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REMARKS ON NEUTRAL CURRENT PHENOMENOLOGYC.H. Llewellyn Smith ^{*)} and D.V. Nanopoulos

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A B S T R A C T

The effects of neutral currents in various leptonic

$(g-2, e^+e^- \rightarrow e^+e^-, e^+e^- \rightarrow \mu^+\mu^-, e^-e^- \rightarrow e^-e^-)$

and semi-leptonic

$(\nu N \rightarrow \nu + \dots, (\mu^e)N \rightarrow (\mu^e)_+ + \dots)$

processes are examined in a unified systematic model-independent way. Existing data are used to bound the effects in

$(\mu^e)N \rightarrow (\mu^e)_+ + \dots$

The bounds turn out to be very weak so that experiments may be worth while even if they cannot approach the accuracy required to detect the effects predicted by the Weinberg-Salam model.

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1. INTRODUCTION

There have been many estimates of the effects of $\Delta S = 0$ weak neutral currents in $e(\mu)N$ and e^+e^- interactions^{1-19)*}. Since the discovery of evidence for neutral currents in νe and νN interactions^{20,21)}, these calculations have been repeated in the framework of the Weinberg-Salam model with the parameter(s) fixed by the data⁵⁻⁹⁾. For deep inelastic e or μ scattering, the results agree closely with the crudest order of magnitude estimate of $10^{-4} Q^2$ (in GeV^2), which would imply that the detection of weak effects in deep inelastic experiments at SLAC, NAL, or the CERN SPS would be very difficult if not impossible.

Even if the gauge strategy is correct, however, there is a widespread theoretical prejudice that the Weinberg-Salam model will not prevail ultimately¹⁷⁾, despite its success in accounting for the data when coupled with the parton paradigm²³⁻²⁶⁾. It is therefore quite possible that the effects of neutral currents are considerably bigger than the "canonical" estimate of $10^{-4} Q^2$ and experimenters should not necessarily be discouraged if they fear that an effect of this order is beyond their grasp. How much bigger could the effects be?

As an example, suppose that the interactions and mass of the (hypothetical) intermediate neutral boson Z conspire to suppress the value of the effective coupling of the neutral current to neutrinos by some factor D^{-1} , relative to its value in the Weinberg-Salam model, while enhancing the coupling to electrons and hadrons by D . This would leave νe and νN scattering unchanged but amplify the weak contribution to the matrix elements for eN and e^+e^- reactions by a factor D^2 . With a modest change, such as $D = 3$, the effects become relatively easy to detect.

The amplification cannot get too big, however, without conflicting with the success of QED to date in $g-2$, Bhabha scattering etc., etc. and even, eventually, the conservation of parity in purely hadronic interactions. But, whether *a priori* plausible or not, could it get big enough to account for the possible evidence for a breakdown of scaling reported in deep inelastic muon scattering at NAL²⁷⁾ [or, even more fancifully, for the large and rising value of $\sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)_{\text{th.}}$ observed at CEA²⁸⁾ and SPEAR²⁹⁾] without conflicting with other experimental facts? To answer such questions, it is necessary to examine the constraints imposed on neutral currents by all existing data in a unified systematic way. This is the purpose of this paper.

*) References 1-9 are on $e(\mu)N$ reactions and Refs. 10-19 on e^+e^- . Reference 9 (which came to our attention after we had finished this work) is the closest to ours; the authors make an analysis similar to ours for the hadronic structure functions but assume the Weinberg-Salam model for leptons.

2. NOTATION

We suppose that there is a neutral intermediate vector boson Z_μ of mass M_Z (with $M_Z \rightarrow \infty$ possibly) and denote its couplings to ν 's, μ 's, and e 's by:

$$\mathcal{L}_{int}^Z = \sqrt{\frac{G \cdot M_Z^2}{\sqrt{2}}}} \left[i g_1 \bar{\nu}_\mu \gamma_\lambda (1-\gamma_5) \nu_\mu + i g_1 \bar{e} \gamma_\lambda (1-\gamma_5) e + \right. \\ \left. + i \bar{\mu} \gamma_\lambda \left[g_L \left(\frac{1-\gamma_5}{2} \right) + g_R \left(\frac{1+\gamma_5}{2} \right) \right] \mu + \right. \\ \left. + i \bar{e} \gamma_\lambda \left[g_L \left(\frac{1-\gamma_5}{2} \right) + g_R \left(\frac{1+\gamma_5}{2} \right) \right] e \right] Z^\lambda \quad (1)$$

where G is Fermi's constant ($\approx 10^{-5} M_p^{-2}$) and in most gauge theories g_1 , g_L , and g_R are of order one. Note that we have assumed μ - e universality; an analysis without this assumption would obviously be more desirable but existing data do not permit this luxury.

We define electromagnetic structure functions in the usual way:

$$\int e^{iq \cdot x} \langle P | [J_\mu^e(x), J_\nu^e(0)] | P \rangle \frac{d^4x}{4\pi} = \quad (2) \\ = - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) W_1(\nu, q^2) + \left(P_\mu - \frac{q_\mu \nu}{q^2} \right) \left(P_\nu - \frac{q_\nu \nu}{q^2} \right) \frac{W_2(\nu, q^2)}{M^2}$$

where J^e is the hadronic part of the electromagnetic current, the states (on which a spin average is implied henceforth) are normalized thus:

$$\langle \vec{P}' | \vec{P} \rangle = 2E \delta^3(\vec{P}' - \vec{P}) (2\pi)^3$$

and otherwise the notation is standard, except that our $\nu = q \cdot p$. We define:

$$\int e^{iq \cdot z} \langle P | [J_\mu^{Z*}(z), J_\nu^Z(0)] | P \rangle \frac{d^4z}{4\pi} = \quad (3) \\ = \frac{G \cdot M_Z^2}{\sqrt{2}} \left[-g_{\mu\nu} \tilde{W}_1 + \frac{P_\mu P_\nu \tilde{W}_2}{M^2} - i \frac{\epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta}{2M^2} \tilde{W}_3 + \dots \right]$$

