Spin dependent structure function $g_1$ at low $x$ and low $Q^2$

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Abstract

Theoretical description of the spin dependent structure function $g_1(x,Q^2)$ in the region of low values of $x$ and $Q^2$ is presented. It contains the Vector Meson Dominance contribution and the QCD improved parton model suitably extended to the low $Q^2$ domain. Theoretical predictions are compared with the recent experimental data in the low $x$, low $Q^2$ region.

1. Introduction

Measurements of polarised deep inelastic lepton–nucleon scattering have determined the cross section asymmetries $A_1$ and spin dependent structure functions $g_1$ of the proton, deuteron and neutron in a wide kinematic range of $Q^2$ and $x$.\footnote{Here, as usual, $x = Q^2/(2pq)$ where $Q^2 = -q^2$ with $q$ and $p$ denoting the four momentum transfer between leptons and the four momentum of the nucleon respectively.} This allowed a verification of sum rules, like e.g. the Bjorken sum rule which is a fundamental relation in the QCD, and the Ellis–Jaffe sum rules. Evaluation of the sum rules requires knowledge of the structure functions $g_1$ over the entire region of $x$ as well as their evolution to a common value of $Q^2$. Since the experimentally accessible $x$ range is limited, extrapolations to $x = 0$ and $x = 1$ are necessary. Of these the former is critical since the small $x$ behaviour of $g_1(x)$ is theoretically not well established and the relevant contribution to the sum rules' integral may in principle be large.

Theoretical predictions for the structure function $g_1$ over a full range of $x$ are even more interesting than for its first moment, especially at low $x$, i.e. at high parton densities, where the new dynamical mechanisms may be revealed. Theoretical and experimental studies at low $x$ in the polarised case are thus awaited for. A possible
future polarising the proton beam at HERA would be a milestone in this field.

In the fixed target experiments the low values of $x$ are reached by lowering at the same time the values of $Q^2$. Theoretical analysis of these data therefore requires a suitable extrapolation of the structure function to the low $Q^2$ region. Low $Q^2$ phenomena and in particular a transition from the perturbative (large $Q^2$) to the nonperturbative (low $Q^2$, including $Q^2=0$) region is actively investigated in the spin insensitive experiments. In spite of a wealth of data and of a wide spectrum of ideas this field is still a major challenge in high energy physics [1]. Among the spin sensitive experiments the only available low $Q^2$ data are from the E143 experiment at SLAC [2] (moderate $x$ and low $Q^2$) and now also from the SMC at CERN [3, 4] (low $x$ and low $Q^2$). In the low $Q^2$ region one can expect that dynamical mechanisms, like the Vector Meson Dominance (VMD), can play an important role. For large $Q^2$ the VMD contribution to $g_1$ gives a power correction term and can usually be neglected. Moreover, the partonic contribution to $g_1$ which controls the structure functions in the deep inelastic domain has to be suitably extended in order to get its extrapolation to the low $Q^2$ region. The latter component will be expressed in terms of the unintegrated (spin dependent) parton distributions and we show that the corresponding representation of $g_1$ can be easily extrapolated to the low $Q^2$ region. The main purpose of our paper is therefore to construct the structure function $g_1(x, Q^2)$ which would include the VMD and the (QCD improved) parton model contributions.

The content of the paper is as follows: in the next Section we present the data on $g_1$ and comment on the Regge model predictions for $g_1$ which are often being used for $x = 0$ extrapolations. In Sec.3 we briefly present a formalism describing $g_1$ in terms of the unintegrated spin dependent parton distributions, incorporating the leading order Altarelli–Parisi evolution and the double logarithmic $\ln^2(1/x)$ resummation at low $x$. In Sec.4 we discuss the Vector Meson Dominance part of the $g_1$ which has to be included in that region since, as it has already been pointed out above, for the fixed target experiments low values of $x$ are correlated with the low values of $Q^2$. Numerical results are also presented there. Finally, in Sec.5 we give a summary of our analysis.

2. The $g_1$ data

Several experiments contributed to the spin structure function $g_1$ measurements on different targets and over different kinematic intervals. As a result, for proton and deuteron, $g_1$ was measured for $0.00006 < x < 0.8$ by the EMC [5], SMC [3, 4], E143 [2], E155 [6] and HERMES [7]. For neutron, $g_1$ was measured for $0.014 < x < 0.8$ by the E142 [8], E154 [9] and HERMES [10]. A summary of $xg_1^{p,d}(x)$ data at the measured $Q^2$ values is presented in Fig.1.
For the SMC data, $\langle x \rangle = 0.0001$ corresponds to $\langle Q^2 \rangle = 0.02$ GeV$^2$. In other experiments $g_1$ was measured with high statistical accuracy for $x \gtrsim 0.01$ and for $Q^2 > 1$ GeV$^2$ only.\footnote{The E143 measured the asymmetry $A_{1p,d,n}^{I=0,1}$ for $0.024 < x < 0.205$ and $0.31 < Q^2 < 1$ GeV$^2$ but $g_1$ was not extracted from those data.} We do not present $g_1^n$ as there are no direct measurements for $x < 0.01$ i.e. in the low $x$ region.

