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COMPUTATION OF $K \ell_4$ FORM FACTORS WITH KAON PCAC AND
VENEZIANO OR WEINBERG MODELS

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A B S T R A C T

We compute the F_1 , F_2 form factors of $K \ell_4$ decay for zero square of the momentum of the lepton pair using kaon PCAC and the $K\pi$ amplitudes of Veneziano or Weinberg models. Agreement with experiment is found to be reasonable for both models.

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By using the Veneziano model, Roberts and Wagner ¹⁾ were able to compute the variation of the $K \ell_4$ form factors. In this note we remark that also their magnitude can be obtained with the help of PCAC for kaons and then using either the Veneziano model or Weinberg's method for the $K \pi$ scattering amplitude. The idea is to write the $K \ell_4$ hadronic current as

$$\begin{aligned} \langle \pi^+(p^+), \pi^-(p^-) | J_\mu | K^+(k) \rangle = & \frac{i}{M_K^3} \epsilon_{\mu\nu\rho\sigma} k_\nu p_\rho^+ p_\sigma^- F_4 + \\ & + \frac{1}{M_K} \{ (p^+ + p^-)_\mu F_1 + (p^+ - p^-)_\mu F_2 + q_\mu F_3 \} \end{aligned} \quad (1)$$

where $q = p_\ell^+ + p_\ell^-$ is the momentum of the lepton pair; p^+ , p^- , k the momenta of π^+ , π^- , K^+ respectively. Using now PCAC for J_μ one gets from (1) that

$$x F_1(x, y, q^2=0) + y F_2(x, y, q^2=0) = C_K M_K T_{K\pi}(x, y, q^2=0) \quad (2)$$

where $x = (p^+ + p^-)q$, $y = (p^+ - p^-)q$, C_K is the kaon decay constant ($\simeq 35.3$ MeV) and $T_{K\pi}$ is the invariant $K \pi$ scattering amplitude with one kaon off-mass shell.

Although the experimental evidence for kaon PCAC is not fully compelling, it is not unreasonable to accept it in the framework of the Veneziano or Weinberg models, as they incorporate, e.g., the soft kaon zeros which are a consequence of PCAC ²⁾.

Let us first consider the Veneziano method. One can write the Veneziano expression for $T_{K\pi}$,

$$T_{K\pi} = \beta \frac{\Gamma(1 - \alpha_p(s)) \Gamma(1 - \alpha_{K^*}(u))}{\Gamma(1 - \alpha_p(s) - \alpha_{K^*}(u))} \quad (3)$$

$s = (p^+ + p^-)^2$, $u = (p^+ + q)^2$, where we take the usual one-term approximation to the "exact" Veneziano amplitude ³⁾, and the quantity β is assumed to carry all dependence on the external masses; as is usually done this dependence is supposed to be very weak and accordingly we set $\beta(q^2=0) = \beta(q^2=M_K^2) = \text{constant}$. The value of β can then be found by normalizing $T_{K\pi}$ either to the experimental K^* width, which gives $\beta \simeq 42$, or to the scattering lengths as calculated "à la Weinberg" ^{*) 4)}, obtaining $\beta \simeq 27$. Also, if one normalizes to the ρ pole, one gets $\beta \simeq 27$, thus substantiating the range of variation $42 \gtrsim \beta \gtrsim 27$ for β .

To compute the F_1 , F_2 form factors one then uses (2) and (3) separating conveniently the soft kaon zero at $q \rightarrow 0$; one gets

$$x F_1 + y F_2 = 0.89 C_K M_K \beta \frac{\Gamma(1 - \alpha_\rho(s)) \Gamma(1 - \alpha_{K^*}(u))}{\Gamma(2 - \alpha_\rho(s) - \alpha_{K^*}(u))} (x+y) \quad (4)$$

with the trajectories given by

$$\begin{aligned} \alpha_\rho(s) &= 0.48 + 0.89s \\ \alpha_{K^*}(u) &= 0.28 + 0.89u \end{aligned} \quad (5)$$

(we use consistently units of GeV unless explicitly stated otherwise).

Expression (4) then allows us to compute F_1 and F_2 at the point $x=y=0$, by differentiating Eq. (4) with respect to x and y , obtaining

$$F_1 = F_2 = 0.89 C_K M_K \beta \frac{\Gamma(1 - \alpha_\rho(M_K^2)) \Gamma(1 - \alpha_{K^*}(M_\pi^2))}{\Gamma(2 - \alpha_\rho(M_K^2) - \alpha_{K^*}(M_\pi^2))} \quad (6)$$

i.e., that $F_1/F_2 = 1$. This result, which also follows from standard current algebra calculations at the soft pion point, is independent

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of smoothness assumptions for $\beta(q^2)$. Furthermore, it is also possible to evaluate F_1 over the whole range of x keeping $y=0$. (It is however not possible to do the same for F_2 as $x=0$ implies $q^2=0$, and hence, $y=0$ too.) The computation is straightforward, and one finds that F_1 varies very little over the region $300 \text{ MeV} \leq \sqrt{s} \leq 400 \text{ MeV}$, (and in fact, over the whole range), which is dominant due to phase space considerations, and there,

$$F_1 = 2.1 \quad (\beta = 42)$$

$$F_1 = 1.4 \quad (\beta = 27)$$

If one accepts the arguments of Ref. 1), according to which F_1 and F_2 have the same variation over the entire range, then Eq. (6) tells us that also $F_2 = F_1$ for $q^2=0$, $y=0$, $300 \text{ MeV} \leq \sqrt{s} \leq 400 \text{ MeV}$.

