On Holography and Cosmology

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Abstract

Starting with the suggestion of Fischler and Susskind, various attempts have recently been made to apply the holographic principle to cosmology. Among these is a generalisation by Bousso which avoids the difficulties of the original proposal in the re-collapsing FLRW models.

Here we take a closer look at the question of holography in cosmology with particular reference to this generalisation. We demonstrate that in general realistic inhomogeneous universes such a proposal would involve extremely complicated - possibly fractal - light sheets. Furthermore, in a real inhomogeneous universe with evolving degrees of lumpiness on a variety of scales, such a light sheet becomes time dependent and cannot be known a priori on the basis of theory. Its construction requires a detailed knowledge of the distribution and growth of inhomogeneities on all scales, and of the resulting caustics in null surfaces. Moreover, the evolution of the universe makes it clear that in general such bounds cannot remain invariant under time reversal and will change with epoch.

We propose a modified version of this proposal in which the light sheets end on the boundary of the past, and hence avoid contact with the caustics. In this way the resulting light sheets and projections can be made much simpler. We discuss the question of operational definability of these sheets within the context of both proposals and conclude that in both cases the theoretical existence of such sheets must be clearly distinguished from their complexity and the difficulty of their construction in practice. This puts into perspective the likely practical difficulties one would face in applying the holographic principle to the real cosmos. These issues may also be of relevance in debates regarding the applications of the holographic principle to other settings such as string theory.
1 Introduction

According to the so called holography principle, the maximum number of degrees of freedom in a volume is proportional to its bounding surface area \[1, 2\]. If true, this would amount to an enormous simplification of the world, as it would enormously reduce the degrees of freedom required to understand it. Furthermore, it would be informative as it could provide a holographic bound on entropy in a variety of physical settings, including cosmology.

There are two ways to approach this question: either phenomenologically or at a fundamental level. Here - in line with other applications of this principle to cosmology - we shall concentrate on the former. We recall that an important motivation for this idea comes from the Bekenstein-Hawking results \[3, 4\] concerning black holes, according to which the entropy of the matter inside a black hole of mass \(S_M\) cannot exceed the Bekenstein-Hawking entropy \(S_{BH}\) given by a quarter of the area \(A\) of its event horizon in Planck units, i.e.

\[
S_M \leq S_{BH} = \frac{A}{4}.
\]  

(1)

The aim of the holographic principle is essentially to generalise this result to more general settings, including cosmology \[5\]. Leaving aside the justification for this enormous extrapolation, this generalisation poses important questions. To begin with, as opposed to the case of the black holes (BH), where appropriate notions of volume and surface are naturally provided by the event horizon, it is not clear whether appropriate analogues of these notions in fact exist in general cosmological settings and if so whether they are unique and how they should be determined. This is particularly true of the choice of surfaces, as even for a fixed volume, the surface is not uniquely defined.

Recently Fischler and Susskind \[5\] have considered the application of holography principle to cosmology. Their proposal may be stated as follows:

**Fischler–Susskind Proposal \[5\]:** Let \(M\) be a four-dimensional spacetime. Let \(\Gamma\) be a spatial region in \(M\) with a two-dimensional spatial boundary \(B\). Let \(L\) be the light surface bounded by \(B\) and generated by the past light rays from \(B\) towards the centre of \(\Gamma\). Then the entropy passing through \(L\) never exceeds the area of the bounding surface \(B\).

In particular, in the case of adiabatic evolution, the total entropy of the matter within the particle horizon\(^1\) must be smaller than the area of the horizon. They have shown that this proposal is compatible with flat and open Friedman-Lemaitre-Robinson-Walker (FLRW) cosmologies, but that it fails for the \(k = +1\) recollapsing models. A number of attempts have subsequently been made to remedy this difficulty \[9\]-\[14\]. In particular, Bousso \[15, 16\] has recently put forward a generalisation of this proposal and has applied it to a number of examples including the recollapsing \(k = +1\) FLRW cosmological models\(^2\).