The lowest $x$ and $Q^2$ region was explored by the SMC due to a high energy of the muon beam and implementation of a dedicated low $x$ trigger. The results of the SMC presented in Fig.1 come from two different analyses [3, 4] which join at $x \sim 0.002$. It should be noted that a direct result of the measurements is the virtual photon–nucleon asymmetry, $A_1$. To get the $g_1$ one has to use the relation $g_1 = A_1 \cdot F_1 \equiv A_1 \cdot F_2/[2x(1 + R)]$, where $F_2 = F_L + F_T$, $R = F_L/F_T$ and $F_T = 2xF_1$ with $F_L$ and $F_T$ denoting the unpolarised nucleon structure functions corresponding to longitudinal and transverse polarisations of the virtual photon respectively. Unfortunately there have been no direct measurements of $F_2$ and $R$ in the kinematic region of the low $x$ and low $Q^2$ SMC data, i.e. for $0.00006 < x < 0.003$ and $0.01 < Q^2 < 1$ GeV$^2$. Thus the SMC used the model [11] for the $F_2$ and a parametrisation of Ref.[12] for $R$ so their results for $g_1$ are model–dependent.

The new low $x$ data of the SMC include the kinematic region where $W^2 = (p + q)^2$ is high, $W^2 \gtrsim 100$ GeV$^2$ and much larger than $Q^2$. Thus one should expect that the Regge model should be applicable there. According to the Regge model, $g_1(x, Q^2) \sim x^{-\alpha}$ for $x \to 0$ and fixed $Q^2$, where $\alpha$ denotes the intercept of the Regge pole trajectory corresponding to axial vector mesons. It is expected that $\alpha \sim 0$ for both $I = 0$ and $I = 1$ trajectories, [13]. This behaviour of $g_1$ should go smoothly to the $W^{2\alpha}$ dependence for $Q^2 \to 0$. Other considerations related to the Regge theory predict $g_1 \sim \ln x$, [14], while the model based on exchange of two nonperturbative gluons gives $g_1 \sim 2\ln(1/x) - 1$, [15]. A perverse behaviour, $g_1 \sim 1/(x\ln^2 x)$, recalled in [14], is not valid for $g_1$, [16].

In the kinematic range of the SMC data $W^2$ changes very little: from about 100 GeV$^2$ at $x = 0.1$ to about 220 GeV$^2$ at $x = 0.0001$, contrary to a quite strong change of $Q^2$ (from about 20 GeV$^2$ to about 0.01 GeV$^2$ respectively). This means that the new SMC measurements cannot test the Regge behaviour of $g_1$ through the $x$ dependence of the latter, without additional assumptions about the $Q^2$ dependence of $g_1$. A model which allows extrapolation of $g_1$ to the low $Q^2$ region is described in the next Section.
3. Partonic contribution to $g_1$

In the region of large values of $Q^2$ the spin dependent structure functions are described by the QCD improved parton model [17]. In this model $g_1 \equiv g_1^{\text{part}}$, where $g_1^{\text{part}}$ is related in a standard way to the polarised quark and antiquark distributions $\Delta q_i$ and $\Delta \bar{q}_i$ corresponding to the quark (antiquark) flavour $i$: 

$$g_1^{\text{part}}(x, Q^2) = \frac{1}{2} \sum_{i=u,d,s} e_i^2 \left[ \Delta q_i(x, Q^2) + \Delta \bar{q}_i(x, Q^2) \right].$$

(1)

In what follows we assume $\Delta \bar{q}_u = \Delta \bar{q}_d$ and set the number of flavours equal 3.

