where the Latin suffix runs over both axial-vector and vector SU(3) indices. The corresponding group structure functions are denoted by h_{ijk} , where $h_{ijk}=0$ if an odd number of its indices are axial-vector, and $h_{ijk}=f_{ijk}$ otherwise.

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Deep-Inelastic Scattering, the Subtraction of Divergent Sum Rules, and Chiral-Symmetry Breaking in the Gluon Model*

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The formal light-cone properties of commutators involving current divergences are studied in the gluon model. Relations are derived which make it possible (in principle) to distinguish the vector- from the (pseudo) scalar-gluon model. In the vector-gluon model these relations provide an experimental determination of the bare quark masses. The additional assumption that the residues of any $\alpha=0$ fixed poles in current scattering amplitudes are polynomials in q^2 makes it possible to relate the σ term in pion-nucleon scattering to convergent integrals over neutrino-scattering structure functions; the polynomial assumption dictates a prescription for subtracting a (linearly) divergent sum rule derived previously. The same technique generates subtracted sum rules for the (neutrino- and spin-dependent) structure functions W_3^- and G_2^- . With the parton-model assumption that the leading scaling behavior of current-divergence and divergence-divergence scattering is given by free-field theory, it is possible to relate fixed-pole residues in ep , en , νp , and νn scattering, deep-inelastic data, the σ term, baryon mass differences, and the bare quark masses; approximate values for the bare quark masses and the parameter μ_0 can be obtained.

I. INTRODUCTION

In this paper we study properties of light-cone commutators involving current divergences in the vector-gluon model. All our considerations are formal; that is to say they are untrue in perturba-

tion theory. It is frequently argued that the scaling observed in the SLAC-MIT inelastic electron scattering experiments implies that formal field theory might be relevant to the real world. This argument is not totally compelling (especially since the data do not exclude $\log Q^2$ terms) but at

least we are in good company in pursuing the formal approach.

Despite the dubious validity of formal field theory, we find it interesting that relations can be obtained which probe the structure of the Hamiltonian. Briefly, we shall derive relations which (although hard to test experimentally) make it possible in principle to distinguish between vector- and (pseudo) scalar-gluon models and determine the bare quark masses in the vector-gluon model. With the further assumption that the residues of any $\alpha=0$ fixed poles in current amplitudes are polynomials in Q^2 , we are able to derive a subtracted sum rule for the σ term in pion-nucleon scattering (σ_π), in place of a sum rule derived previously which diverges linearly according to conventional Regge assumptions. (The same assumption is used here to derive two other "subtracted sum rules" and has other applications.)

Since this paper is rather discursive we have summarized our results explicitly in the last section, to which the reader may wish to turn first. In the rest of this section we shall review some previous work on this subject and explain the plan of our exposition.

In one of their first papers on the subject, Brandt and Preparata¹ pointed out that in the vector-gluon model the most singular terms in operator-product expansions on the light cone would be chirally symmetric. This implies that²

$$\lim_{\nu, -q^2 \rightarrow \infty} \nu W_{4,5} = 0,$$

where ν and q^2 are defined in the usual way (see Sec. II) and W_4 and W_5 are structure functions, defined in Eq. (2.5) below,³ which can (in principle) be measured in neutrino reactions. Subsequently, several authors⁴⁻⁷ have discussed the scaling properties of W_4 and W_5 , which probe the nature of chiral-symmetry breaking.

Mandula *et al.* showed⁴ that the dimension (d) of the operator in the part of the Hamiltonian which breaks chiral symmetry can be bounded in terms of the scaling properties of $W_{4,5}$. However, Ng and Vinciarelli have recently pointed out⁷ that the hope that these scaling properties might therefore be used to measure d is thwarted in the vector-gluon model in which the bound is not saturated (the bound is clearly not saturated in the free-quark model since it allows $\nu^{3/2} W_{4,5}$ to scale; but free-field theory cannot yield fractional powers of ν in the scaling laws).

Ng and Vinciarelli argue that $\nu^2 W_{4,5}$ scale and derive a sum rule for the σ term in pion-nucleon scattering (σ_π) in terms of the scaling functions. (Actually the sum rule of Ref. 7 is for the "light-

cone" σ term, defined at $x^+ = x^0 + x^3 = 0$ rather than at equal times. See Ref. 34 for the relation between the two definitions.) We observe that their sum rule (like the σ -term sum rule in Ref. 6) diverges linearly if conventional Regge behavior is assumed. This led us to reexamine the derivation.

In Sec. II we present a light-cone analysis of commutators involving divergences which determine $W_{4,5}$, and show that they can be used to distinguish models and, in the vector-gluon model, to measure the bare quark masses.

In Sec. III we turn to a momentum-space calculation (which takes account of asymptotic behavior from the start and cannot yield divergent sum rules). We find that σ_π is related to the asymptotic behavior of a subtraction constant. In Sec. IV we show that the latter quantity can be calculated if we accept the assertion that the residues of any $\alpha=0$ fixed poles in current-nucleon scattering amplitudes are polynomials in Q^2 . In Sec. V we show that in the parton model, in which the leading scaling behavior of divergence-divergence and current-divergence scattering are assumed to be given by free-field theory, $\nu^2 W_5$ vanishes asymptotically; our results then make it possible to relate fixed-pole residues in γp and γn scattering, deep-inelastic data, σ_π , baryon mass differences, and bare quark masses; existing data already allow a preliminary investigation of the bare quark masses.

In Sec. VI we use the assumption of polynomial residues to derive two other sum rules.

The reader should be forewarned that the experimental measurement of $W_{4,5}$ to the accuracy required to test our results is almost impossible.⁸ However, the very existence of results such as Eq. (2.14) (which can in principle serve to determine the interaction) is interesting.