\(^1\)Here this is taken to mean the creation light cone, as in \[6\], rather than the set of particles bounding causal connection, as originally defined by Rindler \[7, 8\].

\(^2\)See \[17\] for a recent critique of this scenario.
Here we take a closer look at the question of holography in a generic realistic inhomogeneous cosmological setting. We consider the proposal by Bousso as well as putting forward a modified version, in each case discussing the nature of the resulting light surfaces and the difficulties in their operational definability. In section 2, we briefly look at the application of this principle to FLRW models. Sections 3 and 4 contain a brief discussion of Bousso’s proposal in the inhomogeneous cosmological settings, and the nature of the resulting light surfaces in these settings respectively. In section 5 we put forward a modified version to this proposal and discuss the nature of the resulting light surfaces. Finally section 6 contains our conclusions.

2 Holography and FLRW universes

To begin with, let us briefly recall how the proposal by Fischler and Susskind [5] runs into difficulty in the case of $K = +1$ recollapsing FLRW universes. In this case the metric is given by

$$ds^2 = -dt^2 + a^2(t) \left( d\chi^2 + \sin^2 \chi d\Omega^2 \right),$$

where $a(t)$ is the scale factor, $\chi$ is the azimuthal angle and $d\Omega^2$ is the line element of the 2-sphere at constant $\chi$. Assuming a constant\(^3\) comoving entropy density $\sigma$, the ratio $S/A$ can be readily given as [10]

$$\frac{S}{A} = \sigma \left[ \frac{2\chi_H - \sin 2\chi_H}{4a^2(\chi_H)\sin^2(\chi_H)} \right],$$

where $\chi_H = \int_0^t \frac{dt'}{a(t')}^3$ is the comoving horizon size. This clearly shows that the bound can be violated in this case, on noting that the area (given by the denominator of (3)) becomes zero at the epoch of maximum expansion ($a = a_{\text{max}}$) given by $\chi_H = \pi$.

In order to remedy this shortcoming, Bousso has put forward a generalisation of the Fischler–Susskind Proposal which considers all four light-like directions and selects some according to an additional criterion of non-positive expansion of the null congruences generating the null surfaces orthogonal to the starting surface $B$. More precisely, the idea is as follows:

**Bousso’s Proposal** [16]: Let $M$ be a four-dimensional spacetime which satisfies Einstein’s equations with the dominant condition holding for matter. Let $A$ be the connected area of a two-dimensional spatial surface $B$ contained in $M$. Let $L$ be the connected part of a hypersurface bounded by $B$ and generated by one of the four null congruences orthogonal to $B$ such that the expansion of this congruence, measured in the direction away from $B$, is non-positive everywhere. Let $S$ be the total entropy contained in $L$. Then $S \leq A/4$.

A hypersurface $L$ with the above properties is then referred to as a light sheet (surface) $\mathcal{LS}$ for the surface $B$. The crucial points regarding this proposal are that first it selects which of the four null surfaces orthogonal to $B$ can be considered as light sheets, and second it determines what part of those null surfaces will be included in the light sheet:

\(^{3}\)An assumption that will obviously not be correct in a general inhomogeneous universe.
namely, they start at $B$, and any caustics present in the surface must act as end points to the light sheet [16], if it is extended that far.

Considering the case of FLRW universes, on choosing a surface within the apparent horizon as the surface $B$, this proposal prevents the violation of the holography bound in the contracting phase [16]. We note that in this simple (homogeneous and isotropic) case the light sheet, given optimally by the apparent horizon in a flat radiation dominated universe, is indeed connected as well as being differentiable. Note that while the definition used is time symmetric, the null surfaces in the expanding universe are not invariant under time reversal except at an instant of maximum expansion in a homogeneous universe (which does not correspond to the present day situation), and clearly almost never in an inhomogeneous universe.

