An Improved Direct Measurement of Leptonic Coupling Asymmetries with Polarized $Z^0$'s *

The SLD Collaboration**

Abstract

We report new direct measurements of the $Z^0$-lepton coupling asymmetry parameters $A_e$, $A_\mu$ and $A_\tau$, with polarized $Z^0$'s collected by the SLD detector at the SLAC Linear Collider. The parameters are extracted from the measurement of the left-right-forward-backward asymmetries for each lepton species. The 1996, 1997 and 1998 SLD runs are included in this analysis and combined with published data from the 1993-95 runs. Preliminary results are $A_e = 0.1558 \pm 0.0064$, $A_\mu = 0.137 \pm 0.016$ and $A_\tau = 0.142 \pm 0.016$. If lepton universality is assumed, a combined asymmetry parameter $A_l = 0.1523 \pm 0.0057$ results. This translates into an effective weak mixing angle $\sin^2 \theta_W^{eff} = 0.23085 \pm 0.00073$ at the $Z^0$ resonance.

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INTRODUCTION

The structure of the parity violation in the electroweak interaction can be probed directly in the production and decay of polarized $Z^0$ bosons. The parity violations of all three leptonic states are characterized by the $Z^0$-lepton coupling asymmetry parameters; $A_e$, $A_\mu$, and $A_\tau$. The standard model assumes lepton universality, so that all three species of leptonic asymmetry parameters are expected to be identical and directly related to the effective weak mixing angle, $\sin^2\theta_W^{\text{eff}}$. Measurements of leptonic asymmetry parameters at the $Z^0$ resonance provide an important test of lepton universality and the weak mixing angle \cite{1}.

We report new results on direct measurements of the asymmetry parameters $A_e$, $A_\mu$, and $A_\tau$ using leptonic $Z^0$ decays. The measurements are based on the data collected by the SLD experiment at the SLAC Linear Collider (SLC). The SLC produces polarized $Z^0$ bosons in $e^+e^-$ collisions using a polarized electron beam. The polarization allows us to form the left-right cross section asymmetry to extract the initial-state asymmetry parameter $A_e$. It also enables us to extract the final-state asymmetry parameter for lepton $l$, $A_l$, directly using the polarized forward-backward asymmetry. Experiments at the $Z^0$ resonance without beam polarization \cite{2} have measured the product of initial- and final-state asymmetry parameter, $A_e \cdot A_l$. Those same experiments have also measured the tau polarization \cite{2} which yields $A_e$ and $A_\tau$ separately. The SLC beam polarization enables us to present the only existing direct measurement of $A_\mu$. The polarized asymmetries yield the statistical enhancement on the final-state asymmetry parameter by a factor of about 25 compared to the unpolarized forward-backward asymmetry. In this report, we use the data recorded in 1996-98 at the SLD with the upgraded vertex detector. The obtained results are combined with earlier published results \cite{3}.

There are two principle goals of this study. One is to test lepton universality by comparing the three asymmetry parameters. The other purpose is to complement the weak-mixing-angle result from the left-right cross section asymmetry using the hadronic event sample \cite{4} and to add additional precision to the determination of the weak mixing angle.

THE SLC AND THE SLD

This analysis relies on the Compton polarimeter, tracking by the vertex detector and the central drift chamber (CDC), and the liquid argon calorimeter (LAC). Details about the SLC, the polarized electron source and the measurements of the electron-beam polarization with the Compton polarimeter, can be found in Refs. \cite{5} and \cite{6}. A full description of the SLD and its performance have also been described in detail elsewhere \cite{7}. Only the details most relevant to this analysis are mentioned here.
In the previous measurements [3], the analysis was restricted in the polar-angle range of $|\cos\theta| < 0.7$ because the trigger efficiency for muon-pair events and tracking efficiency fall off beyond $|\cos\theta| = 0.7$. The upgraded vertex detector and new additional trigger system improved the acceptance. The upgraded vertex detector (VXD3) [8], a pixel-based CCD vertex detector, was installed in 1996. The VXD3 consists of 3 layers which enable a self-tracking capability independent of the CDC and provides 3-layer and 2-layer coverage out to $|\cos\theta| = 0.85$ and 0.90, respectively. The self-tracking capability and wide acceptance of VXD3 give significant improvement in solid-angle coverage because high precision VXD3-hit vectors in 3-D are powerful additions to the global track finding capability. The detailed implementation of this new strategy to recover deficiencies in track finding with the CDC alone is already developed, and working well on recent SLD reconstruction data [9]. The new additional trigger for lepton-pair events is the WIC Muon Trigger (WMT). The purpose of the WMT is to trigger muon-pair events passing through the endcaps. The WMT uses data from the endcap Warm Iron Calorimeter (WIC) which consists of inner and outer sections. The WMT requires straight back-to-back tracks in the endcap WIC passing through the interaction point. In order to increase the efficiency of the WMT, only one of back-to-back inner or back-to-back outer tracks are required. Angular coverage of the WMT is $0.68 < |\cos\theta| < 0.95$ with reasonable trigger efficiency covering the lack of leptonic trigger region in the previous analysis.