II. LIGHT-CONE CALCULATIONS

In this section we study the scaling behavior of the structure functions W_4 and W_5 in the vector-gluon and parton models. We use the quark-model light-cone algebra developed by Fritzsche and Gell-Mann⁹ as extended to the vector-gluon model by Gross and Treiman.¹⁰ Identical results may be obtained by studying field theories quantized on the light cone, or almost-equal-time commutators in the Bjorken-Johnson-Low (BJL) limit.¹¹

By means of formal manipulations, based on the canonical equations of motion, Gross and Treiman found that to leading order on the light cone, the vector gluon can be treated as a massless, external, c -number field. That is, to leading order on the light cone the equations of motion

$$[i\not{\partial} - g\not{B}(y) - m]\psi(y) = 0$$

and

$$(\square + \mu^2)B_\nu(y) = g\bar{\psi}(y)\gamma_\nu\psi(y)$$

may be replaced by

$$[i\not{\partial} - g\not{B}(y)]\psi(y) = 0$$

and

$$\square B_\nu(y) = 0,$$

with

$$(x-y)_\mu(x-y)_\nu[B^\mu(x), B^\nu(y)] = 0 \text{ for } (x-y)^2 = 0.$$

Consequently, the free-field results of Fritzsche and Gell-Mann are modified only by the inclusion of a phase in the bilocal operator which modulates the leading free-field, light-cone singularity. The quark-field anticommutator, correct to leading order on the light cone, is given by

$$\begin{aligned} S_0(x, y) &\equiv \{\psi(x), \bar{\psi}(y)\} \\ &= -\not{\partial} D(x-y) \exp\left[-ig \int_y^x B_\mu(\xi) d\xi^\mu\right], \end{aligned} \quad (2.3)$$

where

$$D(x-y) \equiv \frac{1}{2\pi} \delta((x-y)^2) \epsilon(x_0 - y_0)$$

and the integral is taken over a lightlike path from y to x .

For our purposes it is necessary to study the

second-most-singular term in the light-cone expansion of the quark field anticommutator. $S_0(x, y)$ is not an exact solution to Eq. (2.2) because $B_\mu(y)$ is not curlless¹²:

$$\begin{aligned} \frac{\partial}{\partial x^\mu} \int_y^x d\xi^\mu B_\mu(\xi) &= B_\mu(x) - \int_0^1 da (x-y)^\lambda \\ &\quad \times F_{\mu\lambda}(ax + (1-a)y), \\ F_{\mu\lambda} &\equiv \partial_\mu B_\lambda - \partial_\lambda B_\mu. \end{aligned} \quad (2.4)$$

The corrections to $S_0(x, y)$ implied by Eq. (2.4) are of order $D(x-y)$ and therefore less singular on the light cone than $S_0(x, y)$. In the vector-gluon model the first corrections to Eq. (2.3) are rather complicated and are given in part by Gross and Treiman.¹⁰

In the absence of interactions the corrections to the leading light-cone singularity are elementary. The quark-field anticommutator, correct to leading and *next-to-leading* order on the light cone is then

$$S_P(x, y) = (-\not{\partial} + im)D(x-y). \quad (2.5)$$

We take the parton model to be the prescription that the dominant contribution to any current or divergence commutator in the Bjorken limit is given by free-field theory.¹³ In this case Eq. (2.5) is sufficient for the calculation of W_4 and W_5 . It will become apparent how this definition of the parton model differs from canonical quark-gluon models.

To proceed, we define the structure functions for antineutrino scattering as follows:

$$W_{\mu\nu}^{\bar{\nu}, \nu} \equiv \frac{1}{4\pi} \int e^{iq \cdot y} d^4y \langle P | [J_\mu^\dagger(y), J_\nu^\dagger(0)] | P \rangle \quad (2.6)$$

$$\begin{aligned} &= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1^{\bar{\nu}, \nu}(q^2, \nu) + \frac{1}{M^2} \left(P_\mu - \frac{\nu}{q^2} q_\mu\right) \left(P_\nu - \frac{\nu}{q^2} q_\nu\right) W_2^{\bar{\nu}, \nu}(q^2, \nu) \\ &\quad - \frac{i\epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta}{2M^2} W_3^{\bar{\nu}, \nu}(q^2, \nu) + \frac{q_\mu q_\nu}{M^2} W_4^{\bar{\nu}, \nu}(q^2, \nu) + \frac{P_\mu q_\nu + P_\nu q_\mu}{2M^2} W_5^{\bar{\nu}, \nu}(q^2, \nu), \end{aligned} \quad (2.7)$$

where $\nu \equiv P \cdot q$, $q^2 \equiv -Q^2 < 0$, the matrix element is understood to be averaged over the proton's spin,¹⁴ and we have assumed T conservation and set $W_6 = 0$. Throughout this section, we consider only the strangeness-conserving weak current

$$J_\mu^\dagger(y) \equiv \bar{\psi}(y)\gamma_\mu(1 - \gamma_5)\lambda^\dagger\psi(y).$$

Generalization to the $\Delta S = 1$ current is straightforward and the results are stated in Sec. V.

Taking the double divergence of Eq. (2.6) and using

$$\partial_\mu J^{\mu\dagger}(y) = -2im_\phi \bar{\psi}(y)\gamma_5\lambda^\dagger\psi(y),$$

where m_ϕ is the mass of the proton quark (assumed equal to the mass of the neutron quark), we find

$$q^\mu q^\nu W_{\mu\nu}^{\bar{\nu}, \nu} = \frac{m_\phi^2}{\pi} \int e^{iq \cdot y} d^4y \langle P | [\bar{\psi}(y)\gamma_5\lambda^\dagger\psi(y), \bar{\psi}(0)\gamma_5\lambda^\dagger\psi(0)] | P \rangle. \quad (2.8)$$

In the Bjorken limit (ν , $Q^2 \equiv -q^2 \rightarrow \infty$; $x \equiv Q^2/2\nu$ finite) only the leading light-cone singularity of this commutator is required:

$$\langle P | [\bar{\psi}(y) \gamma_5 \lambda^+ \psi(y), \bar{\psi}(0) \gamma_5 \lambda^+ \psi(0)] | P \rangle = \partial^\rho D(y) \langle P | \bar{\psi}(y) \gamma_\rho \lambda^+ \lambda^+ I(y, 0) \psi(0) - \bar{\psi}(0) \gamma_\rho \lambda^+ \lambda^+ I(0, y) \psi(y) | P \rangle$$