where J_μ^Z is the hadronic part of the current to which Z_μ couples, including the coupling constant, and:

$$\begin{aligned} & \frac{1}{2} \int e^{i(q \cdot z)} \frac{d^4 z}{4\pi} \langle P | [J_\mu^Z(z), J_\nu^e(0)] + [J_\mu^e(z), J_\nu^Z(0)] | P \rangle = \\ & = \sqrt{\frac{G \cdot M_Z^2}{\sqrt{2}}} \left[-g_{\mu\nu} W_1^I + \frac{P_\mu P_\nu}{M^2} W_2^I + \frac{i \epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta}{2M^2} W_3^I + \dots \right] \end{aligned} \quad (4)$$

where the dots represent other terms all of which involve explicit factors q^μ or q^ν and therefore play no role in the approximation $m_\mu = m_e = m_\nu = 0$ which we adopt henceforth. In gauge theories $e^2 W_1^I$, $e W_1^I$ and $\tilde{W}_1$ would be expected to be of the same order of magnitude. Generally, in addition to the well known constraints:

$$\begin{aligned} \left(\frac{v^2 + Q^2}{Q^2} \right) W_2 &\geq W_1 \geq 0 \\ \left(\frac{v^2 + Q^2}{Q^2} \right) \tilde{W}_2 &\geq \tilde{W}_1 \geq \frac{\sqrt{v^2 + Q^2}}{Q^2} |\tilde{W}_3| \end{aligned}$$

the interference W_1^I satisfy^{2,8)}

$$\left(\frac{v^2 + Q^2}{Q^2} W_2 - W_1 \right) \left(\frac{v^2 + Q^2}{Q^2} \tilde{W}_2 - \tilde{W}_1 \right) \geq \left(W_1^I - \frac{v^2 + Q^2}{Q^2} W_2^I \right)^2 \quad (5)$$

$$W_1 \cdot \left(\tilde{W}_1 \pm \frac{\sqrt{v^2 + Q^2}}{2} \tilde{W}_3 \right) \geq \left(W_1^I \pm \frac{\sqrt{v^2 + Q^2}}{2} W_3^I \right)^2. \quad (6)$$

where $Q^2 = -q^2 = q^2 - q_0^2$ and we have set $M_p = 1$.

3. LEPTONIC PROCESSES

3.1 g-2

The lowest-order contribution of the Z to g-2 has been calculated many times³⁰⁾ and found to be

$$\alpha_H^Z = \left[\frac{g_H - 2}{2} \right]^2 = \frac{G m_H^2}{12 n^2 \sqrt{2}} [3 g_R g_L - g_R^2 - g_L^2] + O\left(\frac{G m_H^4}{M_Z^2}\right) =$$

$$= (0.77) \cdot 10^{-9} [3 g_R g_L - g_R^2 - g_L^2] \quad (7)$$

in our notation. The agreement between QED and experiment³¹⁾ allows a weak (or unknown) contribution

$$-3 \cdot 10^{-7} \leq \alpha_H^{\text{weak}} \leq 9 \cdot 10^{-7} \quad (8)$$

at the 90% confidence limit, from which we deduce that

$$-390 \leq 3 g_R g_L - g_R^2 - g_L^2 \leq 1200. \quad (9)$$

assuming there is not a cancellation between the Z contribution and some other unknown contribution. We make this assumption throughout our discussion; if it is false our limits on the effects of the Z in $e(\mu)N$ and e^+e^- interactions would become even broader.

The constraints on g_L and g_R are plotted in Fig. 1 (a_e gives a much weaker bound).

3.2 $e^+e^- \rightarrow e^+e^-$

The effects of neutral currents in Bhabha scattering have been calculated before¹⁵⁾. Here we present a trick for writing down the answer almost directly in the case of unpolarized beams which can also be used to get results for Möller scattering $e^-e^- \rightarrow e^-e^-$ (which are new as far as we know). We work in the high-energy limit setting $m_e = 0$ in which approximation both the photon and the Z vertex conserve helicity; hence we can easily construct Table 1 showing the amplitudes to which the s- and t-channel exchanges contribute. The spinor structures of the photon and Z vertices are identical for a given helicity. In terms of the matrix elements S and T for s- and t-channel exchanges in pure QED, the squared matrix element can therefore be written down immediately using Table 1 when there is a Z exchange in addition. The result is:

$$2 [T(1 + \beta g_L g_R)]^2 + 2 [S(1 + \alpha g_L g_R)]^2 +$$

$$+ [S(1 + \alpha g_R^2) + T(1 + \beta g_R^2)]^2 + [S(1 + \alpha g_L^2) + T(1 + \beta g_L^2)]^2 \quad (10)$$

where

$$\alpha = \frac{G \cdot M_Z^2 \cdot s}{\sqrt{2} e^2 (s - M_Z^2)}$$

$$\beta = \frac{G \cdot M_Z^2 \cdot t}{\sqrt{2} e^2 (t - M_Z^2)} \quad .$$
(11)

Using the formula for the differential cross-section Bhabha scattering, which can be found in most textbooks on QED, and identifying the coefficients of s^{-2} , t^{-2} , and $(st)^{-1}$ in the case of QED, we can construct the cross-section including Z contributions:

