Superstring Theory and CP- Violating Phases: Can They Be Related?

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Abstract

We investigate the possibility of large CP-violating phases in the soft breaking terms derived in superstring models. The bounds on the electric dipole moments (EDM’s) of the electron and neutron are satisfied through cancellations occurring because of the structure of the string models. Three general classes of four-dimensional string models are considered: (i) orbifold compactifications of perturbative heterotic string theory, (ii) scenarios based on Hořava-Witten theory, and (iii) Type I string models (Type IIB orientifolds). Nonuniversal phases of the gaugino mass parameters greatly facilitate the necessary cancellations among the various contributions to the EDM’s; in the overall modulus limit, the gaugino masses are universal at tree level in both the perturbative heterotic models and the Hořava-Witten scenarios, which severely restricts the allowed regions of parameter space. Nonuniversal gaugino masses do arise at one-loop in the heterotic orbifold models, providing for corners of parameter space with $O(1)$ phases consistent with the phenomenological bounds. However, there is a possibility of nonuniversal gaugino masses at tree level in the Type I models, depending on the details of the embedding of the SM into the D-brane sectors. We find that in a minimal model with a particular embedding of the Standard Model gauge group into two D-brane sectors, viable large phase solutions can be obtained over a wide range of parameter space.
A central issue to be addressed in supersymmetric theories is the origin and dynamical mechanism of spontaneous supersymmetry breaking. In supersymmetric extensions of the Standard Model (SM) such as the Minimal Supersymmetric Standard Model (MSSM), the effects of the unknown dynamics of supersymmetry breaking are encoded by adding terms to the Lagrangian which break supersymmetry explicitly; these terms depend on a considerable number of parameters which can be considered as independent in the phenomenological analysis of the model. For example, the most general set of soft supersymmetry breaking parameters in the MSSM, which is defined to be the minimal supersymmetric extension of the SM with the standard Higgs sector and conserved R-parity, includes 105 masses, mixing angles, and phases (not counting the gravitino mass and coupling) [1]. From the phenomenological point of view, this large number of parameters can be cumbersome but not otherwise problematic, as it is for experiments to measure and for the underlying theory (for example, superstring theory) to predict the values of these parameters. Phenomenological analyses aid in this process both for experimentalists and theorists by serving as a helpful guide to the allowed regions of parameter space, and are crucial from the experimental side since almost none of the Lagrangian parameters are directly measured.

Due to the large number of parameters of the soft breaking Lagrangian, restricted sets of parameters are often chosen to simplify the analysis. While this approach is sensible, it is important not to exclude possibly allowed regions of parameter space based on potentially misleading theoretical assumptions. An example is the conventional statement of the supersymmetric CP problem, which is that the CP-violating phases in the MSSM (which arise in the soft breaking Lagrangian and in the phase of $\mu$) are individually constrained to be less than $O(10^{-2})$ for sparticle masses at the TeV scale by the experimental upper limits for the electric dipole moments of the electron and the neutron [2–4]. Based on this argument, these CP-violating phases have traditionally been set to zero in phenomenological analyses.

However, a recent reinvestigation of this issue [5,6], see also [7] has demonstrated that cancellations between different contributions to the electric dipole moments can allow for regions of parameter space with phases of $O(1)$ and light sparticle masses that satisfy the phenomenological constraints, contrary to conventional wisdom. If these phases are in fact nonzero (which future experiments will need to determine), they can have important effects on many physical observables, and thus on the extraction of the values of the soft supersymmetry breaking parameters from experimental measurements [8]. A thorough numerical analysis including the seven significant phases [6] indicates that the cancellations can only occur for the large phase solutions if the various soft breaking parameters satisfy particular approximate relations. Such relations may provide clues to the dynamical mechanism of supersymmetry breaking, and hence to the form of the underlying theory.

As superstring theory is the best candidate for the underlying fundamental theory of all interactions, it is desirable to investigate the phase structure of soft supersymmetry breaking terms that can arise in classes of four-dimensional superstring models. In this paper, we address the question of whether the relations among the soft breaking parameters derived in these models allow for large phases that satisfy the electric dipole moment constraints via the cancellations.

CP is a discrete gauge symmetry in string theory, and thus can only be broken spon-
taneously [9]. If this breaking occurs via the dynamics of compactification and/or supersymmetry breaking, then the four dimensional effective field theory will exhibit explicit CP-violating phases. The origin of supersymmetry breaking in string theory remains an unresolved issue, though it is known to be nonperturbative. However, progress in addressing the low energy implications of supersymmetry breaking in string theory can be made by utilizing the phenomenological approach of Brignole, Ibáñez, and Muñoz [10]. In this approach, the degrees of freedom involved in supersymmetry breaking are assumed to be the dilaton $S$ and (untwisted) moduli $T_m$, which are superfields generically present in four-dimensional string models. The effects of the unknown nonperturbative dynamics that break supersymmetry are then encoded in the $F$-component VEV’s of these superfields. These VEV’s are conveniently parameterized in terms of Goldstino angles, which denote the relative contributions of each field to the supersymmetry breaking process. The phases (if nonzero) of the $F$-component VEV’s of the dilaton and moduli provide the main sources for CP-violating phases in the soft terms. Whether or not these VEV’s have sizeable phases is a dynamical question that cannot be addressed within this framework, and hence these phases are treated as independent parameters in the analysis. It is important to note that to obtain the traditional resolution to the supersymmetric CP problem in a natural manner, dynamical principles are required which guarantee that not only the phases of the $F$-component VEV’s but also the phase of $\mu$ (which in principle has a different origin) are zero or negligibly small. While arguments for such principles exist (primarily within the context of perturbative heterotic string models), our strategy has been that it will be experimental information that is likely to play an important role in determining or constraining the values of these parameters.

Within particular classes of four-dimensional string models, the couplings of the dilaton, moduli, and matter fields are calculable, which leads in turn to a specific pattern of soft breaking parameters at the string scale (as a function of the unknown $F$-component VEV’s, which serve as input parameters). In our analysis of the phase structure of the soft breaking terms, we use the renormalization group equations (RGE’s) to obtain the values of the parameters at the electroweak scale and subsequently compute the EDM’s of the electron and neutron. In the analysis, it is important to note that the general results of [6] illustrate that sufficient cancellations among the various contributions to the EDM’s are difficult to achieve unless there are large relative phases in the gaugino masses. Furthermore, the phases of the gaugino mass parameters do not run at one-loop order, and thus at the electroweak scale only deviate from their string-scale values by small two-loop corrections. Therefore, the possibility of viable large phase solutions crucially depends on whether nonuniversal gaugino mass parameters are predicted within a given string model.

Following [10–12], we first consider models derived from weakly coupled heterotic string theory in orbifold compactifications [13], and focus on the case in which $S$ and one “overall modulus” field $T$ participate in the supersymmetry breaking. In these models, the dilaton $F$ term leads to universal gaugino masses at tree-level; however, the modulus field $T$ does provide a nonuniversal contribution at one-loop order (although loop-suppressed). We thus focus on a generalized version of their $O–II$ scenario, with arbitrary phases for $S$ and $T$, and consider the moduli-dominated limit. We find that the phase structure of the soft terms does allow for small regions in the parameter space for which the EDM constraints can be satisfied with large phases, and that the results may depend on the particular (model-dependent) solution employed for the $\mu$ problem. This fact is very encouraging, as it suggests
such analyses may help us learn how the $\mu$ problem is solved.

We next consider the soft breaking terms which arise in newer classes of four-dimensional string models, including the Hořava-Witten scenarios, and models within the general perturbative Type I string picture. In contrast to the weakly coupled heterotic models, the calculational and model-building techniques in each of these scenarios are at early stages, and there is as yet no quasi-realistic model. However, recent studies have indicated that the phenomenological properties of these classes of models, including the patterns of the soft supersymmetry breaking parameters, can be quite distinctive from those of the perturbative heterotic models traditionally studied in superstring phenomenology.