+ terms less singular on the light cone, (2.9)

where

$$I(y, 0) \equiv \exp \left[-ig \int_0^y d\xi^\mu B_\mu(\xi) \right]$$

in the gluon model, or $I(y, 0) \equiv 1$ in the parton model. We now define $\mathcal{F}_\rho^\pm(y \cdot P)$ to be the light-cone restriction of the matrix element of Eq. (2.9):

$$\begin{aligned} \mathcal{F}_\rho^\pm(y \cdot P) &\equiv \langle P | \bar{\psi}(y) \gamma_\rho \lambda^+ \lambda^+ I(y, 0) \psi(0) | P \rangle|_{y^2=0} \\ &= P_\rho F^\pm(y \cdot p) + y_\rho G^\pm(y \cdot P). \end{aligned} \quad (2.10)$$

Combining Eqs. (2.8)–(2.10) and using the Fourier transform representation of

$$\partial^\rho D(y) \equiv \frac{1}{(2\pi)^3} \int d^4k k^\rho e^{-ik \cdot y} \delta(k^2) \epsilon(k_0)$$

we obtain

$$\lim_{B_j} q^\mu q^\nu W_{\mu\nu}^{\bar{\nu}, \nu} = m \phi^2 [\hat{F}^\pm(x) - \hat{F}^\mp(-x)],$$

where $\hat{F}^\pm(x)$ is the Fourier transform of $F^\pm(y \cdot p)$:

$$F^\pm(y \cdot P) \equiv \int dx e^{ixy \cdot P} \hat{F}^\pm(x).$$

The bilocal operator of Eq. (2.10) is identical to that which occurs in the light-cone analysis of the structure function $W_2^{\bar{\nu}, \nu}(q^2, \nu)$, specifically,

$$x[\hat{F}^\pm(x) - \hat{F}^\mp(-x)] = F_2^{\bar{\nu}, \nu}(x).$$

Expressing the double divergence in terms of $W_4^{\bar{\nu}, \nu}$ and $W_5^{\bar{\nu}, \nu}$, we obtain finally

$$\begin{aligned} \lim_{B_j} \left(\frac{\nu^2}{M^4} W_4^{\bar{\nu}, \nu}(q^2, \nu) - \frac{1}{2x} \frac{\nu^2}{M^4} W_5^{\bar{\nu}, \nu}(q^2, \nu) \right) \\ = \frac{m \phi^2}{4x^3 M^2} F_2^{\bar{\nu}, \nu}(x). \end{aligned} \quad (2.11)$$

Proportionality between this combination of W_4 and W_5 and the more easily measured structure function $F_2(x)$ obtains only in models in which chiral symmetry is broken by mass terms and represents, in principle, a test of the interaction. If proportionality is observed the constant measures the bare quark mass.¹⁵

The bare quark mass must therefore be finite in this framework. This is a direct manifestation of the fact that our considerations are invalid in perturbation theory in which the bare mass is of course divergent. In fact the assumptions (a) that $\partial_\mu A^\mu$ can be calculated using the equations of motion (as we did above) and (b) that the equal-time commutator of $\partial_\mu A^{\mu-}$ and $\partial_\mu A^{\mu+}$ defined by the

Bjorken-Johnson-Low limit agrees with the naive canonical definition are sufficient to derive the following sum rule in the vector-gluon model:

$$\lim_{q^2 \rightarrow -\infty} \int_1^\infty \frac{d\omega}{\omega^3} I^-(\omega, q^2) = \frac{m \phi^2}{M^2}, \quad \omega \equiv 1/x$$

where $I^- = I^\nu - I^{\bar{\nu}}$,

$$I(q^2, \nu) \equiv \frac{2\nu^2}{M^4} W_4(q^2, \nu) + \frac{2\nu^3}{q^2 M^4} W_5(q^2, \nu)$$

[given Eq. (2.11), this is a direct consequence of the Adler sum rule]. The integral is expected to converge according to Regge theory in which case m_ϕ must be finite if the limit exists. This is, perhaps, the simplest example of the well-known incompatibility of “formal” field theory and scaling on the one hand, and perturbation theory on the other.

We now turn to the single divergence of $W_{\mu\nu}^{\bar{\nu}, \nu}$:

$$\begin{aligned} q^\mu W_{\mu\nu}^{\bar{\nu}, \nu} \\ = \frac{m \phi}{2\pi} \int e^{iq \cdot y} d^4y \\ \times \langle P | [\bar{\psi}(y) \gamma_5 \lambda^+ \psi(y), \bar{\psi}(0) \gamma_\nu (1 - \gamma_5) \lambda^+ \psi(0)] | P \rangle. \end{aligned} \quad (2.12)$$

A knowledge of the leading term in $\{\psi(y), \bar{\psi}(0)\}$ is not sufficient to evaluate this matrix element. The reason lies in the spin structure of the bilocal operator which multiplies the leading singularity: The term proportional to $\partial^\rho D(y)$ in Eq. (2.3) or (2.5) contributes a term to Eq. (2.12) of the form

$$\partial^\rho D(y) \langle P | \bar{\psi}(y) g_{\nu\rho} \lambda^+ \lambda^+ I(y, 0) \psi(0) | P \rangle.$$

This twist-three bilocal is normalized to some mass. The contribution of the mass term in Eq. (2.5) is of the form

$$m D(y) \langle P | \bar{\psi}(y) \gamma_\nu \lambda^+ \lambda^+ I(y, 0) \psi(0) | P \rangle.$$

Although less singular on the light cone, this term contains a twist-two bilocal normalized to P_ν , and is equally important in the Bjorken limit.

Gluon field connections to Eq. (2.3) are of the same order as the mass term of Eq. (2.5). They make it tedious to calculate the commutator of Eq. (2.12) in the vector-gluon model. Explicit calculation shows that $\nu^2 W_4$ and $\nu^2 W_5$ scale in the Bjorken limit:

$$\begin{aligned} \lim_{\text{Bj}} \frac{\nu^2}{M^4} W_4^{\bar{\nu}, \nu}(q^2, \nu) &= F_4^{\bar{\nu}, \nu}(x), \\ \lim_{\text{Bj}} \frac{\nu^2}{M^4} W_5^{\bar{\nu}, \nu}(q^2, \nu) &= F_5^{\bar{\nu}, \nu}(x). \end{aligned} \quad (2.13)$$

In the parton model (as we have defined it) the leading contribution to Eq. (2.12) is to be computed in free field theory. Explicit calculation shows that the two terms of Eq. (2.5) combine to yield the result that $\nu^2 W_4$ scales while $\lim_{\text{Bj}} \nu^2 W_5 = 0$ (cf. Appendix in Ref. 12). The different behavior of $\nu^2 W_5$ in the vector-gluon and parton models is rather important in the description of chiral symmetry breaking developed in subsequent sections.

