Probing the parton densities of polarized photons at a linear $e^+e^-$ collider
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The present theoretical status of spin-dependent parton densities $\Delta f^\gamma(x, Q^2)$ of circularly polarized photons is briefly reviewed. It is then demonstrated that measurements of the deep-inelastic spin asymmetry $A_{\gamma}^1 \approx g_{\gamma}^1/F_{\gamma}^1$ and of di-jet rapidity distributions at a future linear $e^+e^-$ collider appear to be particularly suited for a determination the spin-dependent photonic quark and gluon densities, respectively. Special emphasis is devoted to a comparison of the different sources of polarized photons at a linear collider: the equivalent photon approximation and backscattered laser (Compton) photons. It is shown that backscattered laser photons are highly favorable, even indispensable, for decent measurements of the $\Delta f^\gamma(x, Q^2)$.

1. $\Delta f^\gamma$: FORMALISM AND PRESENT STATUS

Recent months have seen a substantial amount of new experimental results on unpolarized deep-inelastic electron-photon scattering from LEP [1] which provide a considerably extended kinematical coverage in $x$ and $Q^2$ as compared to all previous results since PEP and PETRA. Complementary information on the partonic structure of photons is provided by photoproduction measurements at HERA, in particular from (di-)jet production data [2], and combining these results should not only vastly improve our knowledge of the photon structure but also seriously challenges the presently available theoretical models.

A similar analysis in longitudinally polarized $e^+e^-$ and $ep$ collisions would be desirable. Measuring the difference between the independent helicity combinations of the incoming particles, $\Delta \sigma = \frac{1}{2} [\sigma(++) - \sigma(+-)]$, instead of the sum, as in unpolarized (helicity-averaged) experiments, would give access to the parton structure of circularly polarized photons, which is completely unknown experimentally so far. These distributions are defined by $\Delta f^\gamma(x, Q^2) \equiv f_{\gamma}^{++}(x, Q^2) - f_{\gamma}^{--}(x, Q^2)$, where $f_{\gamma}^{++}$ ($f_{\gamma}^{--}$) denotes the density of a parton $f$ with helicity `+' (`-') in a photon with helicity `+'.

The densities $\Delta f^\gamma$ contain information different from that contained in the unpolarized ones [defined by taking the sum in (2)], and their measurement is necessary for a thorough understanding of the partonic structure of photons.

The complete next-to-leading order (NLO) QCD framework for the $Q^2$-evolution of $\Delta f^\gamma$ and the calculation of the polarized photon structure function $g_{\gamma}^1$ was recently provided in [3] and will be briefly reviewed here (see also [4]). The $\Delta f^\gamma$ obey the well-known inhomogeneous evolution equations schematically given by

$$\frac{d\Delta q_i^\gamma}{d\ln Q^2} = \Delta k_i + (\Delta P_i \otimes \Delta q_i^\gamma)$$

(any obvious $x$ and $Q^2$ dependence is suppressed here and in what follows), where $q_i^\gamma$ stands for the flavor non-singlet (NS) quark combinations or the singlet (S) vector $\Delta q_S^\gamma \equiv (\Delta \Sigma^\gamma \Delta g^\gamma)$. The symbol $\otimes$ denotes the usual convolution in Bjorken-$x$ space. The evolution equations (3) are most conveniently solved in Mellin-$n$ moment space, where the solutions can be given analytically. The 1-loop (LO) and 2-loop (NLO) contributions to polarized photon-to-parton and parton-to-parton splitting functions, $\Delta k_i$ and $\Delta P_i$, in (3) can be found in [3] and [5], respectively. The solution of (3) can be decomposed into a ‘pointlike’ (inhomogeneous) and a ‘hadronic’ (homogeneous) part,

$$\Delta q_i^\gamma = \Delta q_i^\gamma_{PL} + \Delta q_i^\gamma_{had}$$

($i = \text{NS, S}$) and can be found in [6] (with obvious replacements of all unpolarized quantities by their...
polarized counterparts).