We first consider models based on the Hořava-Witten theory [14] (11-dimensional supergravity compactified on a Calabi-Yau manifold times the eleventh segment) in which the observable sector gauge groups arise from the $E_8$ gauge group on one of the ten-dimensional boundaries, for which the soft terms of the effective supergravity theory [15] have been computed [16,17]. In contrast to the perturbative heterotic case, both the dilaton and modulus fields contribute to the gaugino masses with $O(1)$ coefficients. However, in the limit in which only the overall modulus $T$ contributes to SUSY breaking, the gaugino masses are universal in this scenario. Hence, significant CP-violating phases in the soft terms consistent with the phenomenological bounds are disallowed over the majority of parameter space (the situation is analogous to that of the dilaton-dominated limit of perturbative heterotic orbifold models studied in [18]).

However, within the more general Type I string picture [19-22], there is the possibility of nonuniversal gaugino masses at tree level, which has important implications for the possible CP-violating effects. As an illustrative example of models within this framework, we focus on the four-dimensional Type IIB orientifold models, in which consistency conditions (tadpole cancellation) require the addition of open string (Type I) sectors and Dirichlet branes, upon which the open strings must end. The patterns of the soft supersymmetry breaking terms arising in this class of models crucially depend on the embedding of the SM gauge group into the D-brane sectors. In particular, nonuniversal gaugino masses can be obtained at tree-level if the SM gauge groups are embedded in two distinct D-brane sectors, in direct contrast to the perturbative heterotic orbifolds and the Hořava-Witten scenarios described above. Our analysis indicates that within a minimal model in which $SU(3)$ and $U(1)_Y$ (but not $SU(2)$) arise from the same D-brane sector, the necessary cancellations between the contributions to the EDM’s occur over a wide range of parameter space. The results of this study illustrate that as viable large soft phases depend on how the SM is embedded and how the $\mu$ problem is solved, etc., we may be able to learn about (possibly nonperturbative) Planck scale physics using low energy data.

This paper is structured as follows. In Section II, we briefly review the method and results of the general EDM calculation of [5,6], with an emphasis on issues relevant for our analysis. We present the analysis of the soft breaking terms from the perturbative heterotic orbifold models in Section III. In Section IV, we consider the Hořava-Witten scenarios, and in Section V, we analyze the Type I models. Finally, we present the summary and conclusions in Section VI.
II. ELECTRIC DIPOLE MOMENT CALCULATION

The purpose of this paper is to examine whether the (complex) soft breaking parameters which can arise in classes of string-derived models can satisfy the phenomenological bounds from the EDM’s by cancellations. We build upon the recent calculations of the electric dipole moments presented in [5] and [6]. In this section, we briefly summarize the framework and results of these calculations, and comment upon the issues to be addressed in our analysis of string-motivated models of the soft breaking parameters.

It is well known that in (softly broken) supersymmetric theories with CP-violating phases of \( \mathcal{O}(1) \), superpartner exchange at the one-loop level can lead to contributions to the electric dipole moments of the fermions which can exceed the experimental upper bounds [2–4]. As previously mentioned, the traditional resolution to this problem has been to constrain the phases to be less than \( \mathcal{O}(10^{-2}) \) (which can be interpreted as fine-tuning), or assume heavy sfermion masses (which can violate naturalness). However, the issue was reinvestigated first by [5], and subsequently in [6], in which the EDM’s were computed using an effective theory approach in which the contributions from chargino, neutralino, and gluino loops to the relevant Wilson coefficients were determined numerically. In their work, the main emphasis was on the possibility of cancellations between the various contributions to the Wilson coefficients. This mechanism can allow large values of the phases to give contributions consistent with the experimental bounds on the values of the electric dipole moments \( d_e, d_n \) of the electron and the neutron, respectively. The current limits for the neutron EDM require that [23]

\[
|d_n| < 6.3 \times 10^{-26} \text{ ecm},
\]

at 90% confidence level, and for the electron EDM [24]

\[
|d_e| < 4.3 \times 10^{-27} \text{ ecm},
\]

at 95% confidence level.

In [6], a general set of CP-violating phases is assumed. For simplicity, the phases of the off-diagonal terms in the scalar mass matrices are neglected, as the impact of these phases on physical observables may be suppressed by the same mechanism required to suppress FCNC\(^1\), and in any case are unlikely to modify the results qualitatively. The phases included in the analysis are thus the phases of the gaugino masses \( M_1, M_2, M_3 \), the phases of the \( \mu \) term and the associated \( b = B_\mu \) parameter, and the phases of the \( A \) parameters associated with the trilinear scalar couplings. However, as noted in [25,6,26,27] and references therein, not all of these phases are physical due to additional approximate global \( U(1) \) symmetries of the MSSM Lagrangian which can be promoted to full symmetries by treating the parameters as spurions charged under those symmetries. The result is that there is the freedom to rotate away one of the phases in the gaugino mass sector and also to set \( b \) (and the VEV’s of the

\(^1\)For our purposes, this statement indicates that we do not consider Kähler potentials with off-diagonal metric; for further discussions of this issue see e.g. [10–12].
Higgs doublets) to be real at the electroweak scale without loss of generality. The phase $\varphi_2$ of the $SU(2)$ gaugino mass $M_2$ is set to zero in the parameterization choice of [6], and hence the relevant phases in the analysis are $\varphi_1$, $\varphi_3$ of the $U(1)_Y$ and $SU(3)$ gaugino masses, $\varphi_\mu$, $\varphi_{A_u}$, $\varphi_{A_d}$, $\varphi_{A_t}$ and $\varphi_{A_e}$ (in self-evident notation). Note that in minimal supergravity-inspired models as studied in [5], the gaugino masses can be taken to be real without loss of generality, and then there are only two relevant phases, a common $\varphi_A$ and $\varphi_\mu$.

As $\mu$ and $B$ (and their phases) are relevant in the analysis, the results will in general depend on the solution to the $\mu$ problem. In string models, the “bare” $\mu$ term is absent in the superpotential, since the fields are massless at the string scale, and there are several possibilities for the generation of an effective $\mu$ term (either in the superpotential or in the Kähler potential [28]) without invoking additional gauge singlet matter fields; we refer the reader to [10,29] for further discussions of this issue. The results for the phase of $\mu$ and that of the associated $B$ term strongly depends on which solution (or in fact if both mechanisms are present) is preferred in a given model. Since these issues are highly model-dependent, an additional possibility is to treat the phases of $\mu$ and $B$ as independent parameters; their magnitudes are naturally constrained by the requirement of correct electroweak symmetry breaking. This sensitivity to the way $\mu$ is generated is a very positive feature, since it implies that data on the phases may help determine experimentally how $\mu$ is generated.

The general results of [6] demonstrate that sufficient cancellations among the various contributions to the EDM’s are difficult to achieve unless there are large relative phases in the soft masses of the gaugino sector. This feature is due to the approximate $U(1)_R$ symmetry of the Lagrangian of the MSSM [25], which allows one of the phases of the gaugino masses to be set to zero at the electroweak scale without loss of generality [25,6,26,27]. Furthermore, the phases of the gaugino mass parameters do not run at one-loop order, and thus at the electroweak scale only deviate from the string-scale values by small two-loop corrections. Therefore, if the phases of the gaugino masses are universal at the string scale, they will be approximately zero at the electroweak scale (after the $U(1)_R$ rotation). Cancellations among the chargino and neutralino contributions to the electron EDM are then necessarily due to the interplay between the phases of $A_e$ and $\mu$. The analysis of [6] demonstrates that cancellations are then difficult to achieve as the pure gaugino part of the neutralino diagram adds destructively with the contribution from the gaugino-higgsino mixing, which in turn has to cancel against the chargino diagram. As a result, the cancellations are generally insufficient, and hence in this case over most of the parameter space the phases of the other soft breaking parameters as well as the $\mu$ parameter must naturally be $\lesssim 10^{-2}$ (the traditional bound) [3] unless the sfermion masses are greater than $O$(TeV) $^2$.

Therefore, the possibility of large CP-violating phases in the string-motivated models of soft breaking terms we consider depends significantly on whether the gaugino masses are allowed to have large relative phases (i.e., if they are nonuniversal). It is important to note

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\footnote{We note that this situation is precisely that of the minimal supergravity case studied in [5], as the gaugino masses are universal in this scenario. In this case, a heavy superpartner spectrum is required to exhibit regions of parameter space with large phases consistent with the phenomenological bounds on the EDM’s.}
that the gauginos can be degenerate (or nearly degenerate) in mass at the string scale and have different phases; in practice, we find examples where this holds. In the next sections, this feature will be displayed explicitly in the analysis of the soft breaking parameters in three classes of four-dimensional string models.