At this point it is useful to compare our results with those of Ref. 4 and 7. According to Mandula *et al.*, $\nu^{3/2} W_4$ and $\nu^{3/2} W_5$ may scale in the Bjorken limit in the gluon model. This result is based on the assumption that the bilocal operator which occurs in the expansion of the current-divergence

commutator will in general contain a piece with twist as low as is allowed by the observed scaling of W_2 , i.e., twist 2. In the vector-gluon model such a piece is absent: The bilocal operator $\bar{\psi}(y)\psi(0)$ has twist 3 from which it follows that $\nu^2 W_4$ and $\nu^2 W_5$ scale.

Both Mandula *et al.* and Ng and Vinciarelli derive sum rules relating σ_π to an integral over combinations of $\nu^2 W_4$ and $\nu^2 W_5$. The sum rules follow from the explicit form of Eq. (2.12) in particular models but are linearly divergent in the presence of the expected Regge behavior of W_4 and W_5 . We find that if the sum rules are divergent the derivation is incorrect (in fact, even if they are convergent the sum rules are incorrect unless supplemented by specific assumptions on the nature of $\alpha=0$ fixed-pole residues). In order to proceed we turn to the Bjorken-Johnson-Low expansion where such divergences may be avoided by performing subtractions on dispersion integrals.

III. BJL CALCULATIONS

We now study the single divergence of $\bar{W}_{\mu\nu}^{\bar{\nu}, \nu}$ using the Bjorken-Johnson-Low limit,¹⁶ in which prejudices about asymptotic behavior are incorporated from the start (so that divergent sum rules are avoided) and the presence of possible fixed poles is not excluded *a priori*. We define

$$T_{\mu\nu} = i \int e^{i q \cdot x} d^4 x \theta(x_0) \langle P | [J_\mu^-(x), J_\nu^+(0)] | P \rangle + \text{seagull terms}, \quad (3.1)$$

where the "seagull terms" are polynomials in q which may possibly be present. $T_{\mu\nu}$ can be expanded in terms of structure functions T_i in analogy with Eq. (2.7). The T_i satisfy

$$\text{Im } T_i = 2\pi W_i. \quad (3.2)$$

Assuming conventional Regge behavior, $W_4 \sim \nu$ and $W_5 \sim \text{const}$ as $\nu \rightarrow \infty$ with q^2 fixed, we can therefore write the following dispersion relations:

$$\begin{aligned} T_4(q^2, \nu) &= T_4(q^2, 0) + 2\nu \int_{\nu_0}^{\infty} \frac{W_4^-(q^2, \nu') d\nu'}{\nu'^2 - \nu^2} + 2\nu^2 \int_{\nu_0}^{\infty} \frac{W_4^+(q^2, \nu') d\nu'}{\nu'(\nu'^2 - \nu^2)}, \\ T_5(q^2, \nu) &= 2\nu \int_{\nu_0}^{\infty} \frac{W_5^+(q^2, \nu') d\nu'}{\nu'^2 - \nu^2} + 2 \int_{\nu_0}^{\infty} \frac{\nu' W_5^-(q^2, \nu') d\nu'}{\nu'^2 - \nu^2}, \end{aligned} \quad (3.3)$$

where

$$W_{4,5}^\pm = W_{4,5}^\nu \pm W_{4,5}^{\bar{\nu}}.$$

We consider

$$\begin{aligned} q^\mu T_{\mu\nu} &= \frac{q^2 q_\nu}{M^2} T_4 + \frac{q^2 p_\nu + \nu q_\nu}{2M^2} T_5 \\ &= - \int e^{i q \cdot x} d^4 x \theta(x_0) \langle P | [\partial_\mu J_\mu^-(x), J_\nu^+(0)] | P \rangle + \text{seagull terms} \end{aligned} \quad (3.4)$$

[where the "equal-time" contribution from $\partial_\mu \theta(x^0)$ has been absorbed in the seagull terms]. We now let $q_0 \rightarrow i\infty$ in Eq. (3.4) with $\vec{q}=0$ and P_0 fixed. The right-hand side of Eq. (3.4) is expanded in a series in $1/q^0$ by partial integration. The left-hand side is expanded in powers of $1/q^0$ using the dispersion representation of Eq. (3.3). Finally coefficients of each power of $1/q^0$ on either side are equated. Taking the time component and using the scaling laws derived above, we obtain from the $1/q^0$ term

$$\lim_{q^2 \rightarrow -\infty} q^4 \bar{T}_4(q^2, 0) \equiv \lim_{q_0 \rightarrow i\infty} \left(\frac{q^2 q_0^2 T_4(q^2, 0)}{M^2} - q_0 \times \text{seagull terms} \right) \\ = -i \int d^3x e^{-i\vec{q} \cdot \vec{x}} \langle P | [\partial^\mu J_\mu^-(\vec{x}, 0), J_0^+(0)] | P \rangle. \quad (3.5)$$

The right-hand side of this equation is proportional to the σ term. Explicitly, σ_π is defined by

$$\sigma_\pi = \frac{1}{4iM} \int d^3x \langle P | [\partial^\mu J_\mu^-(\vec{x}, 0), J_0^+(0)] | P \rangle. \quad (3.6)$$

This is the conventional σ term (10–100 MeV ?); the factor $(2M)^{-1}$ occurs because our states are normalized covariantly. In the vector-gluon model

$$\sigma_\pi = \frac{m_\phi}{2M} \left\langle P \left| \bar{\psi}(0) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \psi(0) \right| P \right\rangle. \quad (3.7)$$

Since the seagull is a polynomial in q , Eq. (3.5) relates σ_π to the piece of the subtraction constant $T_4(q^2, 0)$ which behaves as $1/q^4$ as $q^2 \rightarrow -\infty$, which appears theoretically (and experimentally) intractable. However, we shall see in the next section that (given an additional assumption) it can be calculated.