In perturbative QCD the structure function $g_1^{\text{part}}$ is controlled at low $x$ by the double logarithmic $\ln^2(1/x)$ contributions i.e. by those terms of the perturbative expansion which correspond to the powers of $\ln^2(1/x)$ at each order of the expansion [18]. It is convenient to discuss the $\ln^2(1/x)$ resummation using the formalism of the unintegrated (spin dependent) parton distributions $f_j(x', k^2)$ where $k^2$ is the transverse momentum squared of the parton $j$ and $x'$ the longitudinal momentum fraction of the parent nucleon carried by a parton [19, 20, 21]. The conventional (integrated) distributions $\Delta p_j(x, Q^2)$ (i.e. $\Delta q_u = \Delta p_{uv} + \Delta p_u$, $\Delta \bar{q}_u = \Delta p_{\bar{u}}$ etc. for quarks, antiquarks and gluons) are related in the following way to the unintegrated distributions $f_j(x', k^2)$:

$$\Delta p_j(x, Q^2) = \Delta p_j^0(x) + \int k_0^2 \frac{dk^2}{k^2} f_j(x' = x(1 + \frac{k^2}{Q^2}), k^2)$$

(2)

Here $\Delta p_j^0(x)$ denote the nonperturbative parts of the of the distributions, corresponding to $k^2 < k_0^2$ and the parameter $k_0^2$ is the infrared cut-off ($k_0^2 \sim 1 \text{ GeV}^2$). They are treated semiphenomenologically and parametrised in the form used in Refs [19, 20, 21]:

$$\Delta p_j^0(x) = C_j (1 - x)^{\eta_j}$$

(3)

In Eq.(3) we assumed $\eta_{uv} = \eta_{dv} = 3$, $\eta_{u} = \eta_{s} = 7$ and $\eta_{g} = 5$. We also used $k_0^2 = 1 \text{ GeV}^2$. The normalisation constants $C_j$ were determined by imposing the Bjorken sum rule for $\Delta u_v^0 - \Delta d_v^0$ and by requiring that the first moments of all other distributions are the same as those determined from the QCD analysis of [22].

The unintegrated distributions $f_j(x', k^2)$ are the solutions of the integral equations [19, 20, 21] which embody both the LO Altarelli-Parisi evolution [23] and the double $\ln^2(1/x')$ resummation at small $x'$. These equations combined with equations (1) and (2) lead to approximate $x^{-\lambda}$ behaviour of the $g_1^{\text{part}}$ in the $x \to 0$ limit, with $\lambda \sim 0.3$ and $\lambda \sim 1$ for the nonsinglet and singlet parts respectively which is more singular at
low $x$ than that generated by the (nonperturbative) Regge pole exchanges \(^3\). The double $\ln^2(1/x)$ effects are presumably not important in the $W^2$ range of the fixed target experiments (cf. Fig.2 in [19] and Fig. 6 in [21]) but they significantly affect $g_1$ in the low $x$ region which may be probed at the polarised HERA, [19, 20, 21]. However the formalism based on the unintegrated distributions employed here is very suitable for extrapolating $g_1$ to the region of low $Q^2$ at fixed $W^2$ [19].

Formulae (1) and (2) define partonic contribution to the structure function $g_1(x, Q^2)$. Since $x(1+k^2/Q^2) \to k^2/W^2$ for $Q^2 \to 0$ in the integrand in Eq. (2) and since $k^2 > k_0^2$ there, the $g_{1\text{part}}^j(x, Q^2)$ defined by Eqs (1) and (2) can be smoothly extrapolated to the low $Q^2$ region, including $Q^2 = 0$. In that limit, the $g_1$ should be a finite function of $W^2$, free from any kinematical singularities or zeros. The extrapolation, valid for fixed and large $W^2$, can thus be done for the $g_{1\text{part}}^j(x, Q^2)$ given by Eqs (1) and (2) provided that nonperturbative parts of the parton distributions $\Delta p_0^j(x)$ are free from kinematical singularities at $x = 0$, as in the parametrisations defined by Eq. (3). If $\Delta p_0^j(x)$ contain kinematical singularities at $x = 0$ then one may replace $\Delta p_0^j(x)$ with $\Delta p_0^j(\tilde{x})$ where $\tilde{x} = x(1+k_0^2/Q^2)$ and leave remaining parts of the calculation unchanged. After this simple rearrangement the structure function $g_{1\text{part}}^j(x, Q^2)$ can be extrapolated to the low $Q^2$ region (for fixed $W^2$) including the point $Q^2 = 0$. Possibility of extrapolation to $Q^2 = 0$ is an important property of the formalism based on the unintegrated parton distributions.