**III. SOFT BREAKING TERMS IN PERTURBATIVE HETEROTIC SUPERSTRING MODELS**

**A. Theoretical Framework**

In the analysis of [10,11], the primary assumption is that supersymmetry is broken by a combination of the dilaton field $S$ and the moduli $T_m$ present in generic four-dimensional string models. These fields have a vanishing (perturbative) scalar potential and gravitationally suppressed interactions with the fields of the observable sector, and thus are natural candidates to play a role in the breakdown of supersymmetry.

In classes of four-dimensional models derived from perturbative heterotic superstring theory, the Kähler potential $K$, gauge kinetic function $f_a$ (where $a$ labels the gauge groups), and superpotential $W$ are calculable (generally to one-loop order) in string perturbation theory. The calculational techniques have been particularly well developed for the case of orbifold compactifications (see for example [13,30]). However, the nonperturbative contributions to the Kähler potential and the superpotential, which play a crucial role in supersymmetry breaking, remain uncertain. In the absence of the knowledge of how supersymmetry is broken, the authors of [10] proposed an efficient parameterization of the soft breaking terms in terms of the (unknown) $F$-component VEV’s of $S$ and $T_m$. For example, for the case in which the fields which break supersymmetry are just $S$ and the “overall modulus” $T$ associated with the radius of the compactification manifold, the $F$-component VEV’s can be expressed as follows (assuming no mixing among their kinetic terms):

\[
F^S = \sqrt{3} m_{3/2} (S + S^*) e^{i\alpha_S} \sin \theta \\
F^T = m_{3/2} (T + T^*) e^{i\alpha_T} \cos \theta,
\]

in which $m_{3/2}$ is the gravitino mass and $\alpha_S, \alpha_T$ denote the (in this parameterization arbitrary) phases. $\theta$ is the Goldstino angle, which measures the relative contributions of $S$ and $T$.

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3 Due to the holomorphicity of the superpotential and the (Wilsonian) gauge kinetic function, non-renormalization theorems imply these functions do not receive higher-loop corrections. However, the Kähler potential does receive loop corrections, and thus is the least well-determined function of the string theory effective action. For further details, see [30] and references therein.

4 As in [10], we consider the overall modulus case both for simplicity and because the $T$ modulus is always present in generic four-dimensional string models. We comment later about the implications for the purposes of this study of relaxing this assumption, and refer the reader to [11] for a discussion of the multimoduli case in the perturbative heterotic orbifold models.
$T$ to the supersymmetry breaking; the $\sin \theta \to 1$ and $\sin \theta \to 0$ limits correspond to dilaton and moduli dominance, respectively. The soft terms in the case of general orbifold models have been computed in [10]. The results demonstrate the advantage of the parameterization (3), as the soft terms take on very simple forms when expressed in terms of these parameters.

We note in passing that in specific scenarios for spontaneous supersymmetry breaking such as gaugino condensation in the hidden sector, the form of the nonperturbative superpotential $W_{np}(S,T)$ is known and the values of $F^S, F^T$ can in principle be determined. However, explicit models typically suffer from generic problems, including that of the runaway dilaton and a nonvanishing, negative cosmological constant. We choose to follow [10] and consider the parameters in (3) as free parameters, allowing in particular for nonzero values of $\alpha_S, \alpha_T$. Our philosophy is that experimental information will determine or constrain the parameters, thereby leading theorists to recognize how supersymmetry is broken. We comment briefly below on the types of parameter ranges for $\alpha_S, \alpha_T$ encountered in the gaugino condensation scenarios, and refer the reader to [10,12] and references therein for more comprehensive discussions.

As discussed in the previous section, large CP effects consistent with the phenomenological bounds on the EDM’s generally require large relative phases in the gaugino mass parameters, which implies nonuniversal gaugino masses. In general, the source for the gaugino masses is the (field-dependent) gauge kinetic function $f(S,T)$, which in perturbative heterotic string theory is independent of $T$ at tree level and given by

$$f_{\text{tree}} = k_a S,$$

in which $k_a$ is the Kač-Moody level of the gauge group. This expression yields universal gaugino masses, since the dilaton couples universally to all gauge groups. However, the one-loop (threshold) corrections to the gauge kinetic function have been computed in orbifold models, and provide for the possibility of nonuniversal gaugino masses. These corrections depend on $T$ as follows:

$$f_{\text{1-loop}} = -\frac{1}{16\pi^2} \left( b_a' - k_a \delta_{GS} \right) \log[\eta(T)]^4,$$

in which $b_a'$ is a numerical coefficient dependent upon the matter content of the model, $\eta(T)$ is the Dedekind function, and $\delta_{GS}$ is a coefficient (a negative integer in most orbifold models) related to the cancellation of duality anomalies in the theory. This coefficient also is important in that it measures the amount of mixing between the kinetic terms of the $S$ and $T$ fields, which occurs in the loop corrections to the Kähler potential.

In the dilaton-dominated limit ($\sin \theta \to 1$), the contributions to the gaugino masses from (4) dominate the one-loop corrections from (5). Thus, the gaugino masses are universal in this limit and hence the phases in the gaugino sector can be taken to vanish without loss of generality. It is therefore unlikely that sufficient cancellations will occur in this limit except at exceptional points in the parameter space depending on the (model-dependent) solution to the $\mu$ problem, and thus the traditional solutions to the supersymmetric CP problem of either small $O(10^{-2})$ phases or heavy squark masses must be invoked to avoid electric dipole moments for the electron and the neutron which violate the experimental bounds. The analysis of the EDM constraints within the dilaton-dominated scenario has recently
been presented in [18]; their results demonstrate explicitly that large phases are disallowed over the majority of the parameter space.

Therefore, we are naturally led to consider the moduli-dominated ($\sin \theta \to 0$) limit and to include one-loop corrections to $f$ (and $K$, for consistency) to obtain the possibility of nontrivial phases for the gaugino masses. The $S - T$ mixing in the Kähler potential requires a slight modification of the parameterization (3) of the $F$-component VEV’s, which amounts to a redefinition of $\alpha_S$, $\alpha_T$, and $\theta$; a thorough discussion of this issue is given in [10], to which we refer the reader for details.

An example of this type was presented in [10] as the $O - II$ scenario, which is a moduli-dominated scenario in which the one-loop mixing between $S$ and $T$ is crucial to avoid the vanishing of the soft mass-squares of the scalar fields in the $\sin \theta \to 0$ limit. In this model, the $A$ terms and scalar mass-squares are universal, while the gaugino masses are nonuniversal. The soft breaking parameters take the form:

$$m_{\tilde{G}}^2 = m_{3/2}^2 (-\delta_{GS}) \epsilon'$$

$$A_{t,e,u,d} = -\sqrt{3} m_{3/2} e^{-i \alpha_S} \sin \theta,$$  

$$(6)$$

$$M_3 = \sqrt{3} m_{3/2} [e^{-i \alpha_S} \sin \theta - (3 + \delta_{GS}) e^{-i \alpha_T} \cos \theta]$$

$$M_2 = \sqrt{3} m_{3/2} [e^{-i \alpha_S} \sin \theta - (1 + \delta_{GS}) e^{-i \alpha_T} \cos \theta]$$

$$M_1 = \sqrt{3} m_{3/2} [e^{-i \alpha_S} \sin \theta - (\frac{-33}{5} + \delta_{GS}) e^{-i \alpha_T} \cos \theta],$$  

$$(7)$$

in which $\epsilon$, $\epsilon'$ are numerical factors which depend on the VEV’s of $S$ and $T$ (their magnitudes will be discussed below).

