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PHYSICAL REVIEW D

VOLUME 4, NUMBER 11

1 DECEMBER 1971

Dynamically Based Coupling Scheme for the Pomeranchukon

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(Received 8 July 1971)

We present a coupling scheme for the Pomeranchukon which relates its couplings to those of the f and f' trajectories. In this scheme the Pomeranchukon couplings factorize and transform as a combination of SU(3) singlet and octet in a prescribed ratio. This enables us to explain the observed deviation of the ratios of SU(3)-related total cross sections from their symmetry values and, qualitatively, the difference between πp and Kp diffraction slopes. A certain "universality principle" concerning the relative importance of Pomeranchukon exchange to f -trajectory exchange emerges in this scheme. This is tested with and supported by the available data on πp , pp , γp , and Kp total cross sections. The scheme accommodates quite naturally the observation of s -channel helicity conservation at high energies in πN scattering and ρ photoproduction. On the other hand, it does *not* predict s -channel helicity conservation in $\pi N \rightarrow A_1 N$. Indeed, we discuss the possibility that the observation of t -channel helicity conservation in this process may be understood on kinematical grounds. We stress that our coupling scheme is compatible with a large class of dynamical models which differ in their descriptions of the detailed nature of the Pomeranchukon. Thus, our successful confrontation with the available data must be credited to the indispensably essential assumptions that are shared by these different dynamical models.

I. INTRODUCTION

A fundamental fact of high-energy experiments is that elastic scattering and other processes which do not involve the exchange of quantum numbers are characterized by approximately constant cross sections at high energies. Such processes are known as diffraction scattering, as the shape of the differential cross section bears a distinct resemblance to that observed in classical diffraction scattering. They may be accommodated in the framework of Regge theory by introducing the

concept of the Pomeranchukon, which is by definition a singularity in the vicinity of $J=1$. The nature of this singularity is, however, unknown. Theorists have proposed moving poles, fixed poles, moving cuts, fixed cuts, double poles, and complex poles,¹ none of which can be decisively ruled out by the present data. The advent of larger machines and the consequent increased interest in multiparticle phenomena and inclusive experiments should eventually clarify the precise J -plane structure. Duality has further underscored the fact that the Pomeranchukon is most likely a qual-

itatively different object from ordinary Reggeons. At present, it is generally believed that diffraction scattering is generated by the shadow of inelastic processes and the constant total cross section is buoyed up by the opening of more and more channels at high energy. In this paper we exploit the essential features embodied in this belief while bypassing questions on the detailed nature of the Pomeranchukon.

If we entertain the assumption that the J -plane structure of the Pomeranchukon is factorizable, it is possible to discuss the properties of couplings at Pomeranchukon-particle-particle vertices. The Pomeranchukon is predominantly a positive-natural-parity object. Beyond this, several conjectures have been made on the spin-parity properties of the Pomeranchukon's couplings. Among these are Morrison's rule,² s -channel helicity conservation,³ and SU(6) selection rules.⁴ Although there is partial experimental support for each of these conjectures, none of them is correct for all reactions and none of them has been derived in the context of any dynamical model.

In this paper we would like to propose a coupling scheme⁵ which is compatible with a large class of dynamical models. The general feature of this coupling scheme is that the Pomeranchukon coupling is related to that of the lower Regge poles with vacuum quantum numbers. This is independent of the detailed J -plane structure of the various models and is therefore subject to immediate experimental tests. These tests provide quantitative as well as qualitative support for our approach.

The quantum numbers of the Pomeranchukon are quite well determined. With some certainty we may say that it has zero isospin and positive G -parity. It is popularly assumed to be an SU(3) singlet as well. However, SU(3)-symmetry-breaking effects are important. For example, $\sigma_{\text{tot}}(\pi N) \neq \sigma_{\text{tot}}(KN)$. As we shall see, we are able to account for the SU(3)-asymmetric Pomeranchukon within our coupling scheme.

In Sec. II we discuss our general hypothesis and show how it can be derived in three different dynamical models. Experimental tests of our coupling scheme are discussed in Sec. III, and a detailed discussion of s -channel helicity conservation is given in Sec. IV. A summary of our results appears in Sec. V.

II. MODELS FOR THE POMERANCHUKON

A. The Duality Hypothesis

The models we shall consider share the common feature that the Pomeranchukon is generated by the shadow of all inelastic processes. In this pic-

ture the imaginary part of the Pomeranchukon is expressed in terms of the 2-particle- n -particle amplitudes (see Fig. 1). Present data do not decisively favor any particular mechanism⁶ for the n -particle production amplitudes, and a large variety of approximations have been employed to calculate them. These various approximations, along with various consistency conditions which may be imposed on the input n -particle production mechanism and the output Pomeranchukon, lead to entirely different J -plane structures in the different models.

What we would like to show here is that in any of the models we consider, if we supplement whatever approximations are made for the $2 \rightarrow n$ amplitudes with the notion of *duality*, a common general form for the Pomeranchukon is obtained. In each case we will apply duality to a *portion* of the sum over the n -particle intermediate states. In each case it will be possible to exhibit the J -plane structure of the imaginary part of the scattering amplitude in the form

$$\text{Im } T(J, t) \equiv A(J, t) = \sum_{ij} \frac{\beta_i(J, t)}{J - \alpha_i(t)} B_{ij}(J, t) \frac{\beta_j(J, t)}{J - \alpha_j(t)} \quad (2.1)$$

for $t \approx 0$, $J \approx 1$. This equation is illustrated in Fig. 2. The sums on i and j run over all natural-parity, $I=0$, $G=+1$ trajectories coupling at the left- and right-hand vertices, respectively. The α 's denote the trajectory functions and the β 's the imaginary parts of the Regge-pole residues. The Pomeranchukon singularity is contained in the function $B_{ij}(J, t)$. Its precise nature will of course depend on the detailed assumptions of the various models, but the form (2.1) will be generally valid. As a consequence of the factorizability of the Regge poles α_i and α_j , the function $B_{ij}(J, t)$ does not depend on the external particles. Thus the Pomeranchukon communicates with the external particles only indirectly - through lower-lying Regge trajectories.

We should emphasize that the *only* new assumption necessary for the derivation of Eq. (2.1) is that *duality may be used as an aid in summing the n -particle intermediate states*. Furthermore, in

FIG. 1. The imaginary part of the amplitude is determined by the sum over all inelastic intermediate states.

FIG. 2. Schematic representation of the dual Pomeronchukon. B_{ij} contains the Pomeronchukon singularity.

models where this duality hypothesis may be applied, Eq. (2.1) will be valid regardless of the precise J -plane structure $B_{ij}(J, t)$. In the following sections we show how Eq. (2.1) may be derived in three distinct models and indicate how in each case the duality hypothesis is completely compatible with the details of the model.

B. The Two-Cluster Model

The simplest possible approximation⁵ for the inelastic intermediate states is to treat them as two distinct clusters of masses $\sqrt{s'}$ and $\sqrt{s''}$ as illustrated in Fig. 3(a). The sum over intermediate states is represented by integrals over phase space (that is, over the momentum transfers t' and t'') and over the masses of the clusters. At high energies we may assume that the clusters themselves are produced by Pomeronchukon exchange. (See Appendix A for the precise form of the amplitude.)

If we now assume that the continuum of clusters of mass $\sqrt{s'}$ can be treated as a sequence of resonances, we can introduce our duality hypothesis. Each side of Fig. 3(a) has the form of a scattering process: Pomeronchukon + particle $\rightarrow$ Pomeronchukon + particle. Duality enables us to replace the sum over resonances on each side by integrals over Regge poles as illustrated in Fig. 3(b).

It is clear that Fig. 3(b) is essentially the same as Fig. 2, and hence equivalent to Eq. (2.1). This equivalence is shown in detail in Appendix A. Alternatively one can obtain Eq. (2.1) from Fig. 3(b) by applying the rules of Gribov's Reggeon calculus.⁷ The Gribov rules must, however, be modified slightly, since the present diagram corresponds to a particular discontinuity of the amplitude rather than the full amplitude. The correct prescription is to use the imaginary parts of the Regge residues; this immediately gives us Eq. (2.1).

The form of $B_{ij}(J, t)$ and the choice of trajectories α_i in Eq. (2.1) will in this model clearly depend on the choice of the input Pomeronchukon of Fig. 3(a). This arbitrariness can be eliminated by imposing the constraint that the output Pomeronchukon possess the same J -plane singularities and carry the same internal quantum numbers as the input Pomeronchukon. These constraints fix

(a)

(b)

FIG. 3. Sum over intermediate states in the two-fireball model. Duality specifies the equivalence of sums over resonances (a) and integrals over cross-channel Regge poles (b).

the J -plane structure⁸ in the form

$$B_{ij}(J, t) = \beta(t) [J - \alpha_P(t)]^a, \quad (2.2)$$

with $a > 0$ and $\alpha_P(0) = 1$, and require the trajectories α_i to have $I = 0$, $G = +1$.

Thus we have shown that the two-cluster model, supplemented by the duality hypothesis, yields Eq. (2.1). Furthermore, our new assumption has altered none of the general features of the model. The degree of compatibility of the duality hypothesis with other features of the model is underscored by closer study.