**THEORY**

$A_{LR}$ and $\tilde{A}_{FB}$

Polarization-dependent asymmetries are easily computed from the tree-level differential cross section for the dominant process $e_{L,R}^− + e^+ \rightarrow Z^0 \rightarrow l^- + l^+$ at $Z^0$ resonance, where $l$ represents either a $\mu$- or a $\tau$-lepton. The differential cross section is expressed as follows:

$$\frac{d\sigma}{d\cos\theta} \propto (1 - PA_e)(1 + \cos^2\theta) + 2 (A_e - P) A_l \cos\theta,$$

where $\cos\theta$ is the angle of the outgoing lepton ($l^-$) with respect to the electron-beam direction. Photon exchange terms and, if final-state leptons are electrons, $t$-channel contributions [10] have to be taken in to account. The leptonic asymmetry parameters which refer to the initial- and final-state lepton appear in this expression as $A_e$ and $A_l$, respectively. Note that the first term, symmetric in $\cos\theta$, exhibits initial-state coupling to the electron by its dependence on $A_e$. The second term, asymmetric in $\cos\theta$, is mostly influenced by $A_l$. $P$ is the signed longitudinal polarization of the electron beam in the convention that left-handed bunches have negative sign [11].

The relationships between the asymmetry parameters and between vector and axial-vector, or left-right
The couplings, are given as follows:

\[ A_l = \frac{2g_L g_A^l}{g_L^2 + g_A^l} = \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2}. \]  

(2)

where \( g_L^l = g_V^l + g_A^l \) and \( g_R^l = g_V^l - g_A^l \). The Standard Model relates the weak mixing angle to the couplings by the expressions \( g_V^l = -\frac{1}{2} + 2\sin^2 \theta_W^f \) and \( g_A^l = -\frac{1}{2} \).

Simple asymmetries can be used to extract \( A_l \) from data; the left-right asymmetry and the left-right-forward-backward asymmetry. The left- and right-handed cross sections are obtained by integrating Eq. (1) over all \( \cos \theta \) giving \( \sigma_L^f \) or \( \sigma_R^f \) for left- and right-handed electron beams, respectively. (For convenience, we drop the superscript in the following discussions since the meaning of the expressions will be clear enough in context.) Parity violation causes \( \sigma_L \) and \( \sigma_R \) to be different. Hence, we define the left-right cross section asymmetry, \( A_{LR} \),

\[ A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}. \]  

(3)

Four cross sections are obtained by integrating forward (F) and backward (B) hemispheres separately, along with left- and right-handed polarization. Here forward (backward) means \( \cos \theta > 0 \) (\( \cos \theta < 0 \)). Based on these four possibilities, we define the polarized forward-backward asymmetry, \( \tilde{A}_{FB} \), as follows:

\[ \tilde{A}_{FB} = \frac{(\sigma_{LF} - \sigma_{LB}) - (\sigma_{RF} - \sigma_{RB})}{(\sigma_{LF} + \sigma_{LB}) + (\sigma_{RF} + \sigma_{RB})}. \]  

(4)

**Leptonic Asymmetry Parameters: \( A_e, A_\mu \) and \( A_\tau \)**

With equal luminosities for left- and right-handed electron beams, the cross sections in Eq. (3) may be replaced with the numbers of events: \( N_L \) and \( N_R \). After integrating Eq. (1) over all angles to get expressions for \( N_L \) and \( N_R \) in terms of \( P, A_e \) and \( A_l \), and after substituting in Eq. (3) for both signs of polarization, what remains is given by

\[ A_e = A_{LR}. \]  

(5)

In a similar fashion, integrating over forward or backward hemispheres, and substituting both signs of polarization in Eq. (4), gives the expression

\[ A_l = \frac{\tilde{A}_{FB}/|P|}{(1 + \frac{x_{max}^2}{3})/x_{max}}, \]  

(6)

where \( x_{max} = \cos \theta_{max} \) is the maximum polar angle accepted by the lepton-event trigger and reconstruction efficiencies.

