]  
(65)

where \( c, c_1 \) and \( c_2 \) are arbitrary \((c = -b)\) and \( l(m) \) is given by Eq. (61).

From Eq. (48), we get the quark momenta

\[ p_i = m_i \dot{X}_i / \sqrt{X_i^2}, \]  
(66)

which, from Eqs. (51), (52), (55), and (59), are equal to

\[ p_i = (l n^0 + (-1)^i l_i n^1) m_i / \sqrt{L^2 - l_i^2}, \]  
(67)

where \( l = l(m) \) is given by Eq. (61) and

\[ n^1 = \pi_p / \sqrt{-\pi_p^2}. \]  
(68)

Now it is not difficult to introduce the quark spins in a consistent way. We add to the r.-h.s of Eq. (65) the spin-velocity term and the spin-constraint term expressed through the quark momenta (67), and, to ensure the spin-constraint conservation, we replace the orbital angular momentum \( L_\mu \) by total angular momentum

\[ J_\mu = \epsilon_{\mu \nu \rho \sigma} p^\nu M^{\rho \sigma}/2m, \]  
(69)

\[ M^{\mu \nu} = r^{[\mu \nu]} + q^{[\mu \nu]} - i \sum \zeta^\mu_i \zeta^\nu_i. \]  
(70)

As a result, we obtain the SQM Lagrangian

\[ L = -(1/2)(pr - rp) - (1/2)(\pi \dot{q} - q \dot{\pi}) - (i/2) \sum \zeta^M_i \dot{\xi}_i - i \sum (p_i \dot{\zeta}_i - m_\xi \dot{\zeta}_i) \lambda_i - c \left( \sqrt{-J^2} - K(l(m)) \right) - c_1 pq - c_2 \pi q. \]  
(71)

11
One can express this Lagrangian through the configuration space variables by means of the inverse Legendre transformation. We shall not use the configuration-space form of the Lagrangian. For the sake of completeness, it is given in Appendix B with an outline of its derivation.

Besides quark-spin terms, a nonstring short-range interaction must enter into the meson Lagrangian. This interaction cannot be fully described at the classical level and will be taken into account in the quantum equations.

The first line of Eq. (71) corresponds to the first differential form of our system which determines the Poisson brackets of the canonical variables \([4]\). Namely, if we denote the variables by \(y_n\) and the first form by \((1/2)\omega_{mn}y_my_n\), then the Poisson brackets are \(\{y_m, y_n\} = \omega^{mn}\), where \(\omega^{mn}\) is inverse of \(\omega_{mn}\). For instance, from Eq. (71),

\[
\{\xi^M, \xi^N\} = ig^{MN}.
\]

(72)

The other brackets have the usual canonical form. In particular, the total spin \(J^\mu\) has zero Poisson brackets with Lorentz scalars, therefore, the spin constraint functions have zero brackets with the Hamiltonian and are conserved. This justifies the choice of \(\mathcal{L}_{ss}\) in the Lagrangian. The gauge constraint functions are in involution with respect to the Poisson brackets, due to properties of the gauge transformations, and are also conserved.

The second line of Eq. (71) is minus Hamiltonian. We can exclude the constants \(c_1\) and \(c_2\) by choosing the gauge conditions

\[
p\pi = 0, \quad \pi^Z = -1.
\]

(73)

Their conservation yields \(c_1 = c_2 = 0\), and we must solve Eqs. (73) together with the corresponding constraints (60). Introducing four orthonormal vectors \(e_\alpha, \alpha = 0, 1, 2, 3\)

\[
e_0 = p/m, \quad e_\alpha e_\beta = g_{\alpha\beta},
\]

(74)

we can write the solution in the form \((a, b, c = 1, 2, 3)\)

\[
\pi = k^{(a)}e_a, \quad q = e_{abc}k^{(a)}l^{(b)}e_c, \quad L = L^{(a)}e_a.
\]

(75)

We shall use space-vector notations for the set \(\{k^{(a)}\}\) and similar sets

\[
\{k^{(a)}\} = \vec{k}.
\]

(76)

From Eqs. (73) and (64)

\[
\vec{k}^2 = 1, \quad \vec{k}L = 0.
\]

(77)

Using expansions

\[
J = J^{(a)}e_a, \quad \xi_i = \xi_i^{(a)}e_a, \quad \xi_i^{(5)} = \xi_i^{(5)};
\]

(78)

we can rewrite the Lagrangian (71) in the form (up to a total derivative)

\[
\mathcal{L} = -p\dot{z} - [\vec{k} \times \vec{L}]\dot{k} - \frac{i}{2} \sum \xi_{iM}q^{(M)}\xi_i - c(\sqrt{\vec{J}^2} - K) + \sum (p_i^{(a)}\xi_{i(a)} - m_i\xi_i^{(5)})\lambda_i;
\]

(79)
where the new string coordinate is

$$z^\mu = r^\mu + \frac{1}{2} \epsilon_{abc} e^a_\mu \frac{\partial e_{bc}^e}{\partial p^\mu} J^e + \frac{i}{m} \sum \xi^{(0)}_i \xi^{(a)}_i e^a_\mu,$$

(80)

and, in 4-vector notations,

$$p_i^{(a)} = (l, (-1)^i l, \vec{k}) m_i / \sqrt{l^2 - l_i^2}.$$

(81)

The variables $\vec{k}$ and $\vec{L}$ are not independent, but using, e.g., spherical angles to solve Eqs. (77), one can easily obtain from the Lagrangian (79) the following nonzero Poisson brackets

$$\{ p^\mu, z^\nu \} = g^{\mu\nu},$$

(82)

$$\{ L^{(a)}, L^{(b)} \} = \epsilon_{abc} L^{(c)}, \quad \{ L^{(a)}, k^{(b)} \} = \epsilon_{abc} k^{(c)}$$

(83)

$$\{ \xi^{(M)}_1, \xi^{(N)}_2 \} = ig^{MN}.$$

(84)

The Hamilton equations of motion can be easily solved. The solution for spinless quarks was described in Sec. 2.

The quantization of our system is now straightforward. We replace $p, z, \vec{L}, \vec{k}$ and $\xi^{(M)}_i$ by operators and their Poisson brackets by commutators or anticommutators for $\xi$'s (multiplied by $-i$), e.g.,

$$[\xi^{(M)}_1, \xi^{(N)}_2]_+ = -g^{MN}, \quad [\xi^{(M)}_1, \xi^{(N)}_2]_- = 0.$$

(85)

Assuming the second quark to be an antiquark, we take the following solution of these equations

$$\xi^{(\mu)}_1 = \frac{1}{\sqrt{2}} \gamma^5 \gamma^\mu \otimes I, \quad \xi^{(s)}_1 = \frac{1}{\sqrt{2}} \gamma^5 \otimes I$$

(86)

$$\xi^{(\mu)}_2 = \xi^{(\mu)c}_1 = I \otimes \frac{1}{\sqrt{2}} \gamma^5 \gamma^\mu, \quad \xi^{(s)}_2 = \xi^{(s)c}_1 = I \otimes \frac{1}{\sqrt{2}} \gamma^5,$$

(87)

where $\gamma^c = C \gamma C^{-1}$ and $C$ is the charge-conjugation matrix.