IV. FIXED-POLE RESIDUES

Several authors have argued^{17, 18} that the residues of any fixed poles with $\alpha=0$ in kinematic-singularity-free (KSF) current-nucleon scattering amplitudes are polynomials in q^2 ; the arguments advanced are most compelling in the context of field theories¹⁸ (constituent models) such as those considered here (we adopt the language of the Regge model although our specific assumption and results may be valid more generally). In this section, we shall pursue the consequences of assuming polynomial residues. We note that this assumption can be tested since it leads to the sum rule of Cornwall, Corrigan, and Norton and of Rajaraman and Rajasekaran¹⁹:

$$2 \int_0^\infty \nu \bar{W}_2^{ep, en}(\nu, q^2) \frac{d\nu}{q^2} = \int_0^\infty \bar{F}_2^{ep, en}(x) \frac{dx}{x^2} \\ = C^{p, n}, \quad (4.1)$$

where

$$C^p = 1 + \frac{1}{2\pi^2 \alpha} \int_0^\infty \bar{\sigma}_{\gamma p}(\nu) d\nu, \\ C^n = \frac{1}{2\pi^2 \alpha} \int_0^\infty \bar{\sigma}_{\gamma n}(\nu) d\nu, \quad (4.2)$$

$$\bar{F}_2(x) \equiv F_2(x) - F_2^R(x),$$

$$\bar{\sigma}(\nu) \equiv \sigma(\nu) - \sigma^R(\nu),$$

and $F_2^R(x)$ [$\sigma^R(\nu)$] are Regge fits to the small- x [large- ν] behavior of $F_2(x)$ [$\sigma(\nu)$] including all contributions (cuts and poles) with $\alpha > 0$.²⁰

Phenomenological analyses^{21, 22} suggest that $C^p \approx 1$. The sum rule of Eq. (4.1) therefore predicts behavior of the type illustrated in Fig. 1. This striking behavior is not ruled out by existing data. If it fails, the ensuing considerations are invalid.

We consider a function $W_4^R(q^2, \nu)$ which fits the asymptotic behavior of $W_4(q^2, \nu)$ and includes all Regge contributions with $\alpha > 0$. An analytic function $T_4^R(q^2, \nu)$ whose discontinuity equals $2\pi W_4^R$ can be constructed from a dispersion relation. Then the residue of any $\alpha=0$ fixed pole in T_4 is defined as follows:

$$C_4(q^2) \equiv \lim_{\nu \rightarrow \infty} [T_4(q^2, \nu) - T_4^R(q^2, \nu)] \\ = T_4(q^2, 0) - 2 \int \bar{W}_4^+(q^2, \nu') \frac{d\nu'}{\nu'},$$

where $\bar{W}_4^+ = W_4^+ - W_4^{+R}$, or in terms of $F_4^+(\omega)$ for large q^2

$$C_4(q^2) \sim T_4(q^2, 0) - \frac{8M^4}{q^4} \int_0^\infty \bar{F}_4^+(\omega) \frac{d\omega}{\omega^3}. \quad (4.3)$$

Similarly for $W_5(q^2, \nu)$ (which Reggeizes like $\nu^{\alpha-1}$ and therefore has a $1/\nu$ term from an $\alpha=0$ fixed pole) we obtain

$$\frac{M^2}{\nu} C_5(q^2) \equiv \lim_{\nu \rightarrow \infty} [T_5(q^2, \nu) - T_5^R(q^2, \nu)] \\ = -\frac{4M^4}{Q^2 \nu} \int_0^\infty \bar{F}_5^+(\omega) \frac{d\omega}{\omega^2}. \quad (4.4)$$

Unfortunately T_4 and T_5 as defined in analogy to Eq. (2.7) are not KSF amplitudes. In terms of

FIG. 1. Illustration of the behavior of F_2^p implied by the assumption that fixed-pole residues are polynomials in Q^2 ($\omega = 1/x$). This assumption yields: Area A – Area B ≈ 1 .

KSF amplitudes $\hat{T}_i$ [defined in Eq. (2.5) of Ref. 8]

$$\begin{aligned} T_4 &= \hat{T}_4 - (M^2/q^2)\hat{T}_1 - (\nu^2/q^4)\hat{T}_2, \\ T_5 &= \hat{T}_5 + (2\nu/q^2)\hat{T}_2. \end{aligned}$$

If the residues of fixed poles in $\hat{T}_i$ are polynomials $[P_i(q^2)]$, then the fixed-poles contributions to T_4 and T_5 are

$$\begin{aligned} C_4(q^2) &= P_4(q^2) - (M^2/q^2)P_1(q^2) - (M^4/q^4)P_2(q^2), \\ C_5(q^2) &= P_5(q^2) + (2M^2/q^2)P_2(q^2). \end{aligned}$$

Assuming $(\nu^2/M^4)T_4(q^2, \nu)$ and $(\nu^2/M^4)T_5(q^2, \nu)$ to scale sets

$$\begin{aligned} C_4(q^2) &= -(M^4/q^4)P_2(0), \\ C_5(q^2) &= (2M^2/q^2)P_2(0). \end{aligned} \quad (4.5)$$

From Eqs. (3.5) and (4.3)–(4.5) we obtain the σ -term sum rule

$$\begin{aligned} \frac{1}{2i} \int d^3x \langle P | [\partial^\mu J_\mu^-(\vec{x}, 0), J_0^+(0)] | P \rangle \\ = 4M^2 \int_0^\infty x dx \left[\bar{F}_4^+(x) - \frac{1}{4x} \bar{F}_5^+(x) \right] \end{aligned} \quad (4.6)$$

and the further relation

$$\begin{aligned} P_2(0) &= \frac{2Q^2}{M^4} \int_0^\infty \bar{W}_5^+(q^2, \nu) d\nu \\ &= 2 \int_0^\infty \bar{F}_5^+(x) dx \\ &= \frac{2}{M^4} \int_0^\infty \nu \bar{W}_2^+(0, \nu) d\nu \\ &= \frac{2}{M^4} \int_0^\infty \left(1 - q^2 \frac{d}{dq^2} \right) \nu \bar{W}_2^+(q^2, \nu) d\nu. \end{aligned} \quad (4.7)$$

Using Eq. (2.11) the σ -term sum rule [Eq. (4.6)] may be rewritten as follows:

$$\sigma_\pi = \frac{M_\phi^2}{2M} \int_0^\infty \frac{dx}{x^2} \bar{F}_2^+(x) + \frac{M}{4} P_2(0).$$