Spin-dependent inelastic electron-photon scattering (DIS) can be parametrized in terms of a new, polarized structure function \( g_1^\gamma (x, Q^2) \), in analogy to \( F_2^\gamma \) and \( F_L^\gamma \) in the helicity-averaged case, and the NLO expression for \( g_1^\gamma \) reads [3]

\[
g_1^\gamma = \frac{1}{2} \sum_{f=u,d,s} e_f^2 \left\{ 2 \Delta f^\gamma + \frac{\alpha_s}{2\pi} \left[ 2\Delta C_q \otimes \Delta f^\gamma \right. \right.
+ \left. \frac{1}{N_f} \left[ \Delta C_g \otimes \Delta g^\gamma \right] \right\} + \frac{3\alpha}{4\pi} \sum_{f=u,d,s} e_f^3 \Delta C_{\gamma f}. \tag{5} \]

The coefficient functions \( \Delta C_q \) and \( \Delta C_g \) can be found in the \( \overline{\text{MS}} \) scheme, e.g., in [5], and \( N_f \) being the number of active flavors. The \( \ln(1-x) \) dependence of the photonic coefficient \( \Delta C_{\gamma f} \) [3] causes perturbative instability problems for \( x \to 1 \) in the \( \overline{\text{MS}} \) scheme. But, as in the unpolarized case [6], one can overcome this ‘problem’ by absorbing \( \Delta C_{\gamma f} \) into the definition of the quark densities [3] (DIS scheme). Only the contribution of the light flavors has been written out in (5). Heavy quark contributions to \( g_1^\gamma \) should be more appropriately included via the relevant fully massive polarized boson fusion subprocesses (see, e.g., [7]).

At present one has to fully rely on theoretical models for the \( \Delta f^\gamma \). The only guidance is provided by the positivity of the helicity dependent cross sections on the r.h.s. of (1) demanding that \( |\Delta \sigma| \leq \sigma \). This can be directly translated into a useful constraint on the densities\(^2\) in terms of the already known unpolarized distributions \( f^\gamma \):

\[
|\Delta f^\gamma (x, Q^2)| \leq f^\gamma (x, Q^2) . \tag{6} \]

In addition, it was shown [8] that the first moment \( \langle n = 1 \rangle \) of \( g_1^\gamma \) vanishes irrespective of \( Q^2 \) [‘current conservation’ (CC)]. This result holds to all orders in perturbation theory and at every twist provided that the fermions in the theory have non-vanishing mass [8]. Due to the properties of the splitting and coefficient functions for \( n = 1 \), CC is automatically fulfilled for the point-like part of \( g_1^\gamma \) and hence can be enforced for the full \( g_1^\gamma \) by demanding a vanishing hadronic quark input for \( n = 1 \) (the gluon is not constrained in the \( \overline{\text{MS}} \) or DIS, scheme since \( \Delta C_g = 0 \) for \( n = 1 \)).

To obtain a realistic estimate for the theoretical uncertainties in \( \Delta f^\gamma \) coming from the unknown hadronic input, we consider two very different scenarios in LO [9,7] based on the positivity bound (6): for the first (‘maximal scenario’) we saturate (6) using the phenomenologically successful unpolarized GRV photon densities [10]

\[
\Delta f^\gamma_{\text{had}} (x, \mu^2) = f^\gamma_{\text{had}} (x, \mu^2) , \tag{7} \]

whereas the other extreme input (‘minimal scenario’) is defined by

\[
\Delta f^\gamma_{\text{had}} (x, \mu^2) = 0 \tag{8} \]

with \( \mu \simeq 0.6 \text{ GeV} \) [10]. Of the two extreme hadronic inputs only (8) satisfies CC. However, if one is interested only in the region of, say, \( x \gtrsim 0.005 \), CC could well be implemented by contributions from smaller \( x \), which do not affect the evolutions at larger \( x \). Rather than artificially enforcing the vanishing of the first moment of the \( \Delta q_0^\gamma_{\text{had}} (x, \mu^2) \) in (7), we therefore stick to the two extreme scenarios as introduced above.