The leptonic asymmetry parameters are particularly potent ways to measure the weak mixing angle precisely because \( A_l \) is expressed as follows:

\[ A_l = \frac{2 \left( 1 - 4 \sin^2 \theta_W^f \right)}{1 + \left( 1 - 4 \sin^2 \theta_W^f \right)^2}. \]  

(7)
making $A_l$ very sensitive to the weak mixing angle:

$$\frac{dA_l}{d\sin^2 \theta_W^{eff}} \simeq -7.9. \quad (8)$$

### The Maximum Likelihood Method

The essence of the measurement is expressed in Eqs. (3) and (4), but instead of simply counting events we perform a maximum likelihood fit, event by event, to incorporate the contributions of all the terms in the cross section and to include the effect of initial state radiation. The likelihood function for muon- and tau-pair events is defined as follows:

$$L(A_e, A_l, x) = \int ds' H(s, s') \left( Z(s', A_e, A_l, x) + Z\gamma(s', A_e, A_l, x) + \gamma(s', x) \right), \quad (9)$$

where $A_e$ and $A_\mu$ are free parameters. $A_e$ and $A_\mu$ ($A_\tau$) are determined simultaneously with the muon-pair (tau-pair) events. The integration over $s'$ is done with the program MIZA [12] to take into account the initial-state radiation from two times the beam energy $\sqrt{s}$ to the invariant mass of the propagator $\sqrt{s'}$ described by the radiator function $H(s, s')$. The spread in the beam energy has a negligible effect. The maximum likelihood fit is less sensitive to detector acceptance as a function of polar angle than the counting method, and has more statistical power. $Z(...), \gamma(...) \text{ and } Z\gamma(...)$ are the tree-level differential cross sections for $Z$ exchange, photon exchange and their interference. The integration is performed before the fit to obtain the coefficients $f_Z, f_{Z\gamma}$ and $f_\gamma$, and the likelihood function becomes

$$L(A_e, A_l, x) = f_Z \cdot Z(A_e, A_l, x) + f_{Z\gamma} \cdot Z\gamma(A_e, A_l, x) + f_\gamma \cdot \gamma(x). \quad (10)$$

These coefficients give the relative sizes of the three terms at the SLC center-of-mass energy.

As for the electron final state, it includes both $s$-channel and $t$-channel $Z^0$ and photon exchanges which gives four amplitudes and ten cross section terms. All ten terms are energy-dependent. We define a maximum likelihood function for electron-pair events by modifying Eqs. (9) and (10) including all ten terms. The integration over $s'$ is performed with DMIBA [13] to obtain the coefficients to give the relative size of the ten terms.

### ANALYSIS

### Data Sample

This study includes the data obtained during the 1996 and 1997-98 SLD runs. Results are combined with published analyses from data taken during the 1993 SLD run and the 1994-95 run. The 1996 data set
consisted of about 50,000 $Z^0$'s with about 77% polarization. The 1997-98 data sample contains about 340,000 $Z^0$'s. The beam polarization for the 1997-98 runs averaged about 73%. The data were recorded at a mean center-of-mass energy of 91.26 GeV and 91.23 GeV during 96-97 and 98 runs, respectively [14]. The branching ratio $Z^0 \rightarrow l^+l^- = 3.4\%$ so that the total branching ratio into all three lepton species combined is about 10%.

**Event Selection**

Leptonic $Z^0$ decays are characterized by their low multiplicity and high momentum charged tracks. Muons and electrons are particularly distinctive as they emerge back-to-back with little curvature from the primary interaction vertex, and tau pairs form two tightly collimated cones directed in well-defined opposite hemispheres. Lepton-pair candidates are chosen on the basis of the momentum of the charged tracks as well as from energy deposited in the calorimeter. The criteria used for the event selection give a high efficiency for finding the signal events while the backgrounds remain sufficiently low as to be almost entirely negligible.