The constraint functions become operators annihilating the wave function. In the representation where $p$ and $k$ are diagonal the internal part of the wave function $\delta(p - p')\Psi_{a'b}(\vec{k})$ satisfies the equations

$$(\hat{p}_1 - m_1) \Psi = 0,$$

(88)

$$\Psi (\hat{p}_2 + m_2) = 0,$$

(89)

$$(\sqrt{J^2} - K - \sum_{n=1}^4 a_n P_n) \Psi = 0.$$

(90)

In the third equation a new term has been introduced to account for the short-range nonstring interaction. In this term, $a_n$ can depend on $J$, and $P_n$ are four independent
operators commuting with the Dirac operators in the first and second equations (for fixed $J \neq 0$ there are four independent states of two particles with spin 1/2).

Since $a_n$ describes a short-range interaction, it cannot increase with $J$. For the majority of mesons we can take $a_n$ as a constant independent of $J$. Only heavy quarkonia demand more complicated $a_n$, containing a decreasing with $J$ contribution [1].

The choice of $P_n$ is connected with the choice of meson states at fixed $J$ and will be discussed after solving Eqs. (88) and (89).

The solution of the Dirac equations (88) and (89) is a $4 \times 4$ matrix

$$\Psi = \frac{1}{\sqrt{(1 + b_i^2)(1 + b_i^2)}} \begin{pmatrix} -b_2 \chi \bar{\epsilon}_k & \chi \\ b_1 b_2 \bar{\epsilon}_k \chi \bar{\epsilon}_k & -b_1 \bar{\epsilon}_k \chi \end{pmatrix}$$

where

$$b_i = \frac{l_i}{l + \sqrt{m_i l_i/a}}, \quad b_i \rightarrow \begin{cases} 1, & m_i \rightarrow 0 \\ 0, & m_i \rightarrow \infty \end{cases}$$

and $\chi$ is an arbitrary normalized $2 \times 2$ matrix.

We shall take

$$\chi = \chi_{jMIS},$$

which are eigenfunctions of $\tilde{j}^2$, $j^{(3)}$, $\tilde{L}^2$ and $\vec{s}$ with eigenvalues $j(j + 1)$, $M$, $l(l + 1)$ and $S(S + 1)$, respectively, where $\tilde{j} = \tilde{L} + \vec{s}$ and $\vec{s}$ is a 2-dimensional quark spin

$$\vec{s} = \frac{1}{2} \vec{\sigma} \otimes 1 + \frac{1}{2} 1 \otimes \vec{\sigma}, \quad \vec{\sigma} = \sigma_2 \sigma_2 = -\vec{\sigma}^*$$

and 1 stands for the $2 \times 2$ unit matrix. These functions can be easily constructed with the help of Clebsch-Gordan coefficients.

The corresponding functions $\Psi$, denoted by $\Psi_{jMIS}$, are eigenfunctions of $\tilde{j}^2$ and $j^{(3)}$ with eigenvalues $j(j + 1)$ and $M$, respectively, and eigenfunctions of space and charge-conjugation parities

$$P \Psi_{jMIS} = -(-1)^l \Psi_{jMIS}, \quad \Psi_{jMIS}^c = (-1)^{l+S} \Psi_{jMIS}.$$  \hspace{1cm} (95)

The parity transformations are defined by

$$P \Psi(\bar{k}) = \gamma^0 \Psi(-\bar{k}) \gamma^0, \quad \Psi^c(\bar{k}) = C \Psi^T(-\bar{k}) C^T,$$

where $C$ is the charge-conjugation matrix. Dirac equations (88), (89) are charge-conjugation-invariant for $m_1 = m_2$.

We shall assume that mesons in the states $\Psi_{jM,j-1,1}$ and $\Psi_{jM,j+1,1}$ do not mix.

This means that mesons with definite $C$ or $G$-parity are described by the wave functions $\Psi_{jMIS} \equiv \Psi_n$, and the operators $P_n$ in Eq.(90) are the projection operators

$$P_n \Psi_m = \delta_{mn} \Psi_n.$$  \hspace{1cm} (97)
The index $n$ takes four (two) values for fixed $j \neq 0(j = 0)$: $n = 0$ for $l = j$, $S = 0$; $n = 1$ for $l = j$, $S = 1$; and $n = \pm$ for $l = j \pm 1$, $S = 1$. Eq. (90) for these states takes the form
\[ \sqrt{j(j + 1)} = K + a_n \]  
which is called the spectral condition. Here $K$ is a function of $m, m_1, m_2$ and $a$, given by Eqs. (36), (33), and (32).

As mentioned before, $a_n$ can be taken independent of $j$ for all mesons except $c\bar{c}$- and $b\bar{b}$-mesons in the states with $n = -$, for which
\[ a_- = A + \left( \frac{8m_1}{m(2j + 1)^2} \right) B, \]
where $A$ and $B$ do not depend on $j$.

For strange, charmed and bottom mesons the states $\Psi_0$ and $\Psi_1$ can mix for $j > 0$, so that the mesons are described by
\[ \Phi_0 = \cos \alpha \Psi_0 + \sin \alpha \Psi_1 \]
\[ \Phi_1 = -\sin \alpha \Psi_0 + \cos \alpha \Psi_1, \]
and in the spectral condition (98) $a_0$ and $a_1$ are replaced by $a_0 - d$ and $a_1 + d$, respectively, where $d$ is a mixing parameter
\[ d = -b \tan \alpha = \frac{1}{2} (a_0 - a_1 \pm \sqrt{(a_0 - a_1)^2 + 4b^2}) \]
and an upper (lower) sign corresponds to $a_0 - a_1 < 0(> 0)$.

4. Meson mass spectrum and model parameters

First comparison of the spectral condition (98) with experiment was made in Ref. [1,2]. Here we compare with more recent data available [7] and make predictions for mesons with spin up to 7.

The light quark current masses give a very small contribution to the spectral condition and cannot be determined from this condition. So, we use the linear chiral SU$_3$ model relations [8]
\[ m_u/m_d = 0.55, \quad m_s/m_d = 20.1 \]
to express them through the strange quark mass which is determined from comparison with experiment.