Figure 1 compares our LO \( u \)-quark and gluon distributions and the structure function \( g_1^\gamma \) for the two extreme scenarios based on (7) and (8) at \( Q^2 = 10 \) and 1000 GeV\(^2 \). It is interesting to note that the pointlike evolution leads to a sizeable negative \( u \)-quark distribution for \( x \) around 0.1 in the ‘minimal’ scenario despite of the vanishing input (8). In the shown, experimentally relevant \( Q^2 \) region the quark densities in the ‘maximal’ scenario mainly have a different sign and are smaller in their absolute size. Hence one should expect spin asymmetries with opposite signs for quark dominated processes like DIS (cf. the results for \( g_1^\gamma \) in Fig. 1 and Fig. 3). On the contrary the gluon distribution remains small in the ‘minimal’ scenario for all values of \( Q^2 \), and one thus expects almost vanishing polarized cross sections and spin asymmetries for gluon dominated processes such as jet production (cf. Fig. 4 below).

\(^2\)In what follows we limit ourselves to LO which is sufficient to estimate the sensitivity of future experiments to the unknown \( \Delta f^\gamma \). Both scenarios can be straightforwardly extended to NLO, see [3] for details.
Figure 1. $x\Delta u^\gamma/\alpha$ and $xg^\gamma/\alpha$ evolved to $Q^2 = 10$ and 1000 GeV$^2$ in LO using the two extreme inputs (7) and (8). Also shown is the structure function $g_1^\gamma$ in LO.

To finish this section let us briefly turn to the two conceivable future experiments: running HERA in a polarized collider mode at some stage in the future [11] or using a linear $e^+e^-$ collider with polarized beams. Measurements of spin asymmetries in, e.g., the photoproduction of large-$p_T$ (di-)jets can then in principle reveal information on the $\Delta f^\gamma$ through the presence of ‘resolved’ photon processes in the first case. This has been extensively studied in recent years and was shown to be feasible [12,13]. Here we focus exclusively on the latter option which has not attracted much interest so far. A future polarized linear $e^+e^-$ collider can provide complementary information on the $\Delta f^\gamma$ by measuring the structure function $g_1^\gamma(x,Q^2)$ or spin asymmetries in resolved two-photon reactions [14].

2. POLARIZED PHOTON SPECTRA

Before turning to a study of possible tests and signatures for $\Delta f^\gamma$ it is worthwhile and important to explore and compare the different sources for polarized photons at a linear collider first. This is because all experimentally accessible spin asymmetries$^3$ can be roughly decomposed as

$$A \equiv \frac{\Delta\sigma}{\sigma} \approx A_{\text{flux}} \times A_{f^\gamma} \times A_{\hat{\sigma}},$$

where $(\Delta)\sigma$ denotes the (polarized) unpolarized cross section under consideration, $A_{\text{flux}}$ is the yield of polarized photons, $A_{f^\gamma}$ is related to certain combinations of $\Delta f^\gamma/f^\gamma$, and $A_{\hat{\sigma}} \equiv \Delta\hat{\sigma}/\hat{\sigma}$ is the partonic subprocess cross section asymmetry. The latter quantity is calculable in perturbative QCD, while $A_{f^\gamma}$ is sensitive to the unknown distributions $\Delta f^\gamma$ we are looking for. Obviously, the photon flux spin asymmetry in (9) should be as large as possible to make an extraction of $\Delta f^\gamma$ feasible, ideally $A_{\text{flux}} \approx 1$ largely independent of different kinematical configurations.

$^3$All spin experiments measure spin asymmetries rather than $\Delta\sigma$ directly because normalization uncertainties, etc., conveniently drop out in the cross section ratio.
The ‘standard’ description for polarized photons is the equivalent photon approximation (EPA) where a circularly polarized photon is collinearly radiated off a longitudinally polarized electron. The EPA spectrum is shown in Fig. 2(a) for a typical set of parameters at a future linear collider, and the flux spin asymmetry is approximately given by the electron-to-photon splitting function ratio

\[ A_{\text{flux}} \simeq \frac{1 - (1 - y)^2}{1 + (1 - y)^2} , \tag{10} \]

where \( y \) is the energy fraction of the photon taken away from the electron. As can be seen from (10) and Fig. 2(a), the flux asymmetry becomes rapidly less and less favorable as \( y \to 0 \). In addition the spectrum is broad, i.e., not monochromatic, which causes experimental problems in DIS since the \( e\gamma \) c.m.s. energy is not known and the momentum fraction \( x \) cannot be reconstructed as in \( ep \) scattering.