**The pre-selection:**

- Lepton-pair candidates are initially selected by requiring the charged multiplicity between two and eight charged tracks to reduce background from hadronic $Z^0$ decays.
- The product of the sums of the charges of the tracks in each hemisphere must be -1. This insures a correct determination of the sign of the scattering angle.
- Requiring that at least one track have momentum greater than 1 GeV reduces two-photon background while leaving candidate events with a high efficiency.

After the pre-selection, additional selection criteria are applied.

**The Bhabha selection:**

- A single additional cut effectively selects $e^+e^-$ final states. We require the sum of the energies associated with the two highest momentum tracks in the event must be greater than 45 GeV as measured in the calorimeter.

**The muon-pair selection:**
• Muon final state selection starts by demanding that the invariant mass of the event, based on charged tracks, be greater than 70 GeV. Tau final states usually fail this selection.

• Since muons deposit little energy as they traverse the calorimeters, we require also that the largest energy recorded in the calorimeter by a charged track in each hemisphere be greater than zero and less than 10 GeV. Electron pairs are removed by this requirement.

The tau-pair selection:

• Tau selection requires that the largest calorimeter energy associated with a charged track in each hemisphere is less than 27.5 GeV and 20.0 GeV for the magnitude of cos θ less than or greater than 0.7, respectively, to distinguish them from $e^+e^-$ pairs.

• We take the complement of the muon event mass cut and require the event mass to be less than 70 GeV.

• At least one track must have momentum above 3 GeV to reduce backgrounds from two-photon events.

• We define the event acollinearity from the vector sums of the momenta of the tracks in the separate hemispheres, and the angle between the resultant momentum vectors must be greater than 160 degrees. This also removes two-photon events.

• Finally, the invariant mass of charged tracks in each hemisphere is required to be less than 1.8 GeV to further suppress hadronic backgrounds.

The results from the event selections are summarized in Table 1. Each event is assigned a polar production angle based on the thrust axis defined by the charged tracks. Our published results based on the 1993-95 data were restricted to the polar-angle range $|\cos \theta| < 0.7$ because of the lepton trigger and the tracking acceptance [3]. In the 1996-98 data sets, we can use a wider polar-angle range $|\cos \theta| < 0.8$ for the analysis.

Polar-angle distributions for electron-, muon- and tau-pair final states from the 1996 through 1998 data sets are shown in Fig. 1. The left-right cross section asymmetries and forward-backward angular distribution asymmetries are clearly seen. The acceptance in cos θ out to ±0.7 is uniform, but falls off at larger cos θ. Since the data are plotted out to cos θ ±0.8, it was necessary to correct the data for the acceptance efficiency in order that the fitted curve could be compared with the data. Similar fits were done for the 1993 and 1994-95 data sets. In all cases the curves fit the data well.
| Sample | Background | Efficiency for $|\cos \theta| < 0.8$ | # of Events |
|--------|------------|-----------------------------------|-------------|
| $e^+e^-$ | 1.2% $\tau^+\tau^-$ | 87.3% | 14803 |
| $\mu^+\mu^-$ | 0.2% $\tau^+\tau^-$ | 85.5% | 11867 |
| $\tau^+\tau^-$ | 0.7% $e^+e^-$ | 2% $\mu^+\mu^-$ | 78.1% | 11266 |
| | | 1.7% $2\gamma$ & 0.8% hadrons | |

**Systematic Effects and Corrections to Asymmetry Parameters**

The maximum likelihood procedure gives an excellent first estimate of the asymmetry parameters and the statistical error on each parameter. However there are several systematic effects which can bias the result:

- Uncertainty in beam polarization;
- Background;
- Uncertainty in beam energy; and
- V-A structure in $\tau$ decay.

We must estimate the systematic errors on these effects. We discuss these effects in this section and summarize in Table 2 and Table 3.

**Effect of polarization asymmetries:**

Asymmetry measurements at SLD rely critically on the time-dependent polarization. SLD has three detectors to measure the polarization, Cherenkov detector (CKV) [6], Polarized Gamma Counter (PGC) [15] and Quartz Fiber Calorimeter (QFC) [16]. Due to beamstrahlung backgrounds produced during luminosity running, only the CKV detector can make polarization measurements during beam collisions. Hence it is the primary detector and the most carefully analyzed. Dedicated electron-only runs are used to compare electron polarization measurements between the CKV, PGC and QFC detectors. The PGC and QFC results are consistent with the CKV result at the level of 0.5%. Details on the polarization measurements are discussed in Ref. [17]. The preliminary estimates of the error on the polarization are given by $\delta P/P = 0.67\%$ and 1.08\% for 1996 and 1997-98, respectively [18].
Table 2: Systematic corrections to $A_e$ and $A_\tau$ from tau and electron final states for 1996-98 data. Corrections to $A_\mu$ can be neglected.