The best known meson Regge trajectories are described by the wave functions $\Psi_-$ and have $P = C = (-1)^j$ and $j_{\min} = 1$. The parameters $a_-$ or $A$ in Eq. (99) depend very weakly on the quark masses: $a_-(d\bar{u}) = 0.88, \quad a_-(c\bar{u}) = 0.90, \quad A(c\bar{c}) = 0.90, \quad A(b\bar{b}) = 0.77, \quad A(b\bar{b}) = 0.77$. This means that the short-range contributions for the strange quarks in these states are the same as for the light quarks: $a_-(s\bar{u}) = a_-(s\bar{s}) = a_-(d\bar{u}), \quad a_-(c\bar{s}) = a_-(c\bar{u})$, and $a_-(b\bar{s}) = a_-(b\bar{u})$. 

15
These trajectories allow one to determine the main parameters of the model [1,2]

$$a = 0.176 \pm 0.002 \text{ GeV}^2$$

$$m_s = 224 \pm 7, \quad m_c = 1440 \pm 10, \quad m_b = 4715 \pm 20 \quad (103)$$

$$m_a = 6.2 \pm 0.2, \quad m_d = 11.1 \pm 0.4$$

(masses in MeV), and the short-range parameters, Table 1C in Appendix C.

A simpler procedure, when one drops the second term in Eq. (99), uses $a_-$ independent of the $c$-quark masses and applies the minimum-$\chi^2$-method [2], gives the same results for the main parameters (103) with good $\chi^2$.

Fig. 3 shows some of these trajectories. For the light- and strange-quark mesons the trajectories are practically linear.

![Regge trajectories](image)

Fig. 3. Regge trajectories for mesons with $P = C = (-1)^j$ and $j_{\text{min}} = 1$. The argument scales are shifted for each group of mesons. The straight lines for $c\bar{u}$- and $b\bar{u}$-mesons are drawn to show the deviation of the trajectories from linear ones.

For the light-heavy-quark mesons the trajectories are not linear but can be made practically linear by replacing the argument $m^2$ by $(m - m_h)^2$ where $m_h$ is the heavy quark mass (Fig. 4).

In the limit

$$\frac{2(m - m_h)}{\pi m_h} \ll 1, \quad \frac{\pi m_l}{2(m - m_h)} \ll 1, \quad (104)$$

where $m_l$ is the light-quark mass, they must be linear with the slope twice as big as for the light-quark mesons. We see the trajectories are practically linear up to $j = 5$ with bigger effective slopes, but the first condition (104) for the limit slope is not fulfilled.
Fig. 4. The same as in Fig. 3 with changed arguments for \(c\bar{u}\)- and \(b\bar{u}\)-mesons.

Fig. 5. The same as in Fig. 3 for heavy quarkonia.

The trajectories for the heavy quarkonia are essentially nonlinear, Fig. 5.

Table 1C in Appendix C represents a detailed comparison of the model with experiment and contains predictions for new mesons and comparison with potential model predictions [5]. The \(B_j^+(5732)\)-meson, found in ALEPH, DELPHI and OPAL experiments at CERN (see page 574 of Ref. [7] for the References) agrees with SQM much better than with PQM, Fig. 3 and Table 1C.

The prediction for the \(b\bar{c}, 1^-\)-meson is made under the simplest assumption \(a_-(b\bar{c}) = a_-(b\bar{u})\). The other assumptions: \(a_-(b\bar{c}) = a_-(b\bar{b})\) or \(a_-(c\bar{c})\) reduce its mass by 100 MeV. The higher-spin \(b\bar{c}\)-mesons do not practically depend on this assumption.
The trajectories for $\Psi_0$ states, $C = -P = (-1)^j$ and $j_{\min} = 0$ are always nonlinear near $j = 0$ due to the square root in the spectral condition (98), Fig. 6.

Fig. 6. Regge trajectories for light-quark mesons with $-P = C = (-1)^j(I)$, for strange mesons with $P = -(1)^j(II)$ and for light-quark (III) and $s\bar{s}$-quark (IV) mesons with $P = C = (-1)^j$ and $j_{\min} = 0$. The straight lines near $j = 0$ are drawn to show the deviation of the trajectories with the universal string tension from linear ones. The argument scales are shifted for each group of mesons.

Only one parameter, the short-range contribution $a_0$, is unknown for each of these trajectories. It is determined from the mass of corresponding spin-0 meson.

We see that the nonlinear trajectory (98) with the universal slope describes quite well the three mesons $\pi, b_1$ and $\pi_2$.

The constant $a_0$ for the light-quark mesons is small. According to the linear chiral $SU_3$ model [8], it must be proportional to the light-quark masses $m_i$. This property does not contradict the spectral condition (98) where $K$ is proportional to $m_i^{3/2}$.

There is not enough data to analyse $\Psi_1$ states, $P = C = (-1)^j$ and $j_{\min} = 1$. So, we assume that

$$a_1 = a_0 \quad (105)$$

for all mesons except $s\bar{u}$- and $s\bar{d}$-mesons for which mixing is important. Experiment confirms this assumption for known mesons composed by light, $s\bar{s}$- and $c\bar{c}$-quarks. If Eq. (105) is fulfilled also for the $b\bar{b}$-mesons, then using the known mass of $\chi_{b_1}(1P)$, we can estimate the mass of a pseudoscalar $b\bar{b}$-meson $\eta_b(1S), 0^{-+}$ to be 9.50 GeV which is bigger than the mass of $\Upsilon(1S)$.

Linearly extrapolating between $a_0(b\bar{u}) = -0.55$ and $a_0(b\bar{b}) = -0.091$, obtained from Eq. (105), we can find $a_0(b\bar{c}) = -0.41$ and estimate the mass of a pseudoscalar $B_c$-meson to be 6.40 GeV, which is 0.13 GeV higher than in the potential model [5].
We take into account the mixing of $\Psi_0$ and $\Psi_1$ states only for strange mesons. The trajectories for $\Phi_0$ and $\Phi_1$ states (100) with a mixing angle $36^\circ$ are shown in Fig. 6. The detailed comparison with experiment and predictions for $\Psi_{0,1}$ ($\Phi_{0,1}$) states are given in Table 2C in Appendix C.

The behaviour of the trajectories for $\Psi_+$ states, $\Gamma = C = (-1)^J$ and $j_{\text{min}} = 0$, is similar to that for $\Psi_0$, Fig. 6.

The $X(1920)$, $??'$-meson, found in GAMC and VES experiments at IHEP, Protvino, agrees quite well with SQM predictions and may be a $2^{++}$ trajectory partner of $a_0(980)$.