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega} \right)_{CM} = & \frac{\alpha^2}{8E^2} \left[\left(\frac{1 + \cos^2 \theta}{2} \right) \cdot \left(\frac{(1 + \alpha g_R^2)^2 + (1 + \alpha g_L^2)^2 + 2(1 + \alpha g_R g_L)^2}{4} \right) + \right. \\ & + \left(\frac{1 + \cos^4 \frac{\theta}{2}}{\sin^4 \frac{\theta}{2}} \right) \left(\frac{(1 + \beta g_R^2)^2 + (1 + \beta g_L^2)^2 + 2(1 + \beta g_R g_L)^2}{4} \right) - \\ & \left. - \frac{2 \cos^4 \frac{\theta}{2}}{\sin^2 \frac{\theta}{2}} \left(\frac{(1 + \alpha g_R^2)(1 + \beta g_R^2) + (1 + \alpha g_L^2)(1 + \alpha g_R^2)}{2} \right) \right]. \end{aligned}$$
(12)

To compare this formula with experiment, we must integrate it over the appropriate angular range. At present the most accurate data³²⁾ has been obtained for $s \leq 9 \text{ (GeV)}^2$; we assume this is small compared to M_Z^2 and accordingly work to leading order in s/M_Z^2 . For an experiment which detects outgoing particles with $|\cos \theta| < x$ we find

$$\frac{\sigma}{\sigma_{Q.E.D}} = 1 + \frac{G \cdot s}{2\sqrt{2} e^2} \cdot \frac{A}{B}$$
(13)

where

$$\begin{aligned}
 A = & - (g_L + g_R)^2 \left[12 \ln \left(\frac{1+x}{1-x} \right) - 6x + x^3 \right] + \\
 & + (g_L^2 + g_R^2) \left[12 \ln \left(\frac{1+x}{1-x} \right) + x^3 - 15x \right] \\
 B = & -24 \ln \left(\frac{1+x}{1-x} \right) + \frac{48x}{1-x^2} + x^3 + 27x.
 \end{aligned} \tag{14}$$

In the experiment referred to above, $x = \frac{2}{3}$ and hence

$$\frac{G}{G_{Q.E.D.}} = 1 - \frac{G.S.}{\sqrt{2} e^2} \cdot \frac{(g_L^2 + g_R^2 + 5.2 g_L g_R)}{12.4} \tag{15}$$

With this parametrization, the data^{*)} restricts the coefficient of s in Eq. (15) to lie between $-2.57 \cdot 10^{-3} \text{ (GeV)}^{-2}$ and $+2.57 \cdot 10^{-3} \text{ (GeV)}^{-2}$ at the 90% confidence level. Hence we obtain

$$|g_L^2 + g_R^2 + 5.2 g_R g_L| \leq 360 \tag{16}$$

The domain of coupling constants allowed by this result is plotted in Fig. 1. Note that although on its own this result would allow arbitrarily large g_L and g_R provided $g_L^2 + g_R^2 + 5.2 g_L g_R = 0$, this would imply arbitrarily large violations of QED for other values of x which have not been observed. Therefore we would be entitled to mentally round off the spiky protuberances were they not already cut off by the results of other experiments.

Another experiment on $e^+ e^- \rightarrow e^+ e^-$ has been performed at CEA at higher energy³⁴⁾ but it gives the weaker 90% confidence level limit

$$-730 \leq [g_L^2 + g_R^2 + 5.1 g_R g_L] \leq 630. \tag{17}$$

3.3 $e^+ e^- \rightarrow \mu^+ \mu^-$

Using the same trick we employed in the discussion of Bhabha scattering, we can immediately write down the modification due to a Z_μ for unpolarized beams in

*) This is the result of a fit including data not yet published³³⁾.

the approximation $m_\mu = 0$ ¹¹⁻¹⁴):

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{QED} \frac{[(1+g_L^2\alpha)^2 + (1+g_R^2\alpha)^2 + 2(1+\alpha g_L g_R)^2]}{4} \quad (18)$$

An experiment which measures $e^+e^- \rightarrow \mu^+\mu^-/e^+e^- \rightarrow e^+e^-$ has been performed at Frascati ³⁵). Using the errors to turn the quoted 95% confidence level limit $\Lambda > 10 M_p$ for a negative metric heavy photon into the 90% confidence level limit $\Lambda > 11.5 M_p$ we get

$$g_R^2 + g_L^2 + 1.3 g_L g_R \leq 320. \quad (19)$$

from this experiment, which is plotted in Fig. 1. The authors do not quote a limit for positive metric [which would give a negative lower bound in Eq. (19)], but luckily this is irrelevant since the combination of coupling constants measured in this experiment is positive semi-definite.

3.4 $e^-e^- \rightarrow e^-e^-$

With the same method we used for $e^+e^- \rightarrow e^+e^-$, and the well known formula for the QED cross-section, we get:

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega}\right)_{c.m.} = & \frac{\alpha^2}{8E^2} \left[\left(\frac{1 + \cos^4 \frac{\theta}{2}}{\sin^4 \frac{\theta}{2}} \right) \left(\frac{(1+\beta g_L^2)^2 + (1+\beta g_R^2)^2 + 2(1+\beta g_L g_R)^2}{4} \right) + \right. \\ & + \left(\frac{1 + \sin^4 \frac{\theta}{2}}{\cos^4 \frac{\theta}{2}} \right) \left(\frac{(1+\gamma g_L^2)^2 + (1+\gamma g_R^2)^2 + 2(1+\gamma g_L g_R)^2}{4} \right) + \\ & \left. + \frac{2}{\sin^2 \frac{\theta}{2} \cdot \cos^2 \frac{\theta}{2}} \left(\frac{(1+\beta g_L^2)(1+\gamma g_L^2) + (1+\beta g_R^2)(1+\gamma g_R^2)}{2} \right) \right] \end{aligned} \quad (20)$$

where

$$\gamma = \frac{G \cdot M_Z^2 u}{\sqrt{2} e^2 (u - M_Z^2)} \quad (21)$$

$$s + t + u \cong 0.$$

The only experiment to check QED in Möller scattering up to now was done at the relatively modest centre-of-mass energy of 1112 MeV ³⁶). It gives the broad limit:

$$-1200 \leq g_L^2 + g_R^2 + 0.82 g_L g_R \leq 4400. \quad (22)$$

which is plotted in Fig. 1.