We do not consider all possibilities for the $\mu$ and $B$ terms, and refer the reader to [10,29] for further discussions of this issue. We instead first analyze the case in which we assume that the $\mu$ problem is solved via an effective coupling in the superpotential of the form $\mu(S,T)H_1 H_2$, (in which $\mu$ depends only weakly on $S$ and $T$), and then treat $\mu$ and $B$ as independent parameters. In the first case, the $B$ term is given by

$$B_\mu = m_{3/2} [-1 - \sqrt{3} e^{-i \alpha_S} \sin \theta - (1 - \frac{\delta_{GS}}{24 \pi^2 Y})^{-1/2} e^{-i \alpha_T} \cos \theta],$$  

$$(8)$$

in which

$$Y = S + S^* - \frac{\delta_{GS}}{8 \pi^2} \log(T + T^*).$$  

$$(9)$$

In the $O - II$ scenario described in [10], the numerical values of $\epsilon$ and $\epsilon'$ are taken to be $\sim O(10^{-3})$, which corresponds to the situation in which the VEV’s of Re$S$ and Re$T$ are $O(1)$. These values are motivated by the minimization of the scalar potential for $S$ and $T$ that can be derived either in the gaugino condensation approach [32,33] or more generally imposing the requirements of $T$-duality on the scalar potential for the modulus field [33,34]. Within our phenomenological approach, we can in principle regard these VEV’s as free parameters and consequently vary $\epsilon$, $\epsilon'$ within reasonable limits; however, we choose in general not to depart significantly from the case in which the VEV’s are $O(1)$.
In addition, a comment about the phases is in order. While $\epsilon'$ is a real parameter by definition

$$
\epsilon' = \frac{1}{24\pi^2Y},
$$

$\epsilon$ is by nature complex [10] if the VEV’s of $S$ and $T$ are complex, which is of course assumed throughout this paper to obtain nontrivial values for $\alpha_S$ and $\alpha_T$. It is also clear that the phase of $\epsilon$ and $\alpha_{S,T}$ are correlated, with the particular relations depending on the nature of the nonperturbative dynamics responsible for the breakdown of supersymmetry.

In orbifold compactifications within the gaugino condensation approach, in which the nonperturbative superpotential for $S$ and $T$ takes the form $W \sim \exp^{-3f(S,T)/2\beta}$ (with $f(S,T)$ the gauge kinetic function and $\beta$ the beta-function of the gauge group of the gaugino condensate), the soft breaking terms have been computed in [32] and an analysis of the CP-violating phases has been carried out explicitly in [35,36]. The conclusion of [35,36] is that the properties of the nonperturbative superpotential (in particular, that the $T$ dependence of the nonperturbative superpotential $W(T) \sim \eta(T)^{-6}$) are such that the CP-violating phases of the resulting soft terms are negligible. In their analysis, the VEV’s of $S$ and $F_S$ are assumed to be real; in principle, the details depend on the mechanism utilized for the stabilization of the dilaton, which is usually achieved either through nonperturbative corrections to the Kähler potential or through multiple gaugino condensates (“racetrack” models). With this assumption and the knowledge of the $T$ dependence of the superpotential from the form of $f(S,T)$, the value of the phase of $\epsilon$ can be determined. The result is that the gaugino masses are strictly real, as of course are the soft terms which depend solely on $\alpha_S$ (the issue of the phases of $\mu$ and $B$ is considerably more complicated and model-dependent). This result may indicate (as is emphasized in [35,36]) that in the gaugino condensation approach the phases may be small due to the properties of the $T$-dependent modular functions. However, it was also noted in [34,36] that in principle the superpotential may depend on other modular invariant functions (such as the absolute modular invariant $j(T)$) for which the conclusions about negligible CP-violating phases may no longer be valid [36]. We prefer to follow [10] and not restrict our consideration to any particular scenario for the supersymmetry breaking, which in turn allows us to explore the possibility of nontrivial phases for $\alpha_{S,T}$ and $\epsilon$, which we can treat as independent parameters in our analysis. The contrast between the approaches illustrates the possibility that a measurement of the soft phases will help determine how supersymmetry is broken and how to relate compactification to observables.

**B. Results**

We start our numerical analysis of the moduli dominated $O-II$ scenario by calculating the soft breaking parameters at the electroweak scale. The boundary conditions of Eq. (6 - 8) are to be implemented at the string scale $M_{\text{String}} \sim 5 \times 10^{17}$ GeV, which in perturbative heterotic string theory is the scale at which the gauge couplings are predicted to unify [37], and all relevant soft breaking parameters are evolved down to the electroweak scale using two-loop renormalization group equations (RGE’s) for the gauge couplings and one-loop
equations for the Yukawa couplings and the soft parameters.

The renormalization group analysis and resulting patterns for the low-energy mass spectrum of the soft terms of the $O-II$ scenario (assuming $\alpha_{S,T} = 0$) have been presented in [10,39,40], to which we refer the reader for further details. As noted in the previous section, the gaugino mass phases do not run at one-loop; this behavior can be disrupted only by higher loop corrections, threshold effects and other possible corrections [41]. However, the trilinear coupling phases $\varphi_{A_u}, \varphi_{A_d}, \varphi_{A_t}$ and $\varphi_{A_e}$ do run, and they evolve away from the single universal string scale value at different rates depending on the relevant Yukawa couplings and gauge group charges. We perform an $R$ rotation at the electroweak scale, which allows us to set the phase of $M_2$ equal to zero. The phase of $\mu$ is then determined by the phase of the $B$ parameter as $\varphi_\mu = -\varphi_B$, so that $B\mu$ is real and the Higgs potential is not affected by the phases at tree level.

As previously mentioned, we regard $m_{3/2}, \delta_{GS}, \theta, \epsilon, \alpha_S$ and $\alpha_T$ as the free parameters of the model. Since $\epsilon$ is $\sim O(10^{-3})$ (assuming the VEV’s of $S$ and $T$ are $O(1)$), it is necessary to consider the small $\theta$ (moduli-dominated) limit for the gauginos to acquire significant relative phases. We consider a range of numerical values of $\theta$ between $10^{-3}$ and $10^{-1}$, which in turn requires $m_{3/2}$ to be typically greater than $O$(TeV) for the soft breaking parameters to have acceptable masses. In addition, if the $B$ term condition (8) is imposed, such a large mass scale for the gravitino could cause the $B_\mu$ parameter to be of the same order, which is disfavored by naturalness arguments. One way to avoid this result is to require $\alpha_T \sim \pi$ for $\cos \theta \simeq 1$ or $\alpha_T \sim 0$ for $\cos \theta \simeq -1$. It is also clear that if $\delta_{GS} >> 5$ or so, the relative gaugino phases will be suppressed producing no interesting CP-violating phenomena. To obtain a large relative phase between gaugino masses $M_i$ and $M_j$, it is necessary that

$$\min(\kappa_i, \kappa_j)|\epsilon| \lesssim |\theta| \lesssim \max(\kappa_i, \kappa_j)|\epsilon|,$$

where $\kappa_1 = (-33/5 + \delta_{GS})$, $\kappa_2 = (-1 + \delta_{GS})$ and $\kappa_3 = (3 + \delta_{GS})$. All of the above relations significantly constrain the possible parameter space and introduce further correlations between the phases. In particular, the seven CP-violating phases entering the calculation of the electric dipole moments are effectively parametrized by a single phase $\alpha_S$ originating in the string sector. Despite this high degree of correlation, we find that it is possible to find

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5 The discrepancy between the string scale and the GUT scale $M_{GUT} \sim 2 \times 10^{16}$ GeV (where the gauge couplings appear to unify from extrapolating the measured values of the electroweak scale couplings to higher scales assuming the MSSM particle content) is a well-known problem in perturbative heterotic string theory with a number of solutions proposed (see e.g. [38]). In practice, this mismatch between the scales introduces a small numerical discrepancy into the analysis (unless as in [10] intermediate scale matter or some other effect is assumed to be present which solves the problem of the unification of the couplings).

6 Despite different approaches, our results are in a sense consistent with the results of [35,36] in the gaugino condensation framework, in which they obtain negligible phases in the soft breaking terms (in particular in the gaugino mass sector) with the assumption of vanishing phases for the scalar and $F$- component VEV’s of the dilaton.
parametric configurations that lead to relatively substantial values of the phases and light superpartner mass spectra while satisfying the electric dipole moment constraints.

