A PHANTOM MENACE?

R. R. CALDWELL

Department of Physics, Princeton University, Princeton, NJ 08544

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ABSTRACT

It is extraordinary that a number of observations indicate that we live in a spatially flat, low matter density Universe, which is currently undergoing a period of accelerating expansion. The effort to explain this current state has focused attention on cosmological models in which the dominant component of the cosmic energy density has negative pressure, with an equation of state \( w \geq -1 \). Remarkably that most observations are consistent with models right up to the \( w = -1 \) or cosmological constant (\( \Lambda \)) limit; it is natural to ask what lies of the other side, at \( w < -1 \). In this regard, we construct a toy model of a “phantom” energy component which possesses an equation of state \( w < -1 \). The current data are taken seriously, such a component is compatible with, if not favored by most classical tests of cosmology, including the recent type 1a SNe data, as well as the cosmic microwave background anisotropy and mass power spectrum. Either the evidence favoring \( w < -1 \) is due to some unanticipated systematic effect, or the dominant component of the cosmic energy density is stranger than anything expected.

Subject headings: cosmology: theory

Arguments have been put forward that we live in a spatially flat, low matter density Universe which is currently undergoing a period of accelerating expansion. If the observational evidence upon which these claims are based are reinforced and strengthened by future experiments, the implications for cosmology will be incredible. It would then appear that the cosmological fluid is dominated by some sort of fantastic energy density, which has negative pressure, and has just begun to play an important role today. No convincing theory has yet been constructed to explain this state of affairs, although cosmological models based on Quintessence (Q: Caldwell, Dave & Steinhardt 1997) and the historical cosmological constant (\( \Lambda \): for a recent review see Sahni & Starobinsky 1999) are leading candidates. At this stage we are led to notice that the parameterization of the dominant energy component as a fluid with an equation of state (defined as the ratio of pressure to energy density) \( w = p/\rho \geq -1 \) leads to the curious situation that most observational constraints are consistent with models that go right up to the \( w = -1 \) border (Wang et al. 1999). It is natural to ask what lies on the other side of this boundary. The focus of this paper is the investigation of cosmological models with a fluid component for which \( w < -1 \).

We begin by constructing a classical cosmology: a spatially flat FRW space-time which is dominated in the present epoch by a cold dark matter (CDM) component, and a “phantom” (P) energy component. (A phantom is something which is apparent to the sight or other senses, but has no actual or substantial existence.) The phantom energy has positive energy density, \( \rho_p > 0 \), but negative pressure, such that \( \rho_p + p_p < 0 \). (It is no fatal flaw for a component to violate the dominant energy condition for a finite time, as can arise from a bulk viscous stress due to particle production (Barrow 1988).) It is immediately recognizable from the equation of state with \( w < -1 \) that the phantom energy density grows with time. If this energy has begun to dominate today, it must have come on like a bolt from the blue. Taking the constant value \( w = -3 \) as an example, \( p_p \propto a^0 \) where \( a \) is the expansion scale factor. So if \( \Omega_m \sim 0.3 \) today, then the Universe contained 90% CDM at \( z \sim 0.4 \) as opposed to \( z \sim 1.8 \) in a \( \Lambda \)-dominated cosmology. Hence, the phantom energy exerts its influence very late. We now turn to quantify the remarkable effect on the cosmology of a component with \( w < -1 \).

\[ \text{Fig. 1.} - \text{The age in units of the Hubble time is plotted versus} \Omega_m \text{ for a series of cosmological models with} w \leq -1. \text{ For a given value of} \Omega_m \text{, decreasing} w \text{ has a diminishing effect.} \]

\[ \text{The expansion scale factor grows rapidly when the phantom energy comes to dominate the Universe, as} \dot{a}/a \text{ to} \text{'expansion age'. When the phantom energy-driven scale factor as} a(t) = a(t_m)|-w + (1 + w)t/t_m|^{2/(1+w)} \text{ for} t > t_m. \text{ So for} w = -3 \text{ then} a \text{ diverges when} t = 3t_m/2. \text{ Unless} \Omega_m < 1, \text{ however,} t_0 < t_m w/(1 + w) \text{ and the limiting cosmological time occurs well after the present day.} \]