Consider, following Hwa,⁹ the $I = 0$ t -channel partial-wave amplitude for $\pi\pi$ scattering. In the two-cluster model this amplitude has the form

$$A(J, t) = A_0(J, t) + A_1(J, t). \quad (2.3)$$

The term A_0 contains non-Pomeronchukon terms such as the f Regge pole, while A_1 is the Pomeronchukon term

$$A_1(J, t) = \beta_P(t) [J - \alpha_P(t)]^a, \quad a > 0. \quad (2.4)$$

Now $A(J, t)$ satisfies a t -channel elastic unitarity equation for $4m_\pi^2 < t < 9m_\pi^2$. Assuming that the unitarity equation may be analytically continued to $t < 0$, and assuming that A_0 contains only the f trajectory, Hwa has obtained the solution (valid for $J \approx 1$)

$$A_1(J, t) = \frac{c(J-1)^a}{[J - \alpha_f(t)][J - \alpha_f(t)]}, \quad (2.5)$$

where c is a real constant. Clearly Eq. (2.5) is a special case of Eq. (2.1) with

$$B_{ij}(J, t) = (J-1)^a \quad (2.6)$$

and α_i and α_j restricted to be the f trajectory only. Thus Eq. (2.1) can be viewed as a generalization¹⁰ of Eq. (2.5) with no direct appeal to duality.

C. The Multiperipheral Model

The two-cluster model may be generalized to allow each cluster to decompose into further clusters as the available energy increases.¹¹ Assuming a multi-Regge production mechanism for the clusters, we arrive at the multiperipheral picture of Fig. 4. The clusters of mass $\sqrt{s_i}$ are produced by multiple Pomanchukon and Regge exchange. Each cluster contains a number of stable particles; hence this picture describes a larger region of phase space than models¹² in which all of the stable particles in the final state are separated by Regge exchange. Indeed our results are similar to those obtained by Chew and Snider¹³ in a model involving only multipion exchanges.

Figure 5(a) illustrates the contribution of the multiperipheral clusters to the elastic discontinuity. The sum runs over the number of produced clusters and the integrals run over the cluster masses and the multiclusur phase space. The duality hypothesis may now be introduced exactly as in the two-cluster model. If each cluster corresponds to a sequence of resonances, then the integral over cluster masses corresponds to a sum over resonances. By duality, the sum over resonances in each leg is equivalent to a sequence of crossed-channel Regge poles. The result is represented diagrammatically by the series of Fig. 5(b). The bubbles $B(J, t)$ include double-Pomanchukon-exchange terms, Pomanchukon-Reggeon terms, and Reggeon-Reggeon terms; the links joining the bubbles are a sequence of $I=0$, $G=+1$ natural-parity Reggeons. It is evident that Fig. 5(b) has the same general form as Fig. 2 and hence that Eq. (2.1) is valid in this model as well.

We can calculate a specific form for $B_{ij}(J, t)$ if we make a few additional technical assumptions:

(a) Include in $B(J, t)$ only the double-Pomanchukon-exchange term. At infinitely high energy this is a reasonable assumption, although at present-day accelerator energies, the Reggeon terms are likely to be important.

FIG. 4. Multiperipheral approximation for n -particle production. The final-state particles come from the decay of resonances produced by multiple-Pomanchukon and -Reggeon exchange.

(b) Include in the sequence of trajectories α_i only those nearest $J=1$: the f and f' Regge poles.

(c) Treat all couplings as SU(3) symmetric. The f and f' are taken to be that mixture¹⁴ of SU(3) singlet and octet that decouples the f' from pions and nucleons. The bubble $B(J, t)$ will couple like an SU(3) singlet. In a fully self-consistent model the SU(3) nature of $B(J, t)$ would of course be determined. The singlet assumption used here could be strictly correct only if the "input" Pomanchukon (which couples to inelastic states) has different SU(3) properties from the "output" Pomanchukon which couples to elastic states. With these SU(3) assumptions the Regge propagator for each internal link in the chain of Fig. 5(b) is given by $R(J, t) + \frac{1}{2}R'(J, t)$, where $R(J, t) = [J - \alpha_f(t)]^{-1}$ is the f propagator and $R'(J, t) = [J - \alpha_{f'}(t)]^{-1}$ is the f' propagator. The coefficients of the f and f' pieces correspond to the decomposition of the singlet

$$\underline{1} = \left(\frac{2}{3}\right)^{1/2} [f + \left(\frac{1}{2}\right)^{1/2} f'] \quad (2.7)$$

The end links in the chain depend upon the quantum numbers of the external particles. In πp scattering the end links can only contain f Reggeons:

$$A_{\pi p}(J, t) = \beta_{f\pi\pi} R \beta_{fpp} + \sum_{n=0}^{\infty} \beta_{f\pi\pi} R [B(R + \frac{1}{2}R')]^n B R \beta_{fpp} \\ = \beta_{f\pi\pi} \left[R + \frac{RBR}{1 - B(R + \frac{1}{2}R')} \right] \beta_{fpp} \quad (2.8)$$

Here $\beta_{f\pi\pi}$ and β_{fpp} denote Regge residues and the J and t dependence of β , B , R , and R' has been dropped. Similarly for Kp scattering we have

$$A_{Kp}(J, t) = \beta_{f\pi\pi} \left[\frac{1}{2}R + \frac{\frac{1}{2}(R + R')BR}{1 - B(R + \frac{1}{2}R')} \right] \beta_{fpp} \quad (2.9)$$

Note that the f' Regge pole does not contribute by itself since it decouples from the nucleon vertex.

FIG. 5. (a) Sum over intermediate states in the multiperipheral model. The summation runs over the number of clusters, and the integration is over phase space and the cluster masses. Duality relates this to (b) in which the resonance sums have been replaced by Reggeons α^i . B includes Pomanchukon-Pomanchukon, Pomanchukon-Reggeon, and Reggeon-Reggeon terms.

Nevertheless the f' does couple at internal vertices.

Clearly Eqs. (2.8) and (2.9) have the same form as Eq. (1) with

$$B_{ij}(J, t) \propto \frac{B(J, t)}{1 - B(J, t)(R + \frac{1}{2}R')} . \quad (2.10)$$

Note that the apparent double pole at $J = \alpha_f(t)$ in Eq. (2.1) is not really present in Eqs. (2.8) and (2.9). Rather there are single poles given by zeros of the denominator function

$$D(J, t) = \frac{2[J - \alpha_f(t)][J - \alpha_{f'}(t)]}{3J - 2\alpha_{f'}(t) - \alpha_f(t)} - B(J, t) . \quad (2.11)$$

Now the "input" f and f' poles at $J = \alpha_{f'}(t)$ were constrained by duality¹⁴ to be ideally mixed and to be exchange-degenerate with the ρ and ϕ trajectories. If the "output" f and f' poles are to share these properties, at least approximately, then we must require that¹⁵

$$B(J, t) \simeq 0 \quad \text{for } J \simeq \alpha_f(t) \text{ and for } J \simeq \alpha_{f'}(t) . \quad (2.12)$$

With this constraint the f pole in Eqs. (2.8) and (2.9) will coincide with the input f pole, and the residue of the f' pole will vanish.

The precise form of the Pomeranchukon singularity in Eq. (2.10) depends on the choice of $B(J, t)$ (the double-Pomeranchukon-exchange bubble). A self-consistent scheme¹³ may be constructed in which the leading singularity of B_{ij} is a pole. The leading singularities¹⁶ of $B(J, t)$ are then given by

$$B(J, t) = \epsilon(t) \int dt' dt'' \frac{\theta(-\Delta)\beta(t')\beta(t'')}{\sqrt{-\Delta}[J - \alpha_P(t') - \alpha_P(t'')] + \dots} , \quad (2.13)$$

where $\epsilon(t)$ is a coupling parameter and $\beta(t)$ is the residue of the Pomeranchukon pole $\alpha_P(t)$. [Δ is defined in Eq. (A2).] Thus, for example, $B(J, 0)$ contains an Amati-Fubini-Stanghellini cut¹⁷ with a logarithmic branch point at $J = 2\alpha_P(0) - 1$. If the residue function is cut off exponentially in t , then the cut discontinuity will fall exponentially in J to give approximate accord with Eq. (2.12). If the input pole $\alpha_P(t)$ is close to one, then the logarithmic branch point in $B(J, t)$ will be close to one and hence $D(J, t)$ will have a zero close to one. Demanding consistency between the positions of the input and output poles gives the bootstrap constraint

$$D(\alpha_P(t), t) = 0 . \quad (2.14)$$

There is also a bootstrap constraint on the residue functions:

$$\beta(t) = \left\{ \frac{\partial}{\partial J} [A^{-1}(J, t)] \right\}_{J=\alpha_P(t)} . \quad (2.15)$$

Equations (2.14) and (2.15) are analogs of the

bootstrap constraints of Chew and Snider.¹³ Following Chew and Snider we can determine the parameters $\alpha_P(t)$ and $\beta(t)$ in terms of $\epsilon(t)$ from Eqs. (2.14) and (2.15). In this manner we can relate the relative strengths of the Pomeranchukon and f poles (c_P/c_f) in Eq. (2.8) to the intercept of the Pomeranchukon pole, $\alpha_P(0)$. Unfortunately, appreciable values of c_P/c_f are obtained only by lowering the Pomeranchukon intercept. For example, even for $\alpha_P(0) \sim 0.9$, $c_P/c_f < 0.3$. As we shall see in Sec. III, experiment favors values closer to unity. It is of course possible that the model we have been discussing is too simple to give a realistic prediction of c_P/c_f . Regge fits with asymptotically vanishing cross sections¹⁸ apparently require strong-cut contributions to the amplitude. Introducing such cut terms would of course change only the details of $B(J, t)$, but the general relation (2.10), which underwrites the Pomeranchukon coupling scheme proposed in this paper, would remain valid.