<table>
<thead>
<tr>
<th>Final State</th>
<th>Background</th>
<th>$\delta A_e \times 10^{-4}$</th>
<th>$\delta A_\tau \times 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^-$</td>
<td>$\tau^+\tau^-$</td>
<td>$4 \pm 4$</td>
<td>$-2 \pm 2$</td>
</tr>
<tr>
<td>$\tau^+\tau^-$</td>
<td>$e^+e^-$</td>
<td>$25 \pm 25$</td>
<td>$18 \pm 18$</td>
</tr>
<tr>
<td>two photon</td>
<td>hadron</td>
<td>$13 \pm 13$</td>
<td>$-130 \pm 29$</td>
</tr>
<tr>
<td>$\tau^+\tau^-$</td>
<td>V-A</td>
<td>$-101 \pm 37$</td>
<td></td>
</tr>
</tbody>
</table>

**Effect of Backgrounds:**

Muon-pair samples are almost background free but tau-pair candidates are contaminated by electron-pairs, two-photon and hadronic events. A small percentage of tau-pairs are identified as electron-pairs. Beam-gas and cosmic ray backgrounds have been estimated and found negligible. Estimates of backgrounds are given in Table 1. These estimates have been derived from detailed Monte Carlo simulations as well as from studying the effect of cuts in background-rich samples of real data. Tau pairs are the only non-negligible backgrounds in the electron- and muon-pair samples. The tau-pairs background in the muon-pair sample is negligible since the world-averaged measurements say $A_\mu$ and $A_\tau$ are consistent within $13 \times 10^{-3}$ and the effect would be smaller than $5 \times 10^{-5}$. For the same reason, the muon-pair background in the tau-pair sample can be neglected. The $t$-channel electron-pair background, the two-photon background and the hadronic background cause small corrections to $A_\tau$.

We estimate how the backgrounds discussed above affect each asymmetry parameter by creating an ensemble of fast simulations. The simulation-data sets are generated from the same formula for the cross-sections used to fit the real data. Trial backgrounds are then superimposed on each data sets, where the shape of the background has been obtained from the shape of the data that form the particular background. Each background is normalized relative to the signal according to detailed Monte Carlo estimates. The effect of each background on each asymmetry parameter is determined from the differences between the fitted parameter values before and after inclusion of the backgrounds. The net corrections due to backgrounds and their uncertainties are given in Table 2.

**Effect of uncertainty of center of mass energy:**

The calculation of the maximum likelihood function depends on the average beam energy $\sqrt{s}$ since the coefficients in the likelihood functions (see Eq. (10)) will depend on the center-of-mass energy. During the
1998 run, a $Z^0$-peak scan [14] was performed to provide a calibration of the SLD energy spectrometer. It shows that the spectrometer measurements had a small bias and that SLD has been running slightly below the $Z^0$ resonance. Hence we redetermine the coefficients in Eq. (10) for 1998 data to correct the effect. The uncertainty due to a $\pm 1\sigma (\sim 50\text{MeV})$ [19] variation of the center-of-mass energy is estimated by computing them for the peak energy as well as for the $1\sigma$ variation.

**V-A structure in tau decays:**

The largest systematic effect for the tau analysis, indicated in Table 2, comes about because we measure not the taus themselves, but their decay products. The longitudinal spin projections of the two taus from $Z^0$ decay are 100% anti-correlated: one will be left-handed and the other right-handed. So, given the V-A structure of tau decay [20], the decay products from the $\tau^+$ and the $\tau^-$ from a particular $Z^0$ decay will take their energies from the same set of spectra. For example, if both taus decay to $\pi\nu$, then both pions will generally be low in energy (in the case of a left handed $\tau^-$ and right handed $\tau^+$) or both will be generally higher in energy. The effect is strong at SLD because the high beam polarization induces very high tau polarization as a function of polar-production angle. And, most importantly, the sign of the polarization is basically opposite for left- and right-handed polarized beam events. So a cut on event mass causes polar-angle dependence in selection efficiency for taus which has the opposite effect for taus from events produced with the left- and right-handed polarized electron beam. Taking all tau decay modes into account, using Monte Carlo simulation, we find an overall shift of $+0.0130 \pm 0.0029$ on $A_\tau$ (where the uncertainty is mostly from Monte Carlo statistics and the value extracted from the fit must be reduced by this amount). $A_e$ is not affected since the overall relative efficiencies for left-beam and right-beam events are not changed much (only the polar angle dependence of the efficiencies are changed).