3.5 ve scattering

The effects of neutral currents in νe scattering have been calculated many times. Recently, De Rujula et al.²⁶⁾ have presented a convenient summary of the constraints imposed by existing data (since, assuming μ -e universality and a purely left-handed neutrino, all the observable cross-sections involve only two unknown parameters, the data is highly overconstrained; this is the essential content of recent papers by Terazawa³⁷⁾ and Sehgal³⁸⁾ who have presented the allowed domains for the cross-sections). De Rujula et al. have plotted the outer boundary of the domain allowed by various experiments for their coupling constants g_A and g_V which are related to ours by

$$\begin{aligned} g_V &= g_1 \left(\frac{g_L + g_R}{2} \right) \\ g_A &= g_1 \left(\frac{g_L - g_R}{2} \right). \end{aligned} \quad (23)$$

For our purposes we are more interested in a lower bound [otherwise we could let $g_1 \rightarrow 0$, with $g_1^2 \tilde{W}_1$ fixed and hence obtain arbitrarily large results in deep inelastic $e(\mu)N$ reactions]. If we assume that the candidate for $\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$ observed in Gargamelle²⁰⁾ is not due to background (for which the probability is 3%), it gives a 90% confidence level limit³⁹⁾:

$$\sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e) \geq 0.026 \cdot 10^{-41} (E_\nu \text{ GeV}) \text{ cm}^2 \quad (24)$$

(in the published data, one event corresponds to $\sigma = 0.22 \times 10^{-41} E_\nu$). This limit will be greatly improved in the near future if more events are found in the large experimental run now underway^{*)}. In our notation

$$\begin{aligned} \sigma_{\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e} &= \frac{G^2 s}{4\pi} g_1^2 \left[g_R^2 + \frac{1}{3} g_L^2 \right] = \\ &= 0.41 \cdot 10^{-41} g_1^2 \left[g_R^2 + \frac{1}{3} g_L^2 \right] E_\nu (\text{GeV}) \text{ cm}^2 \end{aligned} \quad (25)$$

*) One more candidate has now been found (private communication from the Gargamelle group). Evaluation of the exact quantitative significance of this event must obviously await the end of the present phase of the experiment (it will increase the lower bound by a factor of order 5); qualitatively, it gives us courage to believe the limit based on one event to be safe.

so the lower limit gives:

$$g_1^2 \left[g_R^2 + \frac{1}{3} g_L^2 \right] \geq 0.06 \quad (26)$$

4. SEMILEPTONIC INTERACTIONS

4.1 $\nu N \rightarrow \nu + \dots$

Assuming $s \ll M_Z^2$, the cross-section is given by

$$\frac{d^2 \sigma^{\nu \bar{\nu}}}{dx dy} = \frac{g_1^2 G^2 s}{2\pi} \left[(1-y) \tilde{F}_2 + y^2 x \tilde{F}_1 + y \left(1 - \frac{y}{2}\right) x \tilde{F}_3 \right] \quad (27)$$

where

$$\begin{aligned} x &= \frac{-q^2}{2\nu} \\ y &= \frac{\nu}{ME} \\ \tilde{F}_1 &= \tilde{W}_1, \quad \tilde{F}_{2,3} = \frac{\nu \tilde{W}_{2,3}}{M^2} \end{aligned} \quad (28)$$

terms proportional to m_μ^2 have been neglected and so has a piece proportional to $Mxy/2E$, which is a good approximation for large E since $xy < 1$ always. In writing the same structure functions for ν and $\bar{\nu}$ reactions we have assumed that J_μ^Z is hermitean (which is automatic if Z is its own antiparticle).

In order to progress we will be forced, in common with other authors, to assume scaling (i.e. that the $\tilde{F}_1$, and likewise the F_1 and the F_1^I , are functions of x only) although the data we shall analyse is not at particularly large energy. Furthermore, the inevitable paucity of the existing data forces us to assume that the virtual Z interaction is largely longitudinal -- as is known to be approximately the case for electromagnetic interactions and for charged current neutrino interactions. Accordingly we set

$$\begin{aligned} 2 F_1 &= F_2 \\ 2 \tilde{F}_1 &= \tilde{F}_2 \end{aligned} \quad (29)$$

and hence, according to Eq. (6),

$$2 F_1^I = F_2^I \quad (30)$$

Even with these simplifying assumptions, we still face the problem that the data^{*)} has the cut $E_{\text{Hadronic}}^{\text{vis.}} > 1 \text{ GeV}$ applied to it. If the shapes of the ν and $\bar{\nu}$ spectra were the same we could allow for this exactly for our purposes. The cut determines the relative weight of the contributions of $\int F_2 dx$ and $\int xF_3 dx$ to the cross-sections. However, since these integrals are measured for charged currents, we can use the ratio $\sigma^{\nu \rightarrow \mu} / \sigma^{\bar{\nu} \rightarrow \mu}$ with the same cut applied to determine this weight. These integrals can then be obtained for the neutral current F's using this weight factor. This is what we shall do. Although the shapes of the spectra are not in fact exactly the same, we feel that this procedure is good enough in the circumstances and that the results should not be seriously misleading^{**) . Experimentally²¹⁾:}

$$\left. \frac{NC(\nu)}{CC(\nu)} \right|_{E_{\text{had}} > 1 \text{ GeV}} = 0.21 \pm 0.03 \quad (31)$$

$$\left. \frac{NC(\bar{\nu})}{CC(\bar{\nu})} \right|_{E_{\text{had}} > 1 \text{ GeV}} = 0.46 \pm 0.09$$

and, normalized to the same flux⁴⁰⁾:

$$\frac{CC(\bar{\nu})}{CC(\nu)} = 0.26 \pm 0.03 \quad (32)$$

In addition^{40)***)}

$$B = \frac{-\int x F_3^{\nu(n+p)} dx}{\int F_2^{\nu(n+p)} dx} = 0.90 \pm 0.04 \quad (33)$$

$$\frac{1}{2} \int F_2^{\nu n + \nu p} dx = 0.52 \pm 0.05$$

*) Since the data from the Harvard-Pennsylvania-Wisconsin experiment at NAL has not yet been published, we will only use the Gargamelle data. If the NAL signal turns out to be somewhat smaller than that found at CERN, it could be explained if the Z was quite light so that its propagator has a significant damping effect at NAL energies.