D. The Dual-Resonance Model

A couple of years ago Freund and Rivers¹⁹ conjectured that the Pomeranchukon in a dual model should correspond to the zero-quantum-number-exchange quark graph of Fig. 6(a). Subsequently, calculations²⁰ of the corresponding doubly twisted loop diagram in the unitarized dual-resonance model revealed an unanticipated J -plane singularity. This singularity has a $t=0$ position independent of the intercepts of the normal Regge trajectories and a slope half that of the normal slope. These features have naturally reinforced the interpretation of Fig. 6(a) as the Pomeranchukon graph. Heuristically, it is easy to see how Eq. (2.1) will arise from Fig. 6(a). One need only apply duality transformations to Fig. 6(a) to write it in the highly suggestive form Fig. 6(b).

Technically, the realization of Eq. (2.1) is rather more difficult. The singularity obtained in the orig-

FIG. 6. (a) Quark graph corresponding to Pomeranchukon exchange in meson-meson scattering. Duality demands the equivalence of (a) with (b) in which Reggeon dominance of the Pomeranchukon couplings is manifest.

inal calculation²⁰ had an intercept $\frac{1}{3}$ at $t=0$ instead of 1 and was accompanied by an unphysical cut in t . Lovelace²¹ has presented a modified model in which the normal Reggeons have a $t=0$ intercept of 1 and the Pomeranchukon an intercept of 2. The Pomeranchukon is now a pole and there is subsequently no cut in t . The degeneracy of states has been increased, however, and the four-dimensional space of the Veneziano model has been expanded to 26 dimensions. Still the model serves as a useful guide, and making appropriate *ad hoc* modifications to eliminate tachyons and to lower the trajectory intercepts, Lovelace²¹ has used it to formulate a dual-Reggeon calculus. The resulting expressions for the Pomeranchukon couplings coincide with Eq. (2.1). It should be noted that the dual-resonance model describes the full amplitude and not just its discontinuity as was the case in the models of Secs. IIB and IIC.

III. EXPERIMENTAL CONSEQUENCES

We have seen that several dynamical models seem to support the general form (2.1) for the Pomeranchukon's couplings. Independent of the detailed J -plane structure of $B_{ij}(J, t)$, this equation has several readily verifiable experimental consequences. In the applications of Eq. (2.1) discussed below we shall restrict the trajectories α_i to include only the f and f' Reggeons. On the one hand, the f and f' Reggeons are the only well-established trajectories with natural parity, $I=0$ and $G=+1$. On the other hand, contributions from trajectories lying farther below $J=1$ are suppressed by the factors $(J-\alpha_i)^{-1}$ in Eq. (2.1). Hence the qualitative features of Eq. (2.1) are certainly preserved in this approximation, although lower-lying trajectories may provide small quantitative corrections. We shall refer to this approximation loosely as the

f-coupled Pomeranchukon.

As a first application of the f -coupled-Pomeranchukon hypothesis, consider in detail the SU(3) properties of the Pomeranchukon. In the limit of exact SU(3) symmetry, the Pomeranchukon would be an SU(3) singlet coupling through degenerate f and f' trajectories. When SU(3) is broken the f' mass becomes larger than that of the f and hence the f' trajectory lies lower than the f . A simple assumption is to take this effect to be the only manifestation of the broken symmetry. In other words we can assume that masses shift but couplings remain SU(3) symmetric. This assumption is traditionally used in discussion of broken SU(3). Then B_{ij} remains a singlet, coupling with relative strengths one and $\frac{1}{2}$ to the ideally mixed¹⁴ f and f' trajectories.

The explicit dependence on α_f and $\alpha_{f'}$ in Eq. (2.1) will induce an effective symmetry breaking in the Pomeranchukon coupling. This is parameterized by the ratio

$$r(t) = \frac{\alpha_P(t) - \alpha_f(t)}{\alpha_P(t) - \alpha_{f'}(t)}. \quad (3.1)$$

Assuming $\alpha_P(0) \approx 1$, we estimate²² $r(0) \approx 0.6$ from the physical masses of particles on the f - ω and f' - ϕ exchange-degenerate trajectories. This leads to certain experimental consequences, which are summarized in Table I. Consider, for example, $\sigma_{\text{tot}}(Kp)/\sigma_{\text{tot}}(\pi p)$. By the optical theorem this is equal to

$$A(Kp \rightarrow Kp, t=0)/A(\pi p \rightarrow \pi p, t=0).$$

Using Eq. (2.1)

$$A(\pi p) = \frac{\beta_{f\pi\pi}(J, t)}{J - \alpha_f(t)} B_{ff}(J, t) \frac{\beta_{f\phi\phi}(J, t)}{J - \alpha_{f'}(t)} \quad (3.2)$$

and

TABLE I. Deviations from SU(3) symmetry: predictions and experimental figures.

Reactions $R_1 \quad R_2$		$\sigma_{\text{tot}}(R_1)/\sigma_{\text{tot}}(R_2)$		$b(R_1) - b(R_2)$	
		Theory	Experiment	Theory	Experiment
Kp	πp	$\frac{1+r}{2} \approx 0.8$	0.8 ± 0.1	$\frac{r'}{1+r}$	-0.6 ± 0.1
ωp	ρp	1	1.0 ± 0.1	0	0.1 ± 0.8
ϕp	ρp	$r \approx 0.6$	0.45 ± 0.2	r'/r	-1.3 ± 0.2
Λp	$p p$	$\frac{6-4\epsilon+r(3+\epsilon)}{3(3-\epsilon)}$	...	...	...
Σp	$p p$	$\frac{2+r(1-\epsilon)}{3-\epsilon}$	...	...	...
Ξp	$p p$	$\frac{1-\epsilon+2r}{3-\epsilon}$	...	...	...

$$A(Kp) = \frac{\beta_{fKK}(J, t)}{J - \alpha_f(t)} B_{ff}(J, t) \frac{\beta_{fpp}(J, t)}{J - \alpha_f(t)} + \frac{\beta_{f'KK}(J, t)}{J - \alpha_{f'}(t)} B_{f'f}(J, t) \frac{\beta_{fpp}(J, t)}{J - \alpha_f(t)}. \quad (3.3)$$

Now SU(3) relates

$$\beta_{fKK} = \frac{1}{\sqrt{2}} \beta_{f'KK} = \frac{1}{2} \beta_{f\pi\pi},$$

$$B_{f'f} = \frac{1}{\sqrt{2}} B_{ff},$$

so (3.3) may be written using Eq. (3.1) as

$$A(Kp) = \frac{1+r(t)}{2} A(\pi p). \quad (3.4)$$

Therefore at high energies we predict

$$\frac{\sigma_{\text{tot}}(Kp)}{\sigma_{\text{tot}}(\pi p)} \xrightarrow{s \rightarrow \infty} \frac{1+r(0)}{2} \simeq 0.8. \quad (3.5)$$

Similarly we expect $\sigma_{\text{tot}}(\omega p)/\sigma_{\text{tot}}(\rho p) = 1$ and $\sigma_{\text{tot}}(\phi p)/\sigma_{\text{tot}}(\rho p) = r(0) \simeq 0.6$. Experimental numbers for the Kp - πp ratio may be extracted from Regge fits²³⁻²⁷ to total cross-section data; as indicated in Table I the experimental ratio is consistent with our prediction (3.5). As for the vector-meson-nucleon cross sections, the measured values²⁸ of $\sigma_{\text{tot}}(\rho p)$, $\sigma_{\text{tot}}(\phi p)$, and the natural-parity contributions to $\sigma_{\text{tot}}(\omega p)$ are essentially constant (within experimental errors) above about 5 to 8 GeV. Again the ratios of these cross sections are consistent with our predictions. In Table I are included also predictions for hyperon-nucleon scattering. These predictions involve, in addition to the parameter $r \equiv r(0)$, the D/F ratio ϵ of the spin-nonflip coupling of the octet of tensor trajectories (f, A_2, K^{**}) to the baryon octet.

Another sensitive test of the f -coupled-Pomeranchukon hypothesis is provided by the comparison of the ratio of the Pomeranchukon coupling to that of the f for various processes. A summary of the results of this test is presented in Table II. The Pomeranchukon- and f -Regge terms in the Regge fits^{23-27,29} have been parametrized as $c_P \nu^{\alpha_P B(\nu)}$ and $c_f \nu^{\alpha_f}$, where $\nu = \frac{1}{2}(s-u)$ in units of GeV^2 , and c_P and c_f are the couplings of the Pomeranchukon and f singularities, respectively. The function $B(\nu)$ represents various logarithmic factors; its precise form depends upon the choice of $B_{ij}(J, 0)$. In pp scattering, for example, the Pomeranchukon contribution to the imaginary part of the forward amplitude should have the form

$$A^P(pp) = \frac{\beta_{fpp}(J, 0)}{J - \alpha_f(0)} B_{ff}(J, 0) \frac{\beta_{fpp}(J, 0)}{J - \alpha_f(0)} \quad (3.6)$$

according to Eq. (2.1). At high energies, then,

$$A^P(pp) \xrightarrow{s \rightarrow \infty} \frac{\beta_{fpp}(1, 0)}{1 - \alpha_f(0)} \nu B(\nu) \frac{\beta_{fpp}(1, 0)}{1 - \alpha_f(0)} \quad (3.7)$$

for $\alpha_P \simeq 1$. [In Eq. (3.7) the threshold factors m_p^{2J} have been absorbed in the β 's.] On the other hand, f -Reggeon exchange gives a contribution to the imaginary part of the forward amplitude of

$$A^f(pp) \xrightarrow{s \rightarrow \infty} \beta_{fpp}(\alpha_f(0), 0) \beta_{fpp}(\alpha_f(0), 0) \nu^{\alpha_f(0)}. \quad (3.8)$$

If the J dependence of $\beta_{fpp}(J, t)$ can be factorized in the form

$$\beta_{fpp}(J, t) = \bar{\beta}_{fpp}(t) f(J), \quad (3.9)$$

then the ratio $A^f(pp)/A^P(pp)$ will be independent of the labels on the external particles. Thus, for example, we expect that

$$\frac{A^f(pp)}{A^P(pp)} = \frac{A^f(\pi p)}{A^P(\pi p)}, \quad (3.10)$$

TABLE II. Relative strength of f and Pomeranchukon couplings. Numbers in each column (subject to typical errors of 20%) are predicted to be equal.