The strange mesons $K^*_0(1430), 0^+$ and $K^*(1680), 1^-$ are not described by the same wave function $\Psi_+$ (with different $j$). It seems probable that a new strange $1^-$ meson exists with mass 1900 MeV which is a partner of $K^*_0(1430)$, see Table 3C in Appendix C. On the other hand, the $K^*(1680)$-mass, $1717 \pm 17$ MeV, is only half of its width, $322 \pm 110$ Mev, lower than the SQM value 1910 Mev.

We can tentatively conclude that the SQM describes masses and other quantum numbers of about 2/3 of established mesons, the rest being daughter, glueball, or exotic states. The agreement with experiment for the former mesons is, in general, slightly better than that for the PQM. It seems important to continue systematic experimental study of meson mass spectrum where both models give different new predictions.

5. Internal structure of composite mesons

The model allows one to calculate quark velocities and energies and string energy in mesons at rest, Eqs. (61) and (81):

$$v_i = l_i/l, \quad E_i = l(\lambda m_i/l_i)^{1/2}, \quad E_0 = a l \sum \arcsin v_i,$$

(106)

where $l_i$ is given by Eq. (32) and $l$ is a solution of Eq. (61), $l = x/a$ and $x$ is given for each meson in Tables in Appendix C. The results for some mesons are collected in Table 1.

Table 1. Energy distribution inside mesons at rest. $v_i(E_i)$ is velocity in c (energy in MeV) of the $i$-th quark, $E_0$ is energy of the gluon string in MeV and $m_E = m - m_1 - m_2$.

<table>
<thead>
<tr>
<th>Particle, quark content</th>
<th>$v_1$</th>
<th>$v_2$</th>
<th>$E_1$</th>
<th>$E_2$</th>
<th>$E_0$</th>
<th>$E_0/m_1,%$</th>
<th>$E_0/m_E,%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^+, d\bar{u}$</td>
<td>0.98</td>
<td>0.99</td>
<td>53</td>
<td>39</td>
<td>679</td>
<td>88</td>
<td>90</td>
</tr>
<tr>
<td>$\pi^+, d\bar{u}$</td>
<td>0.88</td>
<td>0.93</td>
<td>23</td>
<td>16</td>
<td>99</td>
<td>72</td>
<td>82</td>
</tr>
<tr>
<td>$B^+, b\bar{u}$</td>
<td>0.07</td>
<td>0.99</td>
<td>4727</td>
<td>46</td>
<td>507</td>
<td>9.6</td>
<td>91</td>
</tr>
<tr>
<td>$J/\psi(1S), c\bar{c}$</td>
<td>0.22</td>
<td>0.22</td>
<td>1476</td>
<td>1476</td>
<td>146</td>
<td>4.7</td>
<td>67</td>
</tr>
<tr>
<td>$Y(1S), b\bar{b}$</td>
<td>0.05</td>
<td>0.05</td>
<td>4720</td>
<td>4720</td>
<td>22</td>
<td>0.2</td>
<td>67</td>
</tr>
<tr>
<td>$\chi_{t2}(1P), b\bar{b}$</td>
<td>0.18</td>
<td>0.18</td>
<td>4795</td>
<td>4795</td>
<td>324</td>
<td>3.3</td>
<td>67</td>
</tr>
</tbody>
</table>

We see that the light quarks are relativistic and give noticeable contributions to the meson masses. The main contribution to the mass "excess" of mesons $m_E = m - m_1 - m_2$ is given by the gluon string.
Let us consider a spin structure of mesons, i.e., average values of internal the angular momentum variables. The SQM allows one to calculate the spin structure of each meson on leading trajectories. The result depends on spin, parities and mass of the meson, string tension and current masses of quarks composing the meson. For instance, an average value of the third projection of the $i$-th-quark spin is given by

$$S^{(3)}_i = \langle \Psi, S_i^{(3)} \Psi \rangle = \int S \rho S \sigma S \Psi^+ S_i^{(3)} \Psi d \mathbf{k},$$  

where

$$\vec{S}_i = \frac{1}{2} \vec{\sigma} \otimes I, \quad \vec{S}_2 = \frac{1}{2} I \otimes \vec{\sigma},$$

$$\vec{\Sigma} = \begin{pmatrix} 0 & \sigma & 0 \\ \sigma & 0 & \sigma \\ 0 & \sigma & 0 \end{pmatrix}, \quad \vec{\Sigma}_c = \begin{pmatrix} \sigma_2 \bar{\sigma}_2 & 0 \\ 0 & \sigma_2 \bar{\sigma}_2 \end{pmatrix} = -\vec{\Sigma}^*.$$  

Introduce the notations

$$c = 2(b_1^2 + b_2^2)N, \quad c_1 = 4b_1^2 b_2^2 N, \quad c_2 = 2(b_2^2 - b_1^2)N,$$

$$N = 1/((1 + b_2^2)(1 + b_1^2))$$  

where $b_i$ is given by Eq. (92). In the nonrelativistic limit ($v_i = 0, m_1 + m_2 = m$) all $c$'s vanish. In the ultrarelativistic limit ($v_i = 1, m_i = 0$) $c = c_1 = 1$ and $c_2 = 0$. Then, for a polarized meson with $J^{(3)} = M$, we obtain

$$\Psi_0 = \Psi_{JMj0}, \quad \vec{S}_i^{(3)} = 0,$$

$$\Psi_1 = \Psi_{JMj1}, \quad \vec{S}_i^{(3)} = \frac{M}{2j(j+1)},$$

$$\Psi_- = \Psi_{JM,j-1,1}, \quad \vec{S}_i^{(3)} = M \left( \frac{1}{2j+1} \left( c + c_1 + (-1)^j c_2 \right) \right),$$

$$\Psi_+ = \Psi_{JM,j+1,1}, \quad \vec{S}_i^{(3)} = \frac{M}{2} \left( \frac{1}{j+1} + \frac{1}{2j+1} \left( c + c_1 + (-1)^j c_2 \right) \right).$$

$$\vec{S}^{(3)} = \sum \vec{S}_i^{(3)}, \quad \vec{L}^{(3)} = M - \vec{S}^{(3)}.$$  

In the same way, for squared quantities, we have

$$(\Psi_0, \vec{S}^2 \Psi_0) = c, \quad (\Psi_1, \vec{S}^2 \Psi_1) = 2,$$

$$(\Psi_-, \vec{S}^2 \Psi_-) = 2 - \frac{j}{2j+1} c,$$

$$(\Psi_+, \vec{S}^2 \Psi_+) = 2 - \frac{j+1}{2j+1} c.$$

$$(\Psi_0, \vec{L}^2 \Psi_0) = j(j+1) + c, \quad (\Psi_1, \vec{L}^2 \Psi_1) = j(j+1),$$

120
\[
\begin{align*}
(\Psi_-, \vec{L}^2 \Psi_-) &= j \left( j - 1 + c + \frac{2j + 2}{2j + 1} c_1 \right), \\
(\Psi_+, \vec{L}^2 \Psi_+) &= (j + 1) \left( j + 2 - c - \frac{2j}{2j + 1} c_1 \right).
\end{align*}
\]

\[
\vec{L}^2 = (1/2) \left( j(j + 1) - \vec{L}^2 - \vec{\bar{S}}^2 \right).
\]

The spin structure of some mesons is presented in Tables 2 and 3.