Photon beams can be obtained also via Compton backscattering of laser photons off the linac electrons \[\text{[15]}\]. These \( \gamma\gamma \) colliders, which are experimentally challenging and have not been realized yet, were so far mainly regarded as a novel tool to study, e.g., properties of intermediate mass Higgs bosons, but they are also particularly suited for QCD studies of the (un)polarized photon structure. Polarizing the electron beams and/or laser photons not only provides circularly polarized backscattered photons but also allows one to tailor the energy spectrum of the photon to one’s needs. Colliding like-handed electrons and laser photons results in a broad spectrum and colliding oppositely handed electrons and photons results in a peaked distribution for the backscattered photon as can be seen in Figs. 2(b) and (c), respectively. It is important to notice that in both cases the resulting photons are highly polarized, i.e., \( |A_{\text{flux}}| \simeq 1 \) in (9). The helicity dependent cross sections for such a \( \gamma\gamma \) collider have to be labeled by four helicities, the helicities of both the two electron beams and the two laser photons, instead of two as on the r.h.s. of (1). This allows, of course, more ways to define \( \Delta \sigma \), however, in practice the use of either of the two spectra shown in Fig. 2 seems to be the most promising option. Apart from the helicities the Compton kinematics is controlled by a dimensionless variable \( x \) which is correlated with the maximum energy of the backscattered photons. In Fig. 2 we have chosen the maximum value \( x = 4.8 \) which just prevents reconversion of the high-energetic photon into an \( e^+e^- \) pair [15].

3. \( \Delta f^\gamma \): TESTS AND SIGNATURES

Equipped with the technical framework let us now study possible tests and signatures for the \( \Delta f^\gamma \) at a future linear \( e^+e^- \) collider with a c.m.s. energy of \( \sqrt{s} = 500 \) GeV and choice of either having the EPA or backscattered laser photons as the source for polarized photons.

Figure 3 shows predictions for the DIS spin asymmetry \( A_1^\gamma \simeq g_1^\gamma/F_1^\gamma \) in LO using the two ex-
Figure 3. Predictions for the DIS spin asymmetry $g_1^\gamma/F_1^\gamma$ in LO using the backscattered laser photon spectrum according to Fig. 2(b) and the two scenarios for $\Delta f^\gamma$ as outlined above. The unpolarized structure function $F_1^\gamma$ was calculated using the LO GRV densities [10]. The error bars denote the expected statistical uncertainty for such a measurement assuming an integrated luminosity of 100 fb$^{-1}$, 70% beam polarization, and two bins in $x$ and $Q^2$ per decade.

For simplicity we ignore here experimental problems due to the unknown precise value of the $e\gamma$ c.m.s. energy caused by the tail to low $y$ in the photon flux. The peaked spectrum in Fig. 2(c) also has a tail which has to be cut-off somehow. Specially tailored spectra should lead, however, to qualitatively very similar results as in Fig. 3.

Since (di)-jet production was shown to be a very promising tool to decipher the $\Delta f^\gamma$ at a future polarized HERA $ep$ collider [12,13], we extend these studies here to linear $e^+e^-$ colliders. Di-jet production is particularly suited as it allows to define single (double) resolved photon samples on an experimental basis by demanding that the momentum fractions for one (both) photon(s) are less than some cut, say, $x_{\gamma}^\pm \lesssim 0.8$. The direct photon ‘background’, $\gamma\gamma \rightarrow q\bar{q}$, which is concentrated at $x_{\gamma}^\pm \simeq 1$ is discarded in that way. In the unpolarized case such measurements were performed at LEP2 by OPAL [16] recently.