\[ \text{The expansion age, and similarly the horizon distance in} \]
a phantom energy cosmology are larger than in the analogous Λ model. In Figure 1 we show the age for a sequence of $w \leq -1$ models. In the range of interest, near $\Omega_m \sim 0.3 - 0.4$, we see that the phantom energy can increase the age by up to $\sim 30\%$ over the $w = -1$ age. For a given value of $\Omega_m$, as $w$ becomes very negative the phantom energy becomes important later and later with diminishing effect on the age. In fact, for $\Omega_m = 0.3$, the bound on the age in units of the Hubble time is $H_0 t_0 \leq 1.2$.

The magnitude - red shift relationship is demonstrated in Figure 3. The predictions for several cosmological models have been shown along with the recent results of the Supernova Cosmology Project (Perlmutter et al. 1998). The phantom energy model rises well above the curve for ΛCDM, and even rises above the pure de Sitter (flat with $\Omega_m = 0, \Omega_\Lambda = 1$) curve. Hence, one expects high red shift supernovae to be dimmer in a phantom energy Universe.

Next, the background evolution of the cosmology impacts on the fluctuation spectra. We assume that the matter component of the cosmological fluid carries a spectrum of scale-invariant perturbations generated by inflation. Furthermore, we assume that, similar to Quintessence, the phantom energy itself does not fluctuate on scales well below the Hubble horizon. Then it is interesting to follow the growth rate, $D(a)/a$, for linear perturbations in the CDM and baryons. This is shown in Figure 4, where we see that perturbations grow as $D \propto a$ until very late, owing to the very late time at which the phantom energy begins to dominate. Hence, the evolution is very similar to the standard CDM scenario, with the difference being in the matter density.

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**Figure 2.** The volume - red shift relationship is shown for several cosmological models. The phantom energy models with $w = -3, -3/2$, and $\Omega_m = 0.3$ predict a larger differential number of objects per unit red shift than ΛCDM with $w = -1$, QCDM with $w = -1/2$, and open CDM all with $\Omega_m = 0.3$, and SCDM for which $\Omega_m = 1$.

**Figure 3.** The magnitude - red shift relationship for the type 1a supernovae observed by the SCP group (Perlmutter et al. 1998) are shown with the predictions for several cosmological models. The phantom energy models with $w = -3$ and $\Omega_m = 0.3$ are shown with the thick black line. The phantom model with $w = -1.5$ and $\Omega_m = 0.3$ is shown with the thick grey line. In the bottom panel, the offset is based on the phantom energy model, and in the thick black line, and the ΛCDM is in grey. Both models provide a good fit to the data. The light, dashed lines are for de Sitter, Milne, and Einstein-de Sitter, from top to bottom.

The volume - red shift relationship is demonstrated in Figure 2. For the same matter density, the phantom energy model gives the largest differential number of objects per red shift interval. Although evolutionary effects are important in cosmological tests based on this relationship, if all other features are held fixed, the phantom model will predict more strong gravitationally lensed quasars than Λ or open models.

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**Figure 4.** The growth factor $D(a)/a$ versus the scale factor is shown for various cosmological models, all with $\Omega_m = 0.3$. The volume - red shift relationship is shown for several cosmological models. The phantom energy models with $w = -3$ and $\Omega_m = 0.3$ are shown with the thick black line. The phantom model with $w = -1.5$ and $\Omega_m = 0.3$ is shown with the thick grey line. In the bottom panel, the offset is based on the phantom energy model, and in the thick black line, and the ΛCDM is in grey. Both models provide a good fit to the data. The light, dashed lines are for de Sitter, Milne, and Einstein-de Sitter, from top to bottom.