We explore the possibility of obtaining superpartner mass spectra with the squark masses lying below 500 GeV while simultaneously generating sizeable and experimentally acceptable CP-violating phases at the electroweak scale within the $O-II$ string scenario for a series of parameter sets classified by the value of the Green-Schwarz parameter $\delta_{GS}$. In addition to the general $O-II$ relations (6)-(7), we consider two general cases. First, we impose the $B$-term condition (8), corresponding to a particular solution to the $\mu$ problem (arising from nonperturbative corrections to the superpotential). To consider the case in which $\mu$ and $B$ can receive contributions from other sources, such as from the Giudice-Masiero mechanism, we do not discuss all of the possibilities in detail, but rather relax the expression (8) for $B$ and treat $B$ as a free complex parameter. This approach adds one more CP phase, namely $\varphi_\mu$, to the set of independent parameters. As in the previous case, $\mu$ is still undetermined and hence is regarded as a free parameter. In addition, the restriction on $\alpha_T$, which was dictated by the form of (8), can in principle be relaxed. However, it is important to note that the expression for the $B$ term arising in the Giudice-Masiero mechanism suffers from a similar problem [28], and thus this restriction is likely to be generic (barring possible cancellations in the $B$ term arising from different sources of the $\mu$ term, which is a possibility we do not consider further in this paper). We find in general that the differences between the case in which the $B$ term is determined within the string model through (8) and the case in which $B$ is left as an independent parameter are not very significant. Therefore, in the results which follow, we display the results for the two cases in tandem to emphasize this feature.

We can further determine several general constraints on the parameter space of this model. As the scalar masses are universal at the string scale, the color-neutral scalar particles (i.e., sleptons and sneutrinos) are significantly lighter than the squarks due to smaller RGE running, with masses typically $\lesssim 200$ GeV. The choice of $\delta_{GS}$ and the requirement of light sfermion masses effectively determines the range of values for the gravitino mass parameter $m_{3/2}$ from (6), and we take $\epsilon' \sim 10^{-3}$ (following [10]). We can also estimate the interesting range of $\theta$ and $\epsilon$ providing for light gauginos. To obtain optimally large $\varphi_1$ and $\varphi_3$ we set $|\theta| \sim |\kappa_2\epsilon|$. The values of the universal $A$ parameters (and the $B$ term when (8) is imposed at the string scale) are then fully determined by the choice of $\delta_{GS}$, $m_{3/2}$, $\theta$, $\epsilon$, and $\alpha_S$.

In Figure 1, we plot the regions of the three most important phases $\varphi_\mu$, $\varphi_1$, and $\varphi_3$ for the case of $\delta_{GS} = -2$. Frame a) shows the points allowed by the electron and neutron EDM constraints in the $\varphi_\mu - \varphi_1$ plane, while frame c) delineates the projection of these points onto the $\varphi_\mu - \varphi_3$ plane. In frames b) and d), we display the results for the same sets of parameters but taking $B$ as an additional independent parameter which we set to the value $|B| = 300$ GeV.

The results illustrate a general feature of this model: to obtain an overlap between the neutron EDM allowed and the electron EDM allowed regions in this model, a low value of $\mu$ is required. This restriction implies large gaugino-higgsino mixing, and thus rather

\[ We note that the RGE running of $\varphi_A$, is the same as in supergravity models, and has been discussed for instance in [42]. \]
FIG. 1. Regions allowed by the electron and neutron EDM constraints in the $O - II$ scenario for $m_{3/2} = 4$ TeV, $\delta_{GS} = -2$, $\theta = 0.021$, $\epsilon = 0.007$, $\tan \beta = 2$ and $\mu = 100$ GeV. The dotted areas show allowed regions resulting from the specific form of soft breaking parameters. The red (black) circles denote points allowed by the eEDM and the green (grey) blocks by the nEDM. In frames a) and c), the $B$ term is assumed to originate from an effective coupling in the superpotential (Eq. 8) while in frames b) and d) $B$ is treated as an independent parameter and its magnitude was set to $|B| = 300$ GeV.

Small values of $\varphi_\mu$ are needed to satisfy the electron EDM constraint, as discussed in [6]. In particular, if $\mu$ is increased the electron and neutron regions overlap only in the small phase region where all phases are $< 10^{-2}$. Small values of $\varphi_\mu$ illustrate that the chargino contribution to the electron EDM is generally much larger than the corresponding neutralino contribution, and hence the cancellation between these contributions is not adequate. As a result, both contributions have to be suppressed by small values of $\varphi_\mu$; residual cancellation in the vicinity of $\varphi_\mu \sim \pi$ subsequently ensures that the effects of $\varphi_1$ and $\varphi_3$ also cancel. A similar effect takes place in the case of the neutron dipole moment, with similar restrictions on the corresponding relevant phases. The overlap between the two regions then yields $\varphi_1 \sim \frac{\pi}{3}$ and remarkably, $\varphi_3 \sim \frac{\pi}{3}$.

General arguments show that for $|\delta_{GS}| \lesssim 5$ and $O(1)$ $\varphi_1$ and $\varphi_3$, viable solutions can be
obtained provided $\varphi_\mu$ is close to $\pi$ (or zero). However, the accessible values of $\varphi_\mu$, $\varphi_1$ and $\varphi_3$ are reduced as $|\delta_{GS}|$ increases, and hence it is more difficult to satisfy the EDM constraints with large phases. Figure 2 clearly demonstrates this effect for $\delta_{GS} = -10$; in this case the gaugino masses are approximately universal, and correspondingly the phases of soft breaking parameters are constrained to satisfy the traditional bound. The results demonstrate that the range of values of $\delta_{GS}$ for which the phases of the soft terms are nontrivial is quite restricted (but are within a reasonable range of values determined in explicit orbifold models). Therefore, if the model of this section were the way nature behaved, it would be possible to determine the anomaly cancellation parameter $\delta_{GS}$ by the measurements of the EDM’s.

We close this section with a brief comment about a further generalization of the $O-II$ scenario, in which the assumption that only the single modulus $T$ plays a role in supersymmetry breaking is relaxed. For the purposes of this study, the important feature remains that these individual moduli will contribute to gaugino masses only at one-loop. Therefore, the number of parameters increases substantially; in addition to the need to define extra Goldstino angles (as is done in [11] and will be required in the Type I models discussed below), there will be [12,13] a $\delta_{GS}$, $\epsilon$, etc. for each modulus field involved in the supersymmetry

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**FIG. 2.** Same as Fig. 1 but for $m_{3/2} = 2$ TeV, $\delta_{GS} = -10$, $\theta = 0.06$ and $\epsilon = 0.006$. 

We close this section with a brief comment about a further generalization of the $O-II$ scenario, in which the assumption that only the single modulus $T$ plays a role in supersymmetry breaking is relaxed. For the purposes of this study, the important feature remains that these individual moduli will contribute to gaugino masses only at one-loop. Therefore, the number of parameters increases substantially; in addition to the need to define extra Goldstino angles (as is done in [11] and will be required in the Type I models discussed below), there will be [12,13] a $\delta_{GS}$, $\epsilon$, etc. for each modulus field involved in the supersymmetry
breaking, the details of which will depend on the particular orbifold model under consideration. Due to the additional complications and model-dependence, we do not consider such scenarios further in this paper. We anticipate that in general there can be particular models for which the parameter space for viable large phase solutions will be wider than that of the minimal scenario considered in this paper.

IV. SOFT BREAKING TERMS IN HOŘAVA-WITTEN SCENARIOS

A. Theoretical Framework

We now turn to a newer class of models based on the work of Hořava and Witten [14], who showed that eleven-dimensional supergravity (the conjectured low energy limit of M theory) compactified on a Calabi-Yau three-fold times an orbifold interval along the eleventh dimension gives rise to $E_8 \times E_8$ gauge theories with $N = 1$ supersymmetry in four dimensions, and further proposed that this framework describes the strongly coupled heterotic $E_8 \times E_8$ string theory. In this scenario, the two $E_8$ gauge multiplets reside on two ten-dimensional boundaries, which are separated by the interval corresponding to the eleventh dimension. The phenomenological implications of this scenario display several attractive features which are not present in the case of perturbative heterotic string theory. For example, there is the possibility of reconciling the string scale and the GUT scale, which is an encouraging result for the unification of the gauge couplings. Furthermore, the usual hidden sector mechanism for the breakdown of supersymmetry can be naturally realized in this class of models; supersymmetry can be broken (perhaps via gaugino condensation) on the hidden boundary, and transmitted to the observable sector by the dilaton and moduli fields, which can travel in the bulk (see e.g. [16,17]).