We solved equations for the functions $f_i(x', k^2)$ [19, 20, 21] and calculated the $g_{1\text{part}}^j(x, Q^2)$ from Eqs (1) and (2) using the parametrisation (3). To be precise we solved equations which resummed only the ladder diagrams contributions in that part which corresponded to the double $\ln^2(1/x)$ resummation but this approximation was completely adequate for the values of $W^2$ which are relevant for the fixed target experiments. Let us also remind that equations for the functions $f_i(x, Q^2)$ [19, 20, 21] combined with equations (1,2) are a generalisation of the LO QCD evolution equations [23] for polarised parton densities and for moderately small and large values of $x$ are equivalent to these equations.

As a consequence $g_{1\text{part}}^j$ calculated at $x$ and $Q^2$ values of the SMC measurement gives a reasonable description of the SMC data on $g_{1\text{part}}^{p,d}(x, Q^2)$, cf. Fig.1 (it does not reproduce at the same time other measurements equally well due to differences in $Q^2$ values between the experiments). For the sake of the comparison the calculated $g_{1\text{part}}^j$ was extrapolated to low values of $Q^2$ since all the data with $x \lesssim 0.001$ have $Q^2 < 1$

\(^3\)To be precise the singular $x^{-\lambda}$ behaviour with $\lambda \sim 1$ for singlet and gluon spin dependent distributions does hold in the approximation when only the ladder diagrams are retained [20]. Complete double logarithmic $\ln^2(1/x)$ resummation which includes also the non-ladder bremsstrahlung diagrams generates less singular behaviour of these distributions [21].
GeV. However the (extrapolated) $g_1^{part}$ may not be the only contribution to $g_1$ in the low $Q^2$ domain.

4. Vector Meson Dominance contribution to $g_1$

One expects that in the low $Q^2$ region an important role may be played by the VMD mechanism. The structure function should thus be represented by the sum of the partonic and VMD contributions, i.e.

$$g_1(x, Q^2) = g_1^{VMD}(x, Q^2) + g_1^{part}(x, Q^2)$$  (4)

The VMD contribution to $g_1(x, Q^2)$ can be written as:

$$g_1^{VMD}(x, Q^2) = \frac{pq}{4\pi} \sum_{v=\rho,\omega,\phi} \frac{m_v^4 \Delta\sigma_v(W^2)}{\gamma_v^2(Q^2 + m_v^2)^2}$$  (5)

In this formula the constants $\gamma_v^2$ are determined from the leptonic widths of the vector mesons [24] and $m_v$ denotes the mass of the vector meson $v$. The cross sections $\Delta\sigma_v(W^2)$ are for high energy $W^2$ given as the following combinations of the spin dependent total cross sections:

$$\Delta\sigma_v = \frac{\sigma_{1/2} - \sigma_{3/2}}{2}$$  (6)

where $\sigma_{1/2}$ and $\sigma_{3/2}$ correspond to the total vector meson - nucleon cross sections with the projections of the total spin on the vector meson momentum equal 1/2 and 3/2 respectively [25]. Unfortunately the cross-sections $\Delta\sigma_v$ are unknown. In order to estimate the VMD contribution, $g_1^{VMD}(x, Q^2)$, we assume that the cross sections $\Delta\sigma_v$ are proportional to the appropriate combinations of the nonperturbative contributions $\Delta p^0_{j}(x)$, defined by Eq.(3), to the polarised quark and antiquark distributions. For the proton we assume:

$$\frac{pq}{4\pi} \sum_{v=\rho,\omega} \frac{m_v^4 \Delta\sigma_v}{\gamma_v^2(Q^2 + m_v^2)^2} = C \left[ \frac{4}{9} \left( \Delta u^0_v(x) + 2 \Delta \bar{u}^0(x) \right) + \frac{1}{9} \left( \Delta d^0_v(x) + 2 \Delta \bar{d}^0(x) \right) \right] \frac{m_p^4}{(Q^2 + m_p^2)^2}$$  (7)

$$\frac{pq}{4\pi} \frac{m_v^4 \Delta\sigma^{\phi}_{\phi}}{\gamma_{\phi}^2(Q^2 + m_{\phi}^2)^2} = C \frac{2}{9} \Delta s^0(x) \frac{m_{\phi}^4}{(Q^2 + m_{\phi}^2)^2}$$  (8)

where $\Delta u^0(x) = \Delta p^0_{u}(x)$ etc. All distributions are parton distributions in the proton. The distributions $\Delta p^0_{j}(x)$, Eq. (3), behave as $x^0$ for $x \to 0$. As a result the cross sections $\Delta\sigma_v$ behave as $1/W^2$ at large $W^2$ that corresponds to the assumption that the corresponding Regge trajectories have their intercepts equal to zero. We include exact $x$ dependence of the nonperturbative (spin dependent) parton distributions $\Delta p^0_{j}(x)$ and not only their (constant) $x \to 0$ limits, $C_j$. This gives an extension of the VMD model to the region of moderately small values of $x$. Formally this means that we allow
additional $Q^2$ dependence of the cross-sections $\Delta \sigma_v$ in terms which are non-leading in the large $W^2$ limit, i.e. vanish faster than $1/W^2$.