**Table 2.** Spin structure of some mesons. Average values of internal angular momentum variables are shown for polarized mesons with \( J^{(3)} = M \). \( nr \) is nonrelativistic limit, \( r \) is real case and \( ur \) is ultrarelativistic limit.

<table>
<thead>
<tr>
<th></th>
<th>( S_1^{(3)}/M )</th>
<th>( S_2^{(2)}/M )</th>
<th>( \bar{S}^{(3)}/M )</th>
<th>( \bar{L}^{(3)}/M )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( nr ) ( r ) ( ur )</td>
<td>( nr ) ( r ) ( ur )</td>
<td>( nr ) ( r ) ( ur )</td>
<td>( nr ) ( r ) ( ur )</td>
</tr>
<tr>
<td>( \rho^+, ud )</td>
<td>1/2 0.22 1/6 0.23 1/6 0.42 1/6 0.45 1/3 0.63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K^{++}, u\bar{s} )</td>
<td>0 0.22 0 0.23 0 0.42 0 0.45 0 0.63</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 3.** Continuation of Table 2. Average values do not depend on the meson polarization.

<table>
<thead>
<tr>
<th></th>
<th>( \bar{S}^2 )</th>
<th>( \bar{L}^2 )</th>
<th>( \bar{S}\bar{L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( nr ) ( r ) ( ur )</td>
<td>( nr ) ( r ) ( ur )</td>
<td>( nr ) ( r ) ( ur )</td>
</tr>
<tr>
<td>( \pi^+, ud )</td>
<td>0 0.83 1 0 0.83 1 0 -0.83 -1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K^+, u\bar{s} )</td>
<td>0.79 0.79 1 0.79 1 -0.79 -1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho^+, ud )</td>
<td>2 1.68 5/3 0 1.87 7/3 0 -0.77 -1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K^{++}, u\bar{s} )</td>
<td>1.70 1.70 1.70 1.70 1.17 1.17 -0.43 -1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We see that the spin structure of light-quark mesons (114) is essentially different from the nonrelativistic case: The average quark spin projections are twice as small. The spin structure of \( \rho \)-meson in SQM is similar to the nucleon spin structure measured in the experiment and different from the nonrelativistic quark model predictions.

The spin structure is also different from the ultrarelativistic case, when the light-quark current masses are neglected. Unlike the spectral condition, the spin structure is sensitive to the light-quark current masses. Measurement of the spin structure allows one to estimate the light-quark current masses from experiment.

We see also that the flavour \( SU_3 \) is badly broken in the spin structure of spinning mesons. The average value of the \( \bar{s} \) spin projection in \( K^* \) is 80% bigger than the \( \bar{d} \) spin projection in \( \rho \).

**6. Conclusions**

The gluon string in SQM can account for the quark confinement in mesons.

The string comprises two mechanisms of potential quark models (PQM): confinement potential and constituent quark masses.
Systematic experimental study of meson spectroscopy is important in checking the SQM predictions in comparison with the PQM predictions.

Spin structure of light-quark vector mesons in SQM is different from the nonrelativistic quark model (NQM): for the average light-quark spin projections $\bar{S}_{SQM} \cong \frac{1}{2} \bar{S}_{NQM}$.

The flavour $SU_3$ is badly broken in the spin structure in SQM: for $s$ and $d$ quarks $\bar{S}_s \cong 2\bar{S}_d$.

Experimental study of the spin structure may eventually provide experimental estimation of the light-quark current masses.

The author is sincerely grateful to Professor A. D. Krisch for the kind hospitality at the Spin Center of the University of Michigan.

Appendix A: Lagrangian for a straight-line string with massive spinless quarks at the ends

This Lagrangian must give equations of motion which follow from the full-string Lagrangian with quarks at the ends, $i = 1$ or $2$

$$\left(\partial L/\partial \dot{X}\right)' + (\partial L/\partial X')' = 0,$$  \hfill (124)

$$(-1)^i \left((\partial L(\sigma_i)/\partial \dot{X})\sigma_i - \partial L(\sigma_i)/\partial X'\right) + (\partial L_i/\partial \dot{X}_i)' = 0,$$  \hfill (125)

where $L(L_i)$ is the Nambu-Goto (the $i$-th-quark) Lagrangian, and the dot (prime) stands for the derivative with respect to $\tau$ ($\sigma$). For a straight-line string in the notations of Sec. 3

$$X(\tau, \sigma) = r(\tau) + (x(\tau, \sigma) + z(\tau))n(\tau),$$  \hfill (126)

$$z = r v^1/b,$$  \hfill (127)

$$w = \dot{z} + \dot{z} - \dot{r}n,$$  \hfill (128)

we can rewrite Eq. (124) in the form

$$\left(x'(lv^0 + xv^1)/s\right)' - (w(lv^0 + xv^1)/s + bsn)' = 0,$$  \hfill (129)

$$s = \sqrt{l^2 - x^2}.$$  \hfill (130)

Using four orthonormal vectors $v^0, v^1$ (Eqs. (50)), $n$ (Eq. (41)) and

$$v^2_\mu = \epsilon_{\mu\nu\rho\sigma}v^0_\nu v^3_\rho v^1_\sigma, \quad v^3 = n,$$  \hfill (131)

$$v^a v^b = g^{ab}, \quad a, b = 0, 1, 2, 3,$$  \hfill (132)

we can expand the l.-h.s. of Eq. (129) with respect to these vectors and get three equations (the fourth one, corresponding to the $n$-component, turns out to be an identity)