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At present we use this simplified scheme in order to focus on the basic properties of such a cosmological scenario. Some of the predictions depend on the details of the microscopic model. Nevertheless, an observation which indicates a $w < -1$ component would necessarily suggest radically new physics.

The cosmological spectrum of fluctuations in the phantom field develop in a fashion similar to the case of Quintessence. The Fourier transformed perturbation equation is $\ddot{\delta}\phi + 3H\dot{\delta}\phi + (k^2 - V_{\phi\phi})\delta\phi = -\dot{h}\phi/2$, where $h$ is the synchronous gauge metric perturbation. In this equation, the sign of $V_{\phi\phi}$ is different than in the standard case. Just as for Quintessence, perturbations on small scales, for $k^2 \gg V_{\phi\phi}$, are suppressed. On larger scales, however, the phantom energy develops inhomogeneities in response to the surrounding matter and density perturbations. If the effective mass, $(k^2 - V_{\phi\phi})^{1/2}$, should become imaginary then $\delta\phi$ would develop a growing (tachyon) instability. However, if we implement a constant equation of state, then $V_{\phi\phi} = 1/2(1-w)(H - 3H^2(1+w))$, which not only is negative, but is identical to the Q case, up to an overall sign. This means the form of the perturbation equation for $w < -1$ is identical to the $w > -1$ case, the effective mass is real, and there is no instability for constant $w$.

**Fig. 5.—** The CMB anisotropy spectra for various cosmological models is shown, all with $\Omega_m = 0.3$, $\Omega_b h^2 = 0.02$, and $h = 0.65$. Phantom energy models with $w < -1$ produce slightly larger temperature anisotropy than the analogous $w > -1$ models.

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The main feature that distinguishes the phantom energy case from $\Lambda$ or $Q$ is that the onset of phantom energy dominance happens at the very last moment – so late that most evolutionary effects which occur in $\Lambda$ and $Q$ models are suppressed. Hence, the CMB integrated Sachs-Wolfe effect is almost turned off. The result, displayed in Figure 5, is that the gap between the low-$\ell$ plateau and the Doppler peak is increased, relative to the analogous $\Lambda$ case. Because the CMB spectra are normalized at $\ell \sim 10$, very little of the late ISW affects the amplitude for $w < -1$, so the Doppler peak height changes by very little.

Now that we have some understanding of how $w < -1$ models behave, let’s see what happens to the constraints on the parameter plane near the $w = -1$ boundary.

The magnitude - red shift relation probed by high red shift supernovae is sensitive to the cosmological equation of state. Considering the supernovae constraint, we find that phantom energy models even with very negative $w$ are in accord with the observations. We see in Figure 6 that the contour region is extensive for $w < -1$, and in fact larger for phantom than Quintessence models.

**Fig. 6.—** The $w$ - $\Omega_m$ parameter space as constrained by the Hi-Z (MLCS: Riess et al. 1998) and SCP (fit C: Perlmutter et al. 1998) supernovae. We have traversed the $w = -1$ boundary, marked by the thin line, finding that there are phantom energy models in accord with the observations. The 1$\sigma$ and 2$\sigma$ contours (based on a $\Delta \chi^2 = 1, 4$ test) are represented by the dark and light shaded regions. For the SCP data, the best fit model lies at $w < -1$; for the Hi-Z data, the best fit lies at $w > -1$.

**Fig. 7.—** The constraint on the $\Omega_m - w$ plane due to the rate of strong gravitational lensing is shown. The shaded regions are consistent with the lensing rate (4 in 502) observed by the HST-SSS (Maoz et al. 1993) at the 95% confidence level. The thin line marks the $w = -1$ boundary.