In the following sections we study various aspects of this proposal in inhomogeneous cosmological settings. This is an essential extension of the previous work if it is to be taken as referring to the real universe.

3 Holography and Inhomogeneous universes

Let us consider a general realistic inhomogeneous universe, which may possibly possess a recollapsing phase. To begin with, recall that a fundamental feature of classical self-gravitating systems is that in general they are not in equilibrium states. This instability gives rise to spontaneous creation of structure (lumpiness) which in the physically realistic case increases with time. As an example of this we briefly recall a recent study of this question in the context of Lemaitre–Tolman and Szekeres inhomogeneous cosmological models [18]. Employing as a measure of density contrast (structuration), covariant density contrast indicators in the form

$$
DC = \int_{\Sigma} \left| \frac{h_{ab}}{\rho} \frac{\partial \rho}{\partial x^a} \frac{\partial \rho}{\partial x^b} \right| dV,
$$

it has been shown that in general such structuration varies with time, as expected$^4$. Here $\rho$ is the density, $h_{ab} = g_{ab} + u_a u_b$ projects orthogonal to the unit 4-velocity $u^a$ and $\Sigma$ is a 3-surface. This indicates that density contrast (lumpiness) is likely to change with time in inhomogeneous cosmological models, which is bound to be reflected in the behaviour of the corresponding Ricci and Weyl tensors.

Now the geometry of an arbitrary null surface encodes detailed information about the Ricci and the Weyl tensors encountered by that surface, as shown by the usual optical scalar equation (see e.g. [19])

$$
\frac{d\theta}{d\lambda} = -\frac{1}{2} \theta^2 - \sigma_{ab} \sigma^{ab} + \omega_{ab} \omega^{ab} - R_{ab} \kappa^a \kappa^b
$$

$^4$Moreover, it has been shown that indicators of this kind exist which grow monotonically with time for both ever-expanding and recollapsing models of Lemaitre–Tolman and Szekeres types, simultaneously (see [18] for details).
together with
\[ \kappa^c \nabla_c \sigma_{ab} = -\theta \sigma_{ab} + C_{cbad} \kappa^c \kappa^d \] (6)
\[ \kappa^c \nabla_c \omega_{ab} = -\theta \omega_{ab} \] (7)
where \( \theta, \sigma_{ab} \) and \( \omega_{ab} \) are respectively the expansion, shear and the twist of a congruence of null geodesics with the tangent field \( \kappa^a \). The convergence of the null generators changes according to the null component of the Ricci tensor sampled by the tangent vector and the rate of change of distortion according to the Weyl tensor component sampled by these generators. The shear in turn alters the expansion which determines the local rate of change of area along the generators. The total area entering the inequality in the holography conjecture is a summation of all the resulting infinitesimal area elements of the null surface, and so is a coarse-grained summation of all this information (in which all the fine details are lost).

As the universe evolves and structures form, the gravitational focusing (and caustic\(^5\)) properties in inhomogeneous cosmologies are time dependent, which in turn makes the structure of the light surfaces time dependent in these models, i.e. if we look at light surfaces associated with a spatial surface \( B_2 \) that lies to the future of a surface \( B_1 \) we expect a time dependence in the associated area and entropy.

To fix ideas, consider an inhomogeneous universe possessing \( N \) lumps at a given time \( t \). Let us denote by \( C_i \) the caustics produced by the lump \( i \) in a null surface orthogonal to \( B \) that starts off with non-expanding normals at \( B \). Now given that, according to the above proposal, caustics must act as end points to the light surface \( \mathcal{LS} \), a necessary condition \( \mathcal{LS} \) needs to satisfy to ensure the holography bound according to the above proposal is that it should contain all such caustics and hence
\[ \mathcal{LS} \supset \bigcup_i C_i. \] (8)
This immediately raises a number of fundamental issues concerning the nature of such light surfaces and their operational definability in practice.