Generalizing the above to all SU(3) currents we obtain the following equations for the three independent scalar densities:

$$\begin{aligned} \left\langle P \left| \bar{\psi}(0) \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \psi(0) \right| P \right\rangle &= m_\phi \int \frac{dx}{x^2} \bar{F}_2^{\nu\phi+\bar{\nu}\phi}(x) + \frac{M^2}{2m_\phi} P_2^{\nu\phi+\bar{\nu}\phi}(0), \\ \left\langle P \left| \bar{\psi}(0) \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \psi(0) \right| P \right\rangle &= 6m_\phi \int \frac{dx}{x^2} \bar{F}_2^{\phi\phi-\pi\pi}(x) + \frac{3M^2}{m_\phi} P_2^{\phi\phi-\pi\pi}(0), \\ \left\langle P \left| \bar{\psi}(0) \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \psi(0) \right| P \right\rangle &= m_\lambda \int \frac{dx}{x^2} [q \bar{F}_2^{\phi\phi+\pi\pi}(x) - \frac{5}{2} \bar{F}_2^{\nu\phi+\bar{\nu}\phi}(x)] + \frac{9M^2}{4m_\lambda} P_2^{\phi\phi+\pi\pi}(0) - \frac{5M^2}{4m_\lambda} P_2^{\nu\phi+\bar{\nu}\phi}(0), \end{aligned} \quad (4.9)$$

where $P_2^{\nu\phi}(0)$ and $P_2^{\pi\pi}(0)$ are the fixed-pole residues in the (experimentally impossible) process of scattering of an axial-vector current with the same quantum numbers as the electromagnetic current. In general the presence of the fixed-pole residues, $P_2(0)$, made Eqs. (4.9) of limited value.²³ In the parton model, however, $F_5(x)$ is identically zero and so, by Eq. (4.7), are the residues $P_2(0)$. In this case Eqs. (4.9) relate the matrix elements of scalar densities to measurable integrals over scaling data. The consequences of Eqs. (4.9) in the parton model are enumerated in the following section.

Before specializing to the parton model we consider the possibility of showing that $P_2(0)$ [or equivalently $\int \bar{F}_5(x) dx$] is zero in the vector-gluon model [in which case powerful constraints on the pattern of chiral symmetry breaking would follow from Eqs. (4.9)].

First let us imagine that all moving Regge poles have $\alpha < 0$ and assume that consequently we can

write unsubtracted dispersion relations for T_4 and T_5 (we see below that this is a strong assumption which is likely to be false). In this case we would obtain an unsubtracted form of Eq. (4.6) and the result that $\int F_5(x) dx = 0$. This last result apparently contradicts Eq. (4.7) (which is valid when all moving poles have $\alpha < 0$ in which case $\bar{F}_5 - F_5$) unless $P_2(0) = 0$. However it does not follow that $P_2(0) = 0$; in fact, by writing an unsubtracted dispersion relation for T_5 it is easy to see that we have implicitly assumed that it contains no $\alpha = 0$ fixed poles [which immediately implies $P_2(0) = 0$]. Here we have an example of the fact that we cannot write unsubtracted dispersion relations for current amplitudes, even if they converge, if by so doing we implicitly assume that (allowed) fixed poles are absent.

V. PARTON-MODEL RESULTS

We now turn to the parton model in which $P_2(0) = 0$, and discuss briefly the consequences of Eq. (4.9).²⁴ First we notice that in the chiral limit

$m_i \rightarrow 0$ the nucleon matrix elements of the scalar densities vanish linearly.²⁵ From this it follows that in the $SU(2) \times SU(2)$ limit $m_\phi \rightarrow 0$, $\sigma_\rho \sim m_\phi^2$ while the chiral-symmetry-breaking parameter²⁶

$$C + \sqrt{2} = \frac{3\sqrt{2} m_\phi}{m_\phi + m_\lambda}$$

vanishes linearly. This implies that a "small" value of $C + \sqrt{2}$ implies a "very small" value of σ_π [more quantitatively, using values for the fixed-pole residues and baryon mass differences as discussed below, we find $\sigma_\pi \approx 100(C + \sqrt{2})^2$ MeV]. This result has an interesting consequence for the important parameter μ_0 , given by

$$\mu_0 = \frac{\sum_i m_i}{6M_p} \left\langle P \left| \bar{\psi}(0) \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \psi(0) \right| P \right\rangle$$

which can be written in the approximate form

$$\mu_0 \approx \frac{3\sigma_\pi}{\sqrt{2}(\sqrt{2} + C)} + \frac{1}{4\sqrt{2}C} (2M_\Sigma + M_\Sigma - 2M_p - M_\Lambda) \quad (5.1)$$

in models (such as ours) in which the $SU(3) \times SU(3)$ -breaking part of H_{strong} belongs to the $(3, 3^*) \oplus (3^*, 3)$ representation. From our expression for σ_π above it follows that the first term is of order $200(\sqrt{2} + C)$ MeV, which is small compared to the second term (which is $145/C$ MeV) for $C \approx -\sqrt{2}$. Consequently μ_0 is negative and of order -100 MeV in this model for conventional values of C . It is worth stressing that according to Eq. (5.1) the value of μ_0 is very sensitive to σ_π for $C \approx -\sqrt{2}$ and a negative value of μ_0 may easily occur (this is allowed since the matrix elements which define μ_0 are understood to be connected); however, all previous papers of which we are aware find $\mu_0 > 0$.

Finally we remark that expressing $(m_\phi - m_\lambda) \times \langle P | \bar{\psi}(0) \lambda_i \psi(0) | P \rangle$ in terms of baryon mass differences we can use Eq. (4.9) [with $P_2(0) = 0$] to calculate m_i and $\int dx (1/x^2) \bar{F}_2^{\nu p + \nu n}(x)$ given σ_π , C^p , and C^n . With the "best" experimental values $C^p = 1.0$, $C^n = 0.1$, and $\sigma_\pi = 40$ MeV (e.g.), we find²⁷

$$\int \frac{dx}{x^2} \bar{F}_2^{\nu p + \nu n}(x) \approx 5,$$

$$m_\phi \approx 120 \text{ MeV}.$$

The values of m_i are actually rather sensitive to the value of σ_π but in all cases the fixed-pole residue in neutrino scattering

$$\int dx (1/x^2) \bar{F}_2^{\nu p + \nu n}(x)$$

is quite large (of order 5), which would have striking experimental implications (cf. Fig. 1).