**) It was required that there be no more than one muon candidate in the charged current data. This introduces a bias against high y. However, there is a similar bias against high y in the neutral current events (which must have no muon candidates). Since the neutral current cross-section is considerably smaller than the charged current cross-section, we can hope that this bias cancels out in the ratio to first approximation.

***) We assume that the target, freon, can be treated as a mixture of an equal number of neutrons and protons. This is presumably a good approximation. We have taken all Gargamelle data which we use from the convenient review by Perkins⁴⁰⁾.

From these numbers we deduce bounds on

$$\tilde{B} = \frac{-\int x \tilde{F}_3^{r(n+p)} dx}{\int \tilde{F}_2^{r(n+p)} dx}, \quad (34)$$

and

$$\frac{g_1^2}{2} \int \tilde{F}_2^{r(n+p)} dx$$

which are plotted in Fig. 2.

4.2 $\mu N \rightarrow \mu + \dots$

In the notation of the previous section, neglecting m_μ^2 and $xyM/2E$ again, the cross-section for a given muon (μ^- or μ^+) with a definite polarization denoted by η is given by

$$\begin{aligned} \frac{d^2\sigma_\eta}{dx dy} = & \left(\frac{d^2\sigma}{dx dy} \right)_{EM} \left[1 + \frac{\sqrt{2} G q^2 M_Z^2 g_\eta}{q^2 (q^2 - M_Z^2)} \cdot \frac{[(1-y)F_2^I + y^2 x F_1^I - \mathcal{F}_\eta y (1-\frac{y}{2}) x F_3^I]}{[(1-y)F_2 + y^2 x F_1]} \right] \\ & + \frac{G^2 s}{8\pi} \cdot \frac{M_Z^4 g_\eta^2}{(q^2 - M_Z^2)^2} \cdot \left[(1-y) \tilde{F}_2 + y^2 x \tilde{F}_1 - \mathcal{F}_\eta y (1-\frac{y}{2}) x \tilde{F}_3 \right]. \end{aligned} \quad (35)$$

In the various cases g_η and ξ_η are given by the following table:

Particle	Polarization	g_η	ξ_η
μ^-	L	g_L	+1
μ^-	R	g_R	-1
μ^+	R	g_L	-1
μ^+	L	g_R	+1

The origin of the g_η column is almost obvious since g_L (g_R) multiplies a projection operator which selects left-(right-)handed particles and right-(left) handed antiparticles (just as the $1-\gamma_5$ interaction gives left-handed neutrinos but right-handed antineutrinos). Left-handed particles and antiparticles excite the same mixture of polarization states of the virtual Z; therefore, apart from the change $g_L \leftrightarrow g_R$, the cross-sections for left-handed μ^- 's and left-handed μ^+ 's are the same (and likewise for right-handed μ^- 's and μ^+ 's). In going from a left- to a right-handed μ^- , however, the amplitudes for left- and right-handed virtual Z's are interchanged; hence the sign of the parity violating V-A interference

term ($F_3 \sim \sigma_{Z_L} - \sigma_{Z_R}$) changes. This explains the behaviour of ξ_η . [With $g_R = 0$, the situation resembles neutrino scattering -- there is a cross-section for left-(right-)handed ν 's ($\bar{\nu}$'s) but the F_3 term changes sign in going from ν to $\bar{\nu}$.] We dwell on this detailed exposition to try to answer questions which we have frequently been asked such as -- how can $d\sigma_{\mu^-}$ differ from $d\sigma_{\mu^+}$ if CP is conserved? This is of course allowed, just as $\sigma_{\nu N} \neq \sigma_{\bar{\nu} N}$ is not forbidden by CP which relates μ_L^- cross-sections on particles to μ_R^+ cross-sections on antiparticles.

Assuming scaling for the F_i^I and that the longitudinal contribution is small [Eqs. (29) and (30)], and dropping the presumably much smaller $\tilde{F}_i$ terms we get:

$$\frac{d\sigma}{dx dy} = \left(\frac{d\sigma}{dx dy} \right)_{E.M.} \left[1 + \frac{G \cdot M_Z^2 \sqrt{2} q^2 g_\eta}{e^2 (q^2 - M_Z^2)} \cdot \left(\frac{F_2^I(x)}{F_2(x)} - \frac{3\eta y (1 - y/2) x F_3^I(x)}{(1 - y + y^2/2) F_2(x)} \right) \right] + O(G^2). \quad (36)$$

This functional form of the differential cross-section, which follows from rather general assumptions, could in principle serve to distinguish the non-partly violating effects of neutral currents (e.g. $d\sigma_{\mu^-} \neq d\sigma_{\mu^+}$) from the effects of two-photon exchange. The only other general remark we can make is that the unknown structure functions must satisfy the scaling limits of the remaining inequalities which are

$$\begin{aligned} \tilde{F}_2 &\geq |x \tilde{F}_3| \\ F_2(\tilde{F}_2 \pm x \tilde{F}_3) &\geq (F_2^I \pm x F_3^I)^2. \end{aligned} \quad (37)$$

We now briefly record the results of a simple model calculation, to give an idea of the generally expected order of the effects, before turning to bounds (with this exception we stick to model independent statements; we are therefore limited to inclusive processes since there is not enough data to allow a model-independent discussion of possible violations of the Rosenbluth formula in $ep \rightarrow ep$, etc.).