In [17], the soft supersymmetry breaking parameters were derived within the framework in which the effects of supersymmetry breaking are encoded in the parameterization (3) of the auxiliary component VEV’s of the dilaton $S$ and overall modulus $T$. The results were obtained by determining the form of the Kähler potential, superpotential, and gauge kinetic function to the first subleading order in the M theory expansion of the effective four-dimensional supergravity theory. For the purposes of this study, we note that the gauge kinetic function of the observable sector $E_8$ gauge group takes the form

$$f_{\text{obs}} = S + \alpha T,$$

in which $\alpha$ is a coefficient of $O(1)$. This feature is in direct contrast to the $T$-dependent

\footnote{It is important to note that the many studies of M theory vacua assume the “standard embedding”, in which the spin connection is embedded into one of the $E_8$ gauge groups, although the standard embedding does not play a special role in the construction of these vacua (in contrast to the case of the weakly coupled heterotic string) [15]. While in fact the relaxation of this condition can lead to more general scenarios, the conclusions about nontrivial CP effects are the same for the case in which $S$ and a single $T$ modulus contribute to the supersymmetry breaking.}
piece of the gauge kinetic function in the perturbative case (5), which is suppressed by a loop factor. The soft breaking parameters take the form

\[ M = \frac{\sqrt{3} m_{3/2}}{1 + \epsilon_0} \left( \sin \theta e^{-i\alpha_S} + \frac{\epsilon_0}{\sqrt{3}} \cos \theta e^{-i\alpha_T} \right) \]

\[ m^2 = m_{3/2}^2 - \frac{3 m_{3/2}^2}{(3 + \epsilon_0)^2} (\epsilon_0 (6 + \epsilon_0) \sin^2 \theta + (3 + 2 \epsilon_0) \cos^2 \theta \]

\[ - 2 \sqrt{3} \epsilon_0 \sin \theta \cos \theta \cos (\alpha_S - \alpha_T) \]

\[ A = - \frac{\sqrt{3} m_{3/2}}{3 + \epsilon_0} \left( (3 - 2 \epsilon_0) \sin \theta e^{-i\alpha_S} + \sqrt{3} \epsilon_0 \cos \theta e^{-i\alpha_T} \right), \]

in which \( \epsilon_0 \) is given by

\[ \epsilon_0 = \frac{4 - (S + S^*)}{S + S^*}. \]

As discussed in [17] (to which we refer the reader for an explanation of this point), the standard embedding constrains the range of \( \epsilon_0 \) to \( 0 < \epsilon_0 < 1 \).

It is clear from above relations that in these scenarios, the gaugino mass parameters are universal when consideration is restricted to the case in which the dilaton and the single \( T \) modulus participate in supersymmetry breaking. Therefore, we anticipate that the cancellation mechanism generally will not be adequate, from the general discussion presented in Section II. We note that this conclusion is not likely to hold in more general scenarios in which several individual \( T_m \) moduli associated with the Calabi-Yau manifold are involved in the supersymmetry breaking. In the multimoduli case, it is likely that the \( T_m \)-dependent contributions to the gaugino masses will be gauge-group dependent; if these contributions have \( O(1) \) coefficients as in (12), the gaugino masses will be nonuniversal over a greater range of parameter space than in the \( O-II \) orbifold model discussed in the previous section, and may provide for interesting CP effects. However, we restrict our consideration to the overall modulus case in this paper, and defer the study of more generalized Hořava-Witten scenarios to a future study.

**B. Results**

In our analysis of possible CP effects in the Hořava-Witten scenario we proceed along similar lines as in the \( O-II \) orbifold models. However, in principle there is an important difference in that the string scale is not fixed to the value \( M_{\text{String}} \sim 5 \times 10^{17} \text{GeV} \) as in the heterotic case, but can take any value [14], including \( M_G \) (which we choose for simplicity). We start from the free parameters \( m_{3/2}, \theta, \) and \( \epsilon_0 \), which, in combination with the two independent complex phases \( \alpha_S \) and \( \alpha_T \), determine the soft breaking parameters at the string scale. All soft terms are subsequently RGE evolved from \( M_{\text{String}} \) down to the electroweak scale and particle masses are calculated together with all physical CP-violating phases. We consider \( B \) and \( \mu \) to be independent parameters, although their magnitudes are numerically determined from the requirement of radiative symmetry breaking. The phase of \( \mu \) is varied independently as another free parameter of the model.
FIG. 3. Electron and neutron EDM allowed regions for the Hořava-Witten scenario with $m_{3/2} = 500$ GeV, $\theta = 0.5$, $\epsilon_0 = 0.9$, and $\tan \beta = 2$ shown in the $\varphi_\mu-\varphi_{A_e}$ plane. The dotted area shows the region of phases allowed in this scenario. The red (black) circles and green (grey) blocks denote points allowed by the eEDM and nEDM respectively. The values of $|B|$ and $|\mu|$ are fixed by radiative electroweak symmetry breaking.

The relative phase between the universal values of $M$ and $A$ determines the physical CP-violating phases of the soft $A$ parameters at the string scale. It is obvious that its value is restricted by the allowed ranges of $\epsilon_0$ and $\theta$; for example, in the limiting cases when either $\sin \theta$ or $\cos \theta$ are zero, the relative phase is zero at the string scale. The additional requirement of positive $m^2$ values also restricts the allowed regions of $\epsilon_0$ and $\theta$. As a result, it is difficult to obtain large phases of the $A$ parameters at the electroweak scale as $\alpha_S$ and $\alpha_T$ are varied from zero to $2\pi$. In Figure 3 we show the points allowed by the electron and neutron EDM’s in a typical example of this scenario with $m_{3/2} = 500$ GeV, $\epsilon_0 = 0.9$, and $\theta = 0.5$. Since the gaugino phases are identically zero and the range of $\varphi_{A_e}$ is severely restricted by correlations between $A$ and $M$ at the string scale, the cancellations are insufficient in this particular scenario. Therefore, only a very small fraction of the $\varphi_\mu-\varphi_{A_e}$ parameter space leads to models allowed by the electron EDM, in direct analogy with the dilaton-dominated scenario in the perturbative heterotic models discussed in [18].

V. SOFT BREAKING TERMS IN TYPE I MODELS

A. Theoretical Framework

We now turn to another example of a new class of models, the four-dimensional Type IIB orientifold models with $N = 1$ supersymmetry [19–22]. These models are based on the Type IIB (closed string) theory compactified on orientifolds, which are orbifold compactifications
accompanied by an additional worldsheet parity operation. The consistency of the theory requires the addition of open string (Type I) sectors, with the open strings ending on Dirichlet D-branes. It is important to note that orientifolds are illustrative of a much larger class of models in the Type I picture, containing more general configurations of nonperturbative objects (e.g. D-brane bound states) in more general singular backgrounds (e.g. conifolds [43]).

The number and type of D-branes required in a given model depends on the details of the orientifold group; however, in the most general case for compact Abelian orbifolds there is one set of nine-branes and three sets of five-branes \( (5_i) \), in which \( i \) labels the complex coordinate of the internal space included in the five-brane world-volume. Gauge groups are associated with each set of coincident D-branes. These models are constructed utilizing perturbative techniques. However, due to the Type I-heterotic S-duality, the Type IIB orientifold models have heterotic duals; the heterotic duals are perturbative for orientifold models with only nine-branes (such as the \( \mathbb{Z}_3 \) orientifold [19]), but nonperturbative in the more general case with additional sets of five-branes (such as orientifolds with order-two twists [20]).

The chiral matter fields also arise from open string sectors and can be classified into two categories. The first category are fields which arise from open strings which start and end on the same type of D-branes. These fields are therefore charged under only the (generically non-Abelian) gauge group of that set of branes, typically in the fundamental or antisymmetric tensor representations. The second class of fields originate from open strings which start and end on different types of branes and are hence are charged under the two associated gauge groups. In this case, the states are bifundamental representations under the associated two gauge groups from the two D-brane sectors. In the closed string sector, there are the dilaton \( S \) and moduli fields \( T_i \), as well as the twisted sector moduli, which play a role in the cancellation of the anomalous \( U(1)'s \) generically present in these models [44].