The rate of strong gravitational lensing of quasars by galaxies is sensitive to the cosmological equation of state, through the volume - red shift relation and the luminosity distances. In principle we would expect the HST-SSS (Maoz et al. 1993) to observe $\sim 11,8,7,5$ lensed galaxies if the underlying cosmology is $\Omega_m = 0.3$ and $w = -3, -3/2, -1, -1/2$ respectively, given certain assumptions regarding the properties of the lenses and sources. (For simplicity, our calculation follows model A1 of Bloo-
field Torres & Waga 1996.) The fact that the survey identified only four lens systems tells us that the constraint on $\Omega_m$ becomes tighter as $w$ becomes more negative, as illustrated in Figure 7. Yet there remains a large portion of the $w < -1$ parameter space which is viable.

![observed cluster abundance](image)

**Figure 8.** The cluster abundance constraint is shown for a sequence of $Q$ ($w > -1$), $\Lambda$ ($w = -1$), and phantom ($w < -1$) models with $\Omega_m = 0.3$, $h = 0.65$, $\Omega_b h^2 = 0.02$, $n = 1$. The 2$\sigma$ constraint is in dark grey, while the predictions with 2$\sigma$ uncertainty are in light grey.

Next, we consider the behavior of the mass power spectrum for $w < -1$. The COBE-normalized amplitude of the mass power spectrum, as for Quintessence, is modulated by the strength of the late ISW effect, and the impact of the direct fluctuations in the phantom field. However, the direct fluctuations are negligibly small for $w < -1$, so that the overall shape of the spectrum is well-described by the BBKS parameterization (see Ma et al. 1999). This just leaves a dependence on the late ISW which in turn has a weak effect on the $\ell \sim 10$ moment. Hence, the amplitude does not change much as $w$ decreases. In Figure 8 we show the prediction and observational constraint on $\sigma_8$ for a sequence of models varying in $w$. We see that the prediction and constraint converge for decreasing $w$ in this case. For different values of $\Omega_m$ and $h$, with decreasing $w$ the predicted $\sigma_8$ asymptotes to a value $\sim 10 - 20\%$ higher than the $w = -1$ or $\Lambda$ value. Based on this rough analysis, we estimate that for the matter density parameter in the range $0.2 \lesssim \Omega_m \lesssim 0.6$ there are $w < -1$ models in accord with the measured abundance of x-ray clusters. We note that a constraint which we will not consider in this brief paper, but should prove interesting based on its sensitivity to the rate of growth of perturbations, is the red shift evolution of the abundance of rich clusters.

The constraint on phantom energy models due to the CMB does not differ substantially from the case of $\Lambda$. As we have shown, for $w < -1$ the Doppler peak increases only slightly, by $\lesssim 10\%$, relative to the analogous $w = -1$ model. Hence, we estimate that just as for $\Lambda$ or Q models, with the matter density parameter in the range $0.2 \lesssim \Omega_m \lesssim 0.6$ there are $w < -1$ models in accord with the combined library of CMB data sets (Wang et al. 1999).

In summary, we have investigated the properties of cosmological models in which the dominant energy density component today has an equation of state $w < -1$. We have demonstrated the impact on the cosmological age, the volume - red shift and magnitude - red shift relations, the CMB, and the mass power spectrum, finding broad agreement with current observational constraints. A careful analysis of the data could very well favor $w < -1$ over $w \geq -1$. The best single method for determining if the equation of state is strongly negative appears to be using the SNe $m-z$ relation. We do not want to overlook the distinct possibility that the observational evidence is simply a phantom – that the apparent accelerating expansion is due to more conventional, though unanticipated causes (e.g. dust or evolution for the SNe). If these systematic effects can be eliminated, and the data continue to support $w < -1$ then the implications for fundamental physics would be astounding, since $w < -1$ cannot be achieved with Einstein gravity and a canonical Lagrangian. It is premature to shift attention towards $w < -1$ model building, but it is important to be aware of the properties and implications of models in each direction of the cosmological parameter space. As has been discussed elsewhere, current observations suggest the presence of an energy component with $-1 < w < -0.4$ (Wang et al. 1999), with many constraints pushing towards $w = -1$: the observations are teetering at the edge of a previously unfamiliar boundary.

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**REFERENCES**