4 Nature of the light sheets in inhomogeneous universes

Assuming that the bound defined by \( \mathcal{LS} \) does indeed hold in general inhomogeneous cosmological settings, a number of important questions still need to be addressed. They include:

4.1 Differentiability and connectedness

Given that light surfaces end at caustics, their structures are in general forced to be extremely complex and non-differentiable, with possibly disconnected or even fractal boundaries, depending upon the nature of the inhomogeneous lumpiness and the resultant caustics in the universe.

\(^5\)Clearly focusing does not always lead to caustics.
We recall that a given source (lens) can in principle produce a hierarchy of caustics with a range of intensities. Thus each star will generally cause strong gravitational lensing with associated caustics\(^6\). However if the star is in the core of a galaxy, there will also be much larger scale multiple images and caustics associated with the gravitational field of the galaxy itself; and if that in turn is in the core of a rich cluster of galaxies, the cluster will produce strong lensing with associated arcs and articles at even larger angular scales. In this way, each such a star would contribute to multiple levels of lensing and caustics. Furthermore, strong gravitational lensing is a commonplace phenomenon. Indeed in the real universe we expect such a hierarchical structure, with at least \(10^{22}\) caustics in our past light cone because of lensing caused by all the stars our past light cone intersects in all visible galaxies, with further multiple layers of caustics caused by additional lensing associated with at least some galaxy cores and some rich clusters of galaxies, as just indicated.

At each level, caustics occur that are associated with parts of the past light cone that lie as indentations inside the boundary of the past, and are associated with multiple images of distant objects (see for example [20, 21]). When lensing occurs, the past light rays generating a past light cone self-intersect first non-locally and then locally, as one follows them back from the apex of the light cone. A light ray near a lensing object is deflected inwards by the gravitational field of the lens as it moves near it. It swings back towards the optic axis (the null geodesic from the observer through the centre of the lens) and self-intersects a similar family of geodesics coming from the other side. At this point it moves from generating the outer part of the past light cone (the boundary of the past) to an inner part, folded inside and lying within the past of the apex point\(^7\). It continues till local self-intersection occurs at a cusp; from there on it generates the back part of the folded light cone, which also lies inside the past of the apex point. This is a general structure that results from the nature of the boundary of the past of a set of points in a generic space-time [22]. In the case of non-spherical lenses, multiple caustics due to a single lens can lie inside each other; these complex nullcone geometries have been investigated analytically in the case of elliptical lenses, and numerically in the case of realistic lensing models (see e.g. [23]).

Additionally, it has been shown that the presence of single BH can produce an infinity of caustics associated with light rays that circle the black hole an arbitrary number of times - albeit with rapidly decreasing intensities\(^8\) [24, 25]. Thus even a finite universe with say \(10^{11}\) black holes in the visible region - a very conservative estimate, given that we expect massive black holes at the centres of many galaxies, as well as all those resulting from the collapse of super-massive stars - could give rise to an infinite number of caustics associated with each of these black holes, and hence to an extremely complicated light surface.

In this way the light sheet \(\mathcal{LS}\) (whether the past light cone of single point or not) may be said to light trace the content of the universe on all scales and thus encode its complexity, particularly through its caustic structure. This makes sense, as in contrast

\(^6\)The corresponding multiple images will in general not be detectable because they will lie too close to the apparent surface of the star.

\(^7\)As is implied by Figure 3 of [15].

\(^8\)Note that even though in practice caustics below a certain cut-off intensity may be ignored, in principle they all need to be taken into account.
to the case of BH\(^9\) and completely smooth FLRW cosmological models, for which \(\mathcal{LS}\) is readily given in terms of the small number of parameters which characterise these systems\(^10\), one would in general cosmological settings expect this surface to be complex since no such constraints exist. To describe the detailed structure of a null surface (e.g. a past light cone) in a realistic cosmological setting will require many millions of parameters.