4.3 Model

In simple parton models in which the nucleon consists only of partons (no antipartons), as suggested as a first approximation by the Gargamelle data for charged weak currents, and in addition the partons are assumed to have a common

momentum distribution, $F_2^I(x)$, $F_2(x)$, and $xF_3^I(x)$ would all have the same x -dependence. In this case:

$$\frac{d\sigma}{dx dy} = \left(\frac{d\sigma}{dx dy} \right)_{\text{E.M.}} \left(1 - \frac{\sqrt{2} G q^2 M_Z^2 g_n}{e^2 (q^2 - M_Z^2)} \left[A - F_n \frac{B y (1 - y/2)}{(1 - y + y^2/2)} \right] \right) \quad (38)$$

where A and B are constants (independent of y and x). We would expect this simple form to be a good first approximation. Specializing further to the Weinberg-Salam-Glashow-Iliopoulos-Maiani four-quark parton model (which fits all other deep inelastic data) we find for neutron and proton targets⁵⁾

$$\begin{aligned} A^{\text{proton}} &= \frac{\sqrt{2} (12 \sin^2 \theta_W - 5)}{6}, & B^{\text{proton}} &= \frac{\sqrt{2} \cdot 5}{6} \\ A^{\text{neutron}} &= \sqrt{2} (2 \sin^2 \theta_W - 1), & B^{\text{neutron}} &= \sqrt{2} \end{aligned} \quad (39)$$

with

$$\begin{aligned} g_L &= \sqrt{2} (-1 + 2 \sin^2 \theta_W) \\ g_R &= 2 \sqrt{2} \sin^2 \theta_W \\ M_Z &= \frac{73 \text{ GeV}}{|\sin 2\theta_W|} \end{aligned} \quad (40)$$

Using the best value of θ_W (which is currently $\sim 38^\circ$) these formulae [which agree partial results obtained earlier^{1,7)}] can be used to estimate the effects of weak electromagnetic interference in this model.

4.4 Bounds

It is convenient to rewrite the weak-electromagnetic interference term in the following form so that the kinematic inequalities can easily be applied:

$$\begin{aligned} \frac{d\sigma}{dx dy} &= \left(\frac{d\sigma}{dx dy} \right)_{\text{E.M.}} \left[1 + \frac{G M_Z^2 \sqrt{2} q^2 g_n}{e^2 (q^2 - M_Z^2) F_2(x) (2 - 2y + y^2)} \right. \\ &\quad \cdot \left((F_2^I(x) + x F_3^I(x)) \alpha(\frac{1}{2}, y) + (F_2^I(x) - x F_3^I(x)) \alpha(-\frac{1}{2}, y) \right) \Big] + \\ &\quad + O(G^2) \end{aligned} \quad (41)$$

where

$$\begin{aligned}\alpha(1, y) &= (1-y)^2 \\ \alpha(-1, y) &= 1.\end{aligned}\tag{42}$$

From this formula it is easy to deduce bounds for any desired asymmetry. We illustrate this for the two quantities:

$$A_{\mu_L^+ - \mu_R^-} = \frac{dG_{\mu_L^+} - dG_{\mu_R^-}}{dG_{\mu_L^+} + dG_{\mu_R^-}}\tag{43}$$

and

$$A_{\mu_L^+ - \mu_R^+} = \frac{dG_{\mu_L^+} - dG_{\mu_R^+}}{dG_{\mu_L^+} + dG_{\mu_R^+}}\tag{44}$$

which satisfy

$$A_{\mu_L^+ - \mu_R^-} = \frac{GM_Z^2 \sqrt{2} q^2}{e^2 (q^2 - M_Z^2)} \cdot g_R \cdot \frac{[(F_2^I + \chi F_3^I) - (F_2^I - \chi F_3^I)] y(2-y)}{F_2 (2 - 2y + y^2)}$$

$$\begin{aligned}|A_{\mu_L^+ - \mu_R^-}| &\leq \frac{G\sqrt{2}}{e^2} \left(\frac{Q^2}{1 - \frac{Q^2}{M_Z^2}} \right) \cdot \frac{y(2-y)}{2 - 2y + y^2} \cdot |g_R| \cdot \\ &\cdot \sqrt{\frac{\tilde{F}_2(x)}{F_2(x)}} \left(\sqrt{1 + \frac{\chi \tilde{F}_3(x)}{\tilde{F}_2(x)}} + \sqrt{1 - \frac{\chi \tilde{F}_3(x)}{\tilde{F}_2(x)}} \right)\end{aligned}\tag{45}$$

and

$$\begin{aligned}A_{\mu_L^+ - \mu_R^+} &= \frac{GM_Z^2 \sqrt{2} q^2}{e^2 (q^2 - M_Z^2)} \cdot \left[\frac{(F_2^I + \chi F_3^I)(g_R(1-y)^2 - g_L)}{F_2 (2 - 2y + y^2)} + \right. \\ &\quad \left. + \frac{(F_2^I - \chi F_3^I)(g_R - g_L(1-y)^2)}{F_2 (2 - 2y + y^2)} \right]\end{aligned}\tag{46}$$

$$\begin{aligned}
 |A_{\mu_L^+ - \mu_R^+}| &\leq \frac{G \sqrt{2}}{e^2} \left(\frac{Q^2}{1 - \frac{Q^2}{M_Z^2}} \right) \cdot \frac{1}{2 - 2y + y^2} \cdot \\
 &\cdot \sqrt{\frac{\tilde{F}_2(x)}{F_2(x)}} \left(\left| \sqrt{1 + \frac{x \tilde{F}_3(x)}{\tilde{F}_2(x)}} \right| |g_R(1-y)^2 - g_L| + \right. \\
 &\left. + \left| \sqrt{1 - \frac{x \tilde{F}_3}{\tilde{F}_2(x)}} \right| |g_R - g_L(1-y)^2| \right).
 \end{aligned}
 \tag{46'}$$