Ratio	Experiment				Theory
$\frac{c_f(\pi p)}{c_P(\pi p)}$	0.9	1.0	1.8	1.2	<i>a</i>
$\frac{c_f(pp)}{c_P(pp)}$	1.2	...	1.6	1.3	<i>a</i>
$(1+r) \frac{c_f(Kp)}{c_P(Kp)}$	0.8	0.9	1.6	1.4	<i>a</i>
$\left(1 + \frac{r}{5}\right) \frac{c_f(\gamma p)}{c_P(\gamma p)}$	1.0	...	...	...	<i>a</i>
Type of fit	Poles	Poles	Poles + cut	Regge eikonal	
$\alpha_f(0)$	0.5	0.6	0.4	0.4	
Refs.	23, 29	24, 25	26	27	

or in terms of parameters in the Regge fits of Table II,

$$\frac{c_f(pp)}{c_P(pp)} = \frac{c_f(\pi p)}{c_P(\pi p)} = a. \quad (3.11)$$

The discussion above was simplified by the fact that the f' trajectory decouples at pion and nucleon vertices. For this reason the f' Reggeon is not present in Regge fits to Kp scattering. On the other hand, in Kp scattering the Pomeranchukon can couple to the kaon vertex through the f' as well as the f . The contribution of the f' to the Pomeranchukon coupling is just $r(0)$ times that of the f , so we expect

$$[1 + r(0)] \frac{c_f(Kp)}{c_P(Kp)} = a. \quad (3.12)$$

Similarly, for γp we predict

$$\left[1 + \frac{r(0)}{5}\right] \frac{c_f(\gamma p)}{c_P(\gamma p)} = a \quad (3.13)$$

if we apply SU(3) directly³⁰ to the $f_{\gamma\gamma}$ and $f'_{\gamma\gamma}$ vertices.

In Table II we tabulate the c_f/c_P ratios for the reactions discussed. Different Regge models yield different values of the parameter $a = c_f(\pi p)/c_P(\pi p)$ for different forms of $B_{ff}(J, 0)$ and different choices of $\alpha_f(0)$. In each Regge model, however, independent determinations of the parameter a in various reactions are all consistent. Since the determination of the parameters c_f and c_P in the various fits is subject to considerable error (perhaps 20%), the universality of the parameter a exhibited in Table II is not absolutely convincing. One can only say that the Regge fits are consistent with the hypothesis of an f -coupled Pomeranchukon but not that they prove its validity. A more striking demonstration of the universality of a can be obtained by plotting the ratio

$$\frac{\sigma_{\text{tot}}(\pi^+p) + \sigma_{\text{tot}}(\pi^-p)}{\sigma_{\text{tot}}(pp) + \sigma_{\text{tot}}(\bar{p}p) + \sigma_{\text{tot}}(pn) + \sigma_{\text{tot}}(\bar{p}n)} = R(s). \quad (3.14)$$

TABLE III. Cross-section ratios for πp and pp scattering. Constancy of R tests the hypothesis that $c_f(\pi p)/c_P(\pi p) = c_f(pp)/c_P(pp)$.

p_{lab} (GeV/c)	$\sigma_{\pi^+p} + \sigma_{\pi^-p}$ (mb)	$\sigma_{pp} + \sigma_{\bar{p}p} + \sigma_{pn} + \sigma_{\bar{p}n}$ (mb)	R
6	54.7 ± 0.5	202.0 ± 7.4	0.270 ± 0.013
8	53.0 ± 0.1	195.5 ± 7.0	0.271 ± 0.010
10	51.7 ± 0.1	...	...
12	50.8 ± 0.1	185.3 ± 6.8	0.274 ± 0.010
14	50.1 ± 0.1	183.4 ± 6.9	0.273 ± 0.010
16	49.7 ± 0.1	180.8 ± 6.8	0.275 ± 0.011
18	49.2 ± 0.1	173.6 ± 14.9	0.283 ± 0.024

The combination of cross sections has been chosen to include only even-signature, $l=0$ exchange – the Pomeranchukon and the f . If c_f/c_P is indeed the same for $\pi p \rightarrow \pi p$ and $pp \rightarrow pp$, then $R(s)$ will actually be energy-independent. In Table III we evaluate³¹ $R(s)$ in the energy range $p_{\text{lab}} = 6\text{--}18$ GeV/c. The result is plotted in Fig. 7. We see that our prediction (3.11) is strikingly confirmed.

Thus far, our discussion has been restricted to $t=0$. We will be bolder¹¹ for the remainder of this section and attempt to extend our discussion to $t < 0$. First of all, our hypothesis of the f -coupled Pomeranchukon tells us nothing about the real part of the amplitude. However, it is a known phenomenological fact¹ that for $t \sim 0$ the real part is much smaller than the imaginary part of the amplitude at high energies. Secondly, some of the implicit assumptions going into our derivation of the hypothesis of the f -coupled Pomeranchukon may perhaps be expected to be less valid for $t \neq 0$. Nevertheless, it is safe to assume that if we only attempt to make some statements for t close to zero, neither of these two points would be cause for concern. Our discussion so far has required no knowledge of $B_{ij}(J, t)$ save that at $t=0$ it has a leading singularity at $J = \alpha_P(0)$. To extend our discussion

FIG. 7. Plot of the indicated combinations of total cross sections versus energy. The ratio $R(s)$ is defined in the text. That it is independent of s provides a test of the relation $c_f(pp)/c_P(pp) = c_f(\pi p)/c_P(\pi p)$.

to cover differential cross sections and to inelastic processes calls for some knowledge of the behavior of $B_{ij}(J, t)$ for $t \neq 0$. At present, one would have to rely on specific dynamical models, perhaps some of those discussed in Sec. II, to suggest how $B_{ij}(J, t)$ behaves for $t \neq 0$. In particular, $B_{ij}(J, t)$ may be expected to be damped exponentially in t . In this paper, we shall refrain from appealing exclusively to a specific dynamical model. Instead, in what follows, we shall only discuss those $t \neq 0$ consequences of the f -coupled Pomeranchukon which are independent of the behavior of $B_{ij}(J, t)$ for $t \neq 0$. The discussion will concern diffraction slopes and A_2 production.

Without making any quantitative assumptions about the behavior of $B_{ij}(J, t)$ for $t \neq 0$, we can extend our SU(3) discussion to diffraction slopes. For large s let $2b(s)$ denote the slope of the diffraction peak,

$$2b(s) = \left(\frac{\partial}{\partial t} \ln \frac{d\sigma}{dt} \right)_{t=0, \text{ large } s}. \quad (3.15)$$

The differential cross section is given by $d\sigma/dt = |F_{\text{nonflip}}|^2 + |F_{\text{flip}}|^2$, where F_x denotes the appropriate helicity amplitude. Normally $|F_{\text{flip}}/F_{\text{nonflip}}|^2 = t h(t)$ for some function $h(t)$ and $h(0)$ may or may not vanish. Helicity conservation in the s channel (see Sec. IV) implies precisely that $h(0) = 0$. Hence in processes which conserve s -channel helicity the flip amplitudes do not contribute to the diffraction slope as defined in Eq. (3.15). We shall show in Sec. IV that s -channel helicity is conserved in the elastic scattering of $\pi p, Kp, \phi p, \rho p, \omega p$. If the real part of F_{nonflip} may also be neglected in computing the diffraction slope b , we may compare diffraction slopes of processes related by SU(3). In particular, using Eq. (3.4), we can derive the relation

$$b(Kp) - b(\pi p) = \frac{r'(0)}{1 + r(0)}, \quad (3.16)$$

where

$$r'(t) = \frac{dr}{dt} = \frac{(\alpha_p - \alpha_f)(\bar{\alpha}_f - \bar{\alpha}_p) + (\alpha_p - \alpha_f)(\bar{\alpha}_p - \bar{\alpha}_f)}{(\alpha_p - \alpha_f)^2}, \quad (3.17)$$

with

$$\alpha_i = \alpha_i(t) \quad (3.18)$$

and

$$\bar{\alpha}_i = \frac{d\alpha_i(t)}{dt}. \quad (3.19)$$

If the Pomeranchukon slope is smaller than that of the f and the f' , then $r'(0)$ will be negative. With the usual slope and intercept assignments, $r'(0)$

$\sim \frac{1}{2}(\bar{\alpha}_p - 1)$. Hence $b(Kp) < b(\pi p)$. Experimentally,³² this inequality is clearly valid. It is worth remarking how our result here differs from the discussion of diffraction slopes given by the authors of Ref. 10. Chiu and Hwa make much stronger assumptions and obtain specific values for $b(\pi p)$, $b(Kp)$, and $b(pp)$. [Their result for $b(pp)$ fails to agree with experiments unless an exotic meson is present.] On the other hand, our results are weaker but more general. In particular, we can make no statement about the diffraction slope of one single process, and our result for $b(Kp) - b(\pi p)$ is completely independent of whether the diffraction slope is shrinking or not.