The above-mentioned systematic effects are non-negligible, although small compared with current statistical errors. Other potential corrections are discussed below. Their effect on the asymmetry parameters is deemed negligible for the current measurements.

**Effect of detector asymmetries:**

Since there will generally be no bias in the fit as long the efficiency is symmetric in $\cos \theta$, there will be a problem only if the efficiency for detecting positive tracks is different from that of negative tracks. We estimate this effect by examining the relative numbers of opposite sign back-to-back tracks with positive-positive and negative-negative pairs. The latter will occur whenever one of the two back-to-back tracks in a two-pronged event has a wrong sign of measured charge. Double charge mismeasurement is less likely. The
correction for biases due to charge mismeasurement is found to be negligible.

**Final state thrust angle resolution:**

We have also studied the effect of uncertainty in the thrust axis by smearing the directions of outgoing tracks. Final state QED radiation can affect the determination of the track angle particularly for electrons, although we find the angle to be well-determined in that case as well. The result depends somewhat on how final pairs are selected but this source of correction is also deemed negligible from our studies.

**Summary of systematic errors:**

Table 3 summarizes the systematic errors on the asymmetry parameters due to the contributing factors discussed above. The superscript on each parameter indicates the lepton species from which that particular parameter was determined. For example, \( A_e^{\mu} \) refers to the estimate of \( A_e \) obtained through the dependence expressed in Eq. (1) by analyzing the muon pairs.

<table>
<thead>
<tr>
<th>Source</th>
<th>( A_e^e )</th>
<th>( A_e^\mu )</th>
<th>( A_e^\tau )</th>
<th>( A_\mu^\mu )</th>
<th>( A_\tau^\tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polarization</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Backgrounds</td>
<td>4</td>
<td>–</td>
<td>25</td>
<td>–</td>
<td>22</td>
</tr>
<tr>
<td>Radiative Correction</td>
<td>30</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>V-A</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>29</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td>34</td>
<td>16</td>
<td>30</td>
<td>16</td>
<td>40</td>
</tr>
</tbody>
</table>

**RESULTS**

Preliminary results from fits to the 1996-98 data are summarized below:

\[
A_e(1996 − 98) = 0.1572 ± 0.0069 ± 0.0027 \text{ (from } e^+ e^−, \mu^+ \mu^- \text{ and } \tau^+ \tau^-);\\
A_\mu(1996 − 98) = 0.147 \pm 0.018 \pm 0.002 \text{ (from } \mu^+ \mu^-); \text{ and}\\
A_\tau(1996 − 98) = 0.127 \pm 0.018 \pm 0.004 \text{ (from } \tau^+ \tau^-),
\]

where the first error is statistical and second is due to systematic effects. The numbers have been corrected for the effect of backgrounds and the “V-A effect” for taus. The estimates for \( A_e, A_\mu \) and \( A_\tau \) are obtained by
fitting each lepton sample separately by the maximum likelihood procedure. $A_e$ is obtained from all lepton species combined (combined from $A_e^e$, $A_e^\mu$ and $A_e^\tau$).

Adding our published results from the 1993-95 data, our current best estimates for leptonic asymmetry parameters at SLD are as follows:

$$A_e(1993-98) = 0.1558 \pm 0.0064 \text{ (from } e^+e^-, \mu^+\mu^- \text{ and } \tau^+\tau^-);$$

$$A_\mu(1993-98) = 0.137 \pm 0.016 \text{ (from } \mu^+\mu^-);$$

$$A_\tau(1993-98) = 0.142 \pm 0.016 \text{ (from } \tau^+\tau^-);$$

$$A_l(1993-98) = 0.1523 \pm 0.0057,$$

where statistical and systematic errors are combined. The asymmetry parameters are consistent with lepton universality. The global asymmetry parameter is referred to as $A_l$ (For $A_l$ systematic errors are conservatively taken to be fully correlated between lepton species).