VI. OTHER SUBTRACTED SUM RULES

It is obviously interesting to reexamine the numerous divergent sum rules to be found discarded in the literature and see if they can also be subtracted. In the theoretical framework considered here suitable candidates must satisfy two criteria:

(a) They must not diverge more than linearly.

(b) They must be true in free-field theory, so that their failure in the light-cone approach (parton model) can be attributed to the expected Regge behavior (parton distribution function) which makes them divergent.

Unfortunately, Adler's α and γ sum rules,²⁸ which might seem the most interesting candidates, fail criterion (b) [the α sum rule also fails (a)]. The reason is that dispersion relations for $W_{1,3}$ (Adler's α and γ) need subtractions in free-field theory although unsubtracted dispersion relations obviously converge. In Regge language this is because of $\alpha = 0$ fixed poles (corresponding to Z diagrams in the $P \rightarrow \infty$ method) as discussed in the previous section. However, although the fixed pole spoils the sum rule, the assumption that its residue is a polynomial gives the nontrivial result

$$\int_0^\infty [\bar{W}_3^\nu(q^2, \nu) - \bar{W}_3^{\bar{\nu}}(q^2, \nu)] d\nu = \text{const.} \quad (6.1)$$

The left-hand side is the residue of any $\alpha = 0$ fixed pole and hence the right-hand side is (by assumption) a polynomial—however, it can only be a constant, since the left-hand side is constant in the scaling limit.

A similar result can be obtained if we consider the antisymmetric part of the hadronic tensor which describes inelastic-electron scattering off a polarized target:

$$W_{\mu\nu} = \frac{1}{4\pi} \int e^{iq \cdot y} d^4y \langle P, s | [J_\mu(y), J_\nu(0)] | P, s \rangle,$$

$$\begin{aligned} \frac{W_{\mu\nu} - W_{\nu\mu}}{2} &= i\epsilon_{\mu\nu\alpha\beta} \frac{q^\alpha s^\beta}{M^2} (G_1 + \nu G_2) \\ &\quad + i\epsilon_{\mu\nu\alpha\beta} \frac{P^\alpha q^\beta q \cdot s G_2}{M^4}, \end{aligned}$$

where s (normalized so that $s^2 = -M^2$) is the spin vector and (according to dimensional analysis or an investigation of the light-cone commutator) $\nu G_1/M^2$ and $\nu^2 G_2/M^4$ are expected to scale (note that to reach this result using dimensional analysis it is essential to normalize s so that it is nonsingular as $M \rightarrow 0$). Hence if the residue of any $\alpha = 0$ fixed pole in the amplitude whose imaginary part is G_2 is a polynomial it must be zero:

$$\int_0^\infty \bar{G}_2(\nu, q^2) d\nu = 0. \quad (6.2)$$

An unsubtracted form of this sum rule was first written down by Burkhardt and Cottingham.²⁹ However, they obtained it as a superconvergence relation, implicitly assuming that there are no fixed poles in G_2 [see the discussion before their Eq. (2.9)]. They pointed out that the unsubtracted sum rule diverges if, as is presumably the case, there are Regge cuts (the contributions of factorizing poles to G_2 are suppressed for kinematical reasons); in this case they suggested that the subtracted form of the sum rule would hold (again they obtained it as a standard superconvergence relation overlooking the possibility of fixed poles).

The unsubtracted form of the sum rule was obtained from the quark light-cone algebra by Jackiw, Dicus, and Teplitz³⁰ (its scaling limit was also obtained by Hey and Mandula,³¹ who did not discuss its convergence). However, their derivation fails if there are Regge cuts (so that the sum rule diverges).

A similar regulation procedure may be applied to a sum rule for the scalar densities, $\langle P | \bar{\psi}(0)\psi(0) | P \rangle$, in terms of $F_2(x)$ and $G_L(x)$ [$\lim_{Bj} W_L(q^2, \nu) = F_L(x) + (1/q^2)G_L(x)$] implicit in the work by Jackiw and Schnitzer³² and also suggested by Weisberger.³³ Properly regulated, and modified for neutrino processes, this sum rule provides an expression for $P_2(0)$ in the vector-gluon model and is exploited to further study chiral symmetry breaking in Ref. 22.

Finally we note that sum rules Eq. (6.1) and Eq. (6.2) test the assumptions of polynomial residues and scaling but not the explicit properties of the light-cone algebra (we also remark that similar techniques can presumably be used to salvage divergent sum rules for $t \neq 0$).

VII. CONCLUSIONS AND ANALYSIS

In order for our analysis of current divergences in the gluon model to be valid, it is necessary (a) that the real world is described by (results abstracted from) "formal" field theory; (b) that the relevant field theory is the quark model.

Therefore, our results will be irrelevant unless the well-known quark-model sum rules derived previously (which are much more easily tested) prove to be correct. However, if prerequisite (a) holds but (b) fails, similar results can doubtless be obtained in other models.

Our main results are the following.³⁴

1. In the gluon model⁷

$$\lim_{Bj} (\nu^2/M^4) W_{4,5}(q^2, \nu) = F_{4,5}(x).$$

If the leading scaling behavior is given by free-field theory, as in the parton model, then $F_5 = 0$.

2. If $[4x^3 F_4(x) - 2x^2 F_5(x)]/F_2(x) = \text{const}$ (independ-

ent of x) this would imply that the interaction between the quarks is vector in nature. In this case

$$\left. \frac{4x^3 F_4(x) - 2x^2 F_5(x)}{F_2(x)} \right|_{\Delta S=0} = \left(\frac{m_q}{M_p} \right)^2,$$

$$\left. \frac{4x^3 F_4(x) - 2x^2 F_5(x)}{F_2(x)} \right|_{\Delta S=1} = \frac{m_q m_\lambda}{M_p^2}$$

so that it is possible (in principle) to measure the bare quark masses in the vector-gluon model; they are finite – in contrast to the situation in perturbation theory.

3. The following sum rule for the σ term can be derived:

$$\sigma_\pi = 2M_p \int_0^1 [x F_4^{\nu p \bar{p}}(x) - \frac{1}{4} F_5^{\nu p \bar{p}}(x)] dx.$$

However, it diverges linearly if Regge behavior is assumed. If it diverges the derivation is invalid and we find instead

$$\sigma_\pi = \lim_{q^2 \rightarrow -\infty} \frac{q^2 \bar{T}_4(q^2, 0)}{4M_p^2}$$

[$\bar{T}_4(q^2, 0)$ is equal to $T_4(q^2, 0)$ with pieces which fall less rapidly than $1/q^4$ subtracted].