Fig. 4 shows our expectations for the double resolved di-jet spin asymmetry $A^{2-\text{jet}}$ in LO using similar cuts in transverse momentum $p_T$ and rapidity $\eta_{\text{jet}}$ of the jets as in [16]. $A^{2-\text{jet}}$ is dominated by gluon-gluon initiated subprocesses, and hence an almost vanishing asymmetry is obtained for the minimal scenario as expected from
To actually unfold information on $\Delta f^\gamma$ it is useful to introduce the concept of ‘effective parton densities’ [17]. Although $A^{2-jet}$ is dominated by $gg$ scattering, all QCD subprocesses contribute, which considerably complicates any analysis. In the unpolarized case it was shown [17] that the ratios of dominant subprocesses, those with a $t$-channel gluon exchange, are roughly constant, i.e., $qq'/gg \simeq qq/gg \simeq 4/9$. Hence by introducing a suitable effective parton density $f_{eff} = \sum_q (q + \bar{q}) + 9g/4$ the jet cross section factorizes into these densities times a single subprocess cross section. In the polarized case this factorization is slightly broken as $qq'/gg \neq qq/gg$. However, the approximation still works surprisingly well at a level of $5-10\%$ accuracy, and the appropriate effective densities are given by [14]

$$\Delta f_{\gamma eff}^{q} = \sum_q (\Delta q^\gamma + \Delta \bar{q}^\gamma) + \frac{11}{4} \Delta g^\gamma$$

such that the polarized double resolved jet cross section can be expressed as

$$\Delta \sigma^{2-jet} \sim \Delta f_{\gamma eff}^{q} \otimes \Delta f_{\gamma eff}^\gamma \otimes \Delta \sigma_{qq' \rightarrow qq'}.$$  \hspace{1cm} (12)\n
Having extracted $\Delta f_{\gamma eff}^{q}$ from a measurement of $A^{2-jet}$ exploiting (12) and $\Delta q^\gamma$ from elsewhere, e.g., from DIS, one can determine $\Delta g^\gamma$ using (11).

4. CONCLUDING REMARKS

It was shown that a future linear $e^+e^-$ collider could reveal first information on the presently completely unmeasured parton densities of circularly polarized photons. Studies of the spin asymmetry in DIS and double resolved di-jet production appear to be particularly suited. It was demonstrated that it would be highly favorable, even mandatory in case of DIS, to have backscattered laser photons available for these measurements. Compared to a measurement of the $\Delta f^\gamma$ in resolved photoproduction processes at a polarized HERA, a linear collider would provide a much ‘cleaner’ environment since the spin-dependent proton densities do not enter here and thus do not interfere with the extraction of $\Delta f^\gamma$.  

Figure 4. Double resolved ($x_\gamma^2 \leq 0.8$) di-jet spin asymmetry $A^{2-jet}$ in LO. The unpolarized cross sections were calculated using the GRV densities [10]. For backscattered laser photons the spectrum in Fig. 2(b) was used, while in the lower part the EPA was used as in Fig. 2(a). The statistical error bars assume a luminosity of $10 \text{ fb}^{-1}$ and 70\% beam polarization.  

Fig. 1. Again, backscattered Compton photons are strongly favored compared to the equivalent photon spectrum, but a measurement might be possible also in the latter case here. It turns out that uncertainties due to the choice of the factorization scale, which are expected to be particularly pronounced in LO, cancel to a large extent in the ratio $A^{2-jet}$. For jet production in polarized $ep$ collisions it was shown that LO QCD agrees reasonably well with Monte Carlo (MC) results including parton showering and hadronization [13], however, due to the lack of a MC generator for polarized $\gamma \gamma$ collisions such a study is not possible here for the time being.
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REFERENCES

1. See, e.g., F. Erne (L3), A. Böhrer (ALEPH), and E. Clay (OPAL), these proceedings.
2. See, e.g., B. Andrieu (H1) and Y. Yamazaki (ZEUS), these proceedings.