**SUMMARY**

We report new direct measurements of the $Z^0$-lepton coupling asymmetry parameters $A_e$, $A_\mu$ and $A_\tau$, with polarized $Z^0$'s collected from 1993 through 1998 by the SLD detector at the SLAC Linear Collider. Maximum likelihood fits to the reactions $e^-_L R + e^+ \rightarrow Z^0 \rightarrow e^+e^-, \mu^+\mu^-$ and $\tau^+\tau^-$ are used to measure the parameters. The probability density function used in the fit incorporates all three $s$-channel terms required from the tree-level calculations for the muon- and tau-pair final states. The electron-pair final states are described by both $s$- and $t$-channel $Z^0$ and photon exchange requiring ten cross section terms, all of which are included in the probability density function. Whether three or ten terms, the probability density function used in the fit results from convoluting the energy-dependent cross section formulas with a spectral function. The function incorporates initial state QED radiation, the intrinsic beam-energy spread and the effect of energy-dependent selection criteria. The parameters obtained from these fits require no further corrections for these effects. However, $A_\tau$ is corrected for a bias that results from the V-A structure of tau decays and both tau- and electron-pair events require additional small corrections due to backgrounds. Preliminary results are summarized in the previous section.

Comparison of the $A_e$, $A_\mu$ and $A_\tau$ shows no significant differences in these asymmetry parameters. By assuming lepton universality, the weak mixing angle corresponding to the global asymmetry parameter, $A_l$, is given

$$\sin^2 \theta_W^{eff} = 0.23085 \pm 0.00073.$$  \hspace{1cm} (11)
The weak mixing angle from $A_{LR}$ using hadrons yields [4]

$$\sin^2 \theta_W^{eff} = 0.23101 \pm 0.00029. \quad (12)$$

Those results are consistent and the combined preliminary result of the weak mixing angle at SLD is

$$\sin^2 \theta_W^{eff} = 0.23099 \pm 0.00026. \quad (13)$$

Our result still differs from the most recent combined LEP I result [21] by about 2.7$\sigma$. It is interesting to note that the LEP leptonic average ($\sin^2 \theta_W^{eff} = 0.23151 \pm 0.00033$), from the tau-polarization and the unpolarized forward-backward leptonic asymmetries, is consistent with our result within 1.2$\sigma$. However the LEP hadronic average ($\sin^2 \theta_W^{eff} = 0.23230 \pm 0.00032$), from the $b$-quark and $c$-quark unpolarized forward-backward asymmetries and the hadronic charge asymmetry, is different from our result by 3.1$\sigma$.

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**References**


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[10] The photon s-channel cross section terms are \( \frac{d\sigma}{dx} \left( \frac{v_e}{v_l} \right) \left( 1 + x^2 \right) \) and \( \frac{d\sigma}{dx} \left( \frac{v_e}{v_l} \right) \left( 1 + x^2 \right) \). The dominant t-channel cross section for electrons is \( \frac{d\sigma}{dx} \left( \frac{v_e}{v_l} \right) \left( \frac{4 + (x + 1)^2}{(x - 1)^2} \right) \).

[11] We define the polarization as \( P = (R - L)/(R + L) \) where \( L(R) \) is the number of left-handed (right-handed) electrons.


The latest study by the polarimeter group [4] estimates the errors are $\delta P/P = 0.67\%$ and $0.5\%$ for 1996 and 1997-98, respectively, which are very close to the final numbers. In this analysis, the preliminary numbers are chosen to estimate the uncertainty conservatively.

The uncertainty was obtained by using the center-of-mass energy measured by the WISRD energy spectrometer for events passing the pre-selection cuts. The obtained uncertainty is higher than the value estimated in Ref. [14]. We use the uncertainty of 50 MeV to estimate the systematic error conservatively for now.


The latest LEP-electroweak results are summarized in the LEP Electroweak Working Group Home Page (http://www.cern.ch/LEPEWWG/Welcome.html).
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Figure 1: Polar-angle distributions for $Z^0$ decays to $e$, $\mu$ and $\tau$ pairs for 1996-98 SLD runs. The open(filled) circles are for left(right)-handed electron polarization. The data are corrected for $|\cos\theta| > 0.7$ where the detection efficiency drops with increasing $|\cos\theta|$. For the Bhabha events, the forward-backward asymmetry has the same sign for both polarizations, while for the muon- and tau-pair events, the forward-backward asymmetry changes sign with polarization.