$$(x'/l/s)' + \alpha x'/s - (wl/s)' = 0,$$  \hfill (133)
\[ (x'z/s)' + \alpha z/l/s - (wz/s)' = 0, \]
\[ \beta l + \gamma z = 0, \]  
where
\[ \alpha = -\dot{v}_0^1, \quad \beta = -\dot{v}_0^2, \quad \gamma = -\dot{v}_1^2. \]  
Since \( z \) is the only function which depends on \( \sigma \), we get from Eq. (135)
\[ \beta = \gamma = 0. \]  
Eqs. (133) and (134) coincide
\[ i^z - \alpha(l^2 - x^2) + (\dot{z} - i\dot{n})l = 0 \]  
and give
\[ i = \alpha = \dot{z} - i\dot{n} = 0. \]  
So, Eq. (124) for the straight-line string is equivalent to
\[ \dot{i} = 0, \quad \dot{v}_0 = 0, \quad \dot{v}_1 = -bn, \quad \dot{z} - i\dot{n} = 0. \]  
Let us consider Eq. (125)
\[ (-1)^i \left( \dot{x}_i(lv_0 + x_1v_1)/s + bs, m \right) + m_i \left( \dot{X}_i/\sqrt{X_i^2} \right) = 0, \]  
where the dot means the total derivative with respect to \( \tau \),
\[ s_i = \sqrt{l^2 - x_i^2}, \]  
and
\[ \dot{X}_i = b(lv_0 + x_1v_1) + (\dot{x}_i + \dot{z} - i\dot{n})n. \]  
Using Eqs. (140), we get
\[ \dot{x}_i = 0, \]
\[ (-1)^i as_i - m_i x_i/s_i = 0. \]  
Eq. (144) yields that the quarks cannot move along the straight-line string. Eq. (145) coincides with Eqs. (52) and (32).

It is not difficult to check that the Lagrangian (49), (46), (48) and (51), used in Sec. 3, gives exactly the same equations of motion (140), (144) and (145).

**Appendix B: The SQM Lagrangian in the configuration space**

Neglecting a total \( \tau \)-derivative, we can rewrite the SQM Lagrangian (71) in the form
\[ L = -p\dot{r} - \pi \dot{\pi} - (i/2) \sum \xi_i^M \xi_i^M - H, \]  
\[ 23 \]
\[ H = c \left( J - K + i \sum F_{ia} c^a_i \lambda_i \right) + c_3 p q + c_2 \pi q, \quad (147) \]

where
\[ c^a_i = n^a \xi_i, \quad c^5_i = \xi^5_i, \quad (148) \]

\[ n^0 = p/m, \quad n^1 = \pi_1, \quad n^2 = \varepsilon_{\mu \nu \rho} n^0 \nu n^3 \rho n^{1 \sigma}, \quad (149) \]

\[ n^3 = q_p/\sqrt{-q_p^2}, \quad (150) \]

\[ \pi^\nu_1 = (g^{\mu \nu} - n^0 \mu n^0 \nu + n^3 \mu n^3 \nu) \pi_\nu, \quad ; \quad q_p^\mu = (g^{\mu \nu} - n^0 \mu n^0 \nu) q_\nu, \quad (151) \]

\[ c^{ab}_i = c^a_i c^b_i, \quad c^{ab,ef}_i = c^{ab}_i c^{ef}_i, \quad (152) \]

\[ J = \sqrt{-J^2} = L(1 + t) + i \sum c^{13}_i, \quad t = \frac{1}{2L^2} \sum (c^{12,12}_i + c^{23,23}_i), \quad (153) \]

\[ L = \sqrt{q_p^2 \pi^2_1}, \quad (154) \]

\[ K = \bar{m} - G(\bar{l}), \quad m = G_l(l) \quad (155) \]

\[ F_{l0} = \bar{l} \sqrt{a m_i \bar{l}_i}, \quad F_{l1} = (-1)^i \sqrt{a m_i \bar{l}_i}, \quad F_{l5} = -m_i. \quad (156) \]

Here \( \bar{l} \) is given by Eq. (32) with substitution of \( \bar{l} \) for \( l \) where \( \bar{l} \) is a solution of the second Eq. (155).

The velocity variables are determined by the inverse Legendre transformation
\[ \dot{r} = -H_p, \quad \dot{q} = -H_\pi, \quad (157) \]

where index in brackets means the corresponding derivative. We can use constraints following from Eq. (147) after the differentiation. We get

\[ \dot{r} = ca_n^0 + cy - c_1 q, \quad (158) \]

\[ l_0 = \bar{l} + \delta_0, \quad \delta_0 = -i \sum F_{ia(m)} c^a_i \lambda_i, \quad (159) \]

\[ y = -J_p - i \sum F_{ia} c^a_i \lambda_i, \quad y n^0 = 0, \quad (160) \]

\[ \dot{q} = ca_\pi_1 + c\sqrt{-q^2} \gamma n^2 - c_2 q, \quad (161) \]

\[ \alpha = \sqrt{q^2 \pi^2_1} (1 - t). \quad (162) \]

\[ \gamma = i \sum \left( \frac{1}{J} c^{13}_i + \frac{1}{L} F_{l1} c^{2}_i \lambda_i \right). \quad (163) \]

Eqs. (146), (147), (158) and(161) yield
\[ \mathcal{L} = -c \left( G(\bar{l}) + \delta_0 m + 2Lt + i \sum (c^{13}_i + F_{ia} c^2_i \lambda_i) \right) - (i/2) \sum \xi^M \xi_i M. \quad (164) \]

From Eq. (161), we get

\[ c = b(1 + \delta_0), \quad (165) \]
\[
\begin{align*}
\delta_b &= t - (1/2)\gamma^2, \\
v^1 &= (1 - (1/2)\gamma^2)n^1 + \gamma n^2.
\end{align*}
\]  
(166)  
(167)

Eqs. (158), (160) and (161) make it possible to find out \( l \) and \( v^0 \):

\[
\begin{align*}
l &= \sqrt{l^2} / b = l_0(1 - \delta_t), \\
\delta_l &= -\delta_b + (1/2)\epsilon^2, \\
\epsilon &= (b_2 - b_1\gamma)/l,
\end{align*}
\]  
(168)  
(169)  
(170)

\[
\begin{align*}
b_1 &= \frac{1}{m} \sum \left[ i c_i^{03} - \frac{1}{L} c_{i,j}^{02,12} + i(F_{i0}c_i^1 + F_{ii}c_i^0)\lambda_i \right], \\
b_2 &= \frac{1}{m} \sum \left[ \frac{1}{L} c_{i,j}^{01,12} - c_{i,j}^{03,23} + iF_{i0}c_i^2\lambda_i \right],
\end{align*}
\]  
(171)  
(172)

\[
v^0 = (1 + (1/2)\epsilon^2)n^0 + \epsilon(-\gamma n^1 + n^2).
\]  
(173)

Since \( v^3 = n^3 \) on the constraints surface, we can use Eqs. (167), (173) and (131) to get

\[
v^2 = (1 + (1/2)\epsilon^2 - (1/2)\gamma^2)n^2 - \gamma n^1 + \epsilon n^0.
\]  
(174)