Since the functional forms of the $\tilde{F}$ are unknown, the best we can do at the moment is to assume that $\tilde{F}_2$, $x\tilde{F}_3$ and F_2 have the same shapes. This is true in simple parton models and we believe that it is likely to be a good first approximation in general. With this assumption we can put

$$\frac{\tilde{F}_2(x)}{F_2(x)} = \frac{\int \tilde{F}_2(x) dx}{\int F_2(x) dx}
 \tag{47}$$

and

$$\frac{x \tilde{F}_3(x)}{F_2(x)} = -\tilde{B}.
 \tag{48}$$

Then, using results above [Eq. (26) and Fig. 2] we get

$$\begin{aligned}
 |A_{\mu_L^+ - \mu_R^+}| &\leq 9 \cdot 10^{-4} \cdot \left(\frac{Q^2}{1 - \frac{Q^2}{M_Z^2}} \right) \frac{y(2-y)}{2-2y+y^2} \sqrt{g_R^2(g_R^2 + \frac{1}{3}g_L^2)} \\
 |A_{\mu_L^+ - \mu_R^+}| &\leq 5 \cdot 10^{-4} \left(\frac{Q^2}{1 - \frac{Q^2}{M_Z^2}} \right) \left[\frac{0.95 |g_R(1-y)^2 - g_L| + 1.05 |g_R - (1-y)^2 g_L|}{2-2y+y^2} \right] \cdot \\
 &\cdot \sqrt{g_R^2 + \frac{1}{3}g_L^2}.
 \end{aligned}
 \tag{49}$$

Using Fig. 1 we get the numerical bounds:

$$|A_{\mu_L^+ - \mu_R^-}| \leq 0.35 \left(\frac{Q^2}{1 - \frac{Q^2}{M_Z^2}} \right) \cdot \frac{y(2-y)}{2-2y+y^2} \quad (50)$$

$$|A_{\mu_L^+ - \mu_R^+}| \leq 0.20 \left(\frac{Q^2}{1 - \frac{Q^2}{M_Z^2}} \right) \cdot \frac{0.95(1-y)^2 + 1.05}{2-2y+y^2}.$$

These bounds are extremely weak. In fact asymmetries this large are ruled out already by the observation of scaling in the SLAC-MIT experiments (excepting miracles); in the spin averaged case we get

$$d\bar{\sigma} = (d\bar{\sigma})_{E.M.} \left[1 + \frac{GM_Z^2 \sqrt{2} q^2}{2e^2(q^2 - M_Z^2)} \left\{ \frac{(F_2^I + x F_3^I)(g_L(1-y)^2 + g_R)}{F_2(x)(2-2y+y^2)} + \frac{(F_2^I - x F_3^I)(g_L + g_R(1-y)^2)}{F_2(x)(2-2y+y^2)} \right\} \right] + O(G^2). \quad (51)$$

To deduce bounds on the interference term, we would have to assume that $(d\bar{\sigma})_{em}$ scales exactly and then examine the data in great detail to discover the average y to which the data correspond; this does not seem worth while. Roughly, we can say that since the data come from all over the ν, Q^2 plane, there cannot be a large accidental cancellation between terms with different y -dependence and the interference term must be less than or of the order of scale breaking. Hence, conservatively, the present data imply μ_{LR}^{+-} asymmetries less than or of order $10^{-2} Q^2$ (in GeV^2), which is much smaller than the bounds above. However, this may well change in the future when very much better lower bounds for $\sigma_{\bar{\nu}_{\mu}e \rightarrow \bar{\nu}_{\mu}e}$ exist and there are even better QED tests in e^+e^- since, symbolically, our upper bound on the asymmetry is proportional to

$$\left[\left(1 - \frac{\sigma_{e^+e^- \rightarrow \text{leptons}}}{\sigma_{QED}} \right) \sqrt{\frac{\sigma_{\nu N \rightarrow \nu + \dots}}{\sigma_{\bar{\nu}_{\mu}e \rightarrow \bar{\nu}_{\mu}e}}} \right]_{\max}. \quad (52)$$

5. CONCLUSIONS

We have found that existing data imply very weak limits on the effects of neutral currents in deep inelastic electroproduction. Even experiments which can only study effects an order of magnitude larger than those predicted by the

Weinberg-Salam model would prove completely virgin territory and might therefore be worth while.

We found that at present the best limits on the weak-electromagnetic interference effect come from the remarkable accuracy to which scaling has been tested in the SLAC-MIT experiments. Conversely, neutral currents will be a possible explanation for any scaling violation which may be found if it has the correct q^2 variation; whether *a priori* plausible or not, this is not at present ruled out by other data. (However, unless we indulge in very special pleading, the fanciful idea that neutral currents could account for the behaviour of $\sigma_{e^+e^- \rightarrow \text{Had.}}$ does seem to be ruled out. With $M_Z \sim 5.5$ GeV, and the parton model in mind to connect the space-like and the time-like regions, we could just about arrange a sufficiently big enhancement in e^+e^- without spoiling scaling too badly in the space-like region. However, the remarkable accuracy to which QED has been tested in Bhabha scattering would then require the ratio of Z-e and Z-hadron couplings to satisfy

$$\frac{g_{Ze.}}{g_{Z \text{ Had.}}} \sim \sqrt{\frac{M_p \cdot \sigma_{\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e}}{m_e \cdot \sigma_{\nu N \rightarrow \nu + \dots}}} \ll 1$$

which is hardly possible unless the $\bar{\nu}_\mu e$ events have been misinterpreted.)