We now turn to the Weinberg model. Following Weinberg⁴⁾, we expand the amplitude to lowest order in s' , t' , u' and require the soft pion and kaon meson zeros (here, s' , t' , u' are the usual variables for $K\pi$ scattering; s' is the energy in the $K\pi$ channel). This gives

$$T_{K\pi}^{(+)} = a (s' + u' + 2t' - 2M_\pi^2 - 2M_K^2) + \dots$$

$$T_{K\pi}^{(-)} = b (s' - u') + \dots$$

for the crossing symmetric and antisymmetric amplitudes. Weinberg's formula for the first order small term when $p_{\pi_1} = p_{\pi_2} = 0$ gives $b = 2\pi L/M_\pi$, where L is Weinberg's length, Ref. 4), (with f_π determined by $\pi \rightarrow \mu \nu$, $b = 27.8 \text{ GeV}^{-2}$; with f_π from the Goldberger-Treiman relation, $b = 37.2 \text{ GeV}^{-2}$). To obtain a , it is necessary to contract out one pion and one kaon; this gives $a = -b$. We require the amplitude $T^{(+)} + T^{(-)}$ with an initial positive pion crossed into the final state. Hence,

$$T_{K\pi}(x, y, q^2=0) = 2b(x+y)$$

$$F_1(x=y=q^2=0) = F_2(x=y=q^2=0)$$

$$F_1(x, y=q^2=0) = 2b C_K M_K = 0.97 \text{ or } 1.3$$

where the first value is obtained using f_π from $\pi \rightarrow \mu \nu$ and the second from the Goldberger-Treiman relation.

The comparison with experimental values has been performed by using the fit with constant form factors of Ely et al.⁵⁾ and rejecting fits where F_1 and F_2 are substantially different. We then have

$$\begin{aligned} F_1^{\text{experimental}} &= 1.18 \pm 0.07 \\ F_2^{\text{experimental}} &= 1.6 \pm 0.2. \end{aligned}$$

The fits with varying form factors are qualitatively similar but we did not use them because their slope is opposite to the one in the Veneziano case.

The agreement with experiment is thus reasonable in each case in view of the errors inherent to our computations (see footnote 2 about errors in PCAC, and below for other errors). Incidentally, we remark that the current algebra computation of Weinberg⁶⁾ who relates F_1 and F_2 to K_{l3} gives $F_1 = F_2 = 0.84$ or 0.97 where, as previously, the difference arises from a different determination of f_π .

It is interesting to look for possible corrections to the Veneziano computation. These may come from the off-mass shell extrapolation of β , or from the Veneziano model itself due to contributions from the Pomeranchuk to the quadratic terms in the amplitude. to get an idea of the importance of these contributions we have computed them by calculating the second derivative at threshold of the Pomeranchuk contribution

$$T_P^{(+)}(E) = \frac{E^2 - M_\pi^2}{\pi} \int dE' \frac{\text{Im } T_P^{(+)}(E')}{(E'^2 - M_\pi^2)(E' - E)}, \quad E = \frac{3 - M_\pi^2 - M_K^2}{2 M_K}, \quad t=0$$

where T_P rises steadily from zero at ~ 1 GeV to give a constant cross-section of 20 mb at infinity, obtaining a factor of $\sim 15\%$.

This seems encouraging, as both the PCAC and Pomeranchuk corrections are of the order of magnitude of the discrepancy between theoretical and experimental results but, on the other hand, it shows that any reasonable model incorporating the current algebra zeros and the soft meson scattering lengths is likely to give the right result ⁷⁾.

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FOOTNOTES AND REFERENCES

- 1) R.G. Roberts and F. Wagner, CERN preprint TH. 990, (1969).
- 2) We mention the following tests of PCAC for kaons:
 - a) The analogue of the Goldberger-Treiman relation gives $G_{\Lambda NK}^2/4\pi \simeq 14$. Direct evidence from strong interactions gives $G_{\Lambda NK}^2/4\pi = 4.7 \pm 1.9, 6 \pm 2, 7.4 \pm 1.8, 11.9 \pm 2.7, 13 \pm 3, 16.0 \pm 2.5$.
 - b) PCAC for pions and current algebra gives the Callan-Treiman relation $f_+ + f_- = f_K / \sqrt{2} f_\pi$ ($M_\pi^2 = 0$) for the $K \rightarrow \pi$ form factors. PCAC for kaons gives $f_+ - f_- = f_\pi / \sqrt{2} f_K$ ($M_K^2 = 0$). Neglecting off-mass shell effects and taking $f_\pi = f_K$ to fix the Cabibbo angle gives $f_+ = 1/\sqrt{2} = 0.71$, $f_- = 0$ (experimentally $f_+ = 0.61$, $f_- \sim 0.16$); assuming $f_+ = 1/\sqrt{2}$ to fix θ gives $f_- = 0$, $f_\pi = f_K$ (experimentally $f_- \sim 0.16$, $f_K = 1.22 f_\pi$).
 - c) Costa and Shaw (Rutherford Lab. preprint RDP/A29) have recently tested the soft kaon zero in K^+p scattering and conclude that it is reasonably well satisfied.
- 3) C. Lovelace, Phys. Letters 28B, 265 (1968).
- 4) S. Weinberg, Phys. Rev. Letters 17, 616 (1966).
- 5) R.P. Ely, Jr., et al., UCRL-18626 (1968);
F.A. Berends, A. Donnachie and G.C. Oades, Nuclear Phys. B3, 569 (1967).
- 6) S. Weinberg, Phys. Rev. Letters 17, 336 (1966) and errata Phys. Rev. Letters 18, 1178 (1967).
- 7) As a matter of fact the deviation of the Veneziano expression off a linear approximation is, in the region of interest to us here, of about the same order of magnitude, i.e., $\sim 20\%$.