The next step is to express all the functions of \( \tilde{l} \) as functions of \( l \) using Eqs. (159) and (169),

\[
\begin{align*}
\tilde{l} &= l + l_1, \\
l_1 &= -\delta_0 + l\delta_t,
\end{align*}
\]  
(175)  
(176)

and the property of the Grassmann variables

\[
l_t^1 = 0.
\]  
(177)

Finally, we must express \( c_i^a \) and their products through the velocity variables

\[
\begin{align*}
u_i^a &= v^a\xi_i, & a &= 0, 1, 2, 3, & u_i^a &= \xi_i^a, \\
u_i^{ab} &= u_i^a u_i^b, & u_i^{ab,ef} &= u_i^{ab} u_i^{ef},
\end{align*}
\]  
(178)  
(179)

where \( v^a \) are given by Eqs. (50), (41), (131), (167), (173), (174) and (175). Using the properties of the Grassmann variables, we obtain the Lagrangian (45), (46), (47) and (51)

\[
L = -b \left( G(l) + i \sum F_{ia}u_i^2\lambda_i \right) - (i/2) \sum \xi_i^M \dot{\xi}_i^M + \mathcal{L}_{ss},
\]  
(180)

where

\[
\begin{align*}
\mathcal{L}_{ss} &= -b \left( A + i \sum B_i\lambda_i + C\lambda_1\lambda_2 \right), \\
A &= i \sum u_i^{13} - K^{-1}u_{i,12}^{12,12}, \\
B_i &= \left( \frac{1}{lG'K} F_{i0} + \frac{l}{K^2} F_{i0} \right) u_{i,j}^{012,12} - \frac{i}{K} F_{ii} \left( u_{i,j}^{2,23} - \frac{2}{lG'K} u_{i,j}^{012,0123} \right),
\end{align*}
\]  
(181)  
(182)  
(183)
\[
C = -\frac{1}{G''} \sum_{a,b} F'_{ia} u^a_i F'_{jb} u^b_j + \left( \frac{1}{2G'} F_{i0} F'_{j0} + \frac{1}{K} F_{i1} F'_{j1} \right) u^{2,2}_{i,j} + \left( \frac{i}{K} S \left( \frac{1}{G'} F'_{i1} \right) F'_{j0} + \frac{1}{2G'} F_{i1} F'_{j0} \right) u^{2,023}_{i,j} + \left( \frac{i}{K} S \left( \frac{1}{G'} F'_{i1} \left( \frac{1}{K} F_{j1} - \frac{1}{K} F_{i1} F'_{j1} \right) \right) u^{2,123}_{i,j} - 2F_{i1} F'_{j1} \left( \frac{1}{2G'K^2} u^2_{i,j} + \frac{1}{K^2} u'^{123,123}_{i,j} \right) - \left[ \frac{2}{G'K^2} F_{i1} F'_{j1} + \frac{1}{2G'} S F_{i0} F'_{j0} + \frac{1}{G'nK^2} S F'_{i0} F''_{j0} \right] u'^{012,012}_{i,j} + \left( -\frac{2l^2}{K^3} + \frac{1}{2G''K} \left( \frac{1}{K} - \frac{1}{G'} - \frac{1}{G''} \right) F'_{i0} F''_{j0} \right) u'^{012,012}_{i,j} \right].
\]

Here all the functions depend on \( l \), the prime stands for the derivative with respect to \( l \) and
\[
SX_{ij} = X_{ij} + X_{ji}.
\]

The conserved spin constrains are
\[
\sum_a F_{ia} u^a_i + B_i - iC\lambda_j = 0,
\]
where \( i = 1 \) or 2 and \( j \neq i \).

**Appendix C: Comparison with experiment and potential quark model, parameters and predictions.**

Comparison of the SQM with the experimental meson spectrum and potential quark model, SQM parameters and predictions are collected in Tables 1C, 2C and 3C below for different meson trajectories called in correspondence with their lowest states

- \( q \) stands for quarks composing mesons; \( j^{PC} \) means \( j^{P} \) for mesons not having \( C^{--} \) or \( G^{--} \)-parity;

- \( y (\text{GeV}) \) is the main kinematical parameter of each meson, \( y = al \), where \( l \) is the solution of Eq.(61) and \( a \) is the string tension. Knowing \( y \), one can easily calculate all the other parameters of the meson wave function. In the classical approximation, \( a/y \) is angular velocity of the string;

- \( m (\text{MeV}) \) is the SQM prediction for meson mass;

- \( m_{EX} \) (in MeV) is experimental meson mass from Ref. [7] if no reference is indicated;

- indicates particles that appear in the Meson Summary Tables [7];

- \( m_{P} (\text{MeV}) \) is a potential quark model prediction from Ref. [5];

- \( n \) is a meson name; and question marks stand for experimentally unknown \( j^{PC} \).
Table 1C. Vector trajectories (wave functions $\Psi_{jM,j-1,1}, P = C = (-1)^j, j_{\text{min}} = 1$).