### 4.2 Operational definability

Strictly speaking, to define the holography bound precisely, all \(C_i\) need to be included in the construction of \(\mathcal{LS}\). The problem, however, is that the details of \(C_i\) are not given a priori in terms of theory, but depend on the details of the contents of the universe (including the masses and sizes of the sources and lenses, together with the detailed knowledge of their distributions in space and time), which needs to be specified through observations. The crucial point being that \(\mathcal{LS}\) is constructive rather than theoretically given. This then raises the important question of operational definability of \(\mathcal{LS}\) for the real universe. Now given that all observations possess finite resolutions, only sources, lenses and caustics above certain threshold levels can be observable in practice. In this way, a cut-off (course-graining) is inevitably involved in the definition of \(\mathcal{LS}\). Thus limitations in observational resolution become a barrier to constructing the precise form of \(\mathcal{LS}\) and hence ensuring the bound.

This then raises the interesting question of whether one could formulate an averaged holography principle in terms of the averaged (coarse-grained) light surface \(\mathcal{ALS}\). The problem, however, is that the coarse-graining (averaging) of the content (say matter distribution) does not commute with the averaging of the geometry, mainly due to the nonlinearity of the curvature tensor (see for example \([26]\) and references therein for a detailed discussion of this issue). Worse still, coarse-graining of neither of these two quantities would in general commute with coarse-graining the caustics. For example, a single BH of a given mass can produce an infinite number of caustics; it is not clear that a cut-off on the mass of the lens in general results in a similar cut-off in the area of the resulting caustics.

### 4.3 Time dependence and reversibility

As was pointed out above, in a real universe the number of lumps \(N\) as well as their positions, masses and shapes vary with time, This is interesting as, in addition to the precise distribution of sources, lenses etc, their time evolution is also important for the construction of \(\mathcal{LS}\), which is time dependent as the surface \(B\) is moved to different epochs in the universe’s history. Now given that this evolution must ultimately be related to the question of entropy (whatever its precise formulation may be in the presence of gravitational fields), the surface \(\mathcal{LS}\) seems to encode time-dependent information regarding entropy as well. What has not been done is to show that at later times in a realistic cosmological setting the total entropy encoded this way will be larger than at earlier times. This is one

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\(^9\)Where the presence of the no-hair theorem ‘smoothes’ the information on the event horizon boundary.

\(^{10}\)Namely mass \(M\) in the case of BH and the deceleration and the Hubble parameters \((q_0, H)\) in the case of FLRW models.
of the issues that needs clarifying; if the definition of entropy has the desired properties, this must work out successfully.

Additionally, even though locally (in a spacetime sense) in the neighbourhood of the bounding surface \( B \) one might argue (as is done by Bousso [15]) that the screen definition is invariant under time reversal, the actual surfaces will not be in an expanding universe: a unique direction of time will be picked by the expansion of the universe, and usually this will be marked by a difference in the expansions of the null normals to \( B \). This difference will be enhanced by a major difference between the caustics encountered in the future and the past of \( B \), with the growth of inhomogeneities at quite different evolutionary stages in the two directions of time from \( B \), giving another way in which the geometry of \( \mathcal{LS} \) causes this time symmetry to be violated.

5 A modified proposal

In the previous section it was shown that the light surfaces in Bousso’s proposal are likely to have extremely complicated structures in a real inhomogeneous universe. This is a direct consequence of the fact that in this proposal light surfaces are taken to end at caustics.

Here we put forward a modified proposal which drastically simplifies the structure of these light surfaces. Before doing so, we note that it is important to distinguish carefully between \( \mathcal{P}_B \), the boundary of the past of the 2–surface \( B \) that starts off from \( B \) with converging null geodesics, and the light sheet \( \mathcal{LS} \) suggested by Bousso. The former is a subset of the latter; the boundary \( \mathcal{P}_B \) ends at the first self-intersections of the null sheets orthogonal to \( B \), which will usually be non-local intersections, whereas the light sheet ends at caustics, which are local self-intersections. It is the region between these self-intersections where the complex connectivity occurs.