A recent investigation shows that the phenomenological properties (including the possibilities for gauge coupling unification) [21] of these models are quite distinctive from those of the perturbative heterotic models. In particular, the string scale is not fixed close to \( M_{\text{Planck}} \) as in the weakly coupled heterotic case, but rather can take lower values. The implications for electroweak scale physics also crucially depend on the nature of the embedding of the SM gauge group into the different D-brane sectors.

The soft supersymmetry breaking terms obtained when the dilaton and moduli fields are responsible for the breakdown of supersymmetry can be determined using the parameterization of the \( F \)-component VEV’s (following [11,21]):

\[
F^S = \sqrt{3}(S + S^*)m_{3/2}\sin \theta e^{i\alpha_S} \\
F^i = \sqrt{3}(T_i + T_i^*)m_{3/2}\cos \theta \Theta_i e^{i\alpha_i},
\]

in which \( \Theta_i \) are generalized Goldstino angles (with \( \sum_i \Theta_i^2 = 1 \)). The soft terms can then be computed [21] with the knowledge of the structure of the Yukawa superpotential couplings [19,20] and the tree-level Kähler potential and gauge kinetic functions [21], which have also been determined for this class of models.

For the purposes of the studying the phase structure of the soft terms, we note that the gauge kinetic functions determined in [21] take the form

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\[ f_9 = S \]
\[ f_{5i} = T_i, \]  
(16)

which demonstrate that the dilaton no longer plays a universal role (as the moduli dependence now occurs at the tree-level) as it did in the perturbative heterotic case. In particular, the structure of (16) illustrates a distinctive feature of this class of models, which is in a sense there is a different “dilaton” for each type of brane.

This fact has important implications in this class of models both for gauge coupling unification [21] and the patterns of gaugino masses, which strongly depend on the embedding of the SM gauge group. In the case in which the SM gauge group is embedded in a single D-brane sector, the pattern of the gaugino masses resembles that of the tree-level gaugino masses in the weakly coupled heterotic models studied in the previous section. For example, if the SM gauge group is embedded within the nine-brane sector, this can be seen from the similarity between (16) and the corresponding tree-level expression for \( f \) in the perturbative heterotic models (4); the situation is similar (with the corresponding modulus field \( T_i \) playing the role of the dilaton) if the SM arises from a single 5 \(_i\) brane sector.

However, if the SM gauge groups arise from distinct D-brane sectors, there is the possibility of nonuniversal gaugino masses at the tree-level, which can be seen from (16). This feature was not possible in the perturbative heterotic models discussed in the previous sections, and is interesting from the point of view of obtaining new patterns of nontrivial relative phases in the soft terms.

To explore this possibility, we consider toy models of soft terms derived with the assumption that the \( SU(3) \) and \( SU(2) \) gauge groups arise from different five-brane sectors\(^9\) (for example, \( 5_1 \) and \( 5_2 \)). The possibilities for the embedding of \( U(1)_Y \) are then restricted by phenomenological criteria. For example, an important constraint is that the MSSM particle content contains the quark doublet states, which are charged under all of the gauge groups; this fact restricts \( U(1)_Y \) to arise from the \( 5_1 \) and/or \( 5_2 \) sectors, as the matter fields of these Type I models are at most charged under the gauge groups of two D-brane sectors. In this paper, we choose for simplicity to restrict our consideration to simplified scenarios in which \( U(1)_Y \) resides in either the \( 5_1 \) or the \( 5_2 \) sector \(^{10}\). Depending on the details of the hypercharge embedding, the remaining MSSM states may either be states which (in analogy with the quark doublets) are trapped on the intersection of these two sets of branes, or

\(^9\)We could also assume that one of the gauge groups arises from the nine-brane sector. It was noted in [21] that it may be more difficult to obtain consistent unification of the gauge couplings at the GUT scale in this case. Although this point is not crucial for the purposes of this study, we choose the case of embedding the SM in the five-brane sectors for the sake of definiteness.

\(^{10}\)Although it is not clear if such special cases can be realized in explicit orientifold models, we note that in the models that have been constructed to date in which the SM non-Abelian gauge groups can arise from different D-brane sectors, the hypercharge gauge group is in general a linear combination of gauge groups arising from the two sectors. We thank Gary Shiu for a discussion of this point.
states associated with the single 5, sector which contains $U(1)_Y$. In any event the natural starting point for constructing models with these features are orientifolds which realize identical GUT gauge groups and massless matter on two sets of intersecting 5- branes. The existence of such symmetrical arrangements is often guaranteed by T-duality. For example, Shiu and Tye [22] have exhibited an explicit model which realizes the Pati-Salam gauge fields of $SU(4) \times SU(2)_L \times SU(2)_R$ and identical chiral matter content on two sets of 5- branes. Additional Higgsing and modding by discrete symmetries could then in principle produce the asymmetrical structures outlined above.

In this scenario, the soft scalar masses can take the form (see the general formulae in [21]):

$$m_{5,5}^2 = m_{3/2}^2(1 - \frac{3}{2}(\sin^2 \theta + \cos^2 \theta \Theta^2_3))$$

$$m_{5}^2 = m_{3/2}^2(1 - 3 \sin^2 \theta).$$

(17)

In the case with $U(1)_Y$ and $SU(3)$ from the 5, sector, the $SU(2)$ doublet states clearly arise from open strings stretching between the two D- brane sectors, while the $SU(2)$ singlets can either be states of the same type or states associated with the 5, brane sector only. The gaugino masses and $A$ terms take the form

$$M_1 = \sqrt{3}m_{3/2} \cos \theta \Theta_1 e^{-i\alpha_1} = M_3 = -A_{t,e,u,d}$$

$$M_2 = \sqrt{3}m_{3/2} \cos \theta \Theta_2 e^{-i\alpha_2},$$

(18)

and hence the relations among the phases $\varphi_i$ of the gaugino mass parameters $M_i$ are $\varphi_1 = \varphi_3 \neq \varphi_2$. Similar expressions apply for the case in which $U(1)_Y$ and $SU(2)$ are associated with the same five-brane sector; in this case, the relations among the phases $\varphi$ of the gaugino mass parameters are $\varphi_1 = \varphi_2 \neq \varphi_3$. In these models, the solution to the $\mu$ problem is not certain, and hence $\mu$ and $B$ are free complex parameters in the analysis (although their phases are as usual related by the PQ symmetry of the MSSM superpotential).

Due to the absence of quasi-realistic Type I models as yet (despite continued progress in model-building techniques [19,20,22]), it is not certain whether this type of SM embedding can be realized in an explicit orientifold model. Therefore, we emphasize that these models should be interpreted as toy models which illustrate new possibilities for the patterns of soft breaking terms in this new class of four-dimensional superstring models.

### B. Results

Our numerical analysis of the Type I models closely follows the approach adopted for the Hořava-Witten scenarios. In the Type I models, the string scale is not fixed; for the sake of simplicity, we assume the string scale and the GUT scale coincide, and that the gauge couplings unify at this scale. It is beyond the scope of this paper to consider all possibilities, and we refer the reader to a comprehensive discussion of this issue and its implications for gauge coupling unification in [21]. For the sake of simplicity, we assume the string scale and the GUT scale coincide, and that the gauge couplings unify at this scale. Thus, in the first model we consider with $SU(3)$ and $U(1)_Y$ arising from the same brane sector, the
boundary conditions (17) and (18) are implemented at the GUT scale, and the parameters are subsequently evolved down to the electroweak scale. The sparticle masses and the CP-violating phases depend on the free parameters $m_{3/2}$, $\theta_i$, $i = 1, 2, 3$, which are related by $\Theta_1^2 + \Theta_2^2 + \Theta_3^2 = 1$, as well as the two phases $\alpha_1$ and $\alpha_2$. To avoid negative scalar mass-squares we restrict our consideration to values of $\theta$ which satisfy $\sin^2 \theta < \frac{1}{3}$, and also assume that $\Theta_3 = 0$ (indicating that the modulus $T_3$ associated with the 5$_3$ brane sector plays no role in supersymmetry breaking, and thus is essentially decoupled from the observable sector). We also treat $B$ and $\mu$ as free parameters, as they are not determined in this scenario. In addition, we explore the phenomenologically motivated scenario in which the electroweak symmetry is broken radiatively as a result of RGE evolution of the Higgs masses $m_{H_1}^2$ and $m_{H_2}^2$. As the minimization conditions are imposed at the electroweak scale, the values of $B\mu$ and $|\mu|^2$ can be expressed in terms of $\tan \beta$ and $M_Z$ [45]. However, even under these assumptions $\varphi_\mu$ is still an independent parameter.