<table>
<thead>
<tr>
<th>q</th>
<th>j$^{PC}$</th>
<th>y</th>
<th>m</th>
<th>$m_{\text{EXP}}$</th>
<th>$m_P$</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\bar{u}$</td>
<td>1$^-$</td>
<td>0.2450</td>
<td>771</td>
<td>$770.5 \pm 0.8$</td>
<td>770</td>
<td>$\rho(770)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$781.94 \pm 0.12$</td>
<td>780</td>
<td>$\omega(782)$</td>
</tr>
<tr>
<td></td>
<td>2$^{++}$</td>
<td>0.4196</td>
<td>1319</td>
<td>$1318.1 \pm 0.6$</td>
<td>1310</td>
<td>$a_2(1320)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$1275.0 \pm 1.2$</td>
<td></td>
<td>$f_2(1270)$</td>
</tr>
<tr>
<td></td>
<td>3$^-$</td>
<td>0.5383</td>
<td>1692</td>
<td>$1691 \pm 5$</td>
<td>1680</td>
<td>$\rho_3(1690)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$1667 \pm 4$</td>
<td></td>
<td>$\omega_3(1670)$</td>
</tr>
<tr>
<td></td>
<td>4$^{++}$</td>
<td>0.6346</td>
<td>1994</td>
<td>$2060 \pm 20[9]$</td>
<td>2010</td>
<td>$h/f_4(2050)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$2010 \pm 20[10]$</td>
<td></td>
<td>$a_4(2040)$</td>
</tr>
<tr>
<td></td>
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<td>2256</td>
<td>$2330 \pm 35[11]$</td>
<td>2300</td>
<td>$\rho_5(2350)$</td>
</tr>
<tr>
<td></td>
<td>6$^{++}$</td>
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<td>2490</td>
<td>$2510 \pm 30[12]$</td>
<td>2510</td>
<td>$r/f_6(2510)$</td>
</tr>
<tr>
<td></td>
<td>7$^-$</td>
<td>0.8603</td>
<td>2703</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$s\bar{u}$</td>
<td>1$^-$</td>
<td>0.2600</td>
<td>893</td>
<td>$891.66 \pm 0.26$</td>
<td>900</td>
<td>$K^*(892)^\pm$</td>
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<tr>
<td></td>
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<td>$896.10 \pm 0.28$</td>
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<td>$K^*(892)^0$</td>
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<td>2$^+$</td>
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<td>1418</td>
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<td>1430</td>
<td>$K_2^*(1430)^\pm$</td>
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<td>$1432.4 \pm 1.3$</td>
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<td>$K_2^*(1430)^0$</td>
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<tr>
<td></td>
<td>3$^-$</td>
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<td>1781</td>
<td>$1776 \pm 7$</td>
<td>1790</td>
<td>$K_3^*(1780)$</td>
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<td>$2045 \pm 9$</td>
<td>2110</td>
<td>$K_4^*(2045)$</td>
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<tr>
<td></td>
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<td>$2382 \pm 14 \pm 19$</td>
<td>2380</td>
<td>$K_5^*(2380)$</td>
</tr>
<tr>
<td>$s\bar{s}$</td>
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<td>0.2758</td>
<td>1013</td>
<td>$1019.413 \pm 0.008$</td>
<td>1020</td>
<td>$\phi(1020)$</td>
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<td>1516</td>
<td>$1525 \pm 5$</td>
<td>1530</td>
<td>$f_2^*(1525)$</td>
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<td></td>
<td></td>
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<td>$1854 \pm 7$</td>
<td>1900</td>
<td>$\psi_3(1850)$</td>
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<td></td>
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<td>0.5636</td>
<td>1870</td>
<td></td>
<td>2200</td>
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</tr>
<tr>
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<td>0.6586</td>
<td>2160</td>
<td></td>
<td>2470</td>
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<tr>
<td></td>
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<td>0.8145</td>
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<tr>
<td>$c\bar{u}$</td>
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<td>0.3031</td>
<td>2008</td>
<td>$2010.0 \pm 0.5$</td>
<td>2040</td>
<td>$D^*(2010)^\pm$</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>$2006.7 \pm 0.5$</td>
<td></td>
<td>$D^*(2007)^0$</td>
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<tr>
<td></td>
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<td>0.4996</td>
<td>2460</td>
<td>$2458.9 \pm 2.0$</td>
<td>2500</td>
<td>$D_2^*(2460)^\pm$</td>
</tr>
<tr>
<td></td>
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<td>$2459 \pm 4$</td>
<td></td>
<td>$D_2^*(2460)^0$</td>
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<tr>
<td></td>
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<td>2777</td>
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</tr>
<tr>
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<td>0.7269</td>
<td>3039</td>
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<td>3110</td>
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<td></td>
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<td>0.8127</td>
<td>3269</td>
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</tr>
<tr>
<td>$c\bar{s}$</td>
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<td>$2112.4 \pm 0.7$</td>
<td>2130</td>
<td>$D^*_{\pm \mp}, ?$</td>
</tr>
<tr>
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27
| $^{2+}$ | 0.6116 | 3557 | 3556.17 ± 0.13 | 3550 | $\chi_{c2}(1P)$ |
| $^{3-}$ | 0.7412 | 3825 | | | |
| $^{4++}$ | 0.8415 | 4050 | | | |
| $^{5-}$ | 0.9267 | 4250 | | | |
| $b\bar{u}$ | 1$^-$ | 0.3629 | 5327 | 5324.9 ± 1.8 | 5370 | $B^*$ |
| | 2$^+$ | 0.5717 | 5716 | 5698 ± 12 | 5800 | $B_J(5732), ?$ |
| | 3$^-$ | 0.7131 | 5994 | | 6110 | |
| | 4$^+$ | 0.8262 | 6224 | | 6360 | |
| | 5$^-$ | 0.9228 | 6426 | | | |
| $b\bar{s}$ | 1$^-$ | 0.3875 | 5432 | 5416.3 ± 3.3 | 5450 | $B_s^*$ |
| | 2$^+$ | 0.5920 | 5803 | | 5880 | |
| | 3$^-$ | 0.7311 | 6073 | | 6180 | |
| | 4$^+$ | 0.8427 | 6298 | | 6430 | |
| | 5$^-$ | 0.9383 | 6497 | | | |
| $b\bar{c}$ | 1$^-$ | 0.5169 | 6489 | | 6340 | |
| | 2$^+$ | 0.7292 | 6780 | | 6770 | |
| | 3$^-$ | 0.8681 | 7003 | | 7040 | |
| | 4$^+$ | 0.9781 | 7195 | | 7270 | |
| | 5$^-$ | 1.0717 | 7368 | | | |
| $b\bar{b}$ | 1$^-$ | 0.2274 | 9463 | *9460.37 ± 0.21 | 9460 | $\Upsilon(1S)$ |
| | 2$^{++}$ | 0.8850 | 9912 | *9913.2 ± 0.6 | 9900 | $\chi_{b2}(1P)$ |
| | 3$^{--}$ | 1.0544 | 10106 | | 10160 | |
| | 4$^{++}$ | 1.1791 | 10267 | | 10360 | |
| | 5$^{--}$ | 1.2829 | 10411 | | | |

$$a = 0.176 \pm 0.002 \text{ GeV}^2,$$
$$m_s = 224 \pm 7, m_c = 1440 \pm 10, m_b = 4715 \pm 20,$$
$$m_u = 6.2 \pm 0.2, m_d = 11.1 \pm 0.4,$$
$$a_-(d\bar{u}) = a_-(s\bar{u}) = a_-(s\bar{s}) = 0.88 \pm 0.01,$$
$$a_-(c\bar{u}) = a_-(c\bar{s}) = A(c\bar{c}) = 0.90, B(c\bar{c}) = 1.43,$$
$$a_-(b\bar{u}) = a_-(b\bar{s}) = a_-(b\bar{c}) = A(b\bar{b}) = 0.77, B(b\bar{b}) = 3.14.$$
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<th>$b\bar{b}$</th>
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Table 3C. Scalar trajectories (wave functions $\Psi_{jM,j+1,1}$, $P = C = (−1)^j$, $j_{\text{min}} = 0$.)
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\[ a_+(d\bar{u}) = -0.88 \] , \( (I)a_+(s\bar{u}) = -1.59 \) , \( (II)a_+(s\bar{u}) = -1.0 \),

\[ a_+(s\bar{s}) = -0.52 \] , \( a_+(c\bar{c}) = -1.06 \), \( a_+(b\bar{b}) = -1.35 \)

\[ \frac{3\pi}{2} \]

References


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