Proceeding exactly as in the Kp case, we can also predict that

$$b(\omega p) - b(\rho p) = 0 \quad (3.20)$$

and

$$b(\phi p) - b(\rho p) = r'(0)/r(0). \quad (3.21)$$

The numerical values of the differences (3.20) and (3.21) may be obtained from data³³ on ρ , ω , and ϕ photoproduction; these are given in Table I. From (3.16) and (3.21) we have the prediction

$$\frac{b(\phi p) - b(\rho p)}{b(Kp) - b(\pi p)} = \frac{1 + r(0)}{r(0)} \simeq 2.7. \quad (3.22)$$

The experimental value of this ratio is 2.2 ± 0.7 .

Independent of any assumptions about the behavior of $B_{ij}(J, t)$ for $t \neq 0$ one fact always remains: *The Pomeranchukon couples to particles A and B if and only if the f - A - B or f' - A - B couplings are nonzero.* This fact leads to some nontrivial selection rules when supplemented by exchange-degeneracy arguments. Consider, for example, the Pomeranchukon- π - A_2 vertex. The A_2 certainly couples to ρ - π ; hence, applying the usual arguments of exchange degeneracy to the process $\pi^+ A_2^- \rightarrow \pi^- A_2^+$, we may conclude that the A_2 couples to f - π as well. But if the f - π - A_2 coupling is nonzero, then the Pomeranchukon- π - A_2 coupling is nonzero as well. In particular, there should be a Pomeranchukon component in the process $\pi p \rightarrow A_2 p$. Cross-section data³⁴ for this process at energies of up to ~ 16 GeV are dominated by ρ and f exchange and exhibit a falloff with energy of $\sim \nu^{-1.4}$. At higher energies we predict that this rapid falloff will not be maintained.

The significance of our result for A_2 production is in allowing a facile comparison of other schemes for Pomeranchukon couplings with the one discussed here. Morrison's rule² would forbid A_2 production with an approximately constant total cross section since $\Delta P = -1$ while $\Delta J = 2$. (See Sec. IV.) Likewise, s -channel helicity conservation³ would forbid diffractive A_2 production. Kinematics

requires in this case that the s -channel helicity-conserving amplitude vanish, so if this is the only allowed helicity amplitude, then the A_2 cannot be produced. We see here evidence of the close connection between Morrison's rule and s -channel helicity conservation, and, more importantly, we conclude that neither is valid in the models of Sec. II. Because the A_2 cannot be produced in the forward direction, its production cross section may be strongly damped by $B_{ij}(J, t)$. Hence, it is the energy dependence of the total production cross section rather than its absolute size which will provide a test of our prediction. High-energy experiments on A_2 production should decide this question conclusively in the near future.

While denying that s -channel helicity conservation is a general principle, we are still able to shed some light on those reactions in which it is observed. We discuss this question in Sec. IV.

In closing this section, we should caution the reader that our discussion of the f -coupled Pomeranchukon for $t \neq 0$ is somewhat less trustworthy than our discussion for $t = 0$ for the reasons cited.

IV. HELICITY CONSERVATION IN DIFFRACTIVE PROCESSES

As mentioned in the Introduction, various conjectures have been made on the spin-parity properties of the Pomeranchukon's couplings. Among those more entrenched as folk wisdom are Morrison's rule² and SU(6) selection rules.⁴ According to Morrison, the spin change ΔJ and parity change ΔP at a Pomeranchukon-particle-particle vertex are restricted by $\Delta P = (-1)^{\Delta J}$. Others have proposed that the SU(6) properties of particles coupling to a Pomeranchukon must be the same.

More recently, it has been suggested³ that s -channel helicity is conserved in diffractive processes at high energy. The experimental evidence underwriting this suggestion came from pion-nucleon scattering and from ρ photoproduction. We shall first review this experimental evidence and then discuss what new understanding our picture of Pomeranchukon couplings has to offer. In pion-nucleon scattering the helicity amplitudes are related to the invariant amplitudes as follows

$$f_{+1/2, +1/2}^s(s, t) \underset{s \rightarrow \infty}{\sim} A(s, t) + \frac{s}{2m} B(s, t), \quad (4.1)$$

$$f_{+1/2, -1/2}^s(s, t) \underset{s \rightarrow \infty}{\sim} \frac{\sqrt{-t}}{m} [A(s, t) + mB(s, t)]. \quad (4.2)$$

We use the standard notation

$$T(s, t) = \overline{u}(p') [A(s, t) + (\not{q} + \not{q}') B(s, t)] u(p), \quad (4.3)$$

where p and q denote the incoming nucleon and pion momenta, respectively, p' and q' the outgoing

momenta. If T is dominated by a single Regge pole α , then as $s \rightarrow \infty$,

$$A(s, t) \sim \beta_A s^\alpha \quad \text{and} \quad B(s, t) \sim \beta_B s^{\alpha-1}. \quad (4.4)$$

Thus for large s and fixed t ³⁵ the helicity-nonflip amplitude $f_{+1/2, +1/2}^s$ will dominate the helicity-flip amplitude $f_{+1/2, -1/2}^s$ if the Pomeranchukon contribution to $A(s, t)$ is suppressed by a power of s . With the help of finite-energy sum rules, various authors³⁶ have isolated that part of $A(s, t)$ growing like s and have concluded that it is consistent with zero.

The experimental³⁷ evidence for s -channel helicity conservation in $\gamma p \rightarrow \rho^0 p$ is considerably more direct: Measurement of the density matrices reveals that the produced ρ is predominantly transversely polarized. At incident photon energies of 2.8 GeV and 4.7 GeV, the density matrices are consistent with s -channel helicity conservation for $|t| < 0.4 \text{ GeV}^2$. For $|t| > 0.4 \text{ GeV}^2$ there seems to be some deviation from helicity conservation. The data appear to exclude the possibility of spin conservation in the center-of-mass frame (Adair frame).

What does our Pomeranchukon coupling scheme say about helicity conservation? For our f -coupled Pomeranchukon, the study of the Pomeranchukon's helicity properties reduces to a study of the helicity properties of the f and f' trajectories. If we know something about how the f trajectory couples, from either theoretical or experimental considerations, then we can say something about how the Pomeranchukon couples. In what follows we appeal both to experimental evidence and theoretical models to show that the f trajectory indeed conserves helicity in coupling to nucleons and ρ 's.

For pion-nucleon scattering the abundance of data permits the contribution of the f trajectory to $A(s, t)$ to be isolated by the same finite-energy-sum-rule analysis which determines the Pomeranchukon contribution. The result of such an analysis³⁸ is that the f trajectory does decouple from $A(s, t)$ and hence does conserve s -channel helicity. The simultaneous decoupling of the Pomeranchukon and the f trajectory from $A(s, t)$ is a fact which – as a phenomenological curiosity – has been known for some time. Indeed some authors³⁸ have used this observation to revive the ancient belief that the f meson lies on the Pomeranchukon trajectory. This is not the case in our scheme, which nevertheless maintains an intimate rapport between the Pomeranchukon singularity and the f trajectory.

Although the experimental evidence for the f trajectory decoupling from the A amplitude is fairly strong, it will be instructive to marshal theoretical support of this hypothesis. We classify the arguments under three headings.

(a) If both the Pomeranchukon and the f trajectory decouple from $A(s, t)$, the amplitude will satisfy an unsubtracted dispersion representation. A striking demonstration of this fact is obtained by combining the Adler self-consistency theorem,³⁹ $A(s = m^2, t = \mu^2) = g^2/m$, and the no-subtraction condition to write the sum rule

$$\frac{g^2}{m} = \frac{1}{\pi} \int_{(m+\mu)^2}^{\infty} ds \operatorname{Im} A(s, t=0) \times \left[\frac{1}{s - (m+\mu)^2} + \frac{1}{s - (m-\mu)^2} \right]. \quad (4.5)$$

It turns out that the $N^*(1236)$ saturates this sum rule almost completely.⁴⁰

One may ask whether the f and f' trajectories decouple from the A amplitude for the entire baryon octet. This possibility may be disposed of immediately by looking at sum rules analogous to Eq. (4.5) for $\pi\Sigma \rightarrow \pi\Sigma$, $\pi\Lambda \rightarrow \pi\Lambda$, and $\pi\Xi \rightarrow \pi\Xi$ scattering. One finds⁴¹ that these sum rules are not saturated by low-lying resonances alone. In fact, the sum rules for the A amplitude determine that the tensor nonet couples to the baryon octet with an F/D ratio of about $\frac{1}{3}$. This particular value for the F/D ratio decouples the f trajectory from nucleons in the A amplitude, but not from the strange baryons.

(b) Other theoretical arguments also support the value $F/D = \frac{1}{3}$. In particular, it has been noted⁴² that the imposition of crossing constraints on a dual model of meson-baryon scattering seems to favor such an F/D ratio.