Previous authors have discussed various pieces of neutral current phenomenology in isolation. We have prevented a unified treatment which allows an immediate evaluation of the relative significance of different experiments. Although it turns out that the limit on γ -Z interference from scaling eclipses all others at present, this will change when better data become available; it will be interesting to re-draw Fig. 1 (which has some independent validity as a unified representation of a possible form of QED breakdown) and re-evaluate our bounds in the future.

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Table 1

Initial e^+ e^-	Final e^+ e^-	L	L	R	R
		L	R	L	R
L	L	T			
L	R		S,T	S	
R	L		S	S,T	
R	R				T

Helicity amplitudes (denoted R, L -- for right-, left-handed polarization)
to which S and T-channel exchanges contribute in Bhabha scattering ($e^+e^- \rightarrow e^+e^-$).

REFERENCES

- 1) A. Love, G.G. Ross and D.V. Nanopoulos, Nuclear Phys. B49, 513 (1972).
- 2) E. Derman, Phys. Rev. 7 D, 2755 (1973).
- 3) E. Petronzio, Roma Preprint 411 (1972).
- 4) N.N. Nikolaev, M.A. Shifman and M.Zh. Shmatikov, Zh. ETF Pis. Red. 18(1), 70 (1973); JETP Lett. 18(1), 39 (1973).
- 5) C.H. Llewellyn Smith, to be published in Proc. Conf. on e^+e^- and ep storage rings, DESY (October 1973).
- 6) M. Suzuki, "The charge asymmetry in the $\mu^\pm$ nucleus collision as a test for the neutral weak current", Berkeley LBL Preprint (October 1973).
- 7) S.M. Berman and J.R. Primack, SLAC PUB 1360 (1973).
- 8) W.J. Wilson, Parity violation in deep-inelastic charged lepton-hadron scattering, Harvard Preprint (1974) (we have not seen this paper which is referred to in Ref. 9).
- 9) C.P. Korthals-Altes, M. Perrottet and E. de Rafael, "Inclusive muon production reactions as a test of neutral currents", Marseille Preprint (1974).
- 10) N. Cabibbo and R. Gatto, Phys. Rev. 124, 1577 (1961).
- 11) A. Love, Nuovo Cimento Letters 5, 113 (1972).
- 12) J. Godine and A. Hankey, Phys. Rev. D6, 3301 (1972).
- 13) V.K. Chung, A.K. Mann and E.A. Paschos, Phys. Letters 41 B, 355 (1972).
- 14) R. Budny, Phys. Letters 45 B, 340 (1973).
- 15) R. Budny and A. McDonald, Phys. Letters 48B, 423 (1974).
- 16) M. Kuroda and Y. Yamaguchi, Prog. Th. Phys. 49, 2160 (1973).
- 17) R. Gatto and G. Preparata, Nota Interna 479, University of Rome (1973), and Lett. al Nuovo Cimento 7, 89 (1973).
- 18) D.A. Dicus, Phys. Rev. D8, 338 (1973).
- 19) G.V. Dass, DESY 73/40 (1973).
- 20) F.J. Hassert et al., Phys. Letters 46 B, 121 (1973).
- 21) F.J. Hassert et al., Phys. Letters 46 B, 138 (1973).
- 22) See, for example, S. Weinberg, J. Phys. (France) Colloque C1, suppl. au No. 10, tome 34, p. (C1)45 (1973).
C.H. Llewellyn Smith, CERN TH.1747 (to be published by North Holland Publishing Co., in Proc. 1973 Bonn Conf.).
- 23) L.M. Sehgal, Nuclear Phys. B65, 141 (1973); Phys. Letters 48 B, 133 (1974).
- 24) C. Albright, Phys. Rev. D8, 3162 (1973).
- 25) R.B. Palmer, Phys. Letters 46 B, 240 (1973).

- 26) A. De Rujula, H. Georgi, S.L. Glashow and H.R. Quinn, Harvard Preprint (1973), "Fact and fancy in neutrino physics" (to be published in Rev. Mod. Phys.)
- 27) K.W. Chen, Invited talk at the Chicago APS Meeting, February 1974.
- 28) A. Litke et al., Phys. Rev. Letters 30, 1189 (1973).
G. Tarnopolosky et al., Phys. Rev. Letters 32, 432 (1974).
- 29) B. Richter, Invited talks at the Irvine Conference (December 1973), and the Chicago APS Meeting (February 1974).
- 30) R. Jackiw and S. Weinberg, Phys. Rev. D5, 2396 (1972).
W.A. Bardeen, R. Gastmans and G. Lautrup, Nuclear Phys. B46, 319 (1972).
J.R. Primack, Talk presented at XVI Int. Conf. on High-Energy Physics, NAL (1972).
- 31) J. Bailey et al., Nuovo Cimento, 9 A, 369 (1972).
- 32) M. Bernardini et al., Phys. Letters 45 B, 510 (1973).
- 33) A. Zichichi (private communication).
- 34) H. Newman et al., Phys. Rev. Letters 32, 483 (1974).
- 35) E. Borgia et al., Lett. al Nuovo Cimento 3, 115 (1972).
- 36) W.C. Barber et al., Cornell Preprint CLNS 139 (1971).
- 37) H. Terazawa, Rockefeller Preprint C00-2232 B14 (1973).
- 38) L.M. Sehgal, Phys. Letters 48 B, 60 (1974).
- 39) D.C. Cundy (private communication).
- 40) D.H. Perkins, Oxford Preprint 67/73.

Fig. 1 Domains of the coupling constants g_L and g_R [Eq. (1)] allowed by various experiments (90% confidence level).

Fig. 2 Allowed domain of the quantities $\tilde{B}$ and $g_1^2 \int \tilde{F}_2 dx$ [Eqs. (1), (27) and (34)] allowed by the Gargamelle data (at the one-standard deviation level).