4. If the residue of any fixed $\alpha=0$ Regge pole is a polynomial in q^2 :

$$\sigma_\pi = 2M_p \int_0^\infty [x \tilde{F}_4^{\nu p \bar{p}}(x) - \frac{1}{4} \tilde{F}_5^{\nu p \bar{p}}(x)] dx,$$

where $\tilde{F}_i = F_i - F_i^R$ and F_i^R is a Regge fit to the small- x behavior of F_i including all contributions (poles or cuts) with $\alpha > 0$. (This result is also true in models with a scalar or pseudoscalar interaction.) As discussed in Sec. IV, the assumption of polynomial residues can be tested experimentally using electroproduction data with unpolarized targets.

5. New tests of the polynomial residue assumption are given by the relations

$$\begin{aligned} Q^2 \int_0^\infty \bar{W}_5^+(q^2, \nu) d\nu &= \int_0^\infty \nu \bar{W}_2^+(0, \nu) d\nu \\ &= \int_0^\infty (1 - q^2 d/dq^2) \nu \bar{W}_2^+(q^2, \nu) d\nu, \end{aligned}$$

$$\int_0^\infty \bar{W}_3^-(q^2, \nu) d\nu = \text{const},$$

$$\int_0^\infty \bar{G}_2(q^2, \nu) d\nu = 0.$$

The last relation is the "corrected" version of a sum rule discussed previously in the literature.

6. If $\int \tilde{F}_5(x) dx = 0$ (as in a simple-minded parton model in which $F_5 = 0$) we can obtain relations between fixed-pole residues in ep , en , $\nu p + \nu n$ scattering (which can be expressed in terms of deep-

inelastic [F_2] data) baryon mass differences and bare quark masses. Three interesting consequences of these relations are the following.

(a) σ_π is of second order in $C + \sqrt{2}$ as $C + \sqrt{2} \rightarrow 0$ (e.g., with $C^p \cong 1.0$ and $C^n \cong 0.1$),

$$\sigma_\pi \cong 100(C + \sqrt{2})^2.$$

(b) μ_0 is negative (typically ~ -100 MeV) – a possibility which has not been entertained previously as far as we know.

(c) There is typically a large fixed pole in $\nu p + \nu n$ scattering

$$2 \frac{\partial}{\partial q^2} \int_0^\infty \nu \tilde{W}_2^{\nu p + \nu n}(q^2, \nu) d\nu = \int_0^\infty F_2^{\nu p + \nu n}(x) \frac{dx}{x^2} \cong 5,$$

which would have striking experimental consequences (cf. Fig. 1).

Independent of the particular model considered, and despite the dubious nature of the formal manipulations in which we have freely indulged, we think it an interesting point of principle that relations which probe the structure of the Hamiltonian can be derived without solving the field theory. It is very hard to measure $W_{4,5}$, so most of our results are difficult to test. However, these results illustrate the fact that these structure functions may contain a wealth of information; it will be worth trying to measure them in future generations of experiments.

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²With our definition W_5 is the interference term between the scalar (divergence) and longitudinal components of the current in the forward current-nucleon scattering amplitude. It follows from the Schwarz inequality that νW_5 must vanish in the Bjorken limit in models (such as the quark model) in which $q_L/\sigma_T \rightarrow 0$. [The inequalities are given explicitly by T. D. Lee and C. N. Yang, Phys. Rev. 126, 2239 (1962), and M. G. Doncel and E. de Rafael, Nuovo Cimento 4A, 363 (1971) (see also Ref. 8).]

³Different authors use different definitions for $W_{4,5}$. The definition used in this paper [Eq. (2.5)] is useful for discussions of chiral-symmetry breaking, but it is inconvenient in other cases since it implies that the W_i are not completely independent as $q^2 \rightarrow 0$.

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⁸The formulas which relate the W_i to observables may be found, e.g., in C. H. Llewellyn Smith, Phys. Reports 3C, 261 (1972). (Note that the definition of $W_{4,5}$ in this reference is different from that used here.) These are two difficulties in measuring $W_{4,5}$: (1) They are only determined if both the muon polarization and the double differential cross section are observed in inelastic neutrino reactions. (2) Their contribution vanishes in the approximation $m_\mu = 0$ (they would be more amenable

if heavy leptons existed coupled to ν_μ).

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¹²In an earlier version of this paper [SLAC Report No. SLAC-PUB-1067 (unpublished)] we had incorrectly assumed $B_\mu(y)$ to be curlless. We thank Dr. D. Broadhurst for pointing this out to us and for a helpful correspondence.

¹³Actually, more sophisticated parton models such as that of Landshoff, Polkinghorne, and Short do contain vector-gluon modifications to the leading light-cone singularity, and are not characterized by Eq. (2.5). See Broadhurst, Gunion, and Jaffe (Ref. 23) for a full discussion.

¹⁴States are normalized covariantly: $\langle P | P' \rangle = (2\pi)^3 2E \delta^3(\vec{P} - \vec{P}')$. Our metric and other conventions follow those of J. D. Bjorken and S. D. Drell [*Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964)].

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²⁴A much fuller discussion was given in a preliminary version of this paper (Ref. 12) in which it was incorrectly asserted that $F_5 = 0$ in the gluon model.

²⁵This is a surprising result, and suggests that it would not be possible to prove $P_2(0) = 0$ in the vector-gluon

model. See Ref. 23 for an alternative expression for $P_2(0)$ in the gluon model which need not in general vanish.

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³⁴Recently F. Feinberg [*Phys. Rev. D* (to be published)] has studied chiral-symmetry breaking in the gluon model quantized on the light cone. He does not, however, consider the effect of Regge behavior of $\alpha = 0$ fixed poles on his results, and therefore obtains superficial disagreement with some of our results. Correct inclusion of these effects brings his work into agreement with ours.

FIG. 1. Illustration of the behavior of F_2^{ep} implied by the assumption that fixed-pole residues are polynomials in $Q^2(\omega=1/x)$. This assumption yields: Area A - Area B ≈ 1 .