(c) The hypothesis⁴³ that the stress-energy tensor is dominated by the f and f' meson lends further support to the hypothesis that the f trajectory decouples from $A(s, t)$. Let us write the $fN\bar{N}$ couplings as

$$\langle p' | f_{\mu\nu} | p \rangle = \bar{u}(p') \left[g_1 (\gamma_\mu P_\nu + \gamma_\nu P_\mu) + \frac{g_2}{m} P_\mu P_\nu \right] u(p), \quad (4.6)$$

where $P = p + p'$. The matrix element $\langle p' | \theta_{\mu\nu} | p \rangle$ is assumed to be dominated by the f meson in a manner analogous to the dominance of the matrix element of the electromagnetic current by the vector mesons.⁴⁴ The two "low-momentum-transfer theorems" corresponding to the nucleon's mass and angular momentum require that $g_2 = 0$. If this result may be extrapolated from $t \sim m_f^2$ to $t \leq 0$, then it follows that the f trajectory decouples from the A amplitude in the process $\pi N \rightarrow \pi N$. It must be noted, however, that there is some incompatibility⁴⁵ between the assumptions of tensor dominance of $\theta_{\mu\nu}$, vector dominance of J_μ , and exchange-degenerate vector and tensor trajectories.

To summarize, theoretical arguments support

a picture of the A amplitude in which the tensor nonet couples to the baryon octet with $F/D = \frac{1}{3}$. This F/D ratio decouples the f from nucleons but not from Λ 's, Σ 's, or Ξ 's. Experimentally, this picture is supported by the fact that the A_2 trajectory gives nonvanishing contribution to the A amplitude in various meson-baryon scattering processes. In particular,⁴⁶ in the processes $\pi p \rightarrow \eta p$, $K^- p \rightarrow \bar{K}^0 n$, and $K^0 p \rightarrow K^+ n$, the A_2 trajectory is found to couple dominantly to the s -channel *flip* amplitude. (This coupling parallels that of the ρ trajectory as might be expected from quark-model arguments.)

Since the f does not decouple from $A(s, t)$ in $\pi\Lambda$ and $\pi\Sigma$ scattering, the f - and hence the Pomeranchukon - will not conserve s -channel helicity in these processes. This statement has important consequences. First, as noted already in Sec. III, we find that *our approach is incompatible with the general principle that s -channel helicity is conserved in all diffractive processes*, as some authors^{3,47} have proposed. Since our approach is based on rather general dynamical principles, as illustrated in Sec. II, we feel that this is a rather strong objection. Second, it should be noted that this issue is subject to experimental test. The reader should recall that in the limit $s \rightarrow \infty$, t fixed, the s -channel helicity amplitudes factorize. (A derivation of this fact is reviewed in Appendix B.) Thus, for example, we expect that s -channel helicity is conserved in $pp \rightarrow pp$ ⁴⁸ in consequence of its conservation in $\pi p \rightarrow \pi p$. Similarly, because we predict helicity not to be conserved in $\pi\Sigma \rightarrow \pi\Sigma$, it follows that helicity should not be conserved in $\Sigma p \rightarrow \Sigma p$. This process should become experimentally accessible in the near future with the advent of hyperon beams.

Lack of experimental data for ρ^0 photoproduction precludes as exhaustive a discussion of this process as of πN scattering. Experimentally, the contribution of the f trajectory to the density matrices has not been isolated. Confirmation of our coupling scheme would require that the f conserve helicity along with the Pomeranchukon. Another test of our picture will be provided by the reaction $\gamma p \rightarrow \rho^+ n$. Neither the Pomeranchukon nor the f trajectory can contribute. Nevertheless, if ρ exchange is dominant here, then ρ - f exchange degeneracy implies that s -channel helicity will be conserved in this process as well.

On the theoretical side, we can attempt to construct some simple model for the γ - ρ -Pomeranchukon vertex. Making use of the factorization of s -channel helicity amplitudes, it will suffice to study $\gamma\pi \rightarrow \rho\pi$. Invoking a mild form of vector-meson dominance, then, we will concentrate on the process $\rho\pi \rightarrow \rho\pi$.

In order to discuss helicity conservation in $\pi\rho$ scattering, we find it necessary to introduce an ansatz which has been occasionally mentioned in discussions of duality. Although the ansatz is a plausible one, there is no compelling evidence for it. When applied to $\pi\rho$ scattering, the ansatz states the following: The leading natural-parity trajectories (in $\pi\pi \rightarrow \rho\rho$) are built up by natural-parity resonances in the crossed channel ($\rho\pi \rightarrow \rho\pi$). The reader will notice that the coupling of each natural-parity resonance (of any spin) to $\pi\rho$ is uniquely determined by parity, namely, the coupling must involve the antisymmetric tensor $\epsilon_{\lambda\mu\nu\sigma}$. Hence the helicity structure of the couplings of the ρ and f trajectories in the $\pi\pi \rightarrow \rho\rho$ channel is in this model uniquely determined. Quite remarkably, that structure is such that s -channel helicity is conserved for the $\pi\rho$ scattering process. (See Appendix C for details of the calculation.) We would not conceal from the reader the fact that the hypothesis made above is a strong one. It amounts to saying that in the $\pi\rho$ channel the couplings of high-mass resonances (mass M) of unnatural parity are only $1/M^2$ as strong as the couplings of high-mass resonances (mass M) of natural parity.

The hypothesis of tensor-meson dominance of the stress-energy tensor is also relevant to the present discussion. When applied to the ρ meson,⁴⁹ supplemented by the hypothesis of vector-meson dominance of J_μ , it also leads to helicity conservation at the ρ - ρ - f vertex. In conclusion, then, we feel that present theoretical and experimental evidence on ρ photoproduction is not inconsistent with our model of an f -coupled Pomeranchukon.

The conjecture that s -channel helicity is conserved in all diffractive processes is not satisfied by all the present experimental data. In fact, in the reactions $\pi p \rightarrow A_1 p$ ^{50,51} and $K p \rightarrow K^*(1300) p$,⁵¹ it appears that it is t -channel helicity that is conserved. The history of these reactions has been a turbulent one, with experimental details and theoretical insights largely obscured by the ubiquitous Deck⁵² effect. In what follows we shall discuss A_1 production, proceeding at first as if the Deck effect did not exist. To our chagrin we will find that inclusion of the Deck effect in our discussion makes it impossible to draw any precise conclusions. The following discussion is necessarily rather tortuous.

Suppose that the " A_1 " produced in $\pi p \rightarrow A_1 p$ is a *bona fide* narrow-width particle. The $A_1 \rho \pi$ vertex may be written in the form

$$a\epsilon_A \cdot \epsilon_\rho + b(\epsilon_A \cdot p_\pi)(\epsilon_\rho \cdot p_\pi). \quad (4.7)$$

In terms of a and b the ratio of the amplitude for longitudinal decay to that for transverse decay of $A_1 \rightarrow \rho\pi$ is

$$\frac{g_{0 \rightarrow 0}}{g_{1 \rightarrow 1}} = \frac{3}{2\sqrt{2}} \left(1 - \frac{b m_A^2}{16a} \times \frac{4}{3} \right). \quad (4.8)$$

(We have simplified the kinematics by approximating $m_\pi^2 \approx 0$, $m_A^2 \approx 2m_\rho^2$.) Note the appearance of the factor of 16 in Eq. (4.8); it may be traced back to the small decay momentum $\vec{p}^2 \approx \frac{1}{16} m_A^2$ in the $A_1 \rightarrow \rho\pi$ decay. We are interested in the helicity structure of the coupling of the f trajectory (and of the exchange-degenerate ρ trajectory) at the π - A_1 vertex. A spin- J recurrence of the ρ - f trajectory has a general coupling to πA_1 of the form

$$[a_J \epsilon_A^{\mu_1} + b_J (\epsilon_A \cdot p_\pi) p_\pi^{\mu_1}] p_\pi^{\mu_2} \cdots p_\pi^{\mu_J} \phi_{\mu_1 \cdots \mu_J}, \quad (4.9)$$

where $\phi_{\mu_1 \cdots \mu_J}$ is the polarization tensor describing the recurrence. Using (4.9) we can work out the contribution of the ρ - f trajectory to the s -channel helicity amplitudes for the process $\pi\pi \rightarrow \pi A_1$. (By factorization it suffices to consider this process with no loss of generality.) We find that

$$T_{0 \rightarrow 0}^s \underset{s \rightarrow \infty}{\sim} \frac{1}{2m_A} \{2sa(t) - t[a(t) + sb(t)]\} f(s), \quad (4.10)$$

$$T_{0 \rightarrow 1}^s \underset{s \rightarrow \infty}{\sim} \sqrt{-t} [3a(t) + sb(t)] f(s).$$

For the f -coupled Pomeranchukon, the helicity structure of the diffractive part of the amplitude will have the same form. The functions $a(t)$ and $b(t)$ interpolate a_J and b_J , respectively, with $a(m_\rho^2) = a$, $b(m_\rho^2) = b$. Note that there are two independent couplings so that *a priori* there is no reason to expect that s -channel helicity is conserved in this process. (The same arguments that fixed the helicity structure of the $\pi\rho \rightarrow \pi\rho$ amplitude do not fix the helicity structure of the $\pi\pi \rightarrow \pi A_1$ amplitude.) From Eq. (4.10) we see that s -channel helicity conservation requires particular values of $a(t)$ and $b(t)$, namely, $b(t) = 0$. From the analog of Eqs. (4.10) for the t -channel helicity amplitudes, one finds that the condition for t -channel helicity conservation is that $a(t) = 0$.