We shall vary the parameter $C$ in Eqs (7) and (8) and analyse a dependence of the structure function $g_1(x, Q^2)$ upon the value of this parameter. It should be noted that the VMD part of $g_1$ vanishes at large $Q^2$ as $1/Q^4$ (contrary to the $g_1^{\text{part}}$ which scales \textit{modulo} logarithmic corrections) but it may be a dominant contribution at (very) low $Q^2$ as it is the case for the unpolarised structure functions. For low $Q^2$ we expect a dominance of the VMD part of $g_1$. In analogy with the unpolarised case we expect that it should exhaust about 80 % of the total $g_1$.

A dependence of the structure function $g_1(x, Q^2)$ given by Eqs (1) – (4) on the parameter $C$ in Eqs (7) and (8) is illustrated in Fig.2 where we plot the asymmetries $A_1(x)$ for the proton at the measured $Q^2$ and for $Q^2 < 1 \text{ GeV}^2$. We expect the VMD contribution to be dominant there. This cut selected the SMC data [3, 4] at low values of $x$ and the SLAC E143 measurements [2] at 16.2 GeV incident electron energy at higher $x$. To obtain predictions for the asymmetry $A_1$ rather than for $g_1$ we used the model [11] for the $F_2$ and two different parametrisations [2, 4] for $R$, as employed in the E143 and SMC analyses respectively.

The statistical accuracy of the SMC data is too poor to constraint the value of the coefficient $C$, i.e. of the VMD–type nonperturbative contribution to the structure function $g_1(x, Q^2)$ at low values of $Q^2$. The SLAC E143 data apparently prefer a small negative value of $C$. The model prediction without VMD contribution ($C=0$) is systematically higher than the E143 measurements. The fact that the data prefer negative value of the VMD contribution is consistent with the results obtained from the phenomenological analysis of the sum-rules [25].

Similar analysis performed for the neutron and deuteron structure functions, $g_n^1$ and $g_d^1$, where in the former case data cover narrower kinematic interval and in the latter the statistics at low $x$ is substantially poorer, turned out to be inconclusive.

5. Summary and conclusions

We have analysed the recent $g_1(x, Q^2)$ measurements at low values of $x$ and $Q^2$ within a formalism based on unintegrated spin dependent parton distributions incorporating the leading order Altarelli–Parisi evolution and the double $\ln^2(1/x)$ resummation at low $x$. A VMD–type nonperturbative part was also included since low values of $x$ in

\footnote{The E143 measured $A_1$ for $Q^2 < 1 \text{ GeV}^2$ also at 9.7 GeV incident electron energy. For these data $4 \leq W^2 \lesssim 10 \text{ GeV}^2$, i.e. above the resonance region but too small for our model to be applicable.}
the measurements correlate with low values of $Q^2$. The $\ln^2(1/x)$ effects are not yet important in the kinematic range of the fixed target experiments but the formalism based on unintegrated parton distributions, summarised by Eq.(2), is very suitable for extrapolating $g_1$ to the region of low $Q^2$. The model reproduces a general trend in the data for the proton. The statistical accuracy of the SMC measurements taken at lowest values $x$, $x > 0.00006$, and of the $Q^2$, $Q^2 > 0.01$ GeV$^2$, is however too poor to constraint the VMD contribution. A more accurate data from the SLAC E143 experiment, where $x > 0.02$ and $Q^2 > 0.5$ GeV$^2$ seem to prefer a nonzero and negative contribution of the VMD to $g_1$ of the proton.

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References


Figure 1: Summary of the $xg_1$ measurements for the proton and for the deuteron as a function of $x$ at the measured $Q^2$ obtained with different experiments. The inserted figures show the SMC data for which $Q^2 < 1 $ GeV$^2$. Errors are statistical. The curves, calculated at $x$ and $Q^2$ values at the SMC measurements result from the model described in Sec.3.
Figure 2: The asymmetry $A_1$ for the proton as a function of $x$ at the measured $Q^2$ (marked above the $x$ axis), obtained by the SMC [3, 4] and SLAC E143 [2] (at 16.2 GeV incident energy). Errors are statistical. Curves are calculated according to Eqs (1) – (5) assuming different values of $C$ in Eqs (7) and (8).