In frame a) of Figure 4, we show the results for $m_{3/2} = 150$ GeV, $\theta = 0.4$ and $\Theta_1 = 0.85$. As in the previous case of orbifold models we fix $\tan \beta = 2$, although different values of $\tan \beta$ have been explored 11. In this case we do not impose the condition of correct radiative electroweak symmetry breaking, but rather assume $B$ and $\mu$ take the values $|B| = 100$ GeV and $|\mu| = 600$ GeV. We find here, remarkably, that in order to satisfy the experimental constraints on the electron and neutron EDM's in this model, the large individual contributions from chargino, neutralino and gluino loops do not have to be suppressed by small CP phases. A cancellation between the chargino and neutralino loop contributions thus causes the electron EDM to be acceptably small. As emphasized in [6], the contributions to chargino and neutralino diagrams from gaugino-higgsino mixing naturally have opposite signs and the additional $\varphi_1$ dependence of the gaugino exchange contribution to the neutralino diagram can provide for a match in size between the chargino and neutralino contributions. The importance of the gaugino exchange diagrams increases for large values of $\mu$ and allows the cancellation to take place for a wider range of $\varphi_\mu$ values. In the neutron case, the contribution of the chargino loop is offset by the gluino loop contributions to the electric dipole operator $O_1$ and the chromoelectric dipole operator $O_2$. Since $\varphi_1 = \varphi_3$ in this scenario, the gluino contribution automatically has the correct sign to balance the chargino contribution in the same region of gaugino phases which ensures cancellation in the electron case. This simple and effective mechanism therefore provides extensive regions of parameter space where the electron and neutron EDM constraints are satisfied simultaneously while allowing for $O(1)$ CP-violating phases.

If electroweak symmetry is assumed to be broken radiatively, the resulting value of $|\mu|$ is somewhat smaller. In our particular case with the remaining parameters unchanged $|\mu| \sim 350$ GeV. The ranges of allowed CP-violating phases are shown in Figure 4 b). Here also the electron and neutron EDM allowed regions overlap substantially although the range of $\varphi_\mu$ is slightly reduced. However, the general picture is valid, and low energy models with light superpartner mass spectra and large CP-violating phases can be obtained within this

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11We do not consider large values of $\tan \beta$, as new types of contributions can become important [46].

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FIG. 4. Electron (red (black) circles) and neutron (green (grey) blocks) EDM allowed regions for the Type I orientifold models with $m_{3/2} = 150$ GeV, $\theta = 0.4$, $\Theta_1 = 0.85$ and $\tan \beta = 2$. In frame a) the values of $B$ and $\mu$ are assumed to be independent and their magnitudes are set to $|B| = 100$ GeV and $|\mu| = 600$ GeV. Frame b) shows the results for the case when electroweak symmetry is assumed to be broken radiatively.

framework, even including the constraint of electroweak symmetry breaking.

The cancellation mechanism in this scenario provides a large range of allowed CP-violating soft phases and requires a specific correlation between $\varphi_\mu$ and $\varphi_1 = \varphi_3$ as shown in Figure 4. To demonstrate the coincidence of the regions allowed by the experimental constraints on the EDM’s, we choose $m_{3/2} = 150$ GeV, $\theta = 0.4$, and $\tan \beta = 2$, which leads to a reasonably light superpartner spectrum. In Fig. 5, we plot the allowed regions for both electron and neutron EDM depending on the values of $\Theta_1$ and $\Theta_2 = \sqrt{1 - \Theta_1^2}$ while $\Theta_3$ is set to zero. Frame a), where $\Theta_1 = 0.85$, shows a very precise overlap between the electron and neutron EDM allowed regions. In frame b), the magnitudes of all three gaugino masses are equal but have different phases due to the different origin of $M_1$ and $M_3$ compared to $M_2$. Finally, in frame c) we set $\Theta_1 = 0.55$ so the magnitude of $M_2$ is significantly larger than that of $M_1 = M_3$; in this case the alignment between the EDM allowed regions is spoiled.
FIG. 5. Illustration of the overlap between the regions allowed by the electron EDM (denoted by the red (black) circles) and neutron EDM (denoted by the green (grey) blocks) constraints. We choose $m_{3/2} = 150$ GeV, $\theta = 0.4$ and $\tan \beta = 2$, and impose radiative EW symmetry breaking. Allowed points are shown for a) $\Theta_1 = 0.85$, b) $\Theta_1 = \sqrt{1/2}$, and c) $\Theta_1 = 0.55$.

and only small CP-violating phases are allowed.

It is also interesting to observe that the actual values of the electron and neutron EDM’s for the allowed points in the phase parameter space are typically slightly below the experimental limit and should be within the reach of the next generation of EDM measuring experiments. In Figure 6 we plot the EDM values for the allowed points in the case of $\Theta_1 = 0.85$ with all the other parameters set to the same values as in previous discussion of Figure 4. This indicates that if the CP-violating phases indeed originate from this type of D-brane configuration, non-zero measured values for both EDM’s much bigger than the SM prediction can be expected.

However, the other orientifold model (in which $SU(2)$ and $U(1)_Y$ arise from the $5_1$ brane sector) does not allow for large phase solutions. The reasons for this behavior are similar to that of the Hořava-Witten scenario: we can use the $U(1)_R$ symmetry of the soft terms to put $\varphi_2 = \varphi_1 = 0$, which severely limits the possibility of cancellation between the chargino and neutralino contributions to the electron EDM. The effect of $\varphi_{A_\kappa}$ alone is not enough to
FIG. 6. Range of the electron and neutron EDM values vs. $\varphi_1 = \varphi_3$ predicted by Eqs. (17) and (18) for the parameters of Figure 4a). All of the points are allowed by the experimental bounds on the EDM’s (note the different scales for the eEDM and nEDM).

offset the potentially large chargino contribution and only a very narrow range of values of $\varphi_\mu$ (close to 0, $\pi$, ...) passes the electron EDM constraint. Hence, except at isolated points in the parameter space of this model, the phases must be at or below the traditional bound $\lesssim \mathcal{O}(10^{-2})$ to satisfy the EDM constraints without assuming large sparticle masses.

VI. CONCLUSIONS

In this paper, we have investigated the possibility that the soft breaking terms derived in classes of superstring models have large CP-violating phases that satisfy the phenomenological bounds on the electric dipole moments of the electron and neutron through cancellations. The analysis builds on the work of [5] and [6], who demonstrated that this effect can allow for viable points in the MSSM parameter space with large phases and light superpartner masses, providing for an alternate resolution to the supersymmetric CP problem.

Sufficient cancellations among the contributions to the EDM’s are difficult to achieve unless there are large relative phases in the gaugino mass parameters [6]. This feature strongly depends on the string model under consideration; for example, large phases consistent with the EDM constraints are disfavored in perturbative heterotic string models and
models based on Hořava-Witten theory (in the overall modulus limit), as the gaugino masses are universal at tree level. However, our analysis demonstrated that this scenario can be achieved naturally within Type I string models, where the tree-level gaugino masses may be nonuniversal depending on the embedding of the SM gauge group into the D-brane sectors.

We found that within Type I string models in which $SU(3)$ and $U(1)_Y$ arise from one five-brane sector and $SU(2)$ arises from another set of five-branes, the cancellations among different contributions to the EDM's occur over a remarkably wide range of parameter space. In this scenario, the typical values of the electric dipole moments are not much smaller than the current experimental limits. Equally remarkably, if we alter the SM embedding such that $SU(2)$ and $U(1)_Y$ arise from the same set of branes, the EDM constraints exclude large phase solutions.

The results presented in this paper illustrate that large soft phases are at least consistent with, and perhaps motivated by, some string models. Most importantly, the analysis demonstrates how we may be able to learn about (even nonperturbative) Planck scale physics using low energy data. For example, if the phases of the soft breaking parameters are determined from collider superpartner data, or measured at B factories, and found to be large, we have seen that they may provide guidance as to how the SM is to be obtained from four-dimensional compactifications of string theory.

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