The density matrix for the t -channel helicity amplitudes has the component

$$\rho_{00}^t(t) = \left(1 - \frac{4m_A^2 t}{\{t + m_A^2 - [b(t)/2a(t)](t - m_A^2)^2\}^2} \right)^{-1}. \quad (4.11)$$

[Note that $\rho_{00}(t) = 1$ if $a(t) = 0$.] Experimentally, $\rho_{00}^t(t)$ is essentially unity out to $t \approx -\frac{1}{4}$. This implies that

$$\frac{b(t = -\frac{1}{4}) m_A^2}{a(t = -\frac{1}{4})} \gtrsim 4. \quad (4.12)$$

In principle, we can calculate the ratio $b(t)m_A^2/a(t)$ at $t = m_\rho^2$ from Eq. (4.8), extrapolate to $t \lesssim 0$, and compare the result with Eq. (4.12) as a test

of the f -coupled-Pomeranchukon model. In practice the possible presence of a Deck background in the A_1 production amplitude makes this test unfeasible.

The Deck mechanism is described in Fig. 8. The incoming pion "emits" a ρ meson and is then diffractively scattered off the nucleon. This produces a broad peak (some 300 MeV wide) above the $\rho\pi$ threshold. Because the effect occurs near the $\rho\pi$ threshold, the $\rho\pi$ system will emerge predominantly in the s wave. In the Gottfried-Jackson frame⁵³ the spin of the ρ meson, when projected along the direction of the incoming pion, is zero. Thus we see that if the experimentally observed A_1 peak is purely a Deck effect, then it is natural to find that t -channel helicity is conserved in this process.

The question remains whether the A_1 is purely a Deck peak, purely a resonance, or a mixture of the two effects. This question is one of continuing controversy,⁵⁴ and in our opinion is still completely unresolved. In a partial-wave analysis⁵⁵ of the 3π final state in $\pi p \rightarrow 3\pi p$, however, one does notice a qualitative difference between the A_1 and A_2 . For example, there is a strong interference of the A_2 with background consistent with the phase variation of a Breit-Wigner formula. In contrast, the interference of the A_1 with background cannot be fitted with a Breit-Wigner formula.

We see that the extraction of a and b by means of Eq. (4.8) is problematical because of difficulties in identifying which events contain genuine A_1 resonances. Even if the A_1 's were unambiguously specified, there would remain the problem of interference between the $A_1 \rightarrow \rho\pi$ and $A_1 \rightarrow \sigma\pi$ modes. This effect can be determined only by a dynamical calculation⁵⁶ of $A_1 \rightarrow 3\pi$ in which a and b appear as inputs. If we (unjustifiably) ignore all these subtleties, three determinations of a and b are available. Ballam *et al.*⁵⁷ find $|g_{1 \rightarrow 1}/g_{0 \rightarrow 0}| = 0.48 \pm 0.13$ after subtracting a Deck-type background; Crennell *et al.*⁵⁸ find $|g_{1 \rightarrow 1}/g_{0 \rightarrow 0}| = 0.89^{+0.07}_{-0.06}$, analyzing the whole " A_1 " peak; Ioffredo *et al.*⁵⁵ support the second value. Taken at face value these numbers correspond to $bm_A^2/a \approx 6$ or 18, and $bm_A^2/a \approx 22$ or 2, respectively.

In summary we will reiterate two points. The issue of whether the " A_1 " is purely a Deck peak, purely a resonance, or a mixture, is in our opinion an unresolved question. If, however, the " A_1 " is predominantly a Deck peak, then t -channel helicity conservation in A_1 production is easily understood, not as a consequence of dynamics, but purely on "kinematical" grounds. In any case, it is difficult to make any definitive test of our model in this reaction.

FIG. 8. The Deck effect, which yields a broad peak at the A_1 mass.

V. CONCLUSION

We have presented a dynamically based coupling scheme for the Pomeranchukon. This scheme has been derived in the context of several different dynamical models, which differ in their description of multiparticle production amplitudes and thus exhibit entirely different J -plane structures. Surveying these derivations one sees that two general assumptions are necessary in each case. First, the multiparticle production amplitudes are such that particles in the final state may be grouped in a number of clusters. Second, a summation over the particles in each cluster must yield the dual crossed-channel Regge poles. We have shown that the two-cluster model and the multiperipheral model, as well as the dual-resonance model, are compatible with these assumptions.

We have shown that the f -coupled Pomeranchukon is capable of explaining SU(3) properties of total cross sections and the relative magnitude of Pomeranchukon to Regge-pole couplings in many reactions. It is also consistent with observed quantum-number selection rules and thus provides a dynamical basis for such rules. We stress that we do not expect either Morrison's rule or s -channel helicity conservation to be a property of all diffractive processes.

The experimental tests described in Secs. III and IV all involved the region $t \approx 0$. To extend the region of t to larger values, it is probably necessary to incorporate multiple-scattering terms in the model. This might be accomplished by interpreting the Pomeranchukon of our model as a term in a Regge eikonal or as an overlap function.¹

Note added. After this work was completed we received a report by P. G. O. Freund, H. F. Jones, and R. J. Rivers.⁵⁹ These authors discussed the implications of the f -coupled Pomeranchukon for the reactions $\pi N \rightarrow A_1 N$ and $\pi N \rightarrow A_2 N$ and reached conclusions similar to ours.

ACKNOWLEDGMENTS

We thank Roger Dashen and Henry Abarbanel for useful comments, and Carl Kaysen for the hospitality of the Institute for Advanced Study. One of us (M.B.G.) thanks Geoffrey Chew for a useful dis-

cussion. Another (A.Z.) thanks M. Ioffredo for providing him with experimental information. Yet another (R.C.) acknowledges the support of a National Science Foundation Postdoctoral Fellowship, and thanks the Theory Division of CERN for its hospitality during the writing of this paper.

$$A(s, t) = \frac{1}{16\pi^2 s} \int ds' ds'' \int \frac{dt' dt'' \theta(-\Delta)}{[-\Delta(t, t', t'')]^{1/2}} \beta(s', t') \beta(s'', t'') \beta^*(s', t'') \beta^*(s'', t') \left(\frac{s}{s' s''}\right)^2 \ln^{-2(1+a)} \left(\frac{s}{s' s''}\right). \quad (\text{A1})$$

The cluster masses $\sqrt{s'}$ and $\sqrt{s''}$, and the squared momentum transfers t , t' , and t'' are indicated in Fig. 3(a). The triangle function Δ is defined by

$$\Delta(t, t', t'') = t^2 + t'^2 + t''^2 - 2tt' - 2t't'' - 2t''t, \quad (\text{A2})$$

and our normalization is such that $A(s, 0) = \sigma_{\text{tot}}(s)$. For simplicity we have assumed that the input Pommeranchukon, i.e., the Pommeranchukon exchanged in Fig. 3(a), is a fixed cut of the form $(J-1)^a$, $a > 0$. If the Regge residues $\beta(s, t)$ are damped in t , then the s' and s'' integrations will be cut off at $s' s'' \simeq s$. Furthermore, the duality hypothesis allows each cluster to be replaced by a sequence of Regge poles:

$$\beta(s', t') \beta^*(s', t'') = \sum_i (s')^{\alpha_i} \beta(t') \beta(t''). \quad (\text{A3})$$

Substituting (A3) in (A1) and performing the t' and t'' integrations for $t \sim 0$ gives

$$A(s, t) \simeq \frac{1}{s} \int_{s_0}^s ds' \int_{s_0}^{s/s'} ds'' \gamma_i(t) (s')^{\alpha_i} \gamma_j(t) (s'')^{\alpha_j} \times \left(\frac{s}{s' s''}\right)^2 \ln^{-b-1} \left(\frac{s}{s' s''}\right). \quad (\text{A4})$$

$\gamma_i(t)$ is a certain reduced residue function. The exact power of the logarithm b will depend on the precise details of how the variables t' and t'' are restricted to the neighborhood of zero. Let $r = s' s'' / s$, and (A4) takes the form

$$\begin{aligned} A(s, t) &= \sum_{ij} \gamma_i \gamma_j \int_{s_0/s}^1 dr \int_{s_0}^{sr} ds' (s')^{\alpha_i - \alpha_j - 1} s^{\alpha_j} r^{\alpha_j - 2} \\ &\quad \times \ln^{-b-1}(1/r) \\ &= \sum_{ij} \gamma_i \gamma_j \int_{s_0/s}^1 dr \frac{s^{\alpha_j}}{\alpha_i - \alpha_j} r^{\alpha_j - 2} \\ &\quad \times \ln^{-b-1}(1/r) \left[(s')^{\alpha_i - \alpha_j} \right]_{s_0}^{rs}. \end{aligned} \quad (\text{A5})$$

Fix the mass scale by setting $s_0 = 1$. Then $A(s, t)$ may be written as

$$A(s, t) = \sum_{ij} \gamma_i(t) \gamma_j(t) \frac{K_i(s) - K_i(s)}{\alpha_i - \alpha_j}, \quad (\text{A6})$$

APPENDIX A

In this Appendix we give some of the details of the derivation of Eq. (2.1) in the two-cluster model. The contribution to the imaginary part of the elastic scattering amplitude arising from the production of two clusters is given by

where

$$K_i(s) = s^{\alpha_i} \int_{1/s}^1 dr \frac{r^{\alpha_i - 2}}{\ln^{1+b}(1/r)}. \quad (\text{A7})$$

Transforming (A7) by the substitution $z = (1 - \alpha_i) \ln r$ we obtain

$$K_i(s) = s^{\alpha_i} (1 - \alpha_i)^b \int_{-(1 - \alpha_i) \ln s}^0 dz (-z)^{-b-1} e^{-z}. \quad (\text{A8})$$

From (A8) it is a straightforward matter to derive the asymptotic expansion

$$K_i(s) = \frac{s}{(1 - \alpha_i) \ln^{1+b} s} \left[1 + \frac{(1+b)}{(1 - \alpha_i) \ln s} + \frac{(1+b)(2+b)}{(1 - \alpha_i)^2 \ln^2 s} + \dots \right]. \quad (\text{A9})$$