It therefore makes sense to separate out the part of the light surface which is not part of the boundary of the past of \( B \). We shall call this the inner light sheet (\( \mathcal{ILS} \)) and refer to the rest of the \( \mathcal{LS} \), i.e. the part of the null surface through \( B \) that is also the null boundary of the past of \( B \), as the outer light sheet (\( \mathcal{OLS} \)). Then \( \mathcal{ILS} \) encodes detailed information on the strong lensing that occurs for the null surface, for it bounds the region after self-intersection but before caustics. The number and topology of such components depends on the lensing objects and hence reflects the degree of strong density inhomogeneity. However, weak inhomogeneities will not cause strong lensing and so will not be encoded in \( \mathcal{ILS} \). It is thus this inner light sheet that produces the enormous complexity in the light sheets proposed by Bousso.

Additionally, continuing the null surface beyond the first self-intersections until the caustics actually result in multiple coverings of part of the interior of \( B \) by the null surfaces, which have turned in on themselves. Thus including this part results in excess area being counted, and a much more complex projection of data onto the null boundary than is necessary when setting up the holographic principle. For this purpose, it is only necessary to include data on the outer light surface \( \mathcal{OLS} \); the data on \( \mathcal{ILS} \) is then redundant, having already been counted on \( \mathcal{OLS} \).

We therefore propose a modified version of Bousso’s proposal thus:
**Proposal:** Let $M$ be a four-dimensional spacetime which satisfies Einstein’s equations with the dominant condition holding for matter. Let $A$ be the connected area of a two-dimensional spatial surface $B$ contained in $M$. Let $L$ be the hypersurface bounded by $B$ and generated by one of the four null congruences orthogonal to $B$ such that the expansion of this congruence, measured in the direction away from $B$, is non-positive everywhere, and ending on the boundary of the past of $B$. Let $S$ be the total entropy contained in $L$. Then $S \leq A/4$.

The hypersurface $L$ with the above properties is the outer light surface $OLS$. The important feature of this modified proposal is that it cuts out the $ILS$, together with the caustics and the fractal boundaries arising from them, and therefore has a much simpler light sheet structure. It also covers regions in the interior of $B$ only once. We therefore suggest that, in the case of realistic inhomogeneous cosmologies, this is a better surface to choose for the holographic principle and the associated entropy conjecture.

### 5.1 Nature of the light sheets in the modified proposal

To begin with let us note that the modified covariant entropy conjecture proposed above leaves unchanged all the examples considered by Bousso [16], including the $K = +1$ FLRW model. This is clear since none of the null surfaces in these examples contain self-intersections other than caustics, for example those at the origin of coordinates ($r = 0$) in the light-sheets of 2-surfaces $B$ that are spherically placed about the origin (see Figure 2 of [15].)

In the case of the more complicated inhomogeneous models with both caustics and self-intersections present, on the other hand, replacing $LS$ by $OLS$ enormously simplifies the structure of the light surface. Despite this, there are a number of difficult points that still remain.

Firstly, even though the caustics are removed in this formulation, the non-continuity of the generators of the boundary of the past of $B$ still remain at the self-intersections, which make the $OLS$ surface non-differentiable there. However a simple smoothing over these regions where the outer surface has self-intersections should deal with this adequately in most cases, where the area of the smoothed surface can be arbitrarily close to that of the real surface. When this smoothing cannot be done, new effects may occur and a very careful analysis will be required.

Secondly, since caustics can be locally determined, the end points of the $LS$ in Bousso’s proposal were definable locally - at least in principle. Our proposal relies on the determination of the null boundary of the past of $B$, which cannot be determined locally. So in this sense there are both advantages and disadvantages with this new proposal. It greatly simplifies the shape of the $LS$ - at least in theory, but operationally it is still difficult to determine $OLS$.

### 5.2 Coarse-graining and information loss

Crucial to the whole discussion is the issue of the scale of description used in the space-time model. One can represent the same physical situation at different averaging scales:
thus we can represent the real universe (a) at a smoothed out cosmological scale, where a FLRW model will suffice; (b) a finer scale, where each cluster of galaxies is represented as an inhomogeneity; (c) a finer scale, where each galaxy is represented; (d) a still finer scale where each star and each black hole is individually represented. The nature of the surface $LS$ will be drastically different in these different representations. A coarse-graining procedure will relate them to each other [27] - remembering all the time that these are different geometrical representations of the same physical situation.

Now it is plausible that in most circumstances, the definition of entropy is closely associated with coarse-graining, and with the loss of information that results from coarse-graining [28] (see also [29]). We might therefore expect quite different results for the entropy determined in terms of the areas associated with the null surfaces obtained on different averaging scales for models representing the same physical situation. We regard this as a fundamental issue but will not pursue it further here except for the following remarks.

The area of the smoothed out model can be expected to be close to that obtained in the detailed (lumpy) model for the surface $OLS$. It is the area of $LS$ and of $ILS$ that will be very different in these two cases; indeed the latter will be empty in the smoothed-out case. The area of $ILS$ is associated with strong lensing only, and may perhaps be considered as a measure of the pure gravitational entropy of the solution: the larger that area is, the larger the degree of inhomogeneity (and hence entropy) encoded in the gravitational field. On coarse-graining and consequent smoothing of the matter representation, the corresponding $ILS$ will decrease to zero. The loss of this area represents loss of detailed information on the inhomogeneity structure of the gravitational field resulting from this coarse-graining. The one-way nature of the information loss associated with coarse-graining is reflected in the fact that the area of the fine-grained surface $LS$ is necessarily larger than that of $LS$, the latter being close to the area of the $OLS$ in the smoothed-out (coarse-grained) description.

There are potential parallels here with the presence of reversibility at the level of microphysics and irreversibility at the macroscopic level. What is needed to make the definitions and theory compelling is a comparison of entropy estimates and associated areas at earlier and later times in the history of the expansion of the universe, at different scales of description. We do not attempt this here, but note it as an important problem.

6 Conclusion

We have taken a closer look at the applications of the holography principle to cosmology and in particular the proposal recently put forward by Bousso. We have argued that in a real inhomogeneous universe, the light surfaces defined in his way in order to satisfy the holography principle would be non-differentiable and extremely (in principle infinitely) complicated. Such a light surface can be viewed as a light tracing of the complexity in the universe, projected onto this surface; like a cosmological analogue of the images on the walls of Plato’s cave!

In this way, to satisfy the holography principle in a general inhomogeneous universe requires a detailed knowledge of the contents of the universe and in turn its detailed caustic structure. Furthermore, the inevitable limits to the observational resolution puts
fundamental limits on the operational definability of this surface. Moreover, given the
dynamical (and irreversible) evolutionary nature of light surfaces in general, such bounds
cannot remain invariant under time reversal.

We have introduced an alternative proposal which results in a much simpler light
surface. However, operationally, it is still very difficult to define such surfaces in practice.

This leads us to conclude that in a realistic setting the theoretical existence of such
surfaces must be clearly distinguished from their complexity and operational definability.

It would be extremely useful if an averaged holography principle could be formulated.
Failing this, given the enormous amount of detailed information required for the con-
struction of such light sheets, it is difficult to see how such a principle - formulated
phenomenologically - can prove useful in simplifying the understanding of the cosmos in
practice. This of course does not rule out the possibility of correctness and usefulness of
such a principle in the real world at a fundamental level and that could still be vitally
important. On the other hand, the phenomenological difficulties raised here, including
the complexity, non-differentiability and potential fractality of such surfaces, might have
some relevance in debates regarding the applications of the holographic principle at a
fundamental level in other settings such as string theory.

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