Performing a Mellin transform on (A9) gives

$$\begin{aligned} \tilde{K}_i(J) &= \frac{(J-1)^b}{1 - \alpha_i} \left[1 + \frac{1-J}{1 - \alpha_i} + \left(\frac{1-J}{1 - \alpha_i} \right)^2 + \dots \right] \\ &= \frac{(J-1)^b}{J - \alpha_i}. \end{aligned} \quad (\text{A10})$$

Substituting in (A6) we obtain the J -plane structure for $t \sim 0$,

$$\begin{aligned} \tilde{A}(J, t) &= \sum_{ij} \gamma_i(t) \gamma_j(t) \frac{(J-1)^{b-1}}{\alpha_i - \alpha_j} \left(\frac{1}{J - \alpha_i} - \frac{1}{J - \alpha_j} \right) \\ &= \sum_{ij} \frac{\gamma_i(t) (J-1)^{b-1} \gamma_j(t)}{(J - \alpha_i)(J - \alpha_j)}. \end{aligned} \quad (\text{A11})$$

This is precisely Eq. (2.1)

APPENDIX B

We will remind the reader that the s -channel helicity amplitude factorizes⁶⁰ in the limit $s \rightarrow \infty$ with t fixed. Let us consider $\rho N - \rho N$ for the sake of definiteness. The s -channel helicity amplitudes are related to the t -channel helicity amplitudes through the crossing matrix as follows:

$$\begin{aligned} f_{ab,cd}^s(s, t) &= \sum_{c', b', A, D} d_{aA}^1(\chi_\rho) d_{cc'}^1(\chi_\rho) d_{bb'}^{1/2}(\chi_N) \\ &\quad \times d_{Dd}^{1/2}(\chi_N) f_{c'A, Db'}^t(s, t). \end{aligned} \quad (\text{B1})$$

The indices a and c refer to the ρ , and the indices b and d refer to the nucleon N . Capital letters stand for antiparticles. χ_ρ and χ_N denote the appropriate crossing angles. Dominance by a J -plane singularity $\alpha(t)$ means that

$$f_{cA,Db}^t(s,t) \rightarrow \beta_{cA,Db}(t) d_{c-A,D-b}^{\alpha(t)}(\cos\theta_t) \quad (\text{B2})$$

as $s \rightarrow \infty$ with t fixed. The standard statement of factorization states that $\beta_{cA,Db}(t) = \beta_{cA}(t) \beta_{Db}(t)$. To this we add the additional mathematical statement that $d_{\lambda\mu}^J(z) \rightarrow z^J h_\lambda k_\mu$ as $z \rightarrow \infty$. Putting all these statements together we have that

$$f_{ab,cd}^s(s,t) \rightarrow s^{\alpha(t)} \sum_{c'A} d_{aA}^1(\chi_\rho) d_{cc'}^1(\chi_\rho) \beta_{c'A}(t) h_{c'-A} \times \sum_{b'D} d_{bb'}^{1/2}(\chi_N) d_{Dd}^{1/2}(\chi_N) \beta_{Dd'}(t) k_{D-b'} \quad (\text{B3})$$

$$\rightarrow s^{\alpha(t)} H_{ac}(t) K_{bd}(t). \quad (\text{B4})$$

Hence, if we are interested in the properties of $H_{ac}(t)$ it suffices to consider the simpler process $\pi\rho \rightarrow \pi\rho$. This is a powerful theorem. For instance, s -channel helicity conservation in $\pi N \rightarrow \pi N$ immediately implies that s -channel helicity is also conserved in $NN \rightarrow NN$.

APPENDIX C

In order to have a simple model for the process $\pi\rho \rightarrow \pi\rho$, we assume that the f trajectory exchanged in the t channel is constructed, in the sense of duality, out of natural-parity resonances in the direct channel (see Fig. 9). This is the simplest dynamical model which determines the helicity couplings of the f trajectory completely.

The coupling of a spin- J natural-parity resonance to $\pi\rho$ is uniquely determined up to an overall constant as

$$G([J, P = (-)^J] \rightarrow \pi(q) + \rho(\epsilon, p)) = g_J \epsilon_{\mu\nu\lambda\sigma} \epsilon_\mu p_\nu q_\lambda \phi_{\sigma\sigma_2 \dots \sigma_J} q^{\sigma_2} \dots q^{\sigma_J}, \quad (\text{C1})$$

where $\phi_{\sigma_1 \dots \sigma_J}$ is the spin- J polarization tensor.

FIG. 9. The f -trajectory exchanged in the t channel is assumed to be constructed, in the sense of duality, out of natural-parity resonances in the s channel.

The contribution of s -channel resonances to the imaginary part of the scattering amplitude for the process $\pi(q)\rho(\epsilon, p) \rightarrow \pi(q')\rho(\epsilon', p')$ is then

$$\text{Im} T(q, p \rightarrow q', p') = K \sum_J g_J^2 \epsilon_\mu p_\nu q_\lambda \epsilon'_\alpha p'_\beta q'_\gamma \times f_J(q \cdot q') \epsilon_{\mu\nu\lambda\sigma} \epsilon_{\alpha\beta\gamma\sigma} \delta(s - m_J^2), \quad (\text{C2})$$

where $f_J(q \cdot q')$ is a function of the squared momentum transfer t only, and K is some constant. In the limit $s \rightarrow \infty$, duality requires that (C2) be proportional to an f trajectory exchange. Now all terms in the sum contain the common factor

$$\epsilon_\mu p_\nu q_\lambda \epsilon'_\alpha p'_\beta q'_\gamma \epsilon_{\mu\nu\lambda\sigma} \epsilon_{\alpha\beta\gamma\sigma}. \quad (\text{C3})$$

To determine the relative strengths of the s -channel helicity amplitudes, it suffices to evaluate (C3) for ϵ and ϵ' taken to be helicity vectors in the s -channel center-of-mass system. We find that in the limit t fixed, $s \rightarrow \infty$, the helicity amplitudes satisfy the relations

$$f_{0 \rightarrow 0}^s / f_{1 \rightarrow 1}^s = O(1), \quad (\text{C4})$$

$$f_{0 \rightarrow 1}^s / f_{1 \rightarrow 1}^s = O(1/s), \quad (\text{C5})$$

$$f_{1 \rightarrow 1}^s / f_{1 \rightarrow 1}^s = O(1/s). \quad (\text{C6})$$

Hence the f trajectory conserves s -channel helicity. According to our picture, the Pomanchukon contribution would also conserve s -channel helicity. We wish to remark that these results are nontrivial and depend on the strong dynamical assumption introduced.

*Research sponsored by the National Science Foundation under Grant No. GP-16147.

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PHYSICAL REVIEW D

VOLUME 4, NUMBER 11

1 DECEMBER 1971

Helicity Structure of the P and P' Contributions to Two-Body Scattering Amplitudes*

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(Received 29 July 1971)

We study various tests for the determination of the helicity structure of the P and P' contributions to two-body scattering amplitudes. Most types of existing data are shown to be incapable of distinguishing between s - and t -channel helicity conservation. Duality tests probably indicate s -channel helicity conservation for the Pomeranchuk contribution, but are inconclusive as far as the P' is concerned. The only direct evidence for s -channel helicity conservation by the Pomeranchuk contribution is the measurement of the R parameter in the π^-p elastic scattering. The conclusion about the P' depends on what is assumed about the Pomeranchuk contribution.

I. INTRODUCTION

The helicity structure of the contributions of the P and P' to two-body hadronic scattering amplitudes has been treated in several papers.¹⁻⁴ In this paper we present various quantitative tests of the helicity structure of the P and P' contributions. We define the P' contributions as that part of the high-energy amplitude which has t -channel isospin $I_t = 0$, and is dual to the s - u crossing-symmetric combination of low-energy resonances; that is to say, not the "pure" P' trajectory, but with its accompanying cuts included (the absorbed pole, or a dual absorptive model as proposed lately by Harari⁵).

The Pomeranchuk, or diffraction, contribution is dual to the low-energy background.⁶⁻⁸ We shall assume that for $-1 \leq t \leq 0$ (GeV/c)² its real part may be neglected. Our main purpose is to confront the conjecture of s -channel helicity conservation with the possibility of t -channel helicity conservation, as it seems that the status of existing data is not conclusive on that issue. In Sec. II we treat the Pomeranchuk contribution. We first discuss existing tests, and then proceed to examine some fur-

ther experimental features. In Sec. III we study the P' contribution. We find that most tests are not sufficiently sensitive for determination of the helicity structure of the P' . We conclude, in Sec. IV, with several remarks concerning the possibilities of distinguishing s - from t -channel helicity conservation.

II. THE HELICITY STRUCTURE OF THE POMERANCHUK CONTRIBUTION

A. Studying Differential Cross Sections

1. πN Elastic Scattering

It is well known that, if one assumes s -channel helicity conservation in πN , one obtains the relation $A' \cong \nu B / (1 - t/4m^2)$. But the t -channel helicity amplitudes are obtained from A' and B by multiplying the latter by some kinematic factors. In fact, the relative contribution of the t -channel helicity amplitudes to the cross section is⁹

$$\frac{|F_{-+}^t|^2}{|F_{++}^t|^2} = \frac{(t/4m^2)[s - (m + E_{\text{lab}})^2/(1 - t/4m^2)]|B|^2}{(1 - t/4m^2)|A'|^2}